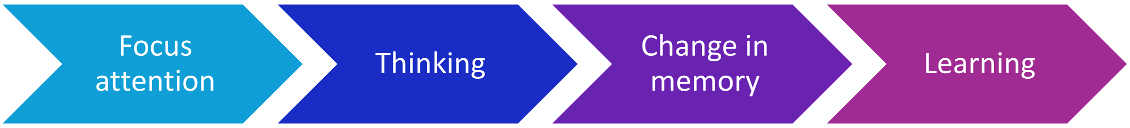
Belief

An important aspect of teaching is to determine what learning is, particularly within the Problem Solving/Reasoning realm. There is an assumption that learning is a permanent change in our long-term memory caused by thinking (**John Sweller and Paul Kirschner 1988 to 2006**), if this is the case, challenging pupils thinking is an important aspect of their learning experience. High expectations on both the level of thinking and pupils' attitude to their work is vital in this process.

Cognitive Load Theory (CLT), developed by John Sweller, explains how the human mind processes information and how this impacts learning. It suggests that our working memory, which handles new information, has limited capacity. CLT aims to minimise cognitive overload by designing instruction that optimises how information is presented, allowing learners to effectively transfer knowledge to long-term memory.

Challenge can be seen as a process whereby educationalists (*Sweller* et al) ensure that, within the mathematics curriculum four elements occur:



Focus Attention: the ability to concentrate on a specific stimulus while filtering out distractions.

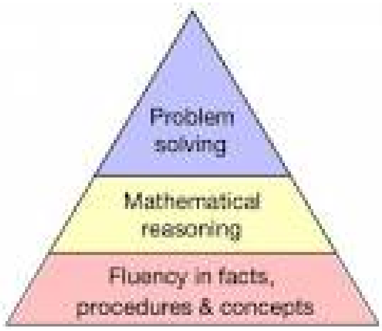
Thinking: the ability to process information, hold attention, store and retrieve memories and select appropriate responses and actions.

Change in Memory: ability to acquire, store, retain, and retrieve information.

Learning: A change in the long-term memory.

Definition of Problem Solving/Reasoning

In mathematics, problem-solving and reasoning are distinct but interconnected skills. Reasoning is the logical thought process used to understand a problem and develop a solution, while problem-solving is the act of applying those reasoning skills to find a specific answer. Essentially, reasoning is the "how" behind problem-solving.



Reasoning in Mathematics:

- **Logical Thinking:** Reasoning involves using logical steps and connections to analyse a situation, identify relationships between concepts, and make deductions.
- **Justification and Explanation:** It's not just about getting the right answer, but also about explaining why it's the right answer, justifying the steps taken, and potentially proving or disproving mathematical statements.
- **Developing Strategies:** Reasoning helps in formulating strategies, comparing different approaches, and adapting known concepts to new situations.

Problem-Solving in Mathematics:

- **Applying Knowledge:** Problem-solving utilises mathematical knowledge, skills, and understanding to tackle specific problems.
- **Following Steps:** It often involves a structured approach, like understanding the problem, devising a plan, carrying out the plan, and examining the solution.
- **Finding Solutions:** The primary goal of problem-solving is to arrive at a specific answer or solution to a given problem.

In essence: When presented with a mathematical problem, pupils use reasoning to make sense of the situation and develop a solution strategy, and then they apply problem-solving skills to implement that strategy and arrive at the answer. Reasoning is the foundation upon which problem-solving is built.

To help clarify this situation so that there is a common understanding of what is meant when we use the term problem solving (**EEF**).

1. Refer to *all* tasks as problems.
2. Define *problem solving* to mean solving *unfamiliar problems*, whereby unfamiliar problems are problems that pupils possess the requisite knowledge to solve but not yet met.
3. Problems are on a spectrum/continuum of familiarity and should form a daily routine.

(*NB: Problems exist on a spectrum of familiarity with pupils positioned at various points along this spectrum – what is familiar to one pupil may not be familiar to another at that current time.)*

4. Familiar problems: the pupil has the requisite knowledge and has met the problem before.
5. Unfamiliar problems: the pupil has the requisite knowledge and has not yet met the problem before.
6. Inaccessible problems: the pupil does not yet have the requisite knowledge.

Note: The more we expose pupils to unfamiliar problems it ceases to become unfamiliar. The more this is practiced by pupils over time, the more familiar problems they have experienced, which in turn supports them to reach further unfamiliar problems.

Mathematics Cognitive Domains

To respond correctly to problem solving and reasoning questions, pupils need to be familiar with the mathematics content being assessed, but they also need to draw on a range of cognitive skills.

The first domain, *knowing*, covers the facts, concepts, and procedures pupils need to know, while the second, *applying*, focuses on the ability of pupils to apply knowledge and conceptual understanding to solve problems or answer questions. The third domain, *reasoning*, goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multistep problems.

Knowing, applying, and reasoning are exercised in varying degrees when pupils display their mathematical competency, which goes beyond content knowledge. Cognitive domains encompass the competencies of problem solving, providing a mathematical argument to support a strategy or solution, representing a situation mathematically (e.g. using symbols and graphs), creating mathematical models of a problem situation, and using tools such as a ruler or a calculator to help solve problems.

Knowing

This domain focuses on the foundational knowledge of mathematical facts, concepts, and procedures. It involves understanding the basic building blocks of mathematics and being able to recall them when needed.

Facility in applying mathematics, or reasoning about mathematical situations, depends on familiarity with mathematical concepts and fluency in mathematical skills. The more relevant knowledge a pupil is able to recall and the wider the range of concepts they understand, the greater the potential for engaging in a wide range of problem-solving situations.

Without access to a knowledge base that enables easy recall of the language and basic facts and conventions of number, symbolic representation, and spatial relations, pupils would find purposeful mathematical thinking impossible. Facts encompass the knowledge that provides the basic language of mathematics, as well as the essential mathematical concepts and properties that form the foundation for mathematical thought.

Procedures form a bridge between more basic knowledge and the use of mathematics for solving problems, especially those encountered by many people in their daily lives. In essence, a fluent use of procedures entails recall of sets of actions and how to carry them out. Pupils need to be efficient and accurate in using a variety of computational procedures and tools. They need to see that particular procedures can be used to solve entire classes of problems, not just individual problems.

Recall	Recall definitions, terminology, number properties, units of measurement, geometric properties, and notation (e.g., $a \times b = ab$, $a + a + a = 3a$).
Recognise	Recognise numbers, expressions, quantities, and shapes. Recognize entities that are mathematically equivalent (e.g., equivalent familiar fractions, decimals, and percents; different orientations of simple geometric figures).
Classify/ Order	Classify numbers, expressions, quantities, and shapes by common properties.
Compute	Carry out algorithmic procedures for +, –, ×, ÷, or a combination of these with whole numbers, fractions, decimals, and integers. Carry out straightforward algebraic procedures.
Retrieve	Retrieve information from graphs, tables, texts, or other sources.
Measure	Use measuring instruments; and choose appropriate units of measurement.

Applying

This domain emphasises the ability to use the knowledge gained in the "Knowing" domain to solve routine problems and answer questions. It involves applying learned concepts and procedures to specific situations.

The applying domain involves the application of mathematics in a range of contexts. In this domain, the facts, concepts, and procedures as well as the problems should be familiar to the pupil. In some items aligned with this domain, pupils need to apply mathematical knowledge of facts, skills, and procedures or understanding of mathematical concepts to create representations. Representation of ideas forms the core of mathematical thinking and communication, and the ability to create equivalent representations is fundamental to success in the subject.

Problem solving is central to the applying domain, with an emphasis on more familiar and routine tasks. Problems may be set-in real-life situations, or may be concerned with purely mathematical questions involving, for example, numeric or algebraic expressions, functions, equations, geometric figures, or statistical data sets.

Determine	Determine efficient/appropriate operations, strategies, and tools for solving problems for which there are commonly used methods of solution.
Represent/ Model	Display data in tables or graphs; create equations, inequalities, geometric figures, or diagrams that model problem situations; and generate equivalent representations for a given mathematical entity or relationship.
Implement	Implement strategies and operations to solve problems involving familiar mathematical concepts and procedures.

Reasoning

This domain goes beyond routine problem-solving and involves tackling unfamiliar situations, complex contexts, and multi-step problems. It requires pupils to think critically, analyse information, and develop logical solutions.

Reasoning mathematically involves logical, systematic thinking. It includes intuitive and inductive reasoning based on patterns and regularities that can be used to arrive at solutions to problems set in novel or unfamiliar situations. Such problems may be purely mathematical or may have real life settings. Both types of items involve transferring knowledge and skills to new situations; and interactions among reasoning skills usually are a feature of such items.

Even though many of the cognitive skills listed in the reasoning domain may be drawn on when thinking about and solving novel or complex problems, each by itself represents a valuable outcome of mathematics education, with the potential to influence learners' thinking more generally. For example, reasoning involves the ability to observe and make conjectures. It also involves making logical deductions based on specific assumptions and rules and justifying results.

Analyse	Determine, describe, or use relationships among numbers, expressions, quantities, and shapes.
Integrate/ Synthesise	Link different elements of knowledge, related representations, and procedures to solve problems.
Evaluate	Evaluate alternative problem-solving strategies and solutions.
Draw Conclusions	Make valid inferences on the basis of information and evidence.
Generalise	Make statements that represent relationships in more general and more widely applicable terms.
Justify	Provide mathematical arguments to support a strategy or solution.

These three domains are not mutually exclusive and are often intertwined during the problem-solving process. For example, a pupil might need to "know" a certain formula (Knowing), "apply" it to a specific problem (Applying), and then "reason" about the best way to interpret the solution within the context of the problem (Reasoning).

Problem-solving in mathematics involves using a systematic approach to find solutions to mathematical tasks or real-world situations. It's a process that requires understanding the problem, devising a plan, carrying out the plan, and evaluating the solution. This process helps pupils develop critical thinking, logical reasoning, and the ability to apply mathematical concepts in various contexts.

Problem-Solving Process:

Here's a breakdown of the key aspects:

Polya's 4-step model: Understanding the problem, devising a plan, carrying out the plan, and looking back. This approach emphasises a structured and thoughtful way to tackle challenges, ensuring a thorough understanding and a robust solution. (**George Polya 1920s**)

Understanding the Problem:

- **Identify the goal:** Clearly define what needs to be found or achieved.
- **Gather information:** Determine what information is given and what is needed to solve the problem.
- **Clarify vocabulary:** Ensure understanding of any unfamiliar mathematical terms.

Devising a Plan:

- **Choose a strategy:** Select appropriate mathematical concepts, formulas, or problem-solving strategies.
- **Develop a plan:** Outline the steps needed to solve the problem.

Carrying Out the Plan:

- **Execute the steps:** Follow the plan systematically, performing calculations and logical reasoning.
- **Record results:** Keep track of calculations, findings, and intermediate steps.

Looking Back (Evaluating the Solution):

- **Check for accuracy:** Verify the solution's correctness using different methods or estimations.
- **Consider reasonableness:** Determine if the solution makes sense in the context of the problem.
- **Generalise the solution:** Explore whether the solution can be applied to similar problems.

5-step approach: Problem identification, analysis, plan development, plan implementation, and plan evaluation.

7-step approach: Define the problem, analyse, develop solutions, evaluate options, select the best option, implement, and measure results.

Importance of Problem Solving in Mathematics:

- **Conceptual understanding:** Problem-solving helps pupils develop a deeper understanding of mathematical concepts and their connections.
- **Mathematical thinking:** It fosters logical reasoning, critical thinking, and the ability to make connections between different mathematical ideas.
- **Real-world applications:** Problem-solving equips pupils with the skills to apply mathematical knowledge to solve practical problems in various contexts.

- **Confidence and motivation:** Successfully solving problems can boost pupils' confidence and motivation in learning mathematics.

Examples of Problem-Solving Strategies:
Mathematical problem-solving can be categorised in various ways, but broadly it involves tasks that challenge one's understanding and application of mathematical concepts. These tasks can range from simple arithmetic to complex, abstract reasoning. Here's a breakdown of different ways to categorize problem-solving in mathematics:

Type of Problem:

- **Memory-based:** These problems test recall of facts and procedures.
- **Skill-based:** These problems require applying learned mathematical skills to solve a problem.
- **Application-based:** These problems involve applying skills in different contexts or situations.
- **Extending skills and theory:** These problems require extending existing knowledge to unfamiliar situations.
- **Abstract vs. Real-world:** Abstract problems are purely mathematical, while real-world problems apply mathematical concepts to practical scenarios.

Complexity and Approach:

- **Simple arithmetic word problems:** These are often used in primary school and involve basic operations.
- **Problems requiring standard techniques:** These problems can be solved using established methods or algorithms.
- **Problems with no obvious method:** These require creative thinking, setting up equations, or using other representations like graphs.
- **Ill-defined problems:** These problems require defining variables, making conjectures, and choosing appropriate representations.
- **Problems requiring mathematical inquiry:** These are problems that mathematicians might consider "real" problems, requiring exploration and specific mathematical thinking.

Mathematical Area:

- **Algebraic problems:** These involve equations, expressions, and functions.
- **Number theory problems:** These focus on properties of numbers.
- **Geometry and measure problems:** These involve shapes, space, and measurement.
- **Probability and statistics problems:** These deal with data analysis and probability.
- **Data handling problems:** These involve collecting, organising, and interpreting data.

By understanding these different ways to categorise problem-solving in mathematics, pupils and educators can better approach and tackle a wide range of mathematical challenges.

A Guide to Solving Problems

Problem Solving
Teaching for Problem Solving
Teaching of Problem Solving
Teaching through Problem Solving

Process

Read the given information and identify the question → Decide on where to start → Be systematic in approach and recording → Review and decide how to continue → Check that the answer works

A comprehensive guide to problem-solving in mathematics involves understanding the problem, devising a plan, executing the plan, and reviewing the solution. Key strategies include identifying given information, defining the goal, choosing appropriate methods (like drawing diagrams, using formulas, or working backward), and checking for accuracy and reasonableness. Often the most common approach is to:

Understand the Problem:

- **Read carefully:** Thoroughly read the problem, identifying what information is given and what needs to be found. Ensure you understand the context and the question being asked.
- **Highlight key information:** Underline or highlight important facts and figures and cross out any unnecessary information.

Clarify the question: Ensure you understand what the problem is asking you to do. Translate the problem's information into the chosen diagram. Label elements clearly and use symbols or colours to highlight key information.

- **Identify the goal:** Determine the end result you are trying to achieve.

Devise a Plan:

- **Choose appropriate strategies:** Consider various problem-solving techniques like:

Using formulas: If applicable, identify relevant formulas to apply.
Formulas are essential tools in mathematical problem-solving, providing a structured approach to finding solutions by applying established rules and relationships between variables. They are used across various mathematical fields like algebra and geometry.

Steps for Solving Problems Using Formulas:
1. Identify the Problem and Relevant Formula: Begin by understanding the problem and identifying which formula(s) apply. This might involve recognising keywords or relationships described in the problem.

2. Extract Information: Identify the known quantities (variables) and their values from the problem statement.

3. Substitute Values: Substitute the known values into the appropriate formula.

4. Solve the Equation: Use algebraic manipulation (like addition, subtraction, multiplication, or division) to isolate the unknown variable and solve for its value.

5. Check the Solution: Ensure the solution makes sense within the context of the problem and verify it by plugging the result back into the formula or original problem.

Example:
Let's say the problem involves finding the area of a rectangle. The formula for the area of a rectangle is: Area (A) = length (l) and width (w).
If the length is 5 units and the width is 3 units, you would substitute these values into the formula: A = 5 x 3 and A = 15 square units.

Importance of Formulas:

- **Efficiency:** Formulas offer a quick and efficient way to solve problems, saving time and effort compared to trial-and-error methods.
- **Accuracy:** Formulas are derived from mathematical principles, ensuring accurate results when applied correctly.
- **Generalisability:** Formulas can be applied to a wide range of similar problems, making them a powerful tool for problem-solving.
- **Foundation for Further Learning:** Understanding formulas lays the groundwork for more advanced mathematical concepts and problem-solving techniques.
- **<u>Working backward</u>:** If the final answer is known, work backward to find the starting point.

To solve a problem by working backwards, we basically want to undo the problem step-by-step. We start at the end of the problem and work through to the beginning. In other words, we do as the name of this solving process suggests - we work backwards!

- **<u>Guess and check</u>:** Make an educated guess and refine it based on the results.
- **<u>Looking for patterns</u>:** Identify repeating patterns or sequences to simplify the problem.
- **<u>Act it out</u>:** Physically acting out the problem can aid in understanding.
- **<u>Creating a table or list</u>:** Organising information in a table or list can reveal relationships and patterns.
- **Break down complex problems:** Divide large problems into smaller, more manageable steps.
- **Trial and error:** Trying different solutions until one works

Carry Out the Plan:

- **Follow your plan step-by-step:** Execute the chosen strategy methodically, showing all your work.
- **Use mathematical tools:** Apply appropriate mathematical operations and techniques.
- **Be flexible:** Don't be afraid to adjust your plan if it's not working as expected.

Look Back and Reflect:

- **Check your answer:** Verify that your answer is reasonable and makes sense in the context of the problem.
- **Use inverse operations:** Check your answer by using the inverse operation (e.g., if you divided, multiply to check).

- **Estimate to check reasonableness:** Round numbers to estimate the answer and see if your calculated answer is close.
- **Check for errors:** Review your calculations and ensure all steps are logically correct.
- **Consider alternative methods:** If possible, try solving the problem using a different approach to confirm your answer.
- **Work the problem one step at a time**

1. Rewrite the given facts in a more organized manner.
2. Given diagrams or charts should show all of the given information.
3. Express the unknown in terms of a variable.
4. Write out each step.

An important problem-solving skill is the ability to distinguish between facts that you know from those you don't know. Write down the required equations and formulas. Break down complex ideas into smaller parts.

- **Analyse the Diagram:**

Look for patterns, relationships, or missing information that can be used to solve the problem. The diagram should guide your thinking and help you develop a plan.

- **<u>Drawing a diagram</u> or Visual Representations:** Visualising the problem can help clarify relationships between quantities.

Drawing diagrams in mathematics helps visualise problems, making them easier to understand and solve. Diagrams act as visual aids that can simplify complex information, allowing for better comprehension and strategic planning. By representing problems visually, pupils can identify relationships between different elements and develop a clearer path towards finding a solution.

- **Simple Drawings:** For spatial problems or problems involving shapes.
- **Number Lines:** For problems involving numbers and their relationships.
- **Venn Diagrams:** For problems involving sets and their intersections.
- **Tables:** For organising data and identifying patterns.
- Creating a visual model of the problem.

Benefits of Using Diagrams:

- **Enhanced Understanding:** Diagrams help visualise abstract concepts and relationships, making them easier to grasp.
- **Improved Problem-Solving Strategies:** They aid in identifying patterns, relationships, and missing information, guiding the problem-solving process.
- **Increased Confidence:** Visualising the problem can boost confidence and reduce anxiety, especially for students who struggle with abstract thinking.

- **Better Communication:** Diagrams can be used to explain solutions to others, making the problem-solving process more transparent.
- **Solve the Problem:** Based on the insights gained from the diagram, apply the appropriate mathematical operations or strategies to find the solution.

By incorporating diagrams into your problem-solving approach, you can unlock a more intuitive and effective way to tackle mathematical challenges.

Common Types (Classification):

- Finding all possibilities.
- Logic problems.
- Finding rules and describing patterns.
- Diagram problems and visual puzzles.

Types of Problem and Appropriate Strategies			
Finding all possibilities	**Logic puzzles**	**Finding rules and describing patterns**	**Diagram problems and visual puzzles**
Have a system for finding the possibilities, e.g. start with the smallest number. Organise the recording of possibilities e.g. in an ordered list or table. Use a method of tracking what has been included and what has not. Have a way of deciding when all the possibilities have been found.	Identify the given facts and prioritise them. Look for any relationships and patterns in the information given. Use one piece of information at a time and see what effect it has, then keep one fixed and test the other. Choose and use a recording system to organise the given information. Check that the answer meets all the criteria.	Decide on the information you need to describe and continue the pattern. Give examples to match a given statement and ones which do not. Describe a rule of a pattern or relationship in words or pictures. Predict the next few terms in a sequence to test the rule. Use a rule to decide whether a given number will be in the sequence or not.	Identify the given information and represent it in another way. Use a systematic approach to solve the problem and a way of recording if necessary. Use drawings or annotations to help visualise the problem using familiar shapes or patterns. Try other possibilities to check the solution.

NB: A word problem is a mathematical exercise which is in the form of a hypothetical question that needs mathematical analysis and equations to be solved.
Word problems can't always be solved by using a formula. Your pupils actually have to understand the concept and be able to apply it with the information given. So often pupils haven't developed the strategies for thinking critically and aren't even sure where to start.

Check Your Solution: Once you have a solution, verify it by reviewing the diagram and the steps taken. Ensure that the solution aligns with the problem's context and makes logical sense.
In mathematics, problem-solving can also be categorised into three main cognitive domains: Knowing, Applying, and Reasoning. These domains represent different levels of mathematical competency and are exercised in varying degrees as pupils engage with problems.

Explanation and Modelling

Worked Examples

A worked example is a problem that has already been solved for the pupil with every step fully explained and clearly shown.

Worked Example Pairs	Incomplete Worked Examples	Correct and Incorrect Worked Examples	Strategy Comparison Worked Examples
Teacher models example, pupils complete theirs. Pupil question is minimally different. Withhold pupil question until worked example is completed. Get pupils to answer on mini whiteboards as check for understanding. Respond accordingly. Use earlier in the learning episode.	Scaffold practice (including retrieval practice). One or more missing step. Gradual reduction/fading more effective than transferring from worked examples to un-scaffolded problems abruptly. Backwards fading (missing the last step) is the simplest form. Use early in the learning episode.	Allows for direct comparison – Correct v Incorrect. Expose misconceptions. No more than one misconception per worked example. Use later in the learning episode. Perfect for exit tickets. Most effective when paired with pupil self-explanation prompts.	Same problem solved in different ways. Deepen conceptual understanding. Identifies similarities and differences. Refer to prior knowledge. Promotes variation. Provides. opportunities to develop structures. and representations. Encourages thinking mathematically.

*NB: **Correct and Incorrect** worked Examples are an ideal way to identify and address any misconceptions in a structured way. This method helps pupils to recognise incorrect procedures, think about the differences between them and provide an opportunity to apply procedures correctly. This is an ideal method to improve conceptual and procedural knowledge. To reduce cognitive load, you may wish to provide a correct example alongside incorrect ones which provide pupils with direct comparison.*

***Strategy Comparison** provides examples of the same problem solved in different ways, so that a comparison of different structures and representations can be made. Comparing different strategies can help to deepen pupils conceptual understanding and identify similarities and differences between them. If problems are presented in different representations (CPA), pupils are also able to identify commonalities and differences in presenting solutions to problems.*

Isolate the Skill

According to Daisy Christodoulou 2017, *'If we want pupils to develop a certain skill, we have to break that skill down into its component parts and help pupils to acquire the underlying mental model.'*

Rather than presenting problems in their entirety continually, which can lead to cognitive overload, the problem should be broken down into smaller components, crafted in a carefully chosen order, that accumulate to achieve greater success with a larger, well-defined outcome.

Analogy can be made in Art. A painter needs the skill of all the component parts in order to produce a painting (*drawing, perspective, colour use, paint technique, brush manipulation etc*).

Atomisation (*Bruno Reddy blog*) is when a problem is broken down into small concise steps. The teacher then needs to identify which steps are known to the pupil (check with practice) and which need to be taught to complete the problem in its entirety.

This process is best identified before any teaching block so the route to achieving the goal is made explicit to the pupils. Lessons become much easier and quicker to plan as teachers can simply work through each step identified.

Goal Free Problems (Sweller)
Problem solving is the key of 'doing' and also learning to do mathematics and takes the fundamental role of developing mathematical knowledge.

What do you see?

What do you know?

What can you tell me?

What else could it be?

A flexible thinker includes the ability to apply knowledge in different contexts including the ability to apply skills to unfamiliar situations rather than simply use it in similar context when it is studied.

Cognitive Load Theory suggests that by reducing extraneous cognitive load during learning could enhance the transference of learning from one context to another.

A goal free problem strategy that is developed based in cognitive load theory have been showed to be effective for transfer learning. The instruction in a goal-free problem directs pupils to 'calculate as many solutions as you can' rather than to calculate a single given goal.

Solving unfamiliar problems can place a strain on working memory, particularly for the 'novice' learner (those with less knowledge and experience of solving problems) who tend to solve problems in an inefficient way.

The complication for the novice learner is that most problems have a goal or endpoint, therefore when faced with an unfamiliar problem pupils tend to work backwards from the goal.

This means that pupils need to consider all of the different means or approaches by which they might be able to achieve the goal/endpoint, while also keeping the goal in mind, potentially causing cognitive overload.

Alternatively - What is the Kipling method?
The Kipling method, also known as the *5W1H method*, is a problem-solving and analytical technique that uses six key questions to gain a comprehensive understanding of a situation. These questions are: 'Who, What, When, Where, Why, and How'.

The Six Questions:
- **Who:** Identifies the individuals or groups involved or affected by the situation.
- **What:** Defines the problem, goal, or subject of analysis.
- **When:** Determines the timing, deadlines, or time-related aspects of the situation.
- **Where:** Specifies the location or context of the situation.

- **Why:** Explores the reasons, motivations, or causes behind the situation.
- **How:** Outlines the process, methods, or steps involved in the situation.

How it Works:
By systematically addressing these questions, the Kipling method helps in:

- **Problem Analysis:** Gaining a deeper understanding of the problem's nature and scope.
- **Creative Thinking:** Generating new ideas and solutions by exploring different angles.
- **Decision Making:** Providing a structured framework for evaluating options and making informed choices.
- **Communication:** Ensuring clarity and accuracy in communication by covering all essential aspects.
- **Information Gathering:** Organising and structuring information in a comprehensive manner.

Here's how the 5W1H method can be applied in mathematics:

- ***What:*** *What is the problem? What are we trying to find or solve? In a mathematical context, this could be identifying the unknown variable, the type of calculation needed, or the specific geometric shape involved.*
- ***Who:*** *Who is involved in the problem? Are there any specific individuals or groups that need to be considered? In mathematics, this might refer to the specific numbers, data sets, or geometric figures involved.*
- ***Where:*** *Where does the problem take place? Is there a specific context or setting? This could be a geometric space, a coordinate plane, or a specific real-world scenario.*
- ***When:*** *When does the problem occur? Is there a specific time frame or sequence of events? In some mathematical problems, the order of operations or the timing of events can be crucial.*
- ***Why:*** *Why is this problem happening? What are the underlying causes or reasons? Understanding the why behind a mathematical problem can help in identifying the correct approach or formula.*
- ***How:*** *How can we solve the problem? What steps or methods are needed? This involves choosing the appropriate mathematical tools, formulas, or techniques to arrive at a solution.*

By answering these questions, you can gain a thorough understanding of the problem and develop effective solutions.

Finally
It is useful to think of Problem Solving as meaning solving unfamiliar problems, problems that pupils have not yet met but have the knowledge, within the spectrum of their understanding, to solve.

We need to challenge pupils (*Sweller*) to solve unfamiliar problems as often as possible, interleaved in daily practice (In much the same way as fluency).

Research suggests that schools should:
- Help pupils to make links with prior knowledge.
- Give pupils hints to aid the retrieval of and make connections to, something they have learned before.
- Get pupils to compare familiar problems with the same mathematical structure but in different contexts to highlight exactly what they have in common (connectivity).
- Share prompts.

- Model how to solve unfamiliar problems, succinctly and accurately. *NB: When planning a lesson factor in what you want the pupils to think about, then design each aspect of your lesson to accommodate this. Be aware of over narration, the longer you take, the longer the pupil believe solving the problem will take. E.g. If a teacher takes 5 minutes to model a question, pupils will believe it will take them, at least, twice as long. Better to model more than one in 5 minutes than over-narrate one.*
- Incorporate representations (Concrete, Pictorial, Abstract) to support understanding and clarification of problem.
    - **Concrete:** Pupils use physical, manipulable objects to represent mathematical concepts. For example, using counters to represent numbers or base ten blocks to understand place value. (*known as computational tools*)
    - **Pictorial:** Pupils move to visual representations like drawings, diagrams, or pictures to represent the same concepts. This stage helps them make connections between the concrete materials and abstract symbols.
    - **Abstract:** Pupils begin using mathematical symbols, numerals, and notation to represent concepts. This stage is reached after students have a solid understanding of the concrete and pictorial representations.
- Scaffold using models and representations.
- Provide multiple strategies to enhance efficiency and effectiveness.
- **Importantly, ensure pupils experience success so they are motivated to learn.** (*Rosenshine 2012 suggests that pupils should have a success rate of around 80%*)