MathsHUBS
Archimedes NE

An Exploration of Mastery

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“What is mastery?”

The Essence of Maths Teaching of Mastery

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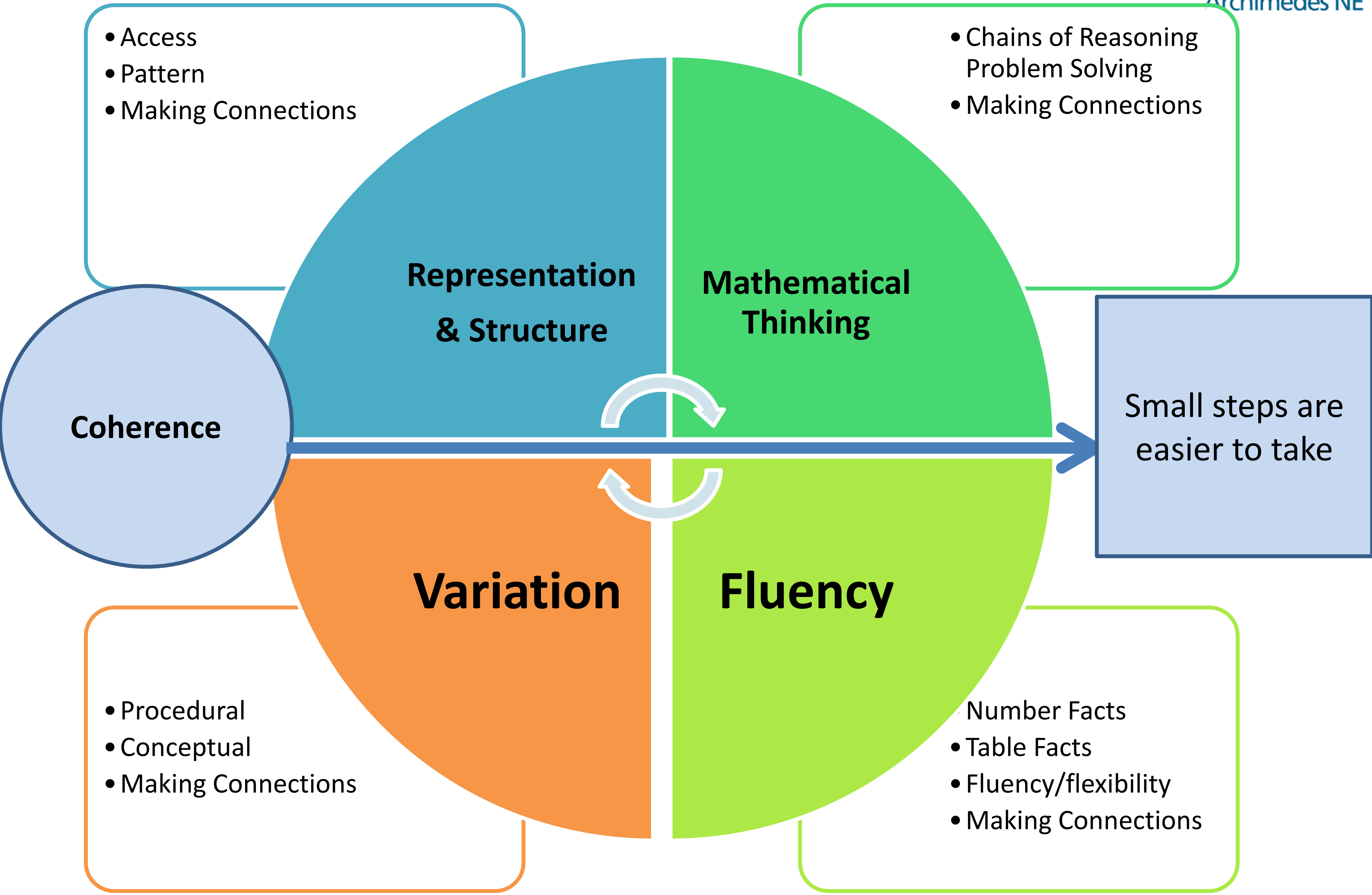
- Pupils are taught through whole-class interactive teaching, where the focus is on **all** pupils working together on the same lesson content at the same time.
- Lesson design identifies the new mathematics that is to be taught, the key points, the difficult points and a carefully sequenced journey through the learning.
- Procedural fluency and conceptual understanding are developed in tandem because each supports the development of the other.
- It is recognised that practice is a vital part of learning, but the practice used is **intelligent practice** that both reinforces pupils' procedural fluency and develops their conceptual understanding.
- Significant time is spent developing deep knowledge of the key ideas that are needed to underpin future learning. The structure and connections within the mathematics are emphasised, so that pupils develop deep learning that can be sustained.

Teaching for Mastery



Coherence

Messages

1. Small steps are easier to take.
2. Focussing on one key point each lesson allows for deep and sustainable learning.
3. Certain images, techniques and concepts are important pre-cursors to later ideas. Getting the sequencing of these right is an important skill in planning and teaching for mastery.
4. When something has been deeply understood and mastered, it can and should be used in the next steps of learning.

Coherence

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A comprehensive, detailed conceptual journey through the mathematics.

A focus on mathematical relationships and making connections

- What are the key conceptual ideas?
- What do they need to know before they can do this?
- Representations
- Anticipate the difficult points
- Variation
- Going deeper

Weekly Overview

Lesson 1 – Unit and non-unit fractions

Lesson 2 – Making a whole

Lesson 3 – Tenths

Lesson 4 – Tenths (2)

Lesson 5 – Fractions on number lines

The unit begins with children understanding what a fraction is and the importance of equal parts and then moves to looking at fractions which go together to make a whole.

In lesson 3, children will build on their understanding of making a whole by looking at how a whole can be split into tenths. Children then look at counting in tenths and what a fraction actually means (division).

Children then end the week by using their previous understanding to recognise fractions as numbers on a number line.

An important part of coherence is to ensure that objectives from previous year groups are secure. It may be worth revisiting these prior to beginning a new unit.

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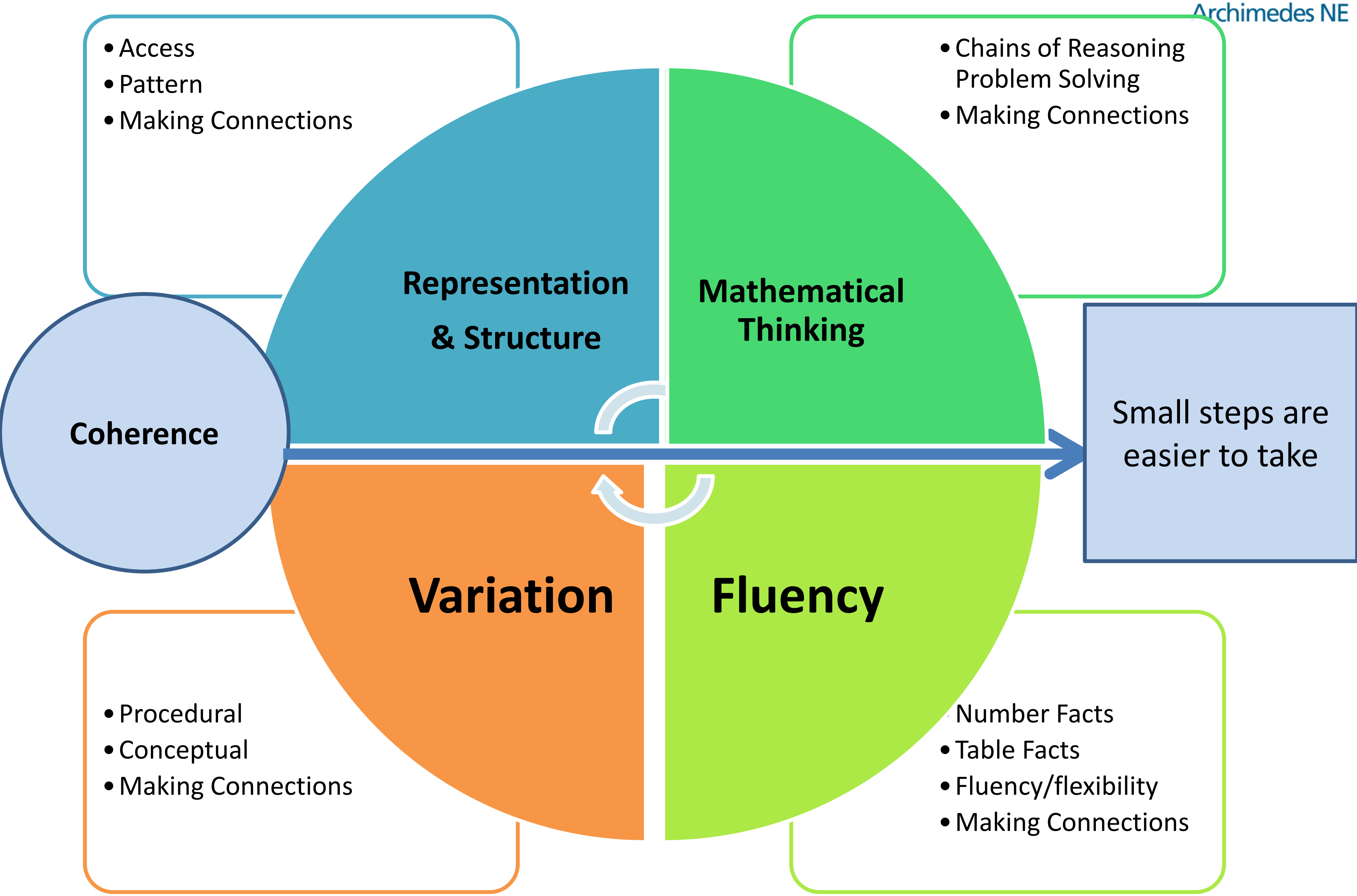

Mathematical thinking

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Messages

1. Mathematical thinking is central to deep and sustainable learning of mathematics.
2. Taught ideas that are understood deeply are not just 'received' passively but worked on by the learner. They need to be thought about, reasoned with and discussed.
3. Mathematical thinking involves:
   - looking for pattern in order to discern structure;
   - looking for relationships and connecting ideas;
   - reasoning logically, explaining, conjecturing and proving.

Mathematical Thinking

The 'Challenge Question'.

A regular part of a lesson

Sometimes it is:

- A challenging question for pupils,
- A "trap" for pupils.
- Very "tricky" which may let the pupils "puzzle" again

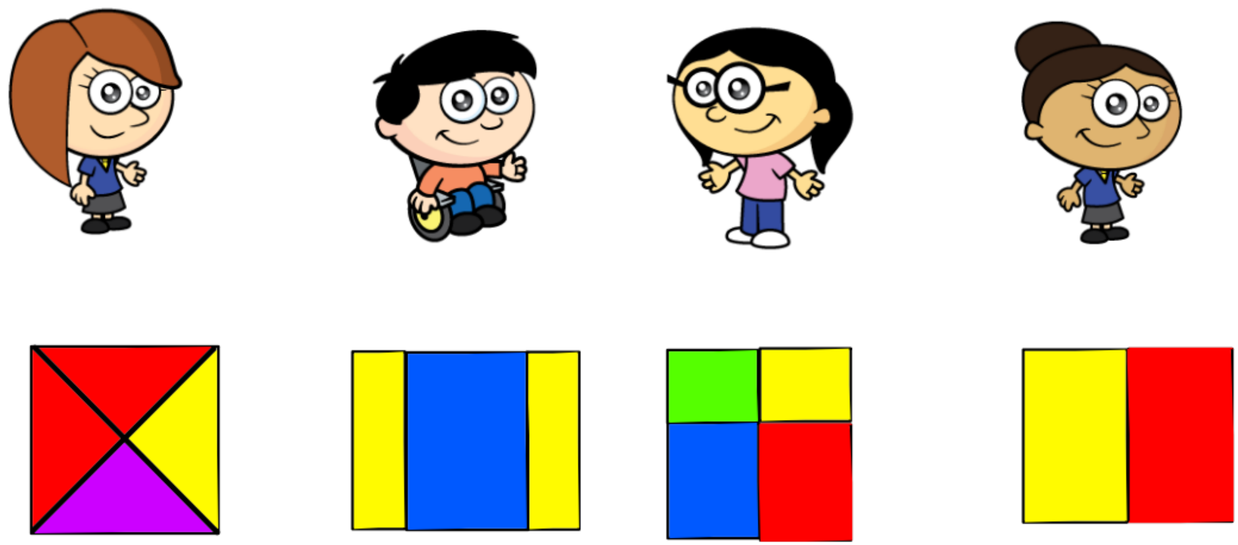
It is an opportunity help pupils think about the knowledge in another way.

Lesson 1

I can identify unit and non-unit fractions.

The children have made patterns.

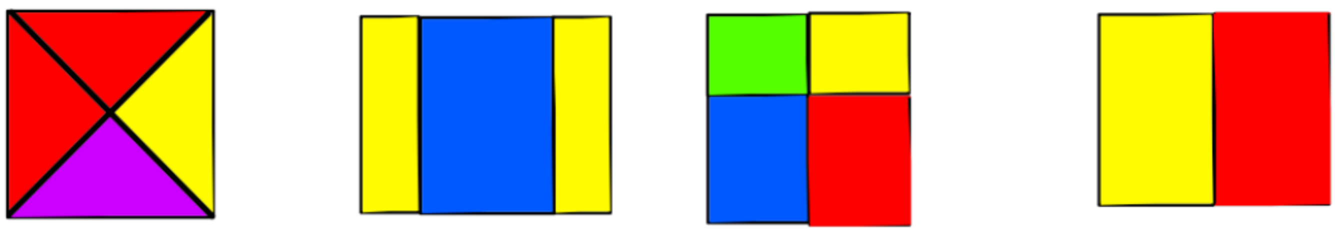
Can you fold or colour the paper shapes to make the patterns?
What do you notice? What do you know?



This question is purposefully open-ended so that the children's understanding can be assessed. Children need to make connections with prior learning e.g. what is and is not a fraction (there are some traps)and some may identify equivalent fractions.

My pattern is $\frac{1}{2}$ red and $\frac{1}{2}$ yellow.

Which one is my pattern?

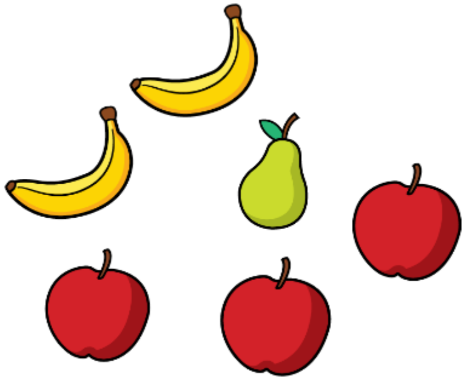


How do you know?

When the whole is divided into ____ equal parts, each part is ________ of the whole.

Through the use of ‘how do you know’, children can demonstrate their understanding of this concept.

Which of these is a unit fraction?



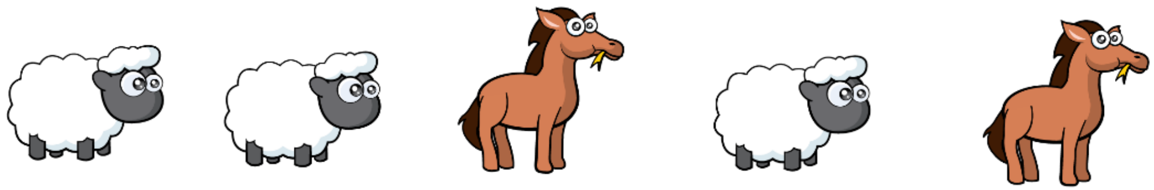
$\frac{1}{2}$ of the fruit are ______________.

__________ of the fruit are bananas.

__________ of the fruit are pears.

The aim here is to recognise fractions of amounts and the relationship between the numerator and the denominator. Some children may identify the equivalent fraction, although this is not the lesson objective.

True or false?



$\frac{3}{5}$ of the animals are sheep.

Is this a unit fraction or a non-unit fraction?

True or false questions are useful for challenging children. Children will be required to explain their mathematical thinking.

Discuss

I ate 4 slices of cake.
What fraction of the cake did I eat?

This challenge is to ascertain if children can recognise that you must know the whole before you can identify the fraction.
This is an open-ended challenge. Some pupils may find one possibility whereas others (i.e. those working at greater depth) may find multiple possibilities.

I ate 4 slices of cake.
What fraction of the cake did I eat?

I need more information.

Is there more than one possible solution?

Here, pupils are provided with a scaffold for their answers to the previous challenge.

Sort the fractions into the table.

	Fractions equal to one whole	Fractions less than one whole
Unit fractions		
Non-unit fractions		

Are there any boxes in the table empty? Why?

$\frac{3}{4}$	$\frac{3}{5}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{2}{2}$	$\frac{4}{4}$	$\frac{2}{5}$	$\frac{1}{2}$

This provides children with an opportunity to demonstrate their understanding in another way.
Children need to reason why there are no unit fractions in the 'equal to one whole' section.
Children could be extended by providing their own examples for each of the sections.

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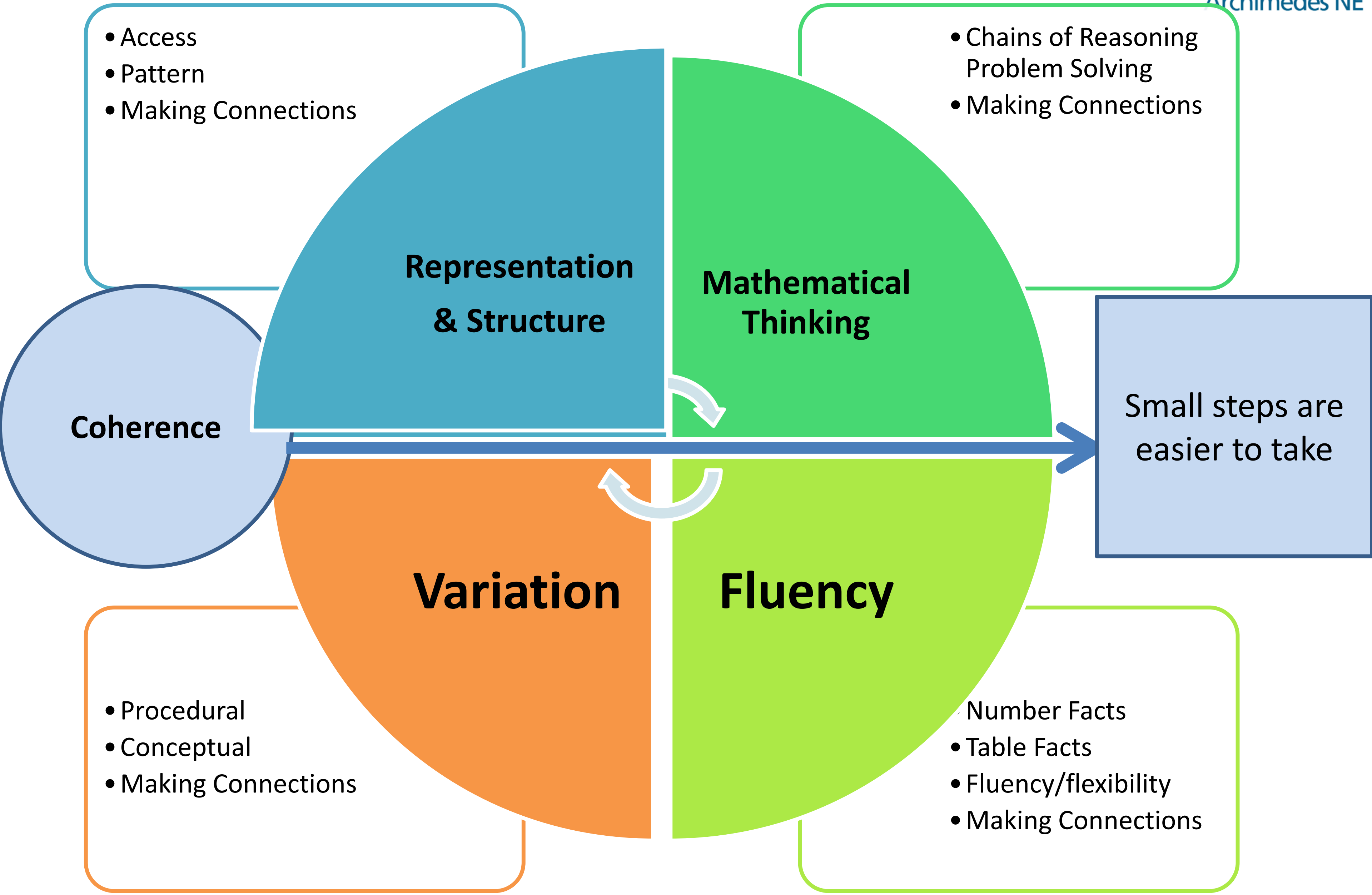

Representation and structure

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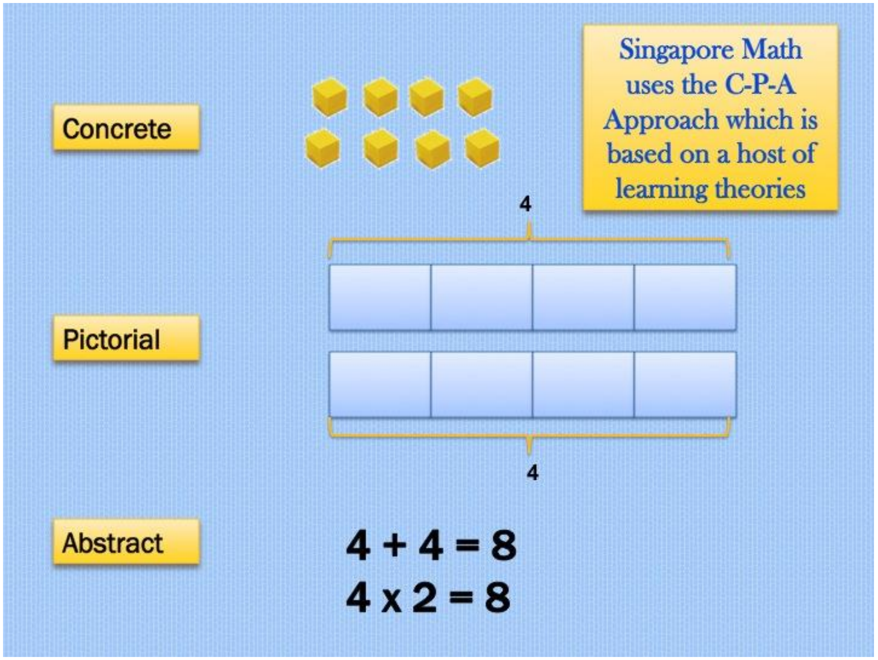
Messages

1. The representation needs to pull out the concept being taught, and in particular the key difficulty point. It exposes the structure.
2. In the end, the children need to be able to do the maths without the representation
3. A stem sentence describes the representation and helps the children move to working in the abstract ("ten tenths is equivalent to one whole") and could be seen as a representation in itself
4. There will be some key representations which the children will meet time and again
5. Pattern and structure are related but different: Children may have seen a pattern without understanding the structure which causes that pattern

Representation and Structure

Mathematics is an abstract subject. Representations have the potential to provide access and develop understanding.



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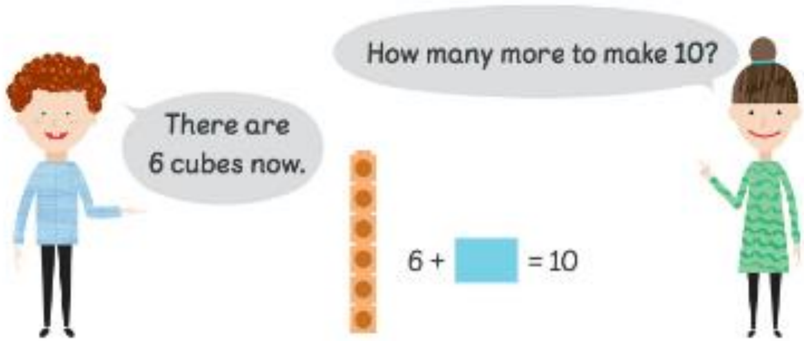
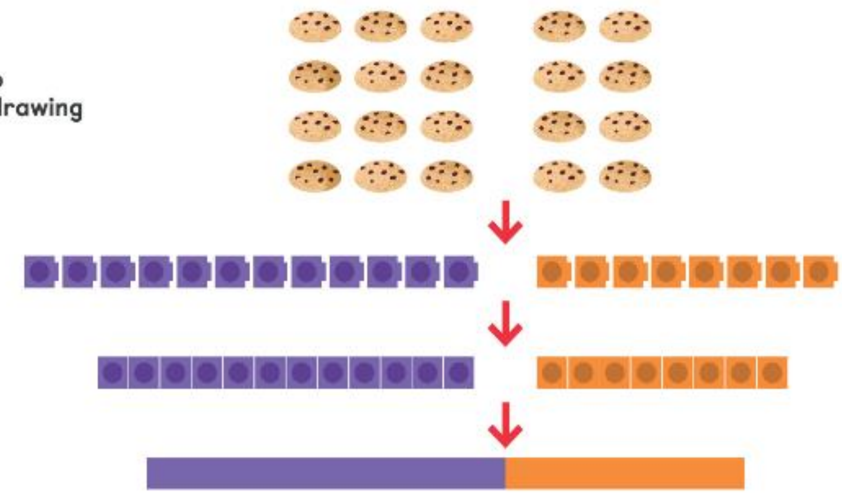


Concrete to
pictorial - drawing

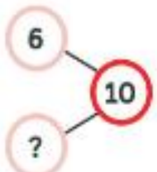


6 + 1 =

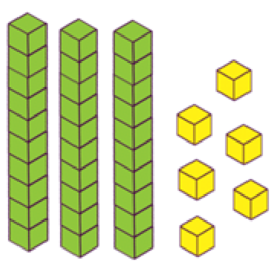
6 + 2 =

6 + 3 =

6 + = 10



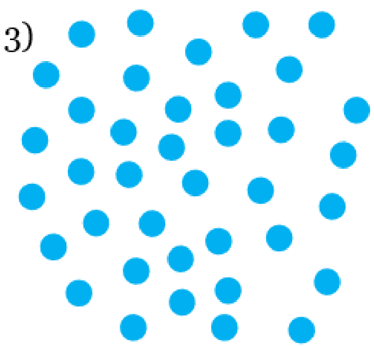
1)



2)

Tens	Ones
3	6

3)



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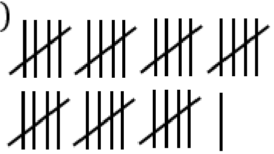

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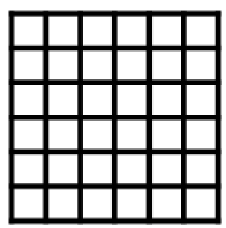
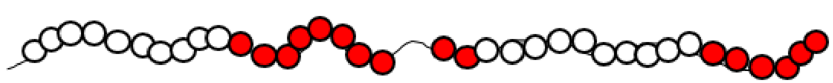


6)



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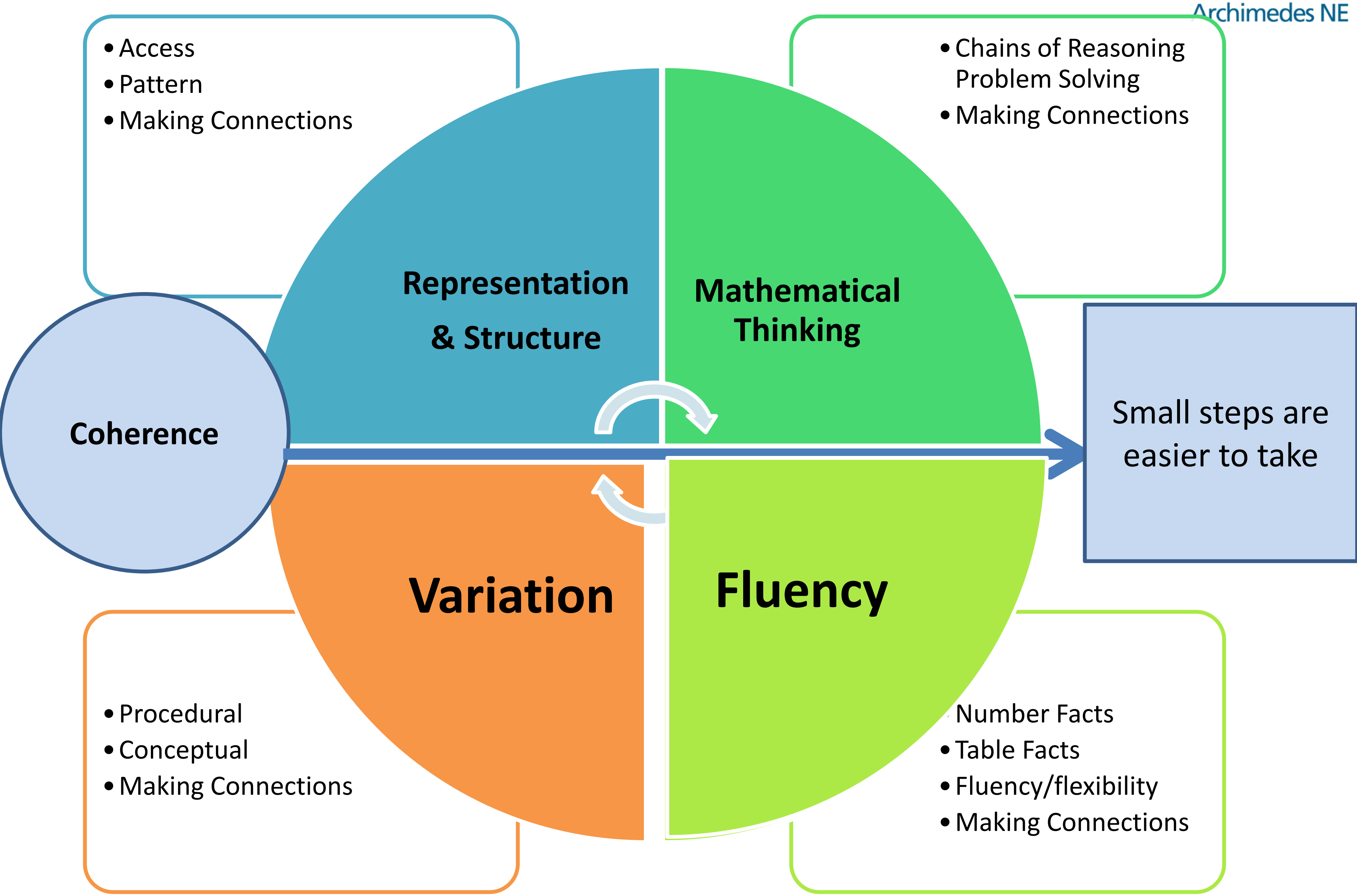

Fluency

Messages

1. Fluency demands more of learners than memorisation of a single procedure or collection of facts. It encompasses a mixture of efficiency, accuracy and flexibility.

2. Quick and efficient recall of facts and procedures is important in order for learners' to keep track of sub problems, think strategically and solve problems.

3. Fluency also demands the flexibility to move between different contexts and representations of mathematics, to recognise relationships and make connections and to make appropriate choices from a whole toolkit of methods, strategies and approaches.

Achieving fluency in the fundamentals of mathematics

Developing fluency requires a balance and connection between conceptual understanding and computational proficiency. On one hand, computational methods that are over-practised without understanding are often forgotten or remembered incorrectly… While, on the other hand, understanding without fluency can inhibit problem solving.

Fluency in Mathematics works through intelligent practice (rather than just mechanical repetition). Once a child has grasped a concept, the idea is that they are exposed to varied fluency activities which develop their understanding.

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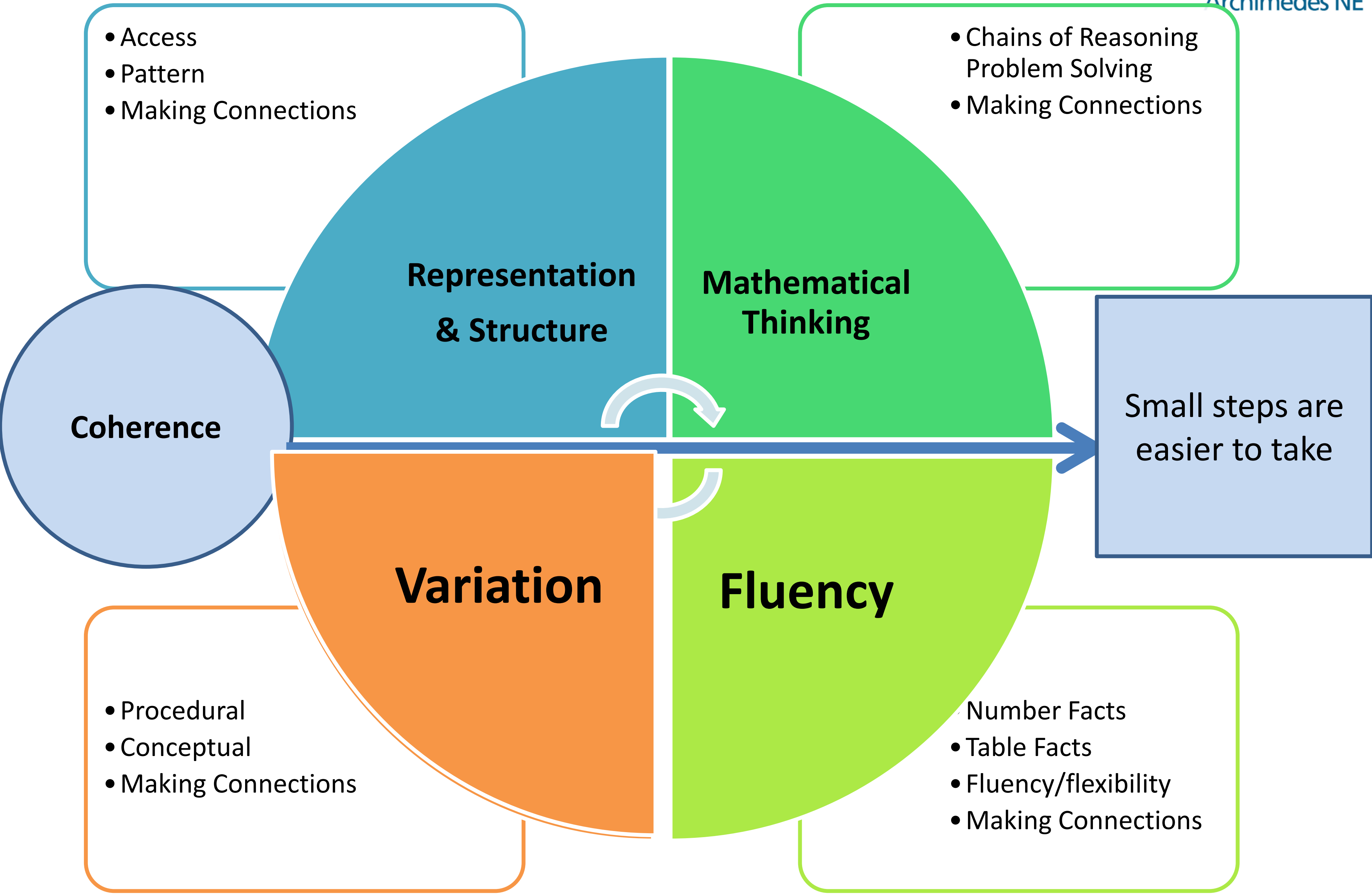

Variation

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Messages

1. The central idea of teaching with variation is to highlight the essential features of a concept or idea through varying the non-essential features.
2. When giving examples of a mathematical concept, it is useful to add variation to emphasise:
    a. What it is (as varied as possible);
    b. What it is not.
3. When constructing a set of activities / questions it is important to consider what connects the examples; what mathematical structures are being highlighted?
4. Variation is not the same as variety – careful attention needs to be paid to what aspects are being varied (and what is not being varied) and for what purpose.

Variation versus variety

- Variety
  - ‘Pick and mix’
  - Most practice exercises contain variety

- Variation
  - Careful choice of WHAT to vary
  - Careful choice of what the variation will draw attention to

2011, Mike Askew, Transforming Primary Mathematics

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'The central idea of teaching with variation is to highlight the essential features of the concept through varying the non- essential features.'

Gu, Huang & Marton, 2004

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Variety

3 + __ = 6

1 + 5 = __

__ + 0 = 6

3 + 3 = __

5 + __ = 6

2 + 4 = __

0 + 6 = __

What do you notice?

What's the same?

What's different?

What structures are exposed?

Variation

6 = 1 + __

6 = __ + 2

6 = __ + 3

6 = 4 + __

6 = __ + 1

6 = __ + 0

__ = 0 + 6

National Centre for Excellence in the Teaching of Mathematics

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Lesson

Lesson Introduction

Learning intention for the lesson

To subtract fractions from one whole.

Context of the lesson

- Children have previously used the bar model to represent fractions within one whole.
- Some children are struggling with the idea of describing how non-unit fractions are made up of several unit fractions. This will be reviewed at the start of the lesson, alongside children's understanding of how to represent and describe fractions equal to one whole.
- Later in the lesson, children will be encouraged to link this prior learning with their new learning.
- The lesson will then move on to looking at real life problems involving subtraction from one whole. Children will be encouraged to use the bar model to represent their subtractions from one whole.

Classroom Context

- This is a year 4 class. There are 29 children in the class and children will be sat in mixed ability pairs.
- Children are used to working through curriculum content at broadly the same pace in a mixed ability class.
- Some children who require extra support in Maths will have support from a TA to help them to access the lesson.

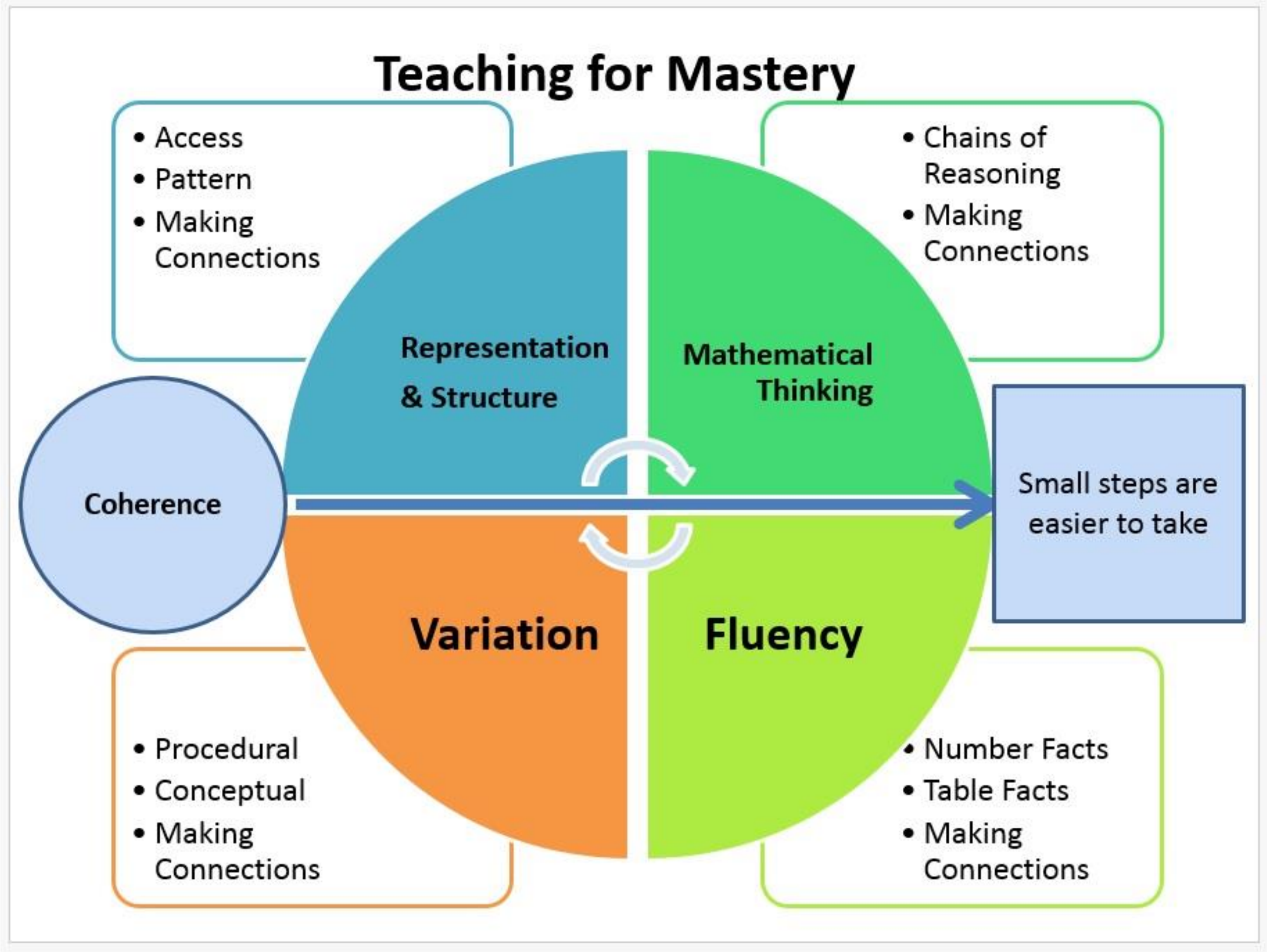
Prior learning

This lesson comes towards the end of a unit of work on fractions.
Prior to this lesson, children have explored the following key concepts:

1. **Parts and whole** – Exploring the part whole relationship and understanding that fractions are made up from one or more equal parts of the whole.
2. **Unit fractions** – Understanding how to identify and represent fractions with a numerator of one.
3. **Non-unit fractions** – Understanding that non-unit fractions are made up from unit fractions
4. **Adding fractions (within 1)** – Adding fractions with the same denominator
5. **Subtracting fractions (within 1)** – Subtracting fractions with the same denominator
6. **Fractions greater than 1** – Representing and expressing fractions greater than 1 as mixed numbers
7. **Adding fractions greater than 1**

Lesson Study

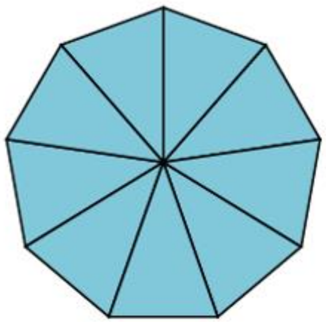
As you are watching the lesson, make notes on your observation sheet linked to the five key ideas. Think about the choices which have been made by the teacher and the impact this has on children's learning.



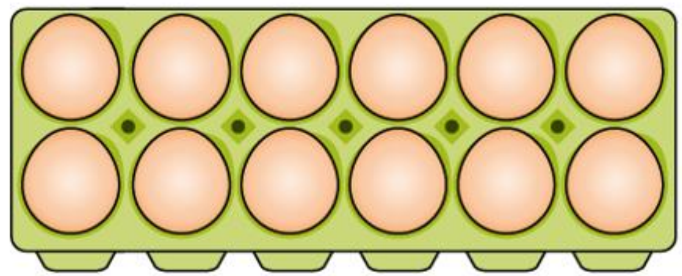
Review

Describe the following fractions.
What do you notice?

$\frac{5}{5}$



$\frac{9}{9}$



$\frac{12}{12}$

The whole has been divided into __ equal parts and I have __ of those parts.
I have __

Review

When the numerator and the denominator are the same the fraction is equivalent to one whole.

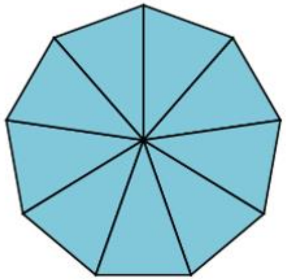
$$1 = \frac{5}{5} = \frac{9}{9} = \frac{12}{12} =$$

Review

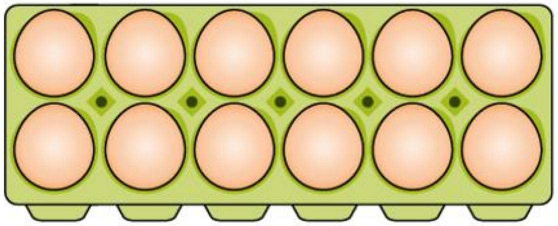
What is a **unit fraction**?

How is this different from a **non-unit fraction**?

$\frac{5}{5}$ is made up of (5) $\mathbf{\frac{1}{5}}$



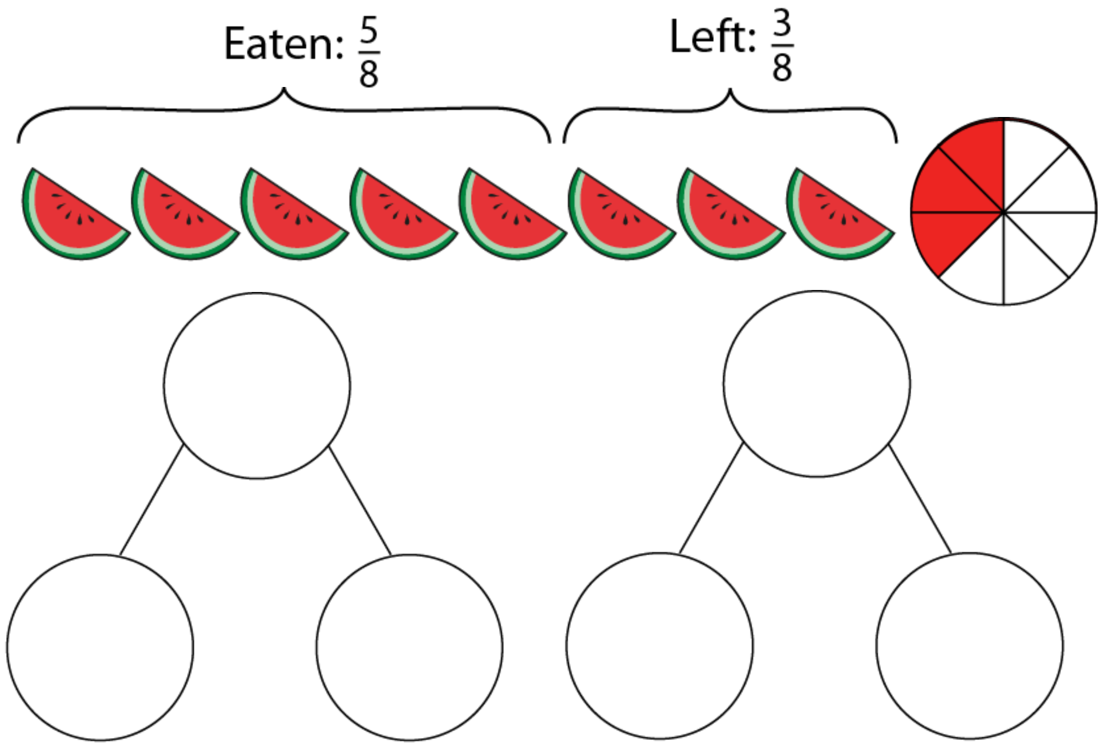
$\frac{9}{9}$ is made up of (9) $\mathbf{\frac{1}{9}}$



$\frac{12}{12}$ is made up of (12) $\mathbf{\frac{1}{12}}$

LI: To subtract fractions from one whole

What fraction of the watermelon is left?



$$\frac{8}{8} - \frac{5}{8} = \frac{3}{8}$$

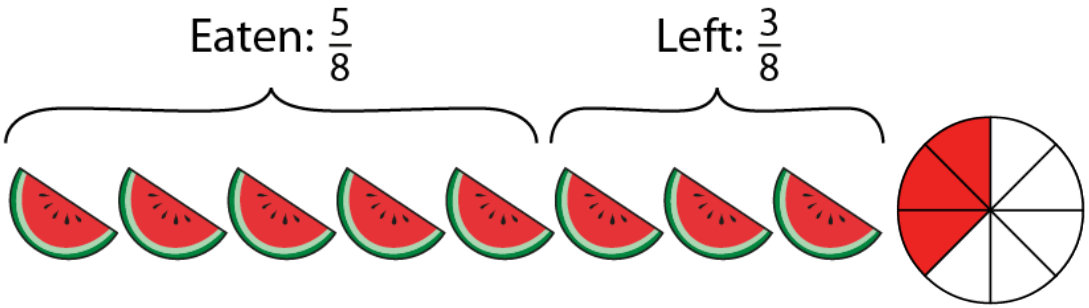
$$1 - \frac{5}{8} = \frac{3}{8}$$

When subtracting fractions with the same denominator, just subtract the numerators.

$$1 - \frac{5}{8} = \frac{3}{8} \qquad \frac{8}{8} - \frac{5}{8} = \frac{3}{8}$$

Subtraction from 1 whole

What fraction of the watermelon is left?

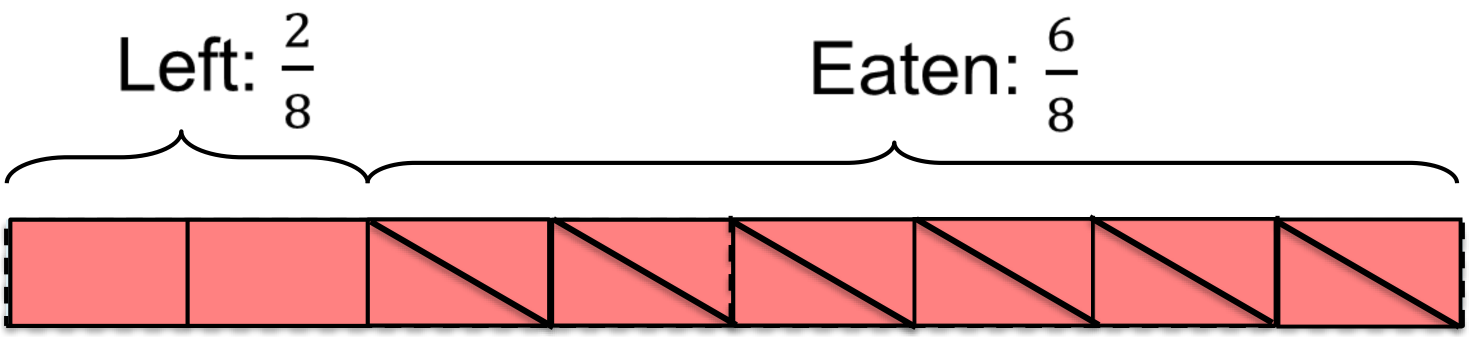


$$(8)\frac{1}{8} - (5)\frac{1}{8} = (3)\frac{1}{8}$$

$$\frac{8}{8} - \frac{5}{8} = \frac{3}{8}$$

A watermelon is cut into 8 equal pieces

$\frac{6}{8}$ of a watermelon is eaten.

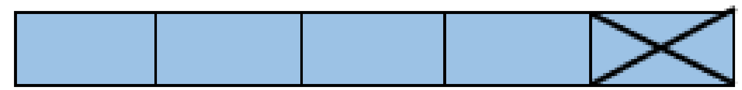
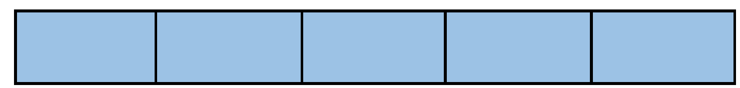
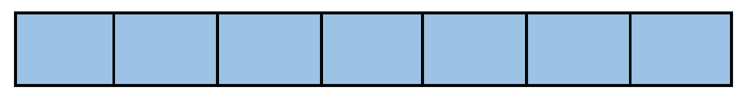
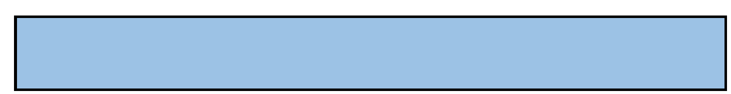


$$1 - \frac{6}{8} = \frac{2}{8}$$

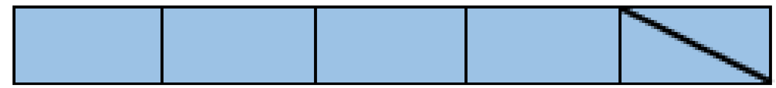
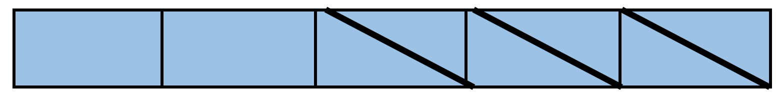
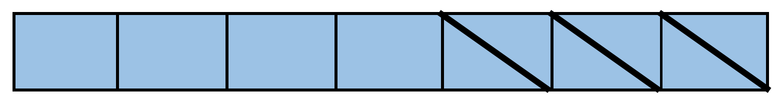
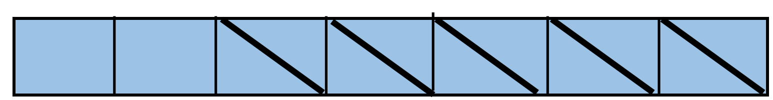
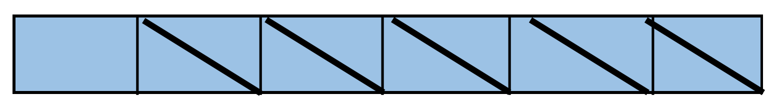
Rewrite the number sentence:

$$\frac{8}{8} - \frac{6}{8} = \frac{2}{8}$$

<u>Section A</u>

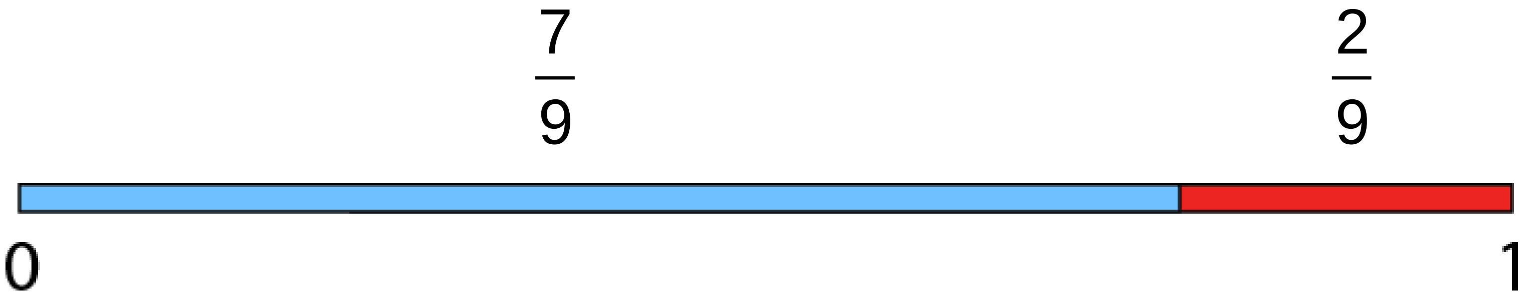
Subtraction	Rewrite the number sentence	Representation
$1 - \frac{1}{5} =$	$\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$	
$1 - \frac{3}{5} =$	$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$	
$1 - \frac{3}{7} =$		
$1 - \frac{5}{7} =$		
$1 - \frac{5}{6} =$		

Section A

Subtraction	Rewrite the number sentence	Representation
$1 - \frac{1}{5} =$	$\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$	
$1 - \frac{3}{5} =$	$\frac{5}{5} - \frac{3}{5} \quad \frac{2}{5}$	
$1 - \frac{3}{7} =$	$\frac{7}{7} - \frac{3}{7} \quad \frac{4}{7}$	
$1 - \frac{5}{7} =$	$\frac{7}{7} - \frac{5}{7} \quad \frac{2}{7}$	
$1 - \frac{5}{6} =$	$\frac{6}{6} - \frac{5}{6} \quad \frac{1}{5}$	

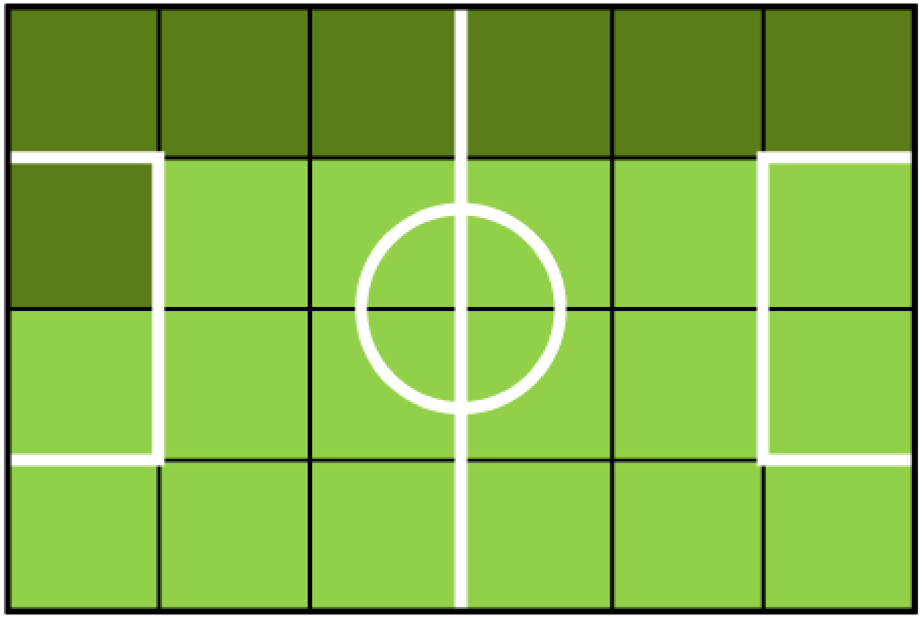
Subtraction from 1 whole

If $\frac{2}{9}$ of the number line is red, what fraction is blue?



$$1 - \frac{2}{9} = \frac{7}{9}$$

Subtraction from 1 whole



$$1 - \frac{7}{24} = \frac{17}{24}$$

True or false?

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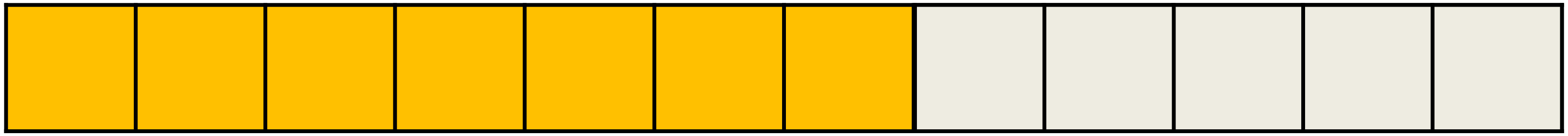
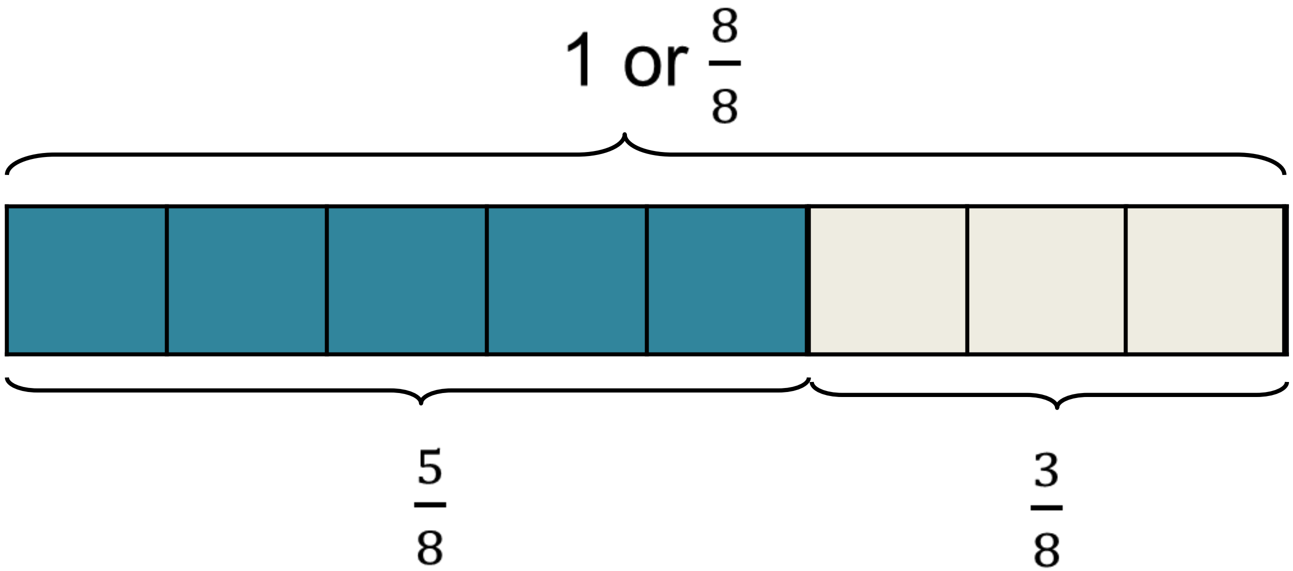
$$1 - \frac{5}{12} = \frac{7}{12}$$ ✓



$$\frac{12}{12} - \frac{5}{12} = \frac{7}{12}$$

True or false?

$$1 - \frac{3}{8} = \frac{3}{8} - \frac{8}{8}$$ ✗



$$1 - \frac{3}{8} = \frac{8}{8} - \frac{3}{8}$$

True or false?

$$1 - \frac{3}{7} - \frac{2}{7} = \frac{5}{7}$$ ✗

$$1 - \frac{3}{7} - \frac{2}{7} = \frac{2}{7}$$

True or false?

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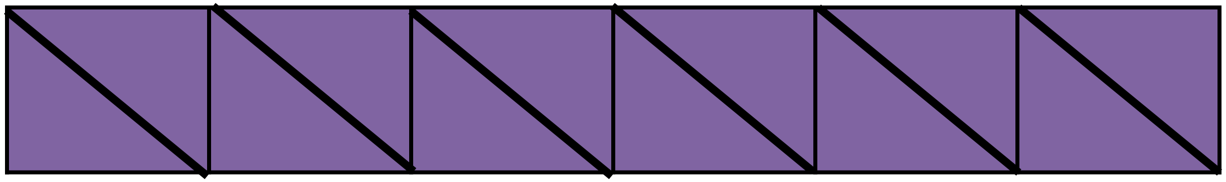
$$1 - \frac{6}{6} = 0$$ ✓

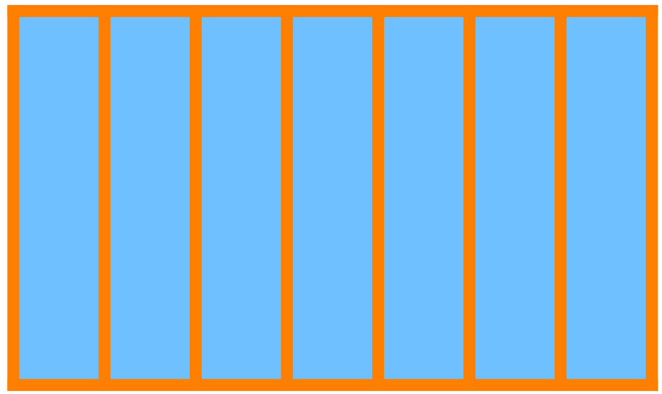


Challenge

Archimedes NE

$$\frac{7}{7} - \frac{3}{4} =$$

Challenge



$$\frac{7}{7} = 1$$

$$\frac{4}{4} = 1$$

$$1 = \frac{7}{7} = \frac{4}{4}$$

Challenge

Archimedes NE

$$\frac{7}{7} - \frac{3}{4} = \frac{1}{4}$$

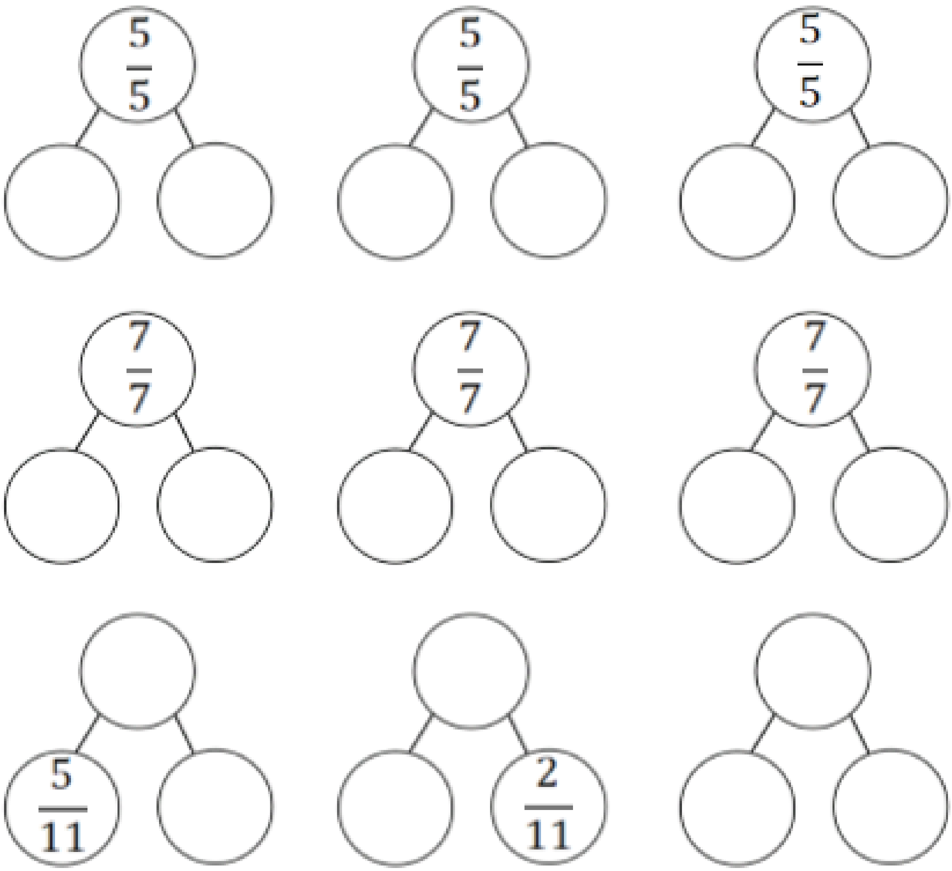
MathsHUBS Archimedes NE

11-3-20

LI: To subtract from one whole

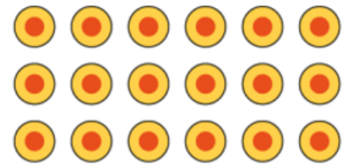
Section B

Use the part-whole model to partition 1 whole in different ways. Can you record the corresponding addition and subtracting number sentences?



Section C

$\frac{11}{18}$ Of a packet of biscuits have been eaten. What fraction of the packet is left?



A car set off with a full tank of fuel. At the end of the journey, there was only $\frac{9}{16}$ left. What fraction of the whole tank of fuel had been used?

Jamie has read $\frac{4}{10}$ of his book. What fraction of the book does he have left?

Challenge

Fill in the missing fractions below.

How many different ways can you find?

$$1 - \frac{\square}{\square} - \frac{\square}{\square} = \frac{2}{9}$$

Teaching for Mastery 5 big ideas

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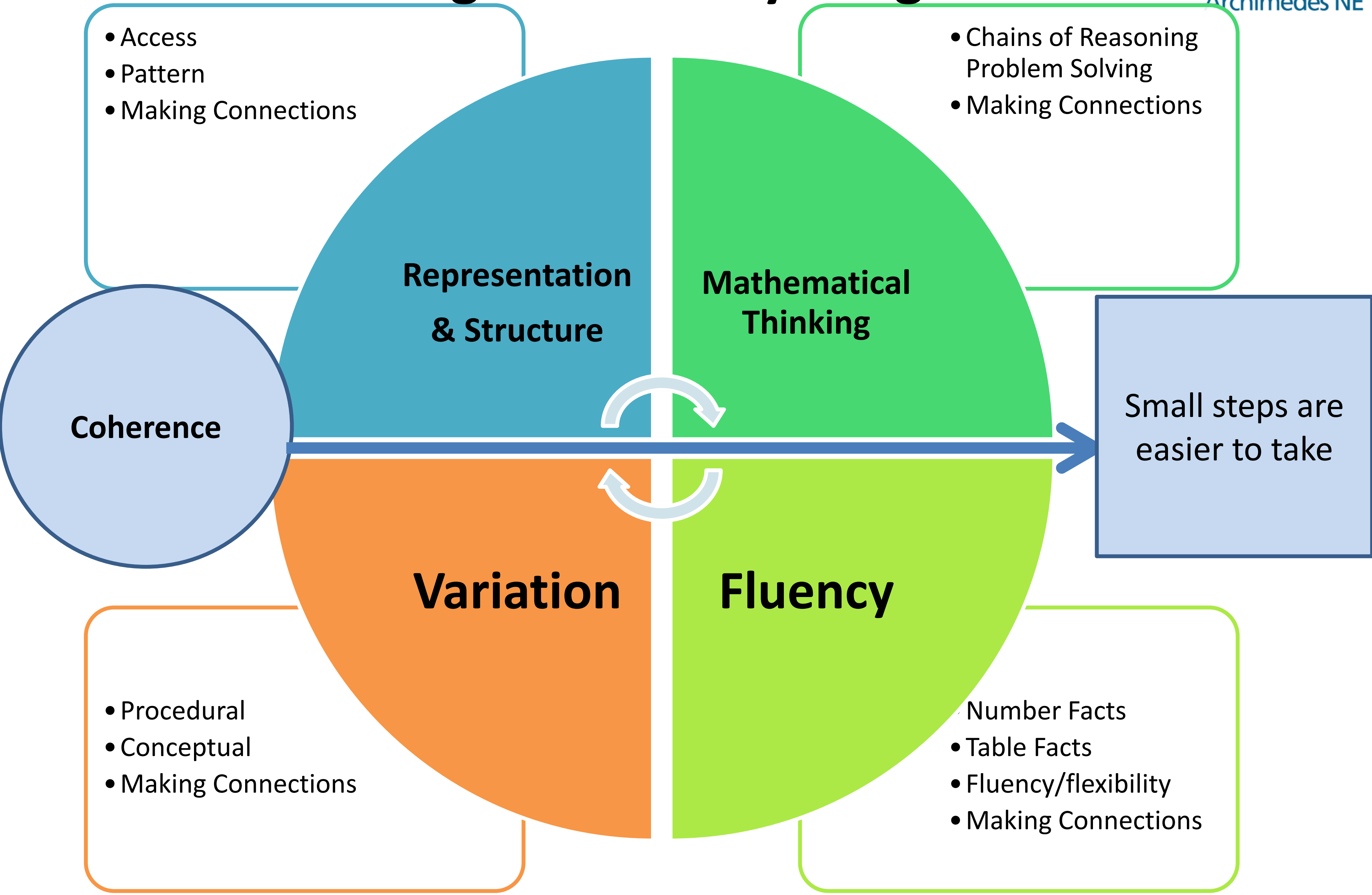
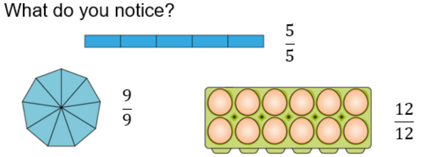


Coherence

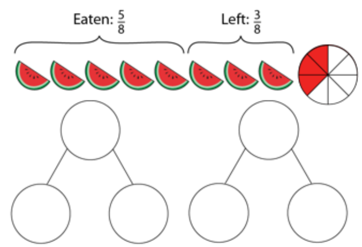
Review

Describe the following fractions.
What do you notice?



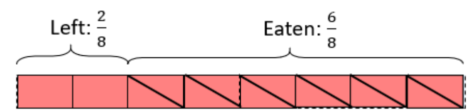
The whole has been divided into __ equal parts and I have __ of those parts.
I have __

What fraction of the watermelon is left?



$\frac{8}{8} - \frac{5}{8} = \frac{3}{8}$ $1 - \frac{5}{8} = \frac{3}{8}$

A watermelon is cut into 8 equal pieces
$\frac{6}{8}$ of a watermelon is eaten.
What fraction of the watermelon is left?



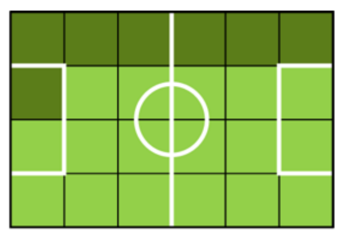
$1 - \frac{6}{8} = \frac{2}{8}$

Rewrite the number sentence:

$\frac{8}{8} - \frac{6}{8} = \frac{2}{8}$

Section A

Subtraction	Rewrite the number sentence	Representation
$1 - \frac{1}{5} =$	$\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$	
$1 - \frac{3}{5} =$	$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$	
$1 - \frac{3}{7} =$		
$1 - \frac{5}{7} =$		
$1 - \frac{5}{6} =$		



$1 - \frac{7}{24} = \frac{17}{24}$

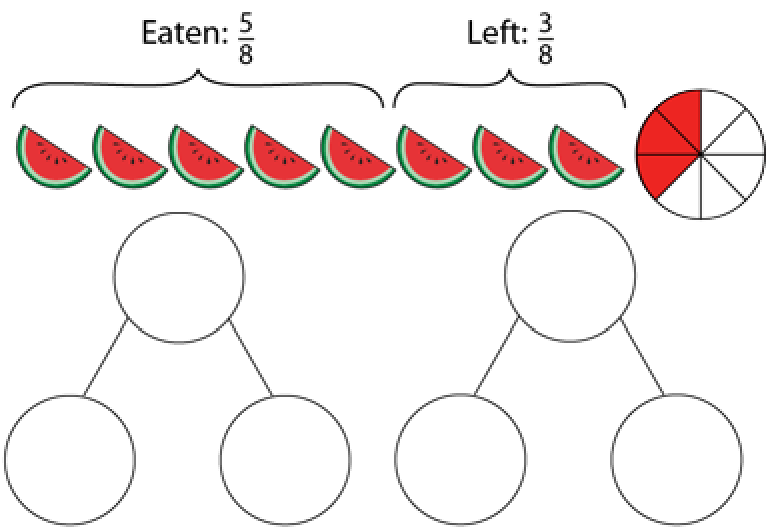
True or false?

$1 - \frac{5}{12} = \frac{7}{12}$ ✓

$\frac{12}{12} - \frac{5}{12} = \frac{7}{12}$

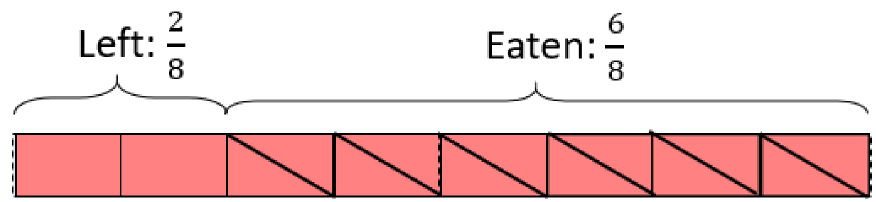
$$\frac{7}{7} - \frac{3}{4} =$$

Representation and structure



$\frac{8}{8} - \frac{5}{8} = \frac{3}{8}$

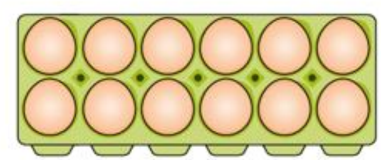
$1 - \frac{5}{8} = \frac{3}{8}$



1 whole or

$\frac{5}{5}$

$\frac{9}{9}$



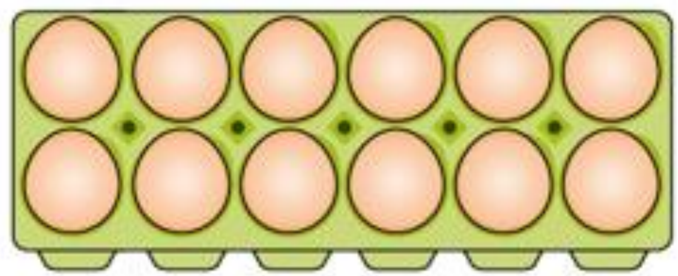
$\frac{12}{12}$

The whole has been divided into __ equal parts
and I have __ of those parts.
I have __

Representation and structure

$\frac{5}{5}$

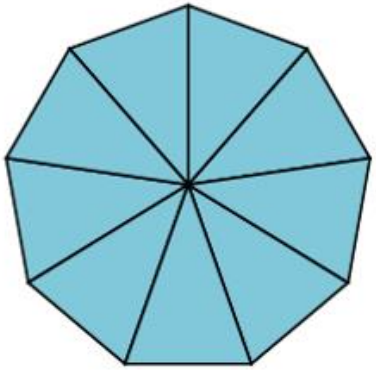
$\frac{9}{9}$



$\frac{12}{12}$

The whole has been divided into __ equal parts and I have __ of those parts.

I have __



How to write fractions:

1^{st} **Fraction bar**

2^{nd} **Denominator**

3^{rd} **Numerator**

$$\frac{9}{9}$$

Numerator

Fraction bar

Denominator

The whole has been divided into __ equal parts and I have __ of those parts.

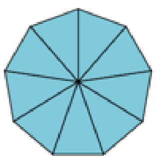
I have __

Fluency

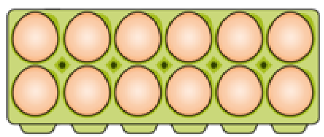
Concrete and pictorial should be used as a vehicle to help children develop fluency with the abstract.

Describe the following fractions.
What do you notice?

$\frac{5}{5}$



$\frac{9}{9}$



$\frac{12}{12}$

$\frac{5}{5}$ is made up of (5) $\frac{1}{5}$

$\frac{9}{9}$ is made up of (9) $\frac{1}{9}$

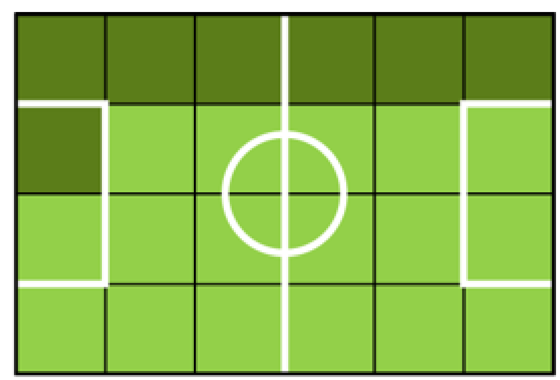
$\frac{12}{12}$ is made up of (12) $\frac{1}{12}$

If $\frac{2}{9}$ of the number line is red, what fraction is blue?

$\frac{7}{9}$ $\frac{2}{9}$

0 1

$1 - \frac{2}{9} = \frac{7}{9}$

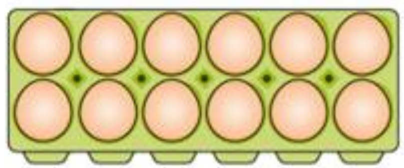


$1 - \frac{7}{24} = \frac{17}{24}$

Conceptual Variation

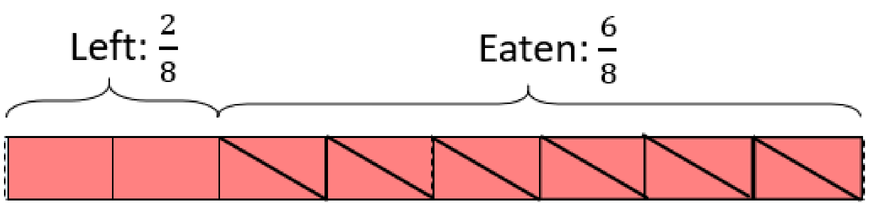
$\frac{5}{5}$

$\frac{9}{9}$

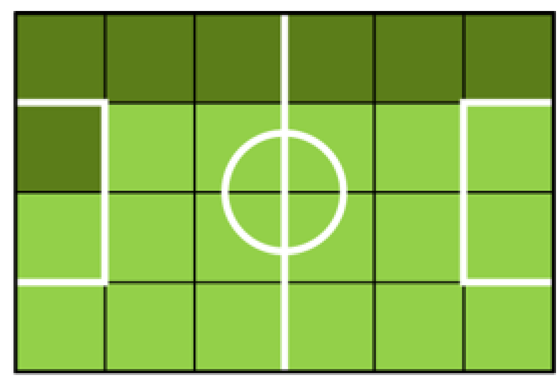


$\frac{12}{12}$

1 whole or $\frac{8}{8}$



1 whole or



$1 - \frac{7}{24} = \frac{17}{24}$

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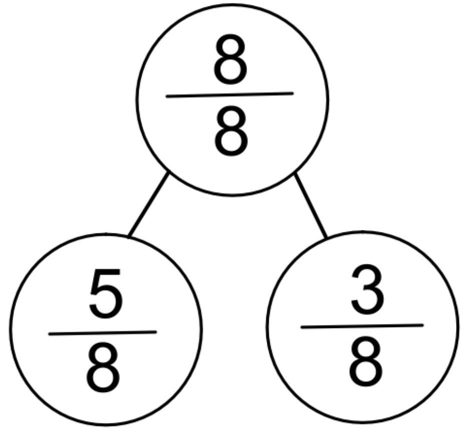
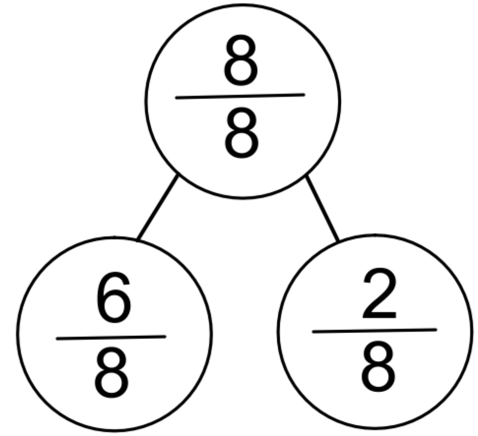
Procedural Variation

<u>Section A</u>

Subtraction	Rewrite the number sentence
$1 - \frac{1}{5} =$	$\frac{5}{5} - \frac{1}{5} = \frac{4}{5}$
$1 - \frac{3}{5} =$	$\frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$
$1 - \frac{3}{7} =$	
$1 - \frac{5}{7} =$	
$1 - \frac{5}{6} =$	





$1 - \frac{5}{8} = \frac{3}{8}$ $\qquad$ $\frac{8}{8} - \frac{5}{8} = \frac{3}{8}$

$1 - \frac{6}{8} = \frac{2}{8}$ $\qquad$ $\frac{8}{8} - \frac{6}{8} = \frac{2}{8}$

Mathematical Thinking

What do you notice?

When the numerator and the denominator are the same the fraction is equivalent to one whole.

$$1 = \frac{5}{5} = \frac{9}{9} = \frac{12}{12}$$

When subtracting fractions with the same denominator, just subtract the numerators.

$$1 - \frac{5}{8} = \frac{3}{8} \qquad \frac{8}{8} - \frac{5}{8} = \frac{3}{8}$$

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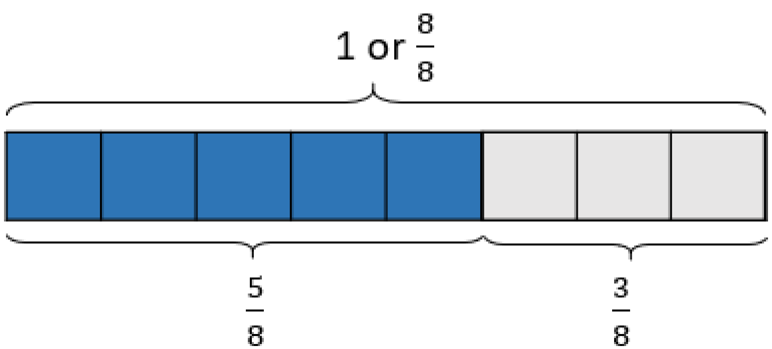
Mathematical Thinking

True or false?

$$1 - \frac{3}{8} = \frac{3}{8} - \frac{8}{8}$$ ✗



$$1 - \frac{3}{8} = \frac{8}{8} - \frac{3}{8}$$

$$\frac{7}{7} - \frac{3}{4} = \frac{1}{4}$$

Questions