

# Aldryngton Primary School



## Calculation Policy for Mathematics

# About our Calculation Policy

The following calculation policy has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school. Please note that early learning in number and calculation in Foundation Stage follows the *Statutory Framework for the Early Years*, and this calculation policy is designed to build on progressively from the content and methods established in the Early Years Foundation Stage.

## Age/stage expectations

The calculation policy is organised according to age expectations as set out in the National Curriculum 2014. **However it is vital that pupils are taught according to the stage that they are currently working at**, being moved onto the next level when they are ready, or working at a lower stage until they are secure enough to move on.

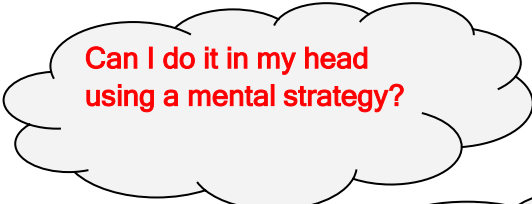
*"Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on." National Curriculum 2014*

## Providing a context for calculation:

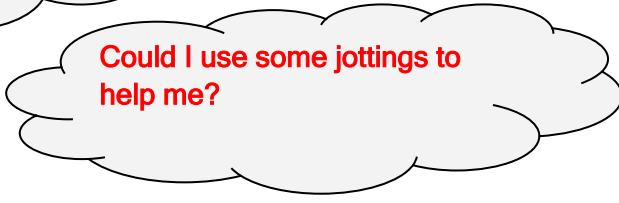
It is important that any type of calculation is given a real life context or problem solving approach, to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons.

## Choosing a calculation method:

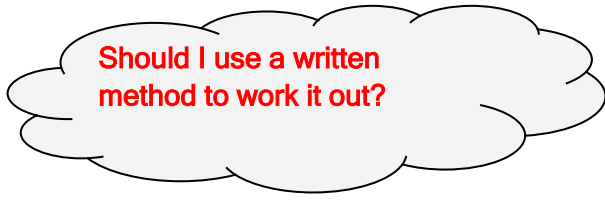
Children need to be taught and encouraged to use the following processes in deciding on the approach they will take to a calculation, to ensure they select the most appropriate method for the numbers involved:



Can I do it in my head  
using a mental strategy?



Could I use some jottings to  
help me?


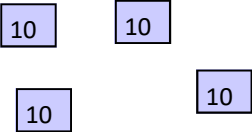
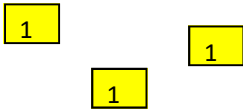


Should I use a written  
method to work it out?

# The Number System and Place Value

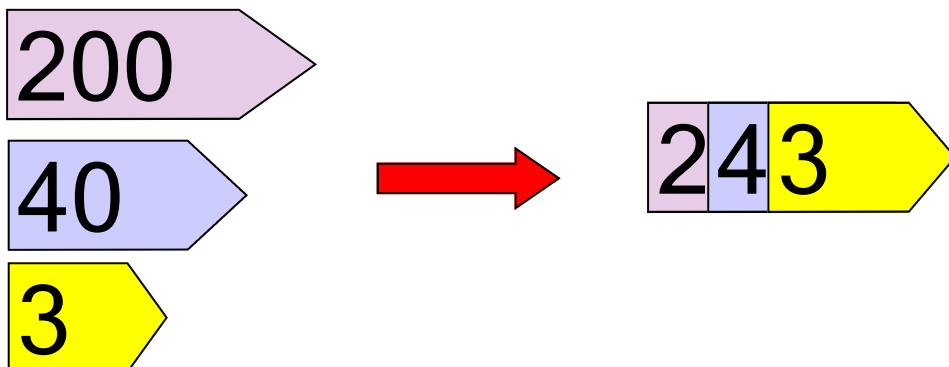
A full understanding of place value within the number system is the foundation on which mathematical development is built. A range of resources are used to develop children's understanding of this fundamental concept.

Place Value Charts used with place value counters

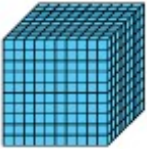
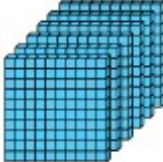
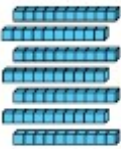

100s	10s	1s
		

2 hundreds	+	4 tens	+	3 units	=	243
200		+ 40		+ 3	=	243

Place Value Arrows



Represent the number 1672

Thousands	Hundreds	Tens	Ones
			
1000	600	70	2

Base 10 or Dienes  
Place value  
equipment

Gattegno  
Place value  
chart

0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900

# Addition

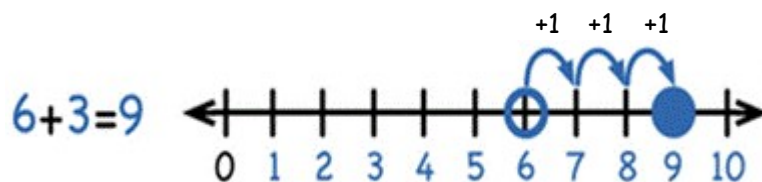
## Year 1

### Add with numbers up to 20



Children should:

- Have access to a wide range of counting equipment, everyday objects, number tracks and number lines, and be shown numbers in different contexts.
- Use **numbered number lines** to add, by counting on in ones. Encourage children to start with the **larger** number and count on.



- Read and write the addition (+) and equals (=) signs within number sentences, understanding that = represents equality:

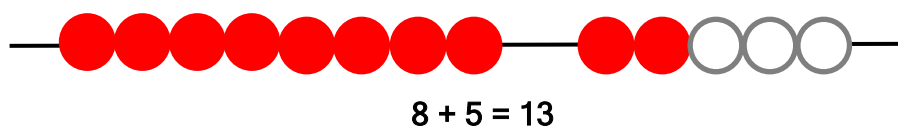
$$1 + 1 = 2 \quad 2 = 1 + 1 \quad 2 + 3 = 4 + 1$$

- Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them:

$$8 + 3 = \square \quad 15 + 4 = \square \quad 5 + 3 + 1 = \square \quad \square + \square = 6$$

This builds on from prior learning of addition by combining two sets of objects into one group (5 cubes and 3 cubes) in Early Years.

*Bead strings or bead bars can be used to illustrate addition, including bridging through ten by counting on 2, then counting on 3.*



**Key vocabulary:** add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line

**Key skills for addition at Year 1:**

- Read and write numbers to 100 in numerals, including 1–20 in words.
- Recall bonds to 10 and 20, and addition facts within 20.
- Count to and across 100.
- Count in multiples of 1, 2, 5 and 10 to 100.
- Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations.

# Addition

## Year 2

### Add with 2-digit numbers



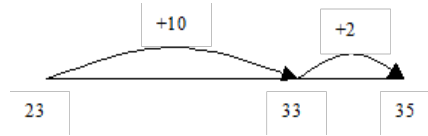
Develop mental fluency with addition and place value involving 2-digit numbers, then establish more formal methods.

A range of representations should be used.

- Continue to use number lines to develop understanding of:

Counting on in tens and ones

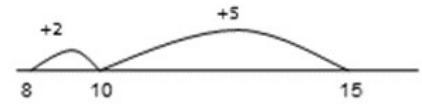
$$\begin{aligned} 23 + 12 &= 23 + 10 + 2 \\ &= 33 + 2 \\ &= 35 \end{aligned}$$



Partitioning and bridging through 10.

The steps in addition often bridge through a multiple of 10  
e.g.  $8 + 7 = 15$

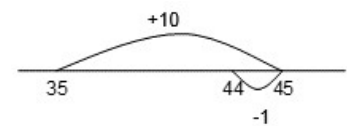
Children should be able to partition the 7 to relate adding the 2 and then the 5.



Adding 9 or 11 by adding 10 and adjusting by 1

e.g.  $35 + 9 = 44$

Add 9 by adding 10 and adjusting by 1



- Add pairs of 2 digit numbers, moving to the partitioned column method when secure adding tens and units:

$$\begin{aligned} 47 + 25 \\ = 60 + 12 = 72 \end{aligned}$$

**23 + 34:**

$$\begin{array}{r} 20 + 3 \\ + 30 + 4 \\ \hline 50 + 7 \\ \hline = 57 \end{array}$$

**STEP 1:** Only provide examples that do NOT cross the tens boundary until they are secure with the method itself.

**STEP 2:** Once children can add a multiple of ten to a 2-digit number mentally (e.g.  $80 + 11$ ), they are ready for adding pairs of 2-digit numbers that DO cross the tens boundary. e.g.  $58 + 43$

$$\begin{array}{r} 50 + 8 \\ 40 + 3 \\ \hline 90 + 11 \\ \hline = 101 \end{array}$$

**STEP 3:** Children who are confident and accurate with this stage should move onto the expanded addition methods with 2 and 3-digit numbers (see Y3)

To support understanding, children may physically make and carry out the calculation with place value counters or other resources, then compare their practical version to the written form, to help them to build understanding.

Missing number problems e.g.  $14 + 5 = 10 + \square$   $32 + \square + \square = 100$   $35 = 1 + \square + 5$

**Key vocabulary:** add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

**Key skills for addition at Year 2:**

- Add a 2-digit number and ones (e.g.  $27 + 6$ ).
- Add a 2-digit number and tens (e.g.  $23 + 40$ ).
- Add pairs of 2-digit numbers (e.g.  $35 + 47$ ).
- Add three single-digit numbers (e.g.  $5 + 9 + 7$ ).
- Show that adding can be done in any order (the commutative law).
- Recall bonds to 20 and bonds of tens to 100 ( $30 + 70$  etc.).
- Count in steps of 2, 3 and 5 and count in tens from any number.
- Understand the place value of 2-digit numbers (tens and ones).
- Compare and order numbers to 100 using  $<$   $>$  and  $=$  signs.
- Read and write numbers to at least 100 in numerals and words.
- Solve problems with addition, using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.

# Addition

## Year 3 Add with 2-digit numbers then establish more formal methods



### Partition into tens and ones

Partition both numbers and recombine.  
Count on by partitioning the second number only  
e.g.  
 $247 + 125 = 247 + 100 + 20 + 5$   
 $= 347 + 20 + 5$   
 $= 367 + 5$

Children need to be secure mentally adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10.  
e.g.  $126 + 10 = 136$

**Missing number problems:** using a range of equations as in Year 2, with appropriate larger numbers.

**Add numbers with up to 3-digits:** The expanded column addition method may be used:

	2	3	6
+		7	3
			9
	1	0	0
	2	0	0
	3	0	9

Add the units first, in preparation for the compact method.

Children need to recognise the value of the hundreds, tens and units without recording the partitioning.

Pupils need to be able to add in columns.



Add units first

$$\begin{array}{r} 236 \\ + \quad 73 \\ \hline 309 \\ \small{1} \end{array}$$

### Introduce the compact column addition method, with 'carrying':

Children who are ready should be taught the **compact column addition** method, being introduced to 'carrying' for the first time. The expanded method can be compared to the compact column method to develop an understanding of the process and the reduced number of steps involved.

"Carry" numbers underneath the bottom line

Remind pupils that the actual value is 'three tens plus seven tens', not three add seven.

**Key vocabulary:** add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, carry, expanded, compact, inverse

### Key skills for addition at Year 3:

- Read and write numbers to 1000 in numerals and words.
- Add 2-digit numbers mentally, including those with a total exceeding 100.
- Add a three-digit number and ones mentally ( $175 + 8$ ).
- Add a three-digit number and tens mentally ( $249 + 50$ ).
- Add a three-digit number and hundreds mentally ( $381 + 400$ ).
- Estimate answers to calculations, using inverse to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition.
- Recognise place value of each digit in 3-digit numbers (hundreds, tens, ones).
- Continue to practise a wide range of mental addition strategies, ie. number bonds, adding the nearest multiple of 10, 100, 100 and adjusting, using near doubles, partitioning and recombining.

# Addition

## Year 4 Add numbers with up to 4 digits



**Mental methods** should continue to develop, supported by a range of models and images, including the number line (see Y2)

**Missing number problems** using a range of equations as in Y2 with appropriate, larger numbers

**Use the compact column method**, adding units first, and 'carrying' numbers underneath the calculation. Also include money and measures contexts.

$$3517 + 396 = 3913$$

Add units first

$$\begin{array}{r} 3517 \\ + 396 \\ \hline 3913 \end{array}$$

Consolidate use of the **compact column addition** method with bigger (4-digit) numbers or with more than two numbers.

Remind pupils that the actual value is 'five hundreds add three hundreds' not five add three.

"Carry" numbers underneath the bottom line

Use and apply this method to contexts such as measures and money.

Extend to:

**Decimals with up to two places**  
(same number of decimal places)

$$\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \\ \hline 1 \quad 1 \end{array}$$

**Adding several numbers**  
(with different numbers of digits).

$$\begin{array}{r} 1421 \\ + 244 \\ \hline 16 \\ \hline 1681 \\ \hline 1 \end{array}$$

**Key vocabulary:** add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, 'carry', expanded, compact, thousands, hundreds, digits, inverse

### Key skills for addition at Year 4:

- Select most appropriate method: mental, jottings or written and explain why.
- Recognise the place value of each digit in a four-digit number.
- Round any number to the nearest 10, 100 or 1000.
- Estimate and use inverse operations to check answers.
- Solve 2-step problems in context, deciding which operations and methods to use and why.
- Find 1000 more or less than a given number.
- Continue to practise a wide range of mental addition strategies, ie. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining.
- Add numbers with up to 4 digits using the formal written method of column addition.
- Solve 2-step problems in contexts, deciding which operations and methods to use and why.
- Estimate and use inverse operations to check answers to a calculation.



# Addition

## Year 5

### Add numbers with more than 4 digits



**Mental methods** should continue to develop, supported by a range of models and images. Children should practise with increasingly large numbers to aid fluency  
e.g.  $12462 + 2300 = 14762$

**Missing number problems** using a range of equations with appropriate, larger numbers.

e.g.

$$\begin{array}{r} 23 \square \\ + 4 \square 8 \\ \hline 655 \end{array}$$

Use the compact column method to add numbers with more than 4 digits including **money, measures** and **decimals** with different numbers of decimal places.

$$\begin{array}{r} \text{£} 23.59 \\ + \text{£} 7.55 \\ \hline \text{£} 31.14 \end{array}$$

The decimal point should be aligned in the same way as the other place value columns, and must be in the same column in the answer.

$$\begin{array}{r} 23481 \\ + 1362 \\ \hline 24843 \end{array}$$

Numbers should exceed 4 digits.

$$\begin{array}{r} 19.01 \\ 3.65 \\ + 0.70 \\ \hline 23.36 \end{array}$$

Pupils should be able to add more than two values, carefully aligning place value columns.

Say 6 tenths add 7 tenths to reinforce place value

Empty decimal places should be filled with zero to show the place value in each column.

**Key vocabulary:** add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, "carry", expanded, compact, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths

#### Key skills for addition at Year 5:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies e.g. add the nearest multiple of 10, 100, 100 and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds.
- Use rounding to check answers and accuracy.
- Solve multi-step problems in contexts, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000.
- Add numbers with more than 4 digits using formal written method of column addition.



# Addition

## Year 6

Add several numbers of increasing complexity



**Mental methods** should continue to develop, supported by a range of models and images, including the number line.

**Missing number problems** using a range of equations with appropriate, larger numbers (see Y5).

	2	3	.	3	6	1
		9	.	0	8	0
	5	9	.	7	7	0
+		1	.	3	0	0
<hr/>						
	9	3	.	5	1	1
	2	1		2		

Empty decimal places should be filled with zero to show the place value in each column.

### **Adding several numbers with different numbers of decimal places**

(including money and measures):

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.
- Zeros could be added into any empty decimal places, to show there is no value to add.

As children become more confident and accurate, they may no longer need to give the decimal point a square of its own.

	8	1	,	0	5	9		
			3	,	6	6	8	
			1	5	,	3	0	1
+			2	0	,	5	5	1
<hr/>								
	1	2	0	,	5	7	9	
		1		1		1		

### **Adding several numbers with more than 4 digits.**

Longer lists of numbers could be separated into 2 or more calculations, with the subtotals added to give a final total.

### **Problem Solving**

Pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding.

**Key vocabulary:** add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, 'carry', expanded, compact, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths

### **Key skills for addition at Year 6:**

- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Pupils understand how to add mentally with larger numbers and calculations of increasing complexity.
- Use negative numbers in context, and calculate intervals across zero.

# Subtraction

## Year 1

### Subtract from numbers up to 20



$$6 - 1 = 5$$

Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc.

$$9 - 2$$

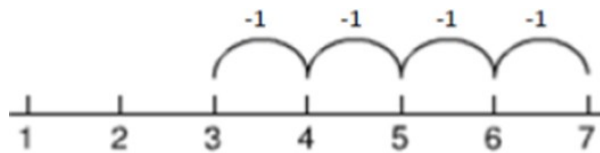


Use Cuisenaire rods or Numicon for subtraction too

#### Subtract by taking away

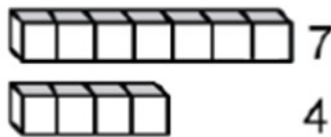
Children are introduced to more formal recording using number lines as below:

**Count back** in ones on a numbered number line to take away, with numbers up to 20:



Model counting back using hundred squares, number tracks/lines and practical apparatus such as cubes.

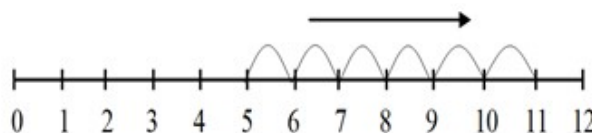
#### Subtract by finding the difference



Seven is three more than four.  
I am three years older than my sister.

This will be introduced practically with the language 'find the difference between' and 'how many more' in a range of familiar contexts. Children will begin by using concrete objects e.g. cubes, that they can move around, before progressing to pictorial representations.

11 is 6 more than 5



#### Mental Subtraction

Children should start recalling subtraction facts up to and within 10 and 20, and should be able to subtract zero.

#### Missing number problems

e.g.  $7 = \square - 9$ ;  $20 - \square = 9$ ;  $15 - 9 = \square$ ;  $\square - \square = 11$ ;  $16 - 0 = \square$

**Key vocabulary:** equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is\_?

#### Key skills for subtraction at Year 1:

- Given a number, say **one more** or **one less**.
- Count to and over 100, **forward and back**, from any number.
- Represent and use **subtraction facts to 20 and within 20**.
- Subtract with **one-digit and two-digit** numbers to 20, including zero.
- Solve one-step problems that involve addition and subtraction, using concrete objects (i.e. bead string, objects, cubes) and pictures, and missing number problems.
- Read and write numbers from 0 to 20 in numerals and words.

# Subtraction

## Year 2

### Subtract with 2-digit numbers

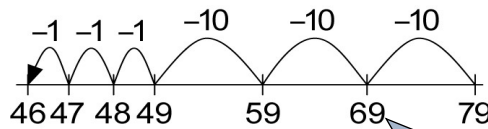
Subtract on a number line to develop mental subtraction skills.

This strategy will be used for:

- 2-digit numbers subtract units (by taking away / counting back) e.g.  $36 - 7$
- 2-digit numbers subtract tens (by taking away / counting back) e.g.  $48 - 30$
- Subtracting pairs of 2-digit numbers (see below:)

#### Subtracting pairs of 2-digit numbers on a number line

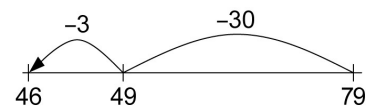
$79 - 33 = 46$  Partition the second number and subtract it in tens and units, as below:



Then subtract units.

Subtract tens first.

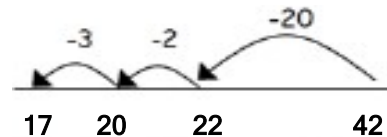
Move towards more efficient jumps back, as below:



Combine methods with use of a hundred square to reinforce understanding

#### Bridging through ten

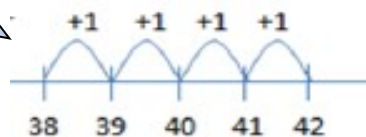
can help children to become more efficient, for example  $42 - 25$ :



Counting on can be used to subtract numbers that are close together:

Start with the smaller number and count on to the larger.

$$42 - 38 = 4$$

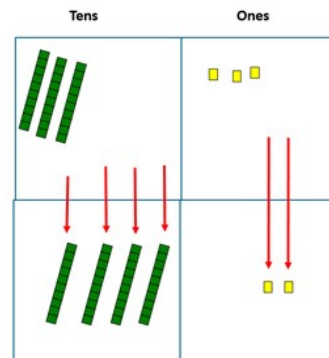


Many mental strategies are taught. Children are taught to recognise that when numbers are close together, it is more efficient to **count on** the difference. They need to be clear about the relationship between addition and subtraction.

#### Towards written methods

Recording addition and subtraction in expanded columns can support understanding of the quantity aspect of place value and prepare for efficient written methods with larger numbers. The numbers may be represented with Dienes apparatus.

E.g.  $75 - 42$



$$\begin{array}{r} 70 \ 5 \\ -40 \ 2 \\ \hline 30 \ 3 \end{array}$$

Missing number problems e.g.  $52 - 8 = \square$ ;  $\square - 20 = 25$ ;  $22 = \square - 21$ ;  $6 + \square + 3 = 11$

**Key vocabulary:** equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is\_? difference, count on, strategy, partition, tens, units

#### Key skills for subtraction at Year 2:

- Recognise the place value of each digit in a two-digit number.
- Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a two-digit number and ones, a two-digit number and tens, and two two-digit numbers.
- Show that subtraction of one number from another cannot be done in any order.
- Recognise and use inverse relationship between addition and subtraction, using this to check calculations and missing number problems.
- Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation, and also applying their increasing knowledge of mental and written methods.
- Read and write numbers to at least 100 in numerals and in words.

# Subtraction

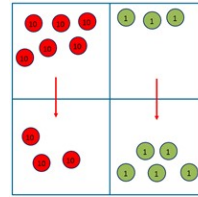
## Year 3 Subtract with 2 and 3-digit numbers.

Missing number problems e.g.  $\square = 43 - 27$ ;  $145 - \square = 138$ ;  $274 - 30 = \square$ ;  
 $245 - \square = 195$ ;  $532 - 200 = \square$ ;  $364 - 153 = \square$

Partitioned column subtraction method.

STEP 1: Introduce this method with examples where **no exchanging** is required.

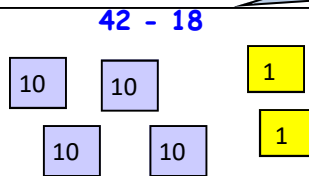
$$\begin{array}{r} 89 - 35 = 54 \\ 80 + 9 \\ - 30 + 5 \\ \hline 50 + 4 \end{array}$$



$$\begin{array}{r} 90 \ 8 \\ - 30 \ 5 \\ \hline 60 \ 3 \end{array}$$

When learning to exchange, explore partitioning in different ways so that children understand that when you exchange, the **VALUE** is the same e.g.  $42 = 40 + 2 = 30 + 12 = 20 + 22$  etc. Emphasise that the value hasn't changed, we have just partitioned it in a different way.

STEP 2: Introduce 'exchanging' through practical subtraction. Make the larger number using place value counters then subtract 18 from it.



$$\begin{array}{r} 30 \cancel{0} + 12 \\ - 10 + 8 \\ \hline 20 + 4 = 24 \end{array}$$

Before subtracting 8 from the 42 blocks, children will need to exchange a ten counter for ten units.

STEP 3: Once pupils are secure with the concept of 'exchanging', they can use the partitioned column method to subtract 2 and 3 digit numbers.

$$\begin{array}{r} 238 - 146 = 92 \\ \begin{array}{r} 100 \\ 200 + 30 + 8 \\ - 100 + 40 + 6 \\ \hline 0 + 90 + 2 \end{array} \end{array}$$

Subtracting money, partition into e.g.

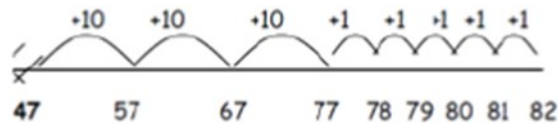
$$\pounds 2 + 30\text{p} + 8\text{p}$$

Counting on in tens is the way we use a hundred square

### Counting on as a mental strategy for subtraction

Continue to reinforce counting **on** as a strategy for **close-together** numbers (e.g. 121–118), and also for numbers that are 'nearly' multiples of 10, 100, 100 or £s, which make it easier to count on (e.g. 102–89, 131–79, calculating change from £1 etc.)

Start with the smaller number and count on in tens first, then count on in units to find the rest of the difference.



**Key vocabulary:** equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is? difference, count on, strategy, partition, tens, units, exchange, decrease, hundreds, value, digit, inverse

### Key skills for subtraction at Year 3:

- Subtract mentally a: 3-digit number and ones, 3-digit number and tens, 3-digit number and hundreds.
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems.
- Find 10 or 100 more or less than a given number.
- Recognise the place value of each digit in a 3-digit number.
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10 (see examples above).
- Read and write numbers up to 1000 in numerals and words.
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or 21), and select most appropriate methods to subtract, explaining why.

# Subtraction

## Year 4

### Subtract with up to 4-digit numbers

Partitioned column subtraction with exchanging (decomposition):

$$\begin{array}{r} 2754 - 1562 = 1192 \\ 2000 + \overset{600}{\cancel{700}} + 50 + 4 \\ - 1000 + 500 + 60 + 2 \\ \hline 1000 + 1000 + 90 + 2 \end{array}$$

As introduced in Y3, but moving towards more complex numbers and values. Use **place value counters** or **Dienes** to reinforce 'exchanging'.

Compact column subtraction

$$\begin{array}{r} 2\overset{6}{\cancel{7}}54 \\ - 1562 \\ \hline 1192 \end{array}$$

To introduce the compact method, show a subtraction calculation with the familiar partitioned column subtraction then display the compact version for that calculation. Ask children to consider how it relates to the method they know; what is similar and what is different, to develop an understanding of it.

Give plenty of opportunities to apply this to money and measures.

Always encourage children to consider the best method for the numbers involved - mental, counting on, counting back or written method.

#### Mental strategies

A variety of mental strategies must be taught and practised, including counting on to find the difference where numbers are closer together, or where it is easier to count on.

#### Missing number/digit problems:

e.g.  $456 + \square = 710$ ;  $1\square7 + 6\square = 200$ ;  $60 + 99 + \square = 340$ ;  
 $200 - 90 - 80 = \square$ ;  $225 - \square = 150$ ;  $\square - 25 = 67$ ;  $3450 - 1000 = \square$ ;  
 $\square - 2000 = 900$

**Key vocabulary:** equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is\_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse

#### Key skills for subtraction at Year 4:

- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.
- Children select the most appropriate and efficient methods for given subtraction calculations.
- Estimate and use inverse operations to check answers.
- Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why.
- Solve simple measure and money problems involving fractions and decimals to two decimal places.
- Find 1000 more or less than a given number.
- Count backwards through zero, including negative numbers.
- Recognise place value of each digit in a 4-digit number.
- Round any number to the nearest 10, 100 or 1000.
- Solve number and practical problems that involve the above, with increasingly large positive numbers.



# Subtraction

## Year 5

Subtract with at least 4-digit numbers including money, measures, decimals.

Compact column subtraction with 'exchanging'

$$\begin{array}{r} \overset{2}{8} \overset{10}{1} \overset{10}{0} \overset{4}{8} \overset{6}{6} \\ - \quad \quad 2 \quad 1 \quad 2 \quad 8 \\ \hline 2 \quad 8, \quad 9 \quad 2 \quad 8 \end{array}$$

Subtract with larger integers

Create lots of opportunities for subtracting and finding differences with money and measures.

$$\begin{array}{r} \overset{6}{7} \overset{10}{1} \overset{6}{6} \overset{8}{9} \cdot \overset{10}{0} \\ - \quad \quad 3 \quad 7 \quad 2 \cdot 5 \\ \hline 6 \quad 7 \quad 9 \quad 6 \cdot 5 \end{array}$$

Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.

Add a zero in any empty decimal places to aid understanding of what to subtract in that column.

**Missing number/digit problems:**

e.g.  $119 - \square = 86$        $1\,000\,000 - \square = 999\,000$        $12\,462 - 2\,300 = \square$

Using the inverse operation:  $600\,000 + \square + 1000 = 671\,000$        $6.45 = 6 + 0.4 + \square$

**Mental methods** should continue to develop, supported by a range of models and images, including the number line (see Y2/3).

**Key vocabulary:** equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is \_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal

**Key skills for subtraction at Year 5:**

- Subtract numbers mentally with increasingly large numbers.
- Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy.
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Count forwards or backwards in steps of powers of 10 for any given number up to 1 million.
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0.
- Round any number up to 1 million to the nearest 10, 100, 1000, 10 000 and 100 000.

# Subtraction

## Year 6

Subtracting with increasingly large and more complex numbers and decimal values.

$$\begin{array}{r}
 \cancel{0}^{\circ} \cancel{5}^{\mu} \cancel{10}^{\eta}, 699 \\
 - \quad 89,949 \\
 \hline
 60,750
 \end{array}$$

Using the **compact column method** to subtract more complex integers

Empty decimal places can be filled with zero to show the place value in each column.

$$\begin{array}{r}
 \cancel{1}^{\circ} \cancel{10}^{\eta} 5 \cdot \cancel{4}^{\circ} 19 \text{ kg} \\
 - \quad 36 \cdot 08 \text{ kg} \\
 \hline
 69 \cdot 339 \text{ kg}
 \end{array}$$

Using the **compact column method** to subtract **money and measures**, including decimals with different numbers of decimal places.

As children become more confident and accurate, they may no longer need to give the decimal point a square of its own.

### Missing number/digit problems:

Make use of inverse operations.

e.g.  $\square$  and  $\#$  each stand for a different number.

$\# = 34$ .  $\# + \# = \square + \square + \#$ . What is the value of  $\square$ ?

What if  $\# = 28$ ? What if  $\# = 21$

$10\,000\,000 = 9\,000\,100 + \square$

$7 - 2 \times 3 = \square$ ;  $(7 - 2) \times 3 = \square$ ;  $(\square - 2) \times 3 = 15$

Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting the most appropriate method to work out subtraction problems.

**Key vocabulary:** equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is...? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, thousandths, decimal point, decimal, decimal place

### Key skills for subtraction at Year 6:

- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Use negative numbers in context, and calculate intervals across zero.
- Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate.



# Multiplication

Year 1

Multiply with concrete objects, arrays and pictorial representations.

X

Understand that multiplication is related to **doubling** and **combining groups** of the same size (repeated addition). Present practical problem solving activities involving counting equal sets or groups of concrete objects, as shown.

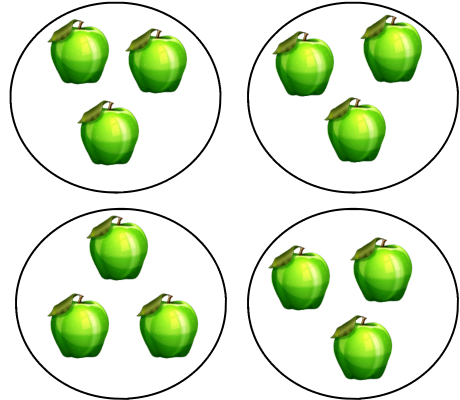
How many legs do 3 teddies have?



$$2 + 2 + 2 = 6$$

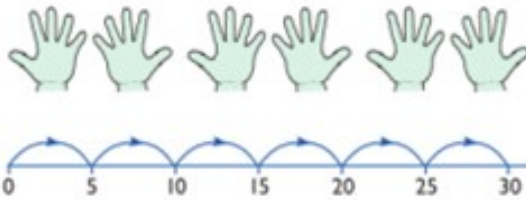
There are 3 apples in one bag.

How many apples in 4 bags altogether?



$$3 + 3 + 3 + 3 = 12$$

Give children experience of counting equal group of objects in 2s, 5s and 10s.



$$5 + 5 + 5 + 5 + 5 + 5 = 30$$

$$5 \times 6 = 30$$

5 multiplied by 6

6 groups of 5

6 hops of 5

Use arrays to understand multiplication can be done in any order (commutative)



$$4 \times 2 = 8$$

$$2 \times 4 = 8$$



$$2 \times 4 = 8$$

$$4 \times 2 = 8$$

2 hops of 4



4 hops of 2

**Key vocabulary:** groups of, lots of, times, array, altogether, multiply, count

**Key skills for multiplication at Year 1:**

- Count in multiples of 2, 5 and 10 to 100.
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays, with the support of the teacher.
- Make connections between arrays, number patterns, and counting in twos, fives and tens. Begin to understand doubling using concrete objects and pictorial representations.

# Multiplication

## Year 2

Multiply using arrays and repeated addition  
(using at least 2s, 5s and 10s)

# X

### Solve missing number problems

using understanding of the inverse and practical resources. E.g.:

$$\begin{array}{ll} 7 \times 2 = \square & \square = 2 \times 7 \\ 7 \times \square = 14 & 14 = \square \times 7 \\ \square \times 2 = 14 & 14 = 2 \times \square \\ \square \times \square = 14 & 14 = \square \times \square \end{array}$$

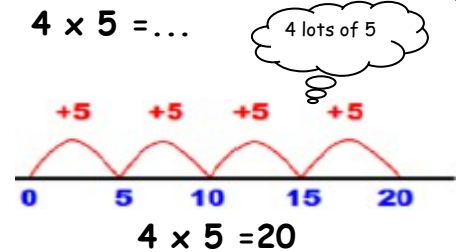
### Use mental recall:

Children should be able to recall multiplication facts up to  $12 \times$  for **2, 5 and 10** multiplication tables.

### Use repeated addition on a number line:

Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using  $\times$  and  $=$  signs.

$$4 \times 5 = \dots$$



### Use arrays:



$$3 \times 5 = 15$$

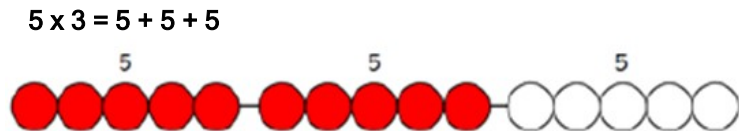
$$5 \times 3 = 15$$

$$5 \times 3 = 3 + 3 + 3 + 3 + 3 = \underline{15}$$

$$3 \times 5 = 5 + 5 + 5 = \underline{15}$$

Use arrays to teach children to understand the commutative law of multiplication, and give examples such as  $3 \times \underline{\quad} = 6$

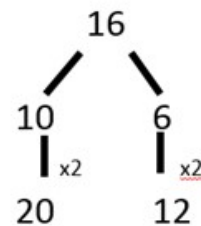
Use practical apparatus:



### Towards written methods:

Use jottings to develop an understanding of doubling two digit numbers.

E.g. double 16



**Key vocabulary:** groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, ... times as big as, once, twice, three times...

### Key skills for multiplication at Year 2:

- Count in steps of 2, 3 and 5 from zero, and in 10s from any number.
- Recall and use multiplication facts from the **2, 5 and 10** multiplication tables, including recognising odds and evens.
- Write and calculate number statements **using the  $\times$  and  $=$  signs**.
- Show that multiplication can be done in any order (commutative).
- Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods, and multiplication facts.
- Pupils use a variety of language to discuss and describe multiplication.

# Multiplication

**Year 3**

Multiply 2 digits by a single digit number

**X**

**Missing Number Problems**

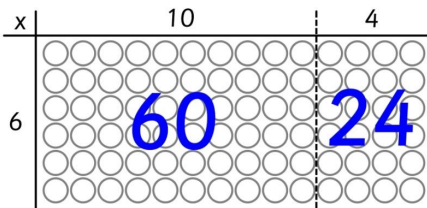
Continue using a range of equations  
e.g.  $\square \times 5 = 20$ ,  $3 \times \square = 18$

**Use mental recall:**

Children should be able to recall multiplication facts for 2, 3, 4, 5, 8 and 10 multiplication tables.

**Introduce the grid method for multiplying 2 digit by single digits:**

Link the layout of the grid to an array initially:



$$14 \times 6 = (10 \times 6) + (4 \times 6)$$

$$= 60 + 24$$

$$= 84$$

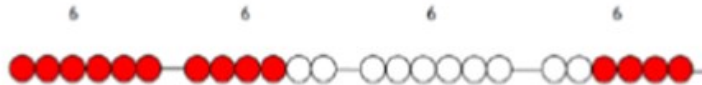
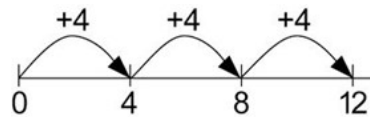
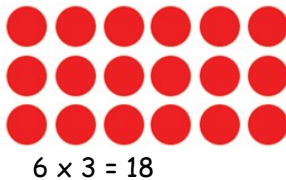
$$35 \times 4 = 140$$

x	30	5
4	120	20

Introduce the grid method with children physically making an array to represent the calculation (e.g.  $23 \times 8$  - make 8 lots of 23 with 10s and 1s place value counters), then translate this to grid method format.

To do this, children must be able to:

- Partition numbers into tens and units
- Multiply multiples of ten by a single digit (e.g.  $20 \times 4$ ) using their knowledge of multiplication facts and place value
- Recall and work out multiplication facts in the 2, 3, 4, 5, 8 and 10 times tables.
- Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are repeated addition using a number line, bead strings and arrays:



**Key vocabulary:** groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, \_times as big as, once, twice, three times..., partition, grid method, multiple, product, tens, units, value, inverse

**Key skills for multiplication at Year 3:**

- Recall and use multiplication facts for the 2, 3, 4, 5, 8 and 10 multiplication tables, and multiply multiples of 10.
- Write and calculate number statements using the multiplication tables they know, including 2-digit x single-digit, drawing upon mental methods, and progressing to reliable written methods.
- Solve multiplication problems, including missing number problems.
- Develop mental strategies using commutative law e.g.  $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$ .
- Solve simple problems in contexts, deciding which operations and methods to use.
- Develop efficient mental methods to solve a range of problems and for missing number problems.

# Multiplication

## Year 4

Multiply 2 and 3 digits by a single digit,  
using all multiplication tables up to 12 x 12

# X

### Missing Number Problems

Continue using a range of equations with appropriate numbers.

E.g.  $\square \times 2 = 160$

### Use mental recall:

Children should be able to recall multiplication facts for all multiplication tables to 12 x 12.

Solve practical problems involving scaling. Relate to known number facts.  
e.g. how tall would a 25cm sunflower be if it grew 6 times taller?

### Developing the grid method:

$$148 \times 4 = 592$$

x	100	40	8
4	400	160	32

400

160

+ 32

592

Encourage column addition to add accurately

Demonstrate the expanded method of short multiplication when children are confident and accurate multiplying 2 and 3-digit numbers by a single digit in a grid and are already confident in "carrying" for written addition. Compare a grid method calculation to the compact method to see how the steps are related. Ensure children see that there are fewer steps in the extended method as the calculation is already laid out for column addition.

$$\begin{array}{r} 148 \\ \times 4 \\ \hline 32 \\ 160 \\ 400 \\ \hline 592 \end{array}$$

When children understand expanded short multiplication they can move on to the compact method (see Y5)

### Children should be able to:

- Approximate before they calculate, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer. e.g:  $346 \times 9$  is approximately  $350 \times 10 = 3500$
- Record an approximation to check the final answer against.
- Multiply multiples of ten and one hundred by a single-digit, using their multiplication tables knowledge.
- Recall all times tables up to 12 x 12

**Key vocabulary:** groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times, as big as, once, twice, three times... partition, grid method, total, multiple, product, sets of, inverse

### Key skills for multiplication at Year 4:

- Count in multiples of 6, 7, 9, 25 and 100.
- Recall multiplication facts for **all multiplication tables up to 12 x 12**.
- Recognise place value of digits in up to 4-digit numbers.
- Use place value, known facts and derived facts to multiply mentally, e.g. multiply by 1, 10, 100, by 0, or to multiply 3 numbers.
- Use commutative law and other strategies mentally  $2 \times 6 \times 5 = 10 \times 6$ ,  $39 \times 7 = 30 \times 7 + 9 \times 7$ .
- Solve problems with increasingly complex multiplication in a range of contexts.
- Solve problems involving scaling e.g. if 2 children have 3 cakes, how many will 6 children have?

# Multiplication

## Year 5

### Multiply up to 4 digits by 1 or 2 digits



#### Using mental methods:

- Use practical resources and jottings to explore equivalent statements (e.g.  $4 \times 35 = 2 \times 2 \times 35$ )
- Recall prime numbers up to 19 and identify prime numbers up to 100

**Missing Number Problems** Continue using a range of equations with appropriate numbers.

e.g.  $0.7 \times \square = 5.6$        $4 \times \square = 320$

**Solve practical problems involving scaling.** Relate to known number facts.

#### Short multiplication for multiplying by a single digit

Children need to be taught to approximate first e.g. for  $93 \times 58$  they will use rounding.  $93 \times 58$  is approximately  $90 \times 60 = 5400$ ; use the approximation to check the reasonableness of their answer.

x	100	40	8	
4	400	160	32	
				400
				160
				+ 32
				<u>592</u>



$$\begin{array}{r} 148 \\ \times 4 \\ \hline 592 \\ 13 \end{array}$$

Work out a given calculation using the grid, then compare to the compact column method. What are the similarities and differences? Unpick the steps and show how it reduces the number of steps.

#### Introduce long multiplication for multiplying by 2 digits

x	40	8	
60	2400	480	
4	160	32	



$$\begin{array}{r} 64 \\ \times 48 \\ \hline 512 \\ 2560 \\ \hline 3072 \end{array}$$

When multiplying by the tens digit, teach to multiply by ten first by putting a zero in the units then think  $4 \times 4 = 16$ ,  $4 \times 60 = 240$ . This will help to make the transition to the compact method easier.

When children understand expanded long multiplication they can move on to the compact method (see Y6)

Use grid method to introduce long multiplication, as the individual steps can be clearly seen.

Children should move on to more complex calculations (e.g.  $1436 \times 18$ ) as they become more confident.

Measures and money should be used regularly as a context for calculations.

**Key vocabulary:** groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, ...times as big as, once, twice, three times..., partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short/long multiplication, 'carry'

#### Key skills for multiplication at Year 5:

- Identify multiples and factors, using knowledge of **multiplication tables to 12x12**.
- Solve problems where larger numbers are decomposed into their factors.
- Multiply and divide integers and decimals by 10, 100 and 1000.
- Recognise and use square and cube numbers and their notation.
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.



# Multiplication

## Year 6

### Short and long multiplication

Multiply decimals with up to 2 decimal places by a single digit



**Mental methods:** Identifying common factors and multiples of given numbers

**Missing number problems:** Continue using a range of equations with appropriate numbers (see Y5).

**Solve practical problems involving scaling:** Relate to known number facts.

#### Short multiplication: compact method

$$\begin{array}{r} 148 \\ \times 4 \\ \hline 592 \\ 13 \end{array}$$

In the units column,  $4 \times 8 = 32$ . Write the 2 in the units column and 'carry' the 3 into the tens.

In the tens column,  $4 \times 4 = 16$  plus 3 tens carried in makes 19.

Write 9 in the tens column and 'carry' 1 into the hundreds column.

In the hundreds column,  $4 \times 1 = 4$  plus the 1 hundred carried from the tens column makes 5.

#### Long multiplication: move from extended to compact method

$$\begin{array}{r} 64 \\ \times 48 \\ \hline 32 \\ 480 \\ 160 \\ 2400 \\ \hline 3072 \end{array}$$



$$\begin{array}{r} 64 \\ \times 48 \\ \hline 512 \\ 2560 \\ \hline 3072 \\ 1 \end{array}$$

##### 64 x 8 on the first row

In the units column,  $8 \times 4 = 32$ : write down the 2 and carry the 3.

In the tens column,  $8 \times 6 = 48$  plus the 3 carried in makes 51.

##### 64 x 40 on the second row

Multiply the answer by 10 first by putting a zero in the units column which moves all the digits one place to the left.

In the tens column,  $4 \times 4 = 16$ ; write down the 6 and carry the 1.

In the hundreds column,  $4 \times 6 = 24$  plus the 1 carried in makes 25.

#### Multiplying with decimal numbers

$$\begin{array}{r} 3.19 \\ \times 8 \\ \hline 25.52 \end{array}$$

Remind children:

- Line up the decimal points in the question and the answer.
- The single digit belongs in the units column.

#### Children should be able to:

- Use rounding and place value to approximate and check answers.
- Use short multiplication to multiply numbers with more than 4 digits, including numbers with up to 2 decimal places by a single digit.
- Use long multiplication to multiply numbers with at least 4 digits by 2 digits.

As children become more confident and accurate, they may no longer need to give the decimal point a square of its own.

**Key vocabulary:** groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times... partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short / long multiplication, 'carry', tenths, hundredths, decimal place, decimal point

#### Key skills for multiplication at Year 6:

- Recall multiplication facts for all times tables up to  $12 \times 12$  (as Y4 and Y5).
- Multiply multi-digit numbers, up to 4-digit x 2-digit using long multiplication.
- Perform mental calculations with mixed operations and large numbers.
- Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods.
- Estimate answers using round and approximation and determine levels of accuracy.
- Round any integer to a required degree of accuracy.

# Division

## Year 1 Group and share small quantities

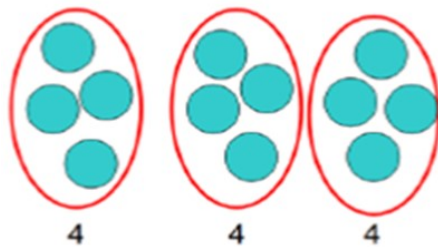
Using objects, diagrams and pictorial representations to solve problems involving both grouping and sharing.

Grouping



How many groups of 4 can be made with 12 stars? = 3

Sharing



12 shared between 3 is 4

Pupils should:

- Use lots of practical apparatus, arrays and picture representations
- Be taught to understand the difference between 'grouping' objects (How many groups of 2 can you make?) and 'sharing' (Share these sweets between 2 people)
- Be able to count in multiples of 2s, 5s and 10s.
- Find half of a group of objects by sharing into 2 equal groups.

**Example division problem in a familiar context:**

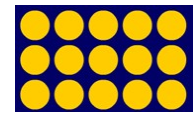
There are 6 pupils on this table and there are 18 pieces of fruit to share between us. If we share them equally, how many will we each get?

Can they work it out and give a division statement... ?

"18 shared between 6 people gives you 3 each."



Arrays



$15 \div 3 = 5$  There are 5 groups of 3.  
 $15 \div 5 = 3$  There are 3 groups of 5.

**Key Vocabulary:** share, share equally, one each, two each..., group, groups of, lots of, array

**Key number skills needed for division at Year 1:**

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations, arrays, with the support of the teacher.
- Through grouping and sharing small quantities, pupils begin to understand division, and finding simple fractions of objects, numbers and quantities. ( $\frac{1}{2}$ ,  $\frac{1}{4}$ )
- They make connections between arrays, number patterns, and counting in twos, fives and tens.



# Division

## Year 2

Group and share using the  $\div$  and  $=$  sign Use objects, arrays, diagrams, physical resources and pictorial representations to explore sharing and grouping. Use a number line.

$\div$  = signs and missing numbers

$$6 \div 2 = \square$$

$$\square = 6 \div 2$$

$$6 \div \square = 3$$

$$3 = 6 \div \square$$

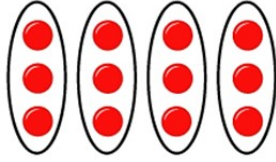
$$\square \div 2 = 3$$

$$3 = \square \div 2$$

$$\square \div \triangle = 3$$

$$3 = \square \div \triangle$$

Arrays:



$$12 \div 3 = 4$$

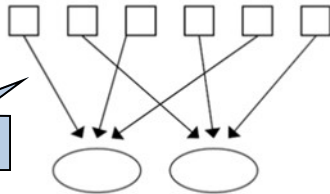
This represents  $12 \div 3$ , posed as how many groups of 3 are in 12?

Pupils should also show that the same array can represent  $12 \div 4 = 3$  if grouped horizontally.

### Know and understand sharing and grouping

6 sweets shared between 2 people; how many do they each get?

Grouping



Sharing

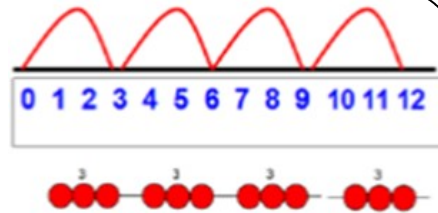
There are 6 sweets, how many people can have 2 sweets each?



Children should be taught to recognise whether problems require sharing or grouping.

### Grouping using a number line

Group from zero in equal jumps of the divisor to find out 'how many groups of  $\_$  in  $\_$  ?'. Pupils could use a bead string or practical apparatus to work out problems like 'A CD costs £3. How many CDs can I buy with £12?'



This is an important method to develop understanding of division as grouping.

$$12 \div 3 = 4$$

Express  $12 \div 3$  as 'How many groups of 3 are in 12?'

**Key Vocabulary:** share, share equally, one each, two each..., group, groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over

### Key number skills needed for division at Year 2:

- Count in steps of 2, 3 and 5 from 0.
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the  $\times$ ,  $\div$  and  $=$  signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

# Division

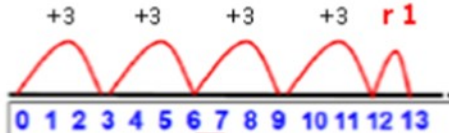
## Year 3

Divide 2 digit numbers by a single digit.

**÷ = signs and missing numbers**

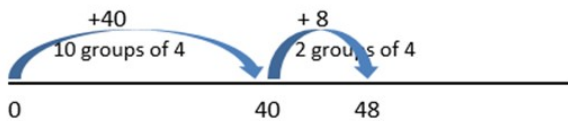
Continue using a range of equations (as in year 2) with appropriate numbers.

$$13 \div 3 = 4 \text{ r } 1$$



**STEP 1:** Children continue to work out unknown division fact by grouping on a number line from zero. They are also now taught the concept of remainders, as in the example. This should be introduced practically and with arrays (see Y1/Y2), as well as being translated to a number line.

$$48 \div 4 = 12$$



**STEP 2:** Children need to become more efficient using a number line by being able to partition the dividend (the number being divided) in different ways.

**Short division:** Limit numbers to NO remainders in the answer OR carried (each number must be a multiple of the divisor)

$$\begin{array}{r} 32 \\ 3 \overline{) 96} \end{array}$$

**STEP 3:** Once the children are secure with division as grouping and can demonstrate this using number lines, short division for larger 2 digit numbers should be introduced, initially with carefully selected examples requiring no calculation of remainders at all.

Remind children of correct place value (96 is 90 + 6) but work across each column:

In the 10s column, how many 3s in 9? Record in 10s column.

In the 1s column, how many 3s in 6? Record in 1s column.

**Knowledge of tables facts makes this much easier!**

**Short division:** Limit numbers to NO remainders in the final answer, but remainders occurring within the calculation.

$$\begin{array}{r} 18 \\ 4 \overline{) 72} \end{array}$$

**STEP 4:** Once children demonstrate an understanding of the short division method, introduce the concept of remainders with examples where the remainder occurs within the calculation (e.g.  $72 \div 4$ ). Teach the children to 'carry' the remainder to the next digit.

Only introduce Step 4 when children can calculate remainders mentally.

Real life contexts need to be used routinely to help pupils gain the ability to recognise division and understand how to apply it to problems.

**Key Vocabulary:** share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, dividend

**Key number skills needed for division at Year 3:**

- Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through doubling, connect the 2, 4 and 8s).
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplication and division.
- Pupils develop efficient mental methods, for example, using multiplication and division facts (e.g. using  $3 \times 2 = 6$ ,  $6 \div 3 = 2$  and  $2 = 6 \div 3$ ) to derive related facts ( $30 \times 2 = 60$ , so  $60 \div 3 = 20$  and  $20 = 60 \div 3$ ).
- Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and progressing to the formal written method of short division.

# Division

## Year 4

### Divide up to 3-digit numbers by a single digit

#### $\pm$ = signs and missing numbers

Continue using a range of equations (see Y2) with appropriate numbers.

#### Sharing, Grouping and using a number line

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding. Children should progress in their use of written division calculations:

- Using tables facts with which they are fluent
- Experiencing a logical progression in the numbers they use, for example:
  1. Dividend (number to be divided) just over 10x the divisor, e.g.  $84 \div 7$
  2. Dividend just over 10x the divisor when the divisor is a teen number e.g.  $173 \div 15$
  3. Dividend over 100x the divisor, e.g.  $840 \div 7$
  4. Dividend over 20x the divisor, e.g.  $168 \div 7$

All the above stages should include calculations with remainders as well as without. Remainders should be interpreted according to the context (i.e. rounded up or down to relate to the answer to the problem)

e.g.  $840 \div 7 = 120$

#### *Jottings*

$$7 \times 100 = 700$$

$$7 \times 10 = 70$$

$$7 \times 20 = 140$$



#### Continue to develop short division:

$$\begin{array}{r} 218 \\ 4 \overline{) 872} \end{array}$$

Once children are secure with Stage 3 expectations, they can move onto dividing numbers with up to 3 digits by a single digit. Problems should not result in a final remainder at this stage.

$$\begin{array}{r} 037 \\ 5 \overline{) 185} \end{array}$$

When the answer for the first column is zero (e.g.  $1 \div 5$ ), children could write a zero above to acknowledge the place value. The number must be carried over to the next digit as a remainder.

Include money and measures contexts when children are confident with the short division method.

**Key Vocabulary:** share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, dividend, divisible by, factor

#### Key number skills needed for division at Year 4:

- Recall multiplication and division facts for all numbers up to  $12 \times 12$ .
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number.
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example  $200 \times 3 = 600$  so  $600 \div 3 = 200$ .
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

# Division

## Year 5

Divide up to 4 digit numbers by a single digit, including those with remainders

÷ = signs and missing numbers

Continue using a range of equations with appropriate numbers e.g.  $135 \div \square = 15$

**Sharing, grouping and using a number line**

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding (as in Y4).

**Short division, including remainder answers:**

$$\begin{array}{r} 0663 \text{ r } 5 \\ 8 \overline{) 5309} \end{array}$$

**Short division with remainders:** Now that children are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where **pupils consider the meaning of the remainder and how to express it**, i.e. as a fraction, a decimal, or as a rounded number or value, depending upon the context of the problem.

The answer to  $5309 \div 8$  could be expressed as:

- $663 \frac{5}{8}$
- $663 \text{ r } 5$
- $663.625$
- $663$
- or  $664$

depending on the problem involved.

Continuing short division to give a decimal answer is covered in Year 6.

**Key Vocabulary:** share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisor, dividend, divisible by, factor, inverse, **quotient, prime number, prime factors, composite number (non-prime)**

**Key number skills needed for division at Year 5:**

- Recall multiplication and division facts for all numbers up to  $12 \times 12$  (as in Year 4).
- Multiply and divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
- Use multiplication and division as inverses.
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding:  
e.g.  $98 \div 4 = 24 \text{ r } 2 = 24 \frac{1}{2} = 24.5$ .
- Solve problems involving combinations of all four operations, including understanding of the equals sign, and including division for scaling by different fractions and problems involving simple ratios.

# Division

## Year 6

Divide at least 4 digit numbers by both single digit and 2 digit numbers (including decimal numbers and quantities)

÷ = signs and missing numbers

Continue using a range of equations with appropriate numbers.

### Sharing and Grouping and using a number line

Children will continue to explore division as sharing and grouping, and to represent calculations on a number line as appropriate.

### Short division, for dividing by a single digit e.g. $6497 \div 8$

**Short division with remainders:** Children should continue to use this method, but with numbers up to at least 4 digits. They should understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

$$\begin{array}{r} 8 \overline{)6497.000} \\ \underline{8} \phantom{00} \\ 64 \phantom{00} \\ \underline{64} \phantom{00} \\ 97 \phantom{00} \\ \underline{96} \phantom{00} \\ 100 \phantom{00} \\ \underline{100} \phantom{00} \\ 1000 \phantom{00} \\ \underline{1000} \phantom{00} \\ 10000 \phantom{00} \\ \underline{10000} \phantom{00} \\ 0 \phantom{00} \end{array}$$

A zero can be used as a place holder, as it is here, but is not encouraged once children are confident with this method.

**Calculating a decimal remainder:** In this example, rather than expressing the remainder as r 1, a decimal point is added after the units because there is still a remainder, and the one remainder is carried on to zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved.

$$\begin{array}{r} 22 \\ 32 \overline{)704} \\ \underline{64} \phantom{0} \\ 64 \phantom{0} \\ \underline{64} \phantom{0} \\ 0 \end{array}$$

### Introduce long division for dividing by 2 digits using expanded method

Children are taught to work across the place value columns, asking themselves each time, 'What is the biggest multiple of the divisor I can take away?'. Write the size of the 'chunk' of the divisor on the answer line, then subtract the multiple. Once the subtraction is completed, bring down the next digit from the answer and repeat the process. As with short division, answers can be given as a whole number with a remainder, as a mixed number or with a decimal depending on the context.

Using mental strategies e.g. repeated doubling or tables facts, will speed up the calculation process.

**Key Vocabulary:** share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple, divisible by, divisor, dividend, factor, inverse, quotient, prime number, prime factors, composite number (non-prime), common factor, divisibility

### Key number skills needed for division at Year 6:

- Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations.
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.