

For this exam <https://www.aqa.org.uk/subjects/mathematics/as-and-a-level/further-mathematics-7366>

For the A-Level exam in 2025 <https://www.aqa.org.uk/subjects/mathematics/as-and-a-level/further-mathematics-7367>

### Revision List

Two 90-minute papers – Paper 1 is on Pure; Paper 2 is Mechanics and Statistics – Two 45-minute papers  
Attached is the AS Specification – this is the revision list. It can be linked to the AS Guidance Document, coming soon

### Useful revision resources

#### Websites

- TLMaths ([tllmaths.com](http://tllmaths.com))
- MathsGenie ([www.mathsgenie.co.uk](http://www.mathsgenie.co.uk))

#### Recommended Revision Guides

CGP A-Level Mathematics Revision Guide and Workbook (Edexcel version is suitable for AQA also)  
CGP Textbook for Mathematics that you have  
Kerboodle Textbook for AQA Further Mathematics

#### Recommended Calculators

Casio Classwiz EX-991 CW - available on Wisepay, CG-50 also available

#### Maths Sets

We have a Maths Sets available on Wisepay and are priced at £2, these come in an exam-friendly transparent pencil case

### Revision Tips

Revision for Mathematics is based upon practice (and more practice). You need to be confident at the skills and concepts that make up the course in order to be able to work through the more challenging problems. Revision should be interactive, not just reading notes

Students can work through guidance document to identify areas that they are good at, alongside areas they are struggling with and need to work on

A potential plan of action would be

- Work through the guidance document – RAG the topics to indicate your level of understanding
- Work through maths problems and past papers.
- Do not just read your notes/revision guides as you need to practice your Maths skills.

## 3 Subject content

The subject content in sections A to H is compulsory for all students. Students must study two of the optional applications. The optional applications are mechanics (MA to MD), statistics (SA to SH) and discrete (DA to DG).

### 3.1 Overarching themes

AS specifications in further mathematics must require students to demonstrate the overarching knowledge and skills contained in sections **OT1**, **OT2** and **OT3**. These must be applied, along with associated mathematical thinking and understanding, across the whole of the detailed content set out in sections **A** to **DG**.

Appendix A sets out the mathematical notation that students are required to understand for this qualification. Appendix B sets out the mathematical formulae and identities students are required to use in this qualification. Further information is provided in the appendices.

#### 3.1.1 OT1: Mathematical argument, language and proof

|       | Content  |
|-------|--|
| OT1.1 | Construct and present mathematical arguments through appropriate use of diagrams; sketching graphs; logical deduction; precise statements involving correct use of symbols and connecting language, including: constant, coefficient, expression, equation, function, identity, index, term, variable. |
| OT1.2 | Understand and use mathematical language and syntax as set out in the content.   |
| OT1.3 | Understand and use language and symbols associated with set theory, as set out in the content.   |
| OT1.5 | Comprehend and critique mathematical arguments, proofs and justifications of methods and formulae, including those relating to applications of mathematics.  |

#### 3.1.2 OT2: Mathematical problem solving

|       | Content   |
|-------|---|
| OT2.1 | Recognise the underlying mathematical structure in a situation and simplify and abstract appropriately to enable problems to be solved. |
| OT2.2 | Construct extended arguments to solve problems presented in an unstructured form, including problems in context.                        |
| OT2.3 | Interpret and communicate solutions in the context of the original problem.   |

|       | Content  |
|-------|--|
| OT2.6 | Understand the concept of a mathematical problem solving cycle, including specifying the problem, collecting information, processing and representing information and interpreting results, which may identify the need to repeat the cycle. |
| OT2.7 | Understand, interpret and extract information from diagrams and construct mathematical diagrams to solve problems, including in mechanics.   |

### 3.1.3 OT3: Mathematical modelling

|       | Knowledge/skill   |
|-------|---|
| OT3.1 | Translate a situation in context into a mathematical model, making simplifying assumptions.   |
| OT3.2 | Use a mathematical model with suitable inputs to engage with and explore situations (for a given model or a model constructed or selected by the student).    |
| OT3.3 | Interpret the outputs of a mathematical model in the context of the original situation (for a given model or a model constructed or selected by the student). |
| OT3.4 | Understand that a mathematical model can be refined by considering its outputs and simplifying assumptions; evaluate whether the model is appropriate.        |
| OT3.5 | Understand and use modelling assumptions.   |

## 3.2 Compulsory content

### 3.2.1 A: Proof

|    | Content   |
|----|---|
| A1 | Construct proofs using mathematical induction; contexts include sums of series, divisibility, and powers of matrices. |

### 3.2.2 B: Complex numbers

|    | Content   |
|----|---|
| B1 | Solve any quadratic equation with real coefficients; solve cubic or quartic equations with real coefficients (given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics). |

|    | Content   |
|----|---|
| B2 | Add, subtract, multiply and divide complex numbers in the form $x + iy$ with $x$ and $y$ real; understand and use the terms 'real part' and 'imaginary part'. |

|    | Content   |
|----|---|
| B3 | Understand and use the complex conjugate; know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs.   |
|    | Knowledge/skill   |
| B4 | Use and interpret Argand diagrams.  |
|    | Content   |
| B5 | Convert between the Cartesian form and the modulus-argument form of a complex number (knowledge of radians is assumed).                       |
|    | Content   |
| B6 | Multiply and divide complex numbers in modulus-argument form (knowledge of radians and compound angle formulae is assumed).                   |
|    | Content   |
| B7 | Construct and interpret simple loci in the Argand diagram such as $ z - a  > r$ and $\arg(z - a) = \theta$ (knowledge of radians is assumed). |

### 3.2.3 C: Matrices

|    | Content   |
|----|---|
| C1 | Add, subtract and multiply conformable matrices; multiply a matrix by a scalar.   |
|    | Content   |
| C2 | Understand and use zero and identity matrices.  |
|    | Content   |
| C3 | Use matrices to represent linear transformations in 2D; successive transformations; single transformations in 3D (3D transformations confined to reflection in one of $x = 0$ , $y = 0$ , $z = 0$ or rotation about one of the coordinate axes) (knowledge of 3D vectors is assumed). |
|    | Content   |
| C4 | Find invariant points and lines for a linear transformation.  |
|    | Content   |
| C5 | Calculate determinants of $2 \times 2$ matrices.  |

|    | Content  |
|----|--|
| C6 | Understand and use singular and non-singular matrices; properties of inverse matrices.<br>Calculate and use the inverse of non-singular $2 \times 2$ matrices. |

### 3.2.4 D: Further algebra and functions

|    | Content   |
|----|---|
| D1 | Understand and use the relationship between roots and coefficients of polynomial equations up to quartic equations. |

|    | Content  |
|----|--|
| D2 | Form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation (of at least cubic degree). |

|    | Content  |
|----|--|
| D3 | Understand and use formulae for the sums of integers, squares and cubes and use these to sum other series. |

|    | Content   |
|----|---|
| D4 | Understand and use the method of differences for summation of series. |

|    | Content   |
|----|---|
| D6 | Recognise and use the Maclaurin series for $e^x$ , $\ln(1+x)$ , $\sin x$ , $\cos x$ , and $(1+x)^n$ , and be aware of the range of values of $x$ for which they are valid (proof not required). |

|    | Content  |
|----|--|
| D8 | Inequalities involving polynomial equations (cubic and quartic). |

|    | Content  |
|----|--|
| D9 | Solving inequalities such as $\frac{ax+b}{cx+d} < ex+f$ algebraically. |

|     | Content  |
|-----|--|
| D12 | Graphs of rational functions of form $\frac{ax+b}{cx+d}$ ; asymptotes, points of intersection with coordinate axes or other straight lines; associated inequalities. |

|     | Content  |
|-----|--|
| D13 | Graphs of rational functions of form $\frac{ax^2+bx+c}{dx^2+ex+f}$ , including cases when some of these coefficients are zero; asymptotes parallel to coordinate axes. |

|     | Content  |
|-----|--|
| D14 | Using quadratic theory (not calculus) to find the possible values of the function and coordinates of the stationary points of the graph for rational functions of form $\frac{ax^2+bx+c}{dx^2+ex+f}$ . |

|     | Content   |
|-----|---|
| D15 | Sketching graphs of curves with equation $y^2 = 4ax$ , $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , $xy = c^2$ including intercepts with axes and equations of asymptotes of hyperbolas. |

|     | Content   |
|-----|---|
| D16 | Single transformations of curves involving translations, stretches parallel to coordinate axes and reflections in the coordinate axes and the lines $y = \pm x$ . |

### 3.2.5 E: Further calculus

|    | Content  |
|----|--|
| E2 | Derive formulae for and calculate volumes of revolution. |

|    | Content   |
|----|---|
| E3 | Understand and evaluate the mean value of a function. |

### 3.2.6 F: Further vectors

|    | Content  |
|----|--|
| F1 | Understand and use the vector and Cartesian forms of an equation of a straight line in 3D. |

|    | Content   |
|----|---|
| F3 | Calculate the scalar product and use it to calculate the angle between two lines. |

|    | Content  |
|----|--|
| F4 | Check whether vectors are perpendicular by using the scalar product. |

|    | Content   |
|----|---|
| F6 | Find the intersection of two lines.<br>Calculate the perpendicular distance between two lines and from a point to a line. |

### 3.2.7 G: Polar coordinates

|    | Content  |
|----|--|
| G1 | Understand and use polar coordinates and be able to convert between polar and Cartesian coordinates. |

|    | Content  |
|----|--|
| G2 | Sketch curves with $r$ given as a function of $\theta$ , including use of trigonometric functions. |

### 3.2.8 H: Hyperbolic functions

|    | Content  |
|----|--|
| H1 | Understand the definitions of hyperbolic functions $\sinh x$ , $\cosh x$ and $\tanh x$ , and be able to sketch their graphs. |

|    | Content  |
|----|--|
| H3 | Understand and be able to use the definitions of the inverse hyperbolic functions. |

|    | Content   |
|----|---|
| H4 | Derive and use the logarithmic forms of the inverse hyperbolic functions. |

|    | Content  |
|----|--|
| H6 | Understand and use $\tanh x \equiv \frac{\sinh x}{\cosh x}$<br>Understand and use $\cosh^2 x - \sinh^2 x \equiv 1$ |

## 3.3 Optional application 1 – mechanics

### 3.3.1 MA: Dimensional analysis

|     | Content   |
|-----|---|
| MA1 | Finding dimensions of quantities; checking for dimensional consistency. |

|     | Content   |
|-----|---|
| MA2 | Prediction of formulae; finding powers in potential formulae. |

### 3.3.2 MB: Momentum and collisions

|     | Content   |
|-----|---|
| MB1 | Conservation of momentum for linear motion and cases where velocities are given as one or two dimensional vectors (resolving will not be required). |

|     | Content  |
|-----|--|
| MB2 | Coefficient of restitution and Newton's Experimental Law. Use in direct collisions and impacts with a fixed smooth surface (resolving will not be required). |

|     | Content   |
|-----|---|
| MB3 | Impulse and its relation to momentum (in one- or two-dimensions) (resolving will not be required).<br>Use of $Ft = mv - mu$ |

|     | Content  |
|-----|--|
| MB4 | Impulse for variable forces. One dimension only. Use of $I = \int Fdt$ . |

### 3.3.3 MC: Work, energy and power

|     | Content   |
|-----|---|
| MC1 | Work done by a force acting in the direction of motion or directly opposing the motion. |

|     | Content   |
|-----|---|
| MC2 | Gravitational potential energy. Use in conservation of energy problems. |

|     | Content   |
|-----|---|
| MC3 | Kinetic energy. Use in conservation of energy problems. |

|     | Content   |
|-----|---|
| MC4 | Hooke's Law including using modulus of elasticity.<br>Use of $T = kx$ or $T = \frac{\lambda}{l}x$ |

|     | Content  |
|-----|--|
| MC5 | Work done by a variable force. Use of $WD = \int Fdx$ .<br>Use in conservation of energy problems. |

|     | Content   |
|-----|---|
| MC6 | Elastic potential energy using modulus of elasticity. Use of $EPE = \frac{kx^2}{2}$ and $EPE = \frac{\lambda x^2}{2l}$ .<br>Use in conservation of energy problems. |

|     | Content   |
|-----|---|
| MC7 | Power (resolving will not be required). Use of $P = Fv$ |

### 3.3.4 MD: Circular motion

|     | Content   |
|-----|---|
| MD1 | Motion of a particle moving in a circle with constant speed (knowledge of radians assumed). |

|     | Content  |
|-----|--|
| MD2 | Understand the definition of angular speed.<br>Use both radians and revolutions per unit time. |

|     | Content   |
|-----|---|
| MD3 | Relationships between speed, angular speed, radius and acceleration. Use of $v = r\omega$ , $a = r\omega^2$ and $a = \frac{v^2}{r}$ |

## 3.4 Optional application 2 – statistics

### 3.4.1 SA: Discrete random variables (DRVs) and expectation

|     | Content  |
|-----|--|
| SA1 | Understand DRVs with distributions given in the form of a table or function. |

|     | Content                           |
|-----|-----------------------------------|
| SA2 | Evaluate probabilities for a DRV. |

|     | Content   |
|-----|---|
| SA3 | Evaluate measures of average and spread for a DRV to include mean, variance, standard deviation, mode and median. |

|     | Content  |
|-----|--|
| SA4 | Understand expectation and know the formulae: $E(X) = \sum x_i p_i$ ; $E(X^2) = \sum x_i^2 p_i$ ;<br>$\text{Var}(X) = E(X^2) - (E(X))^2$ |

|     | Content   |
|-----|---|
| SA5 | Understand expectation of linear functions of DRVs and know the formulae:<br>$E(aX + b) = aE(X) + b$ and $\text{Var}(aX + b) = a^2 \text{Var}(X)$ |

|     | Content  |
|-----|--|
| SA6 | Know the discrete uniform distribution defined on the set $\{1, 2, \dots, n\}$ . Understand when this distribution can be used as a model. |

|     | Content  |
|-----|--|
| SA7 | Proof of mean and variance of discrete uniform distribution. |

### 3.4.2 SB: Poisson distribution

|     | Content   |
|-----|---|
| SB1 | Understand conditions for a Poisson distribution to model a situation. Understand terminology $X \sim \text{Po}(\lambda)$ . |

|     | Content   |
|-----|---|
| SB2 | Know the Poisson formula and calculate Poisson probabilities using the formula or equivalent calculator function. |

|     | Content   |
|-----|---|
| SB3 | Know mean, variance and standard deviation of a Poisson distribution.<br>Use the result that, if $X \sim \text{Po}(\lambda)$ then the mean and variance of $X$ are equal. |

|     | Content  |
|-----|--|
| SB4 | Understand the distribution of the sum of independent Poisson distributions. |

|     | Content   |
|-----|---|
| SB5 | Formulate hypotheses and carry out a hypothesis test of a population mean from a single observation from a Poisson distribution using direct evaluation of Poisson probabilities. |

### 3.4.3 SC: Type I and Type II errors

|     | Content  |
|-----|--|
| SC1 | Understand Type I and Type II errors and define in context. Calculate the probability of making a Type I error from tests based on a Poisson or Binomial distribution. |

### 3.4.4 SD: Continuous random variables (CRVs)

|     | Content   |
|-----|---|
| SD1 | Understand and use a probability density function, $f(x)$ , for a continuous distribution and understand the differences between discrete and continuous distributions. |

|     | Content   |
|-----|---|
| SD2 | Find the probability of an observation lying in a specified interval. |

|     | Content  |
|-----|--|
| SD3 | Find the median and quartiles for a given probability density function, $f(x)$ . |

|     | Content   |
|-----|---|
| SD4 | Find the mean, variance and standard deviation for a given pdf, $f(x)$ . Know the formulae<br>$E(X) = \int xf(x)dx$ , $E(X^2) = \int x^2f(x)dx$ , $\text{Var}(X) = E(X^2) - (E(X))^2$ |

|     | Content   |
|-----|---|
| SD5 | Understand the expectation and variance of linear functions of CRVs and know the formulae:<br>$E(aX + b) = aE(X) + b$ and $\text{Var}(aX + b) = a^2\text{Var}(X)$<br>Know the formula $E(g(X)) = \int g(x)f(x)dx$<br>Find the mean, variance and standard deviation of functions of a continuous random variable such as<br>$E(5X^3)$ , $E(18X^{-3})$ , $\text{Var}(6X^{-1})$ |

|     | Content  |
|-----|--|
| SD8 | Know that if $X$ and $Y$ are independent (discrete or continuous) random variables then $E(X + Y) = E(X) + E(Y)$ and $\text{Var}(X + Y) = \text{Var}(X) + \text{Var}(Y)$ |

### 3.4.5 SE: Chi squared tests for association

|     | Content  |
|-----|--|
| SE1 | Construction of $n \times m$ contingency tables. |

|     | Content   |
|-----|---|
| SE2 | Use of $\sum \frac{(O_i - E_i)^2}{E_i}$ as an approximate $\chi^2$ statistic with appropriate degrees of freedom. |

|     | Content  |
|-----|--|
| SE3 | Know and use the convention that all $E_i$ should be greater than 5. |

|     | Content  |
|-----|--|
| SE4 | Identification of sources of association in the context of a question. |

### 3.4.6 SH: Confidence intervals

|     | Content   |
|-----|---|
| SH1 | Construct symmetric confidence intervals for the mean of a normal distribution with known variance.                       |
|     | Content   |
| SH2 | Construct symmetric confidence intervals from large samples, for the mean of a normal distribution with unknown variance. |
|     | Content   |
| SH3 | Make inferences from constructed or given confidence intervals.   |