**GCE A level Mathematics (9AM0) – Paper 2**

**Pure Mathematics 2**

**October 2020 student-friendly mark scheme**

**Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn’t show follow-through marks (marks that are awarded despite errors being made) or special cases.**

**It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.**

**This document is intended for guidance only and may differ significantly from the final mark scheme published in December 2020.**

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| **Guidance on the use of codes within this document** |
| M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.  A1 – accuracy mark. This mark is generally given for a correct answer following correct working.  B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.  Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer). |

**Question 1 (Total 5 marks)**

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| --- | --- | --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** | | |
| (a) | *h* = 5 | B1  1.1a | This mark is given for stating *h* = 5 | | |
| {0.5774 + 0.8452 + 2(0.7071 + 0.7746 + 0.8165)} | | | M1  1.1b | This mark is given for a method to use the trapezium rule |
| 1.50 | A1  1.1b | This mark is given for the correct answer only | | |
| (b) | 3 × 1.50 = 4.50 | B1  2.2a | This mark is given for a correct estimate only | | |
| (c) | The answer is accurate to 2 significant figures | B1  3.2b | This mark is given for a correct statement about the accuracy | | |

**Question 2 (Total 3 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  | = **q** – **p**,  = **r** – **p**,  = **r** – **q** | M1  1.1b | This mark is given for a method to find at least two of the relevant vectors |
| **r** – **q** **=** 2(**q** – **p**) | M1  3.1a | This mark is given for using the information written in vector form |
| **r** – **q** **=** 2**q** – 2**p**  2**p** + **r** = 3**q**  **q** = (**r** + 2**p**) | A1  2.1 | The mark is given for fully correct working leading to the given answer |

**Question 3 (Total 5 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | 2 log (4 – *x*) = log (4 – *x*)2 | B1  1.2 | This mark is given for stating that 2 log (4 – *x*) = log (4 – *x*)2 |
| 2 log (4 – *x*) = log (*x*+ 8)  log (4 – *x*)2 = log (*x*+ 8)  (4 – *x*)2 = (*x* + 8) | M1  1.1b | This mark is given for a method to eliminate logs to form a quadratic equation |
| 16 + 8*x* + *x*2 = *x* + 8  *x*2 – 9*x* + 8 = 0 | A1  2.1 | The mark is given for fully correct working leading to the given answer |
| (b)(i) | (*x* – 1)(*x* – 8) = 0  *x* = 1, *x* = 8 | B1  1.1b | This mark is given for the correct answer only |
| (b)(ii) | 8 is not a solution since log (4 – 8) cannot be found | B1  2.3 | This mark is given for a correct reason stated |

**Question 4 (Total 3 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  | 7C4 *a*3 (2*x*)4 | M1  1.1b | This mark is given for selecting the correct term |
| *a*3 × 24 = 15120 | M1  2.1 | This mark is given for a complete method to find the value of *a* |
| *a* = 3 | A1  1.1b | This mark is given for the correct answer only |

**Question 5 (Total 6 marks)**

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| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  | 15 – 2*x* + 1 = 3 × 2*x* | B1  1.1b | This mark is given for finding 15 – 2*x* + 1 = 3 × 2*x* |
| 15 – 2 × 2*x* = 3 × 2*x*  2*x* = *c* | M1  1.1b | This mark is given for a method to obtain an equation with 2*x*as the subject |
| *x* = log2 *c* | M1  1.1b | This mark is given for a full method to find the ­x­-coordinate |
| *x* = log2 3 | A1  1.1b | This mark is given for the correct answer only |

**Question 6 (Total 7 marks)**

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| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | *x*2 + 8*x* – 3 = (*Ax* + *B*)(*x* + 2) + *C* | M1  1.1b | This mark is given for a method to find the values of *A*, *B* and *C* |
| = (*x* + 6) – 15  *A* = 1, *B* = 6, *C* = –15 | A1  1.1b | This mark is given for two of three correct values *A*, *B* and *C* |
| A1  1.1b | This mark is given for a a third correct value *A*, *B* and *C* |
| (b) | d*x* =  d*x* | M1  1.1b | This mark is given for integrating an expression of the form |
| = *x*2 + 6*x* – 15 ln (*x* + 2) + *c* | A1  1.1b | This mark is given for a correct integration of  + *Bx* + *C* |
| d*x* =  = (18 + 36 – 15 ln 8) – (0 + 0 – 15 ln 2)  = 18 + 36 – (15 – 45) ln 2 | M1  2.1 | This mark is given for a method to use limits |
| =54 – 30 ln 2 | A1  1.1b | This mark is given for a correct answer only |

**Question 7 (Total 10 marks)**

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| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | = | B1  1.1a | This mark is given for the derivative of ln *x* seen |
| 2 ×  +  × | M1  1.1b | This mark is given for a method to differentiate |
| A1  1.1b | This mark is given for a correct differentiation |
| = 3√*x* +  –  = | A1  2.1 | The mark is given for fully correct working leading to the given answer |
| (b) | 12*x*2 + *x* – 16√*x* = 0 | M1  1.1b | This mark is given for a method to set  = 0 and divide by √*x* |
| 12 = 16 – √*x* | M1  1.1b | This mark is given for a method to make  the subject |
| =  –  ⇒ *x* = | A1  2.1 | The mark is given for fully correct working leading to the given answer |
| (c) | *x*2 = | M1  1.1b | This mark is given for a method to use the iterative formula with *x*1 = 2 |
| *x*2 = 1.13894 | A1  1.1b | This mark is given for a correct value for *x*2 |
| *x* = 1.15650 | A1  2.2a | This mark is given for a correct value for the *x*-coordinate of *P* |

**Question 8 (Total 6 marks)**

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| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
|  | f(*x*) = 2*x*3 + *ax*2 – 23*x* + *c* | M1  1.1b | This mark is given for a method to integrate f ′(*x*) |
| A1  1.1b | This mark is given for a fully correct integration (including +*c*) |
| *c* = –12 | B1  2.2a | This mark is given for using the *y‑*intercept to find the value of *c* |
| f(–4) = 0  ⇒ 2 × (–4)3 + *a*(–4)2 – 23(–4) – 12 = 0 | M1  3.1a | This mark is igvne for a method to find *a* by setting f(–4) = 0 |
| *a* = 6 | M1  1.1b | This mark is given for a correct value for *a* |
| f(*x*) = 2*x*3 + 3*x*2 – 23*x* – 12 | A1  2.1 | This mark is given for the correct answer only |

**Question 9 (Total 6 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| --- | --- | --- | --- |
| (a) | *t* = 0, *θ* = 18  18 = *A* – *B* | M1  1.1b | This mark is given for method to use the model with *t* = 0, *θ* = 18 |
| *t* = 10, *θ* = 44  44 = *A* – *B*e–0.7 | M1  3.1b | This mark is given for method to use the model with *t* = 10, *θ* = 44 |
| *A* = 69.6, *B* = 18 | M1  3.1a | This mark is given for solving a pair of simultaneous equations to find *A* and *B* |
| *θ* = 69.6 – 51.6e0.07*t* | A1  3.3 | This mark is given for an equation of the model |
| (b) | The maximum temperature is 69.6 °C | B1  3.4 | This mark is given for identifying the value of *A* as the boiling point of the model |
| The model is not appropriate since 69.6 °C is much lower than 78 °C | B1  3.5a | This mark is given for a correct statement about the accuracy of the model |

**Question 10 (Total 8 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| --- | --- | --- | --- |
| (a) | cos 3*A* = cos (2*A* + *A*)  = cos 2*A* cos *A* – sin 2*A* sin *A* | M1  3.1a | This mark is given for a method to use the compound angle formula for cos (2*A*+ *A*) |
| = (2 cos2 *A* – 1) cos *A –* (2 sin *A* cos *A*) sin *A* | M1  1.1b | This mark is given for a method to use double angle identities for cos 2*A* and sin 2*A* |
| = (2 cos2 *A* – 1) cos *A –* 2 cos (1 – cos2 *A*) | M1  2.1 | This mark is given for a method to get all in terms in cos *A* |
| = 4 cos3 – 3 cos *A* | A1  1.1b | The mark is given for fully correct working leading to the given answer |
| (b) | 1 – cos 3*x* = sin2 *x*  cos2 *x* + 3 cos *x* – 4 cos3 *x* = 0 | M1  1.1b | This mark is given for a method to get all in terms in cos *x* |
| cos *x* (4 cos2 *x* – cos *x* – 3) = 0  cos *x* (4 cos *x* + 3)(cos *x* – 1) = 0 | M1  3.1a | This mark is given for finding a quadratic equation to be solved |
| –90°, 0°, 90°, 139° | A1  1.1b | This mark is given for any two correct answers |
| A1  2.1 | This mark is given for the other two correct answers |

**Question 11 (Total 7 marks)**

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| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | *x* = –4, *y* = –5 | B1  1.1b | This mark is given for one coordinate found |
| *P*(–4, –5) | B1  2.2a | This mark is given for a correct answer only |
| (b) | 3*x* + 40 = –2(*x* + 4) – 5 | M1  1.1b | This mark is given for a method to solve the equation |
| *x* = –10.6 | A1  2.1 | This mark is given for a correct answer only |
| (c) | *a* > 2 | B1  2.2a | This mark is given for a deduction that *a*> 2 |
| *a* = | M1  3.1a | This mark is given for finding the value of *a* using *P*(­4, ­5) and *O* |
| {*a* : *a* ≤ 1.25} ∪ {*a* : *a* > 2} | A1  2.5 | This mark is given for a correct answer only |

**Question 12 (Total 11 marks)**

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| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a)(i) | d*t* = | M1  1.2 | This mark is given for a method to findd*t* |
| Using sin 2*t* = 2 sin *t* cos *t* | M1  1.1b | This mark is given for a method to use sin 2*t* = 2 sin *t* cos *t* |
| Area = | A1  2.1 | The mark is given for fully correct working leading to the given answer |
| (a)(ii) | = –20 cos3 *t* | M1  1.1b | This mark is given for a method to integrate |
| A1  1.1b | This mark is given for a correct integration |
| Area =  = 0 – (–20) = 20 | A1  2.1 | The mark is given for fully correct working leading to the given answer |
| (b) | 5 sin 2*t* = 4.2  sin 2*t* = | M1  3.4 | This mark is given for a method to find values for *t* |
| *t* = 0.4986, *t* = 1.072 | A1  1.1b | This mark is given for correct values for *t* |
| *t* = 0.4986 ⇒ *x* = 2.869  *t* = 1.072 ⇒ *x* = 5.269 | M1  3.4 | This mark is given for a method to find values of *x* for both values of *t* |
| A1  1.1b | This mark is given for correct values for *x* |
| Width of path = 2.40 metres | A1  3.2a | This mark is given for a correct answer only |

**Question 13 (Total 6 marks)**

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| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | *k* = e2 | B1  2.2a | This mark is given for a correct deduction |
| (b) | (g(*x*)) =  = | M1  1.1b | This mark is given for a method to use the quotient rule to differentiate |
| A1  2.1 | This mark is given for a correct differentiation |
| *x* > 0 and (ln *x* – 2) is squared, so g′(*x*) > 0 | A1  2.4 | This mark is given for a correct statement |
| (c) | 3 ln *x* – 7 > 0 and ln *x* – 2 < 0 | M1  3.1a | This mark is given for a method to solve either 3 ln *x* – 7 > 0 or ln *x* – 2 < 0 |
| 0 < *a* < e2, *a* > | A1  2.2a | This mark is given for a correct answer only |

**Question 14 (Total 7 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | | **Notes** | |
| (a) | (*x* – *r*)2 + (*y* – *r*)2 = *r*2  or  *x*2 + *y*2 – 2*rx* – 2*ry* + *r*2 = 0 | B1  2.2a | | This mark is given for deducing an equation of *C* | |
| *x*2 + (12 – 2*x*)2 – 2*rx* – 2*r*(12 – 2*x*)+ *r*2 = 0 | M1  1.1b | | This mark is given for an attempt to solve *y* = 12 – 2*x* and *x*2 + *y*2 – 2*rx* – 2*ry* + *r*2 = 0 | |
| *x*2 + 144 + 24*x* + 4*x*2 – 2*rx* – 24*r* + 4*rx* + *r*2 = 0  ⇒ 5*x*2 + (2*r* – 48)*x* + (*r*2 – 24*r* + 144) = 0 | | A1  2.1 | | The mark is given for fully correct working leading to the given answer |
| (b) | *b*2 – 4*ac* = 0  *a* = 5, *b* = 2*r* – 48, *c* = *r*2 – 24*r* + 144 | M1  3.1a | | This mark is given for a method to find *b*2 – 4*ac* = 0 | |
| (2*r* – 48)2 – 4 × 5 × (*r*2 – 24*r* + 144) = 0 | A1  1.1b | | This mark is given for a correct equation in *r* | |
| *r*2 – 18*r* + 36 = 0  (*r* – 9)2 – 81 + 36 = 0 | M1  1.1b | | This mark is given for a method tosolve to find *r* | |
| *r* = 9 ± 3√5 | A1  1.1b | | This mark is given for a correct answer only | |

**Question 15 (Total 8 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | *Sn* = *a* + *ar* + *ar*2 + … + *arn* – 1 | B1  1.2 | This mark is given for writing the sum of *Sn* |
| *rSn* = *ar* + *ar*2 + *ar*3 … + *arn* | M1  2.1 | This mark is given for a method to use the sum of *rSn* |
| *Sn* – *rSn* = *a* – *arn* | A1  1.1b | This mark is given for subtracting *rSn* from *Sn* |
| *Sn*(1 – *r*) = *a*(1 – *rn*) ⇒ *Sn* = | A1  2.1 | The mark is given for fully correct working leading to the given answer |
| (b) | = 4 ×  (1 – *r*10) = 4(1 – *r*5) | M1  3.1a | This mark is given for a method to find an equation in terms of *r*10 and *r*5 |
| *r*10 – 4*r*5 + 3 | A1  1.1b | This mark is given for a correct equation in terms of *r*10 and *r*5 |
| (*r*5 – 1)(*r*5 – 3) = 0 | M1  2.1 | This mark is given for rearranging to find a quadratic in *r*5 to be solved |
| *r* =  only (reject *r* = 1) | A1  1.1b | This mark is given for the correct answer only |

**Question 16 (Total 4 marks)**

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| **Part** | **Working or answer an examiner might expect to see** | **Mark** | | **Notes** | |
|  | Natural numbers are 3*k*, 3*k* + 1, 3*k* + 2 | M1  3.1a | | This mark is given for realising that all natural numbers can be written as 3*k*, 3*k* + 1 and 3*k* + 2 | |
| (3*k*)2  = 9*k*2  (3*k* + 1)2 = 9*k*2 + 6*k* + 1  (3*k* + 2)2 = 9*k*2 + 12*k* + 4 | | M1  1.1b | | This mark is given for a method to find a squared expression for all three expressions for the natural numbers |
| (3*k*)2  = 3 × (3*k*2)  (3*k* + 1)2 = 3(3*k*2 + 2*k*) + 1  (3*k* + 2)2 = 3(3*k*2 + 4*k* + 1) + 1 | A1  2.1 | | This mark is for showing algebraically that each case is either a multiple of three or a multiple of three, plus one | |
| Each case is either a multiple of three or a multiple of three, plus one | A1  2.4 | | This mark is given for a conclusion stated | |