

# Introduction

The following calculation policy has been devised to meet the requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school.

Children should work through the progressions outlined in this policy, in order that they know and understand a compact standard method for each numerical operation by the end of Year 6.

This process should not be rushed. Children should be moved on towards more compact methods when they are ready. As a result, it is likely that there will be children at different stages of learning in a particular class or group.

Judgements will need to be made as to whether pupils possess sufficient skills to progress to a particular stage, as different prerequisite skills are needed for each operation.

If at any time, children are making significant errors, return to the previous stage in calculation.

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons.

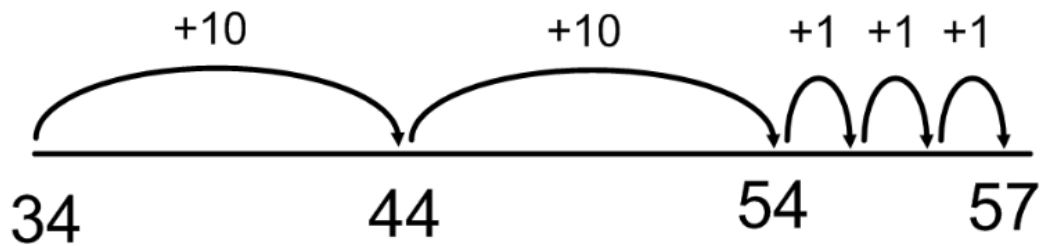
## Aims:

- To ensure a consistent approach to the teaching of written calculation methods to enhance children's progression and understanding.
- To ensure that children develop an efficient, reliable, formal written method of calculation for all four operations.
- To inform parents/carers of the progression in written calculation methods that are taught in school.

# Addition

## Stage 1: Use a number line

$$34 + 23$$

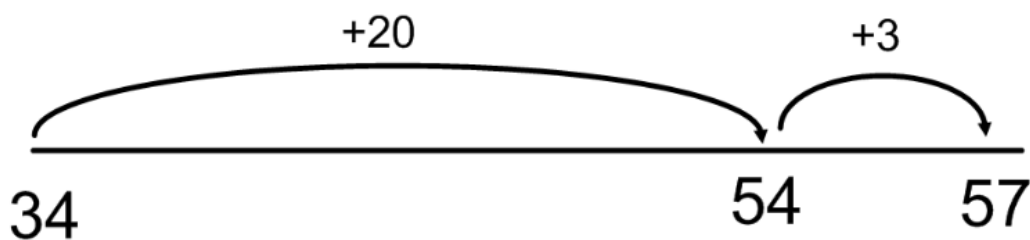


### **Notes:**

- *Always put the biggest number first (34)*

## Stage 2: Progress to more effective jumps

$$34 + 23$$



### **Notes:**

- *Put the biggest number first (34) and then partition the smaller number (20 + 3)*

**Stage 3: Partitioning**

$43 + 25$

4	3	+	2	5			
4	0	+	3	2	0	+	5
4	0	+	2	0	=	6	0
	3	+	5	=	8		
6	0	+	8	=	6	8	

**Notes:**

- **Partition the numbers in to tens and units.**
- **Add the tens together and add the units together.**
- **Recombine to give the answer.**
- **Always model and encourage the use of one digit per square.**

**Stage 4: Expanded methods****236 + 73 using partitioning**

	2	0	0	+	3	0	+	6					
+					7	0	+	3					
	2	0	0	+	1	0	0	+	9	=	3	0	9

**Notes:**

- **Partition the numbers in to hundreds, tens and units.**
- **Add the hundreds, tens and units together.**
- **Recombine to give the answer.**
- **Always model and encourage the use of one digit per square.**

**Progressing to setting out without partitioning**

	2	3	6									
+		7	3									
			9	(	6	+	3	)				
	1	0	0	(	3	0	+	7	0	)		
	2	0	0	(	2	0	0	+	0	0	0	)
	3	0	9									

**Notes:**

- **Add the least significant digits (units) first, then the tens, then the hundreds.**
- **Use the language of place value to ensure understanding. Six add three is nine. Thirty add seventy is one hundred (NOT three add seven). Two hundred add zero is two hundred.**
- **Always model and encourage the use of one digit per square.**

**An example involving decimals**

$$27.2 + 24.3$$

	2	7	.	2									
+	2	4	.	3									
		0	.	5	(	0	.	2	+	0	.	3	)
	1	1	.	0	(	7	+	4	)				
	4	0	.	0	(	2	0	+	2	0	)		
	5	1	.	5									

**Notes:**

- **Use the language of place value to ensure understanding.**
- **Ensure that the decimal points line up.**
- **Always model and encourage the use of one digit per square.**

### Examples involving money

$$£4.38 + £3.53$$

	£	4	.	3	8					
+	£	3	.	5	3					
		0	.	1	1	(8 <sub>p</sub>	+	3 <sub>p</sub> )		
		0	.	8	0	(3	0 <sub>p</sub>	+	5	0 <sub>p</sub> )
		7	.	0	0	(£	4	+	£	3)
	£	7	.	9	1					

	£	4	.	3	8					
+	£	3	.	5	3					
		0	.	1	1	(0.0	8	+	0.0	3)
		0	.	8	0	(0.3	0	+	0.5	0)
		7	.	0	0	(4.0	0	+	3.0	0)
	£	7	.	9	1					

#### Notes:

- **Use the language of place value to help children understand that 0.08 is equivalent to 8p.**
- **Tenths, hundredths, thousandths etc... should be correctly aligned.**
- **Always model and encourage the use of one digit per square.**

**Stage 5: Standard compact method**

$436 + 347$

	4	3	6
+	3	4	7
	<hr/>		
	7	8	3
		1	
	<hr/>		

**Notes:**

- **Use the language of place value to ensure understanding. So... 6 add 7 equals 13, which is made up of one ten and three units. Write 3 in the units column and 'carry' 1 across in to the tens column to represent the ten. Any numbers carried should be written underneath the line. Next, 30 + 40 + the ten that we carried = 80 (DO NOT SAY 3 + 4 + 1 = 8). Three tens add four tens add one ten is acceptable. Then,**
- **400 + 300 = 700**
- **Always model and encourage the use of one digit per square.**

$$2363 + 1298$$

	2	3	6	3
+	1	2	9	8
	<hr/>			
	3	6	6	1
		1	1	

**Notes:**

- *Discuss similarities between this formal method and the expanded methods shown previously.*

**Examples using decimals**

$$27.2 + 24.3$$

	2	7	.	2
+	2	4	.	3
	<hr/>			
	5	1	.	5
	1			

**Notes:**

- *The decimal point should be aligned in the same way as the other place value columns, and must remain in the same place in the answer row.*

$$35.26 + 27.28$$

	3	5	2	6
+	2	7	2	8
	<hr/>			
	6	2	5	4
	<hr/>			
	1		1	

**Notes:**

- ***Tenths, hundredths, thousandths etc... should be correctly aligned.***
- ***Always model and encourage the use of one digit per square.***

**An example using money**

$$£239.17 + £176.38$$

	£	2	3	9	1	7
+	£	1	7	6	3	8
	<hr/>					
	£	4	1	5	5	5
	<hr/>					
		1	1		1	

**Adding more than two numbers together**

$$35.262 + 27.280 + 62.500$$

	3	5	.	2	6	2
	2	7	.	2	8	0
+	6	2	.	5	0	0
<hr/>						
	1	2	5	.	0	4
						2
	1	1	1			

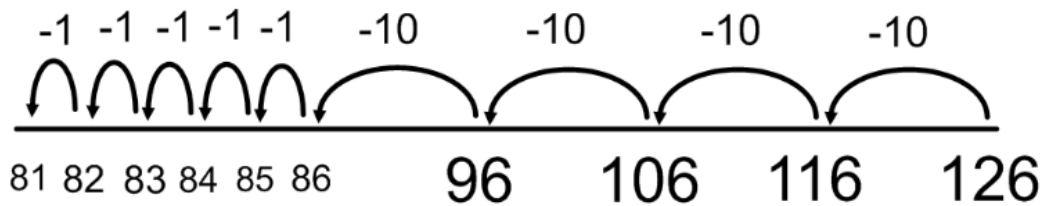
**Notes:**

- **Empty decimal places should be filled with zero to show the place value of each column.**

# Subtraction

## Stage 1: Use a number line to count back

$$126 - 45$$



## Progressing to more efficient jumps:

$$126 - 45$$



### **Notes:**

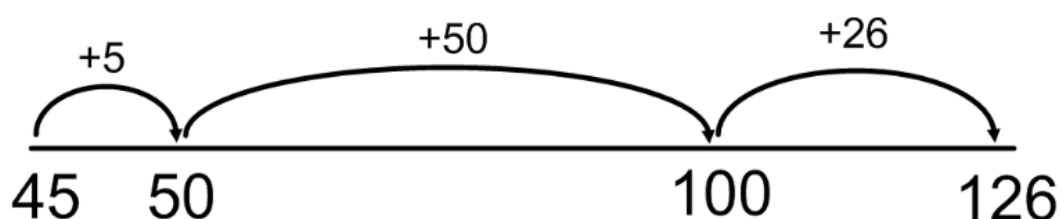
- **Partition the second number and subtract the tens first.**
- **Encourage the most efficient jumps.**

**Stage 2: Using a blank number line to count on (finding the difference)**

**Notes:**

- ***As children progress and their understanding of the link between subtraction and difference is more secure, they should be developing the idea of using an empty number line to count on from the smaller number.***

$$126 - 45$$



$$= 5 + 50 + 26$$

$$= 81$$



**Stage 3: Expanded methods****Using partitioning without the need for decomposition****77 - 24**

	7	0	7				
-	2	0	4				
	5	0	3	=	5	3	

**Notes:**

- *Always model and encourage the use of one digit per square.*
- *Leave a space between the partitioned numbers to make calculations clear.*
- *Ensure the bigger number is placed on top of the smaller number.*

**Progressing to partitioning with 'exchange'****84 - 47**

	70		14				
	<del>8</del>	<del>0</del>	<del>4</del>				
-	4	0	7				
	3	0	7	=	3	7	

852 - 427

				40	12					
	8	0	0	<del>5</del>	<del>0</del>	<del>2</del>				
-	4	0	0	2	0	7				
	4	0	0	2	0	5	=	4	2	5

**Notes:**

- ***The use of language is very important to ensure consistency. We will not use the term 'borrow' a ten/hundred. We will speak of 'taking' a ten/hundred and 'exchanging' it for tens/units.***
- ***When learning to 'exchange', explore partitioning in different ways so that pupils understand that when you exchange, the VALUE is the same (e.g: 852 = 800 + 50 + 2 or 800 + 40 + 12 etc...***
- ***Exchange could be introduced through practical subtraction using Base 10 equipment.***
- ***Horizontal crossing through rather than diagonal.***

**Stage 4: Standard compact method (without the need for 'exchange')**

$$77 - 24$$

	7	7
-	2	4
<hr/>		
	5	3
<hr/>		

**Notes:**

- Children who are still not secure with number facts and place value will need to remain on stage 3 (using partitioning) until ready for the compact method.
- Create lots of opportunities for subtracting and finding differences with money and measures.

**Stage 5: Standard compact method (with 'exchange')**

$$84 - 47$$

	7	14
	<del>8</del>	<del>4</del>
-	4	7
<hr/>		
	3	7
<hr/>		

**Notes:**

- **Horizontal crossing through rather than diagonal.**
- **Use the language of place value to ensure understanding.**
- **See earlier notes about 'exchange'.**
- **Discuss similarities between this formal method and the expanded methods shown previously.**

**852 - 427**

		4	12
	8	<del>5</del>	<del>2</del>
-	4	2	7
	4	2	5

**An example using decimals**  
**35.26 - 27.28**

	2	14	11	16
	<del>3</del>	<del>5</del>	<del>2</del>	<del>6</del>
-	2	7	2	8
	0	7	9	8

**Notes:**

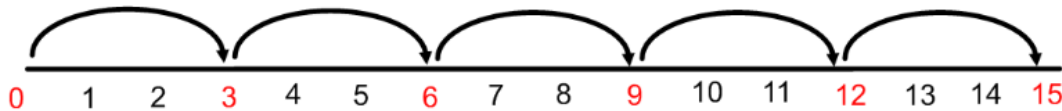
- ***The decimal point should be aligned in the same way as the other place value columns, and must remain in the same place in the answer row.***



# Multiplication

## Stage 1: Using a number line to show equal jumps

Jumps of 3 = 0, 3, 6, 9, 12 etc...

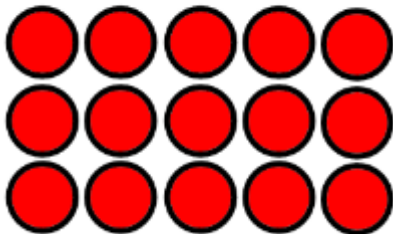


### **Notes:**

- **Emphasise the importance of learning multiplication and division facts, starting with the 2x, 5x and 10x tables.**

## Stage 2: Repeated addition and arrays

**3 x 5 or 5 x 3**



### **Notes:**

- **Show the link between multiplication and repeated addition. So  $3 + 3 + 3 + 3 + 3 = 15$  and  $5 \times 3 = 15$**
- **Vary language used (lots of / groups of).**
- **Use arrays to show the reversibility of multiplication. So  $5 \times 3 = 15$  and  $3 \times 5 = 15$ .**

**Stage 3: Grid method****24 x 6**

X		2	0		4				
6	1	2	0	2	4	=	1	4	4

**342 x 7**

X		3	0	0		4	0		2					
7	2	1	0	0	2	8	0	1	4	=	2	3	9	4

**43 x 76**

	X			4	0			3					
7	0	2	8	0	0	2	1	0					
	6		2	4	0		1	8	=	3	2	6	8

**Notes:**

- **Partitioning the number vertically is acceptable and may help children to add up the answer.**
- **For more difficult sums encourage a written method for addition to add accurately (see addition policy).**
- **Ensure the correct use of language and understanding of place value for sums such as 7 x 40.**

**Stage 4: Short multiplication (expanded)**

**72 x 8**

	7	2				
x		8				
<hr/>						
	1	6	(8 x 2)			
5	6	0	(8 x 70)			
<hr/>						
5	7	6				

**272 x 5**

	2	7	2				
x			5				
<hr/>							
		1	0	(5 x 2)			
	3	5	0	(5 x 70)			
1	0	0	0	(5 x 200)			
<hr/>							
1	3	6	0				

**Notes:**

- **Multiply the least significant digits first.**
- **Using brackets to record working out reinforces understanding of place value.**
- **Refer back to the addition policy where it is necessary to 'carry'.**

### An example involving money / decimals

£65.72 x 4

	£	6	5	.	7	2				
x						4				
				0	.	0	8	(4 x	£0.02)	
				2	.	8	0	(4 x	£0.70)	
		2	0	.	0	0		(4 x	£5.00)	
		2	4	0	.	0	0	(4 x	£60.00)	
£	2	6	2	.	8	8				

### Stage 5: Short multiplication (compact)

24 x 6

342 x 7

	2	4
x		6
	1	4
	1	4
	2	

	3	4	2
x			7
	2	3	9
	2	3	9
	2	1	

$$2741 \times 6$$

	2	7	4	1
X				6
<hr/>				
1	6	4	4	6
	4	2		

**Notes:**

- **Children could be asked to compare this method to the expanded method, looking for similarities and differences.**
- **Once children reach this stage, they will need to understand why they must multiply the least significant digits first.**

**An example involving money / decimals**

$$£65.72 \times 4$$

	£	6	5	.	7	2
X						4
<hr/>						
£	2	6	2	.	8	8
		2	2			

**Notes:**

- **Emphasise the need for the decimal point to hold its position.**
- **Reinforce place value. So the 7 in this example represents 70p.**

**Stage 6: Long multiplication (expanded)****58 x 36**

		5	8				
x		3	6				
		4	8	(6 x 8)			
	3	0	0	(6 x 50)			
	2	4	0	(30 x 8)			
1	5	0	0	(30 x 50)			
	2	0	8	8			
	1						

**Notes:**

- **Multiply the least significant digits first.**
- **Using brackets to record working out reinforces understanding of place value.**
- **Refer back to the addition policy where it is necessary to 'carry'.**

**Stage 7: Long multiplication (compact)****58 x 36**

		5	8
x		3	6
<hr/>			
	3	4	8
1	7	4	0
<hr/>			
2	0	8	8
<hr/>			
1			

**Notes:**

- **Only move on to this stage when children are absolutely secure in all of the previous stages. This process should not be rushed.**
- **Do not tell the children to 'add a zero' at the start of the second line of working out. An understanding of the previous stage will ensure that children see this as  $30 \times 8 = 240$  and not  $3 \times 8 = 24$ .**

### An example involving money / decimals

£234.93 x 24

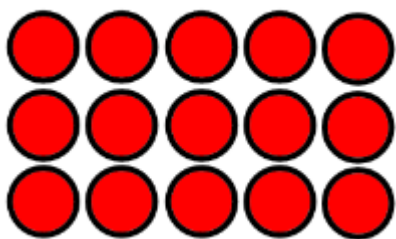
	£	2	3	4	.	9	3
	x					2	4
£		9	3	9	.	7	2
		<sup>1</sup>	<sup>1</sup>	<sup>3</sup>		<sup>1</sup>	
£	4	6	9	8	.	6	0
			<sup>1</sup>				
£	5	6	3	8	.	3	2
		<sup>1</sup>	<sup>1</sup>	<sup>1</sup>	<sup>1</sup>		

#### Notes:

- **Emphasise the need for the decimal point to hold its position.**
- **Reinforce place value. So the 3 in this example represents 3p.**

# Division

## Stage 1: Using arrays



$15 \div 5 = 3$  (or 15 divided by 5 = 3)

$15 \div 3 = 5$  (or 15 divided by 3 = 5)

How many groups of 3?

How many groups of 5?

15 counters shared between 3 people is 5 each

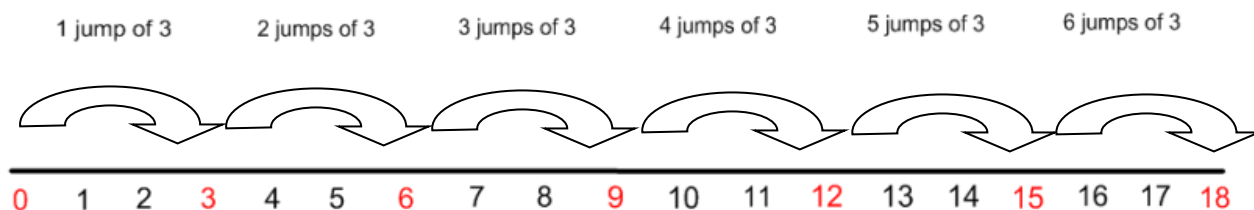
15 counters shared between 5 people is 3 each

## Stage 2: Using an empty number line

$18 \div 3$

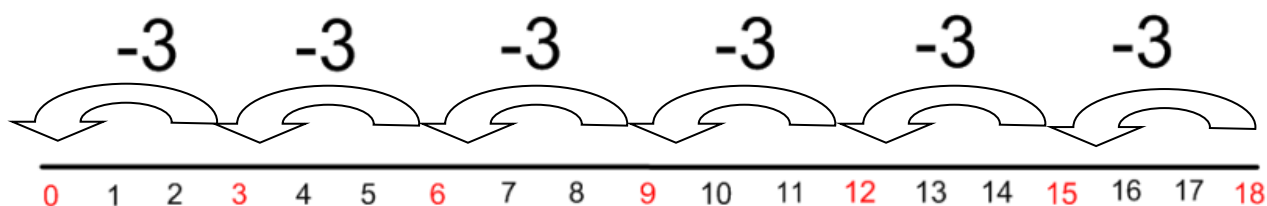
...means how many 3s in 18? (or how many groups of 3 in 18?)

Using an empty number line to count forwards...



So the answer is 6.

**Also jump back from 18 to make the link with repeated subtraction...**



**So the answer is 6.**

**Progressing to using the formal layout for division using multiplication/division facts that the children know.**

$$18 \div 3$$

$$\begin{array}{r} 6 \\ 3 \overline{) 18} \end{array}$$

**The link to arrays can be shown here**

$$\begin{array}{r} 6 \\ 3 \overline{) 18} \end{array}$$

The number 18 in the division layout is represented by a 3x6 grid of red circles.

**Stage 3: Division using partitioning****98 divided by 7****98 can be partitioned in to 70 and 28****70 divided by 7 = 10****28 divided by 7 = 4****So 98 divided by 7 = 14****Progressing to a more formal way of setting out...**

98 can be partitioned in to 70 and 28

$$\begin{array}{r} 10 + 4 \\ 7 \overline{) 70 + 28} \end{array}$$

So  $98 \div 7 = 14$ **Notes:**

- **Children will need to practise partitioning in a variety of ways.**

### **Stage 4: Short Division (without remainders)**

#### **General notes for short division:**

- **Always remind children of correct place value. For example 69 is equal to 60 and 9, but in short division pose:**
  - **How many 3s in 6? And record it above the 6 tens**
  - **How many 3s in 9? And record it above the 9 units.**
- **Base Ten equipment and place value counters can be used to demonstrate exactly how and why this method works.**

		2	3						
3		6	9						
				So	69	÷	3	=	23

#### **Notes:**

**Begin by using carefully selected examples requiring no calculation of remainders at all (so each digit must be a multiple of the divisor).**

**Progressing to...**

$$\begin{array}{r} 14 \\ 7 \overline{) 98} \end{array}$$

So  $98 \div 7 = 14$

**Notes:**

- ***Once children demonstrate a full understanding of remainders, they can be taught how to use the short division method when remainders do occur within the calculation. In such sums, they will need to 'carry' the remainder on to the next digit.***
- ***As mentioned above, Base Ten equipment and place value counters can be used to demonstrate exactly how and why this method works.***

**Stage 5: Short Division (with remainders)**

$432 \div 5$

	0	8	6	r	2				
5	4	3	2						
	<sup>4</sup>	<sup>3</sup>							

So  $432 \div 5 = 86 \text{ r}2$

**Notes:**

- **When the answer for the first column is zero, children should write a zero above to acknowledge its place and then must 'carry' the number (4) over to the next column as a remainder.**

$5327 \div 4$

	1	3	3	1	$\frac{3}{4}$				
4	5	3	2	7					
	<sup>1</sup>	<sup>1</sup>							

So  $5327 \div 4 = 1331.75$

**Notes:**

- **Now that pupils are introduced to examples that give remainders, division needs to have a real life problem solving context, where pupils consider the meaning of the remainder and how to express it (e.g. as a fraction, a decimal or as a rounded number). The answer needs to be expressed as appropriate to the problem.**

**Stage 6: Expanded Long Division**

**$432 \div 15$**

			2	8	r	1	2		
1	5		4	3	2				
			3	0	0			20 lots of 15 = 300	
			1	3	2				
			1	2	0			8 lots of 15 = 120	
				1	2				

So  $432 \div 15 = 28$  remainder 12



**Notes:**

- **Multiples of the divisor (15) have been subtracted from the dividend (432). 20 (lots of 15) + 8 (lots of 15) = 28. 12 is the remainder.**
- **Teach pupils to write a 'useful list' first at the side that will help them decide what chunks to use.**
  - **$1x = 15$**
  - **$10x = 150$**
  - **$20x = 300$**
  - **$100x = 1500$**
- **As children become more confident with the process, encourage more efficient chunks to get the answer quicker and expand on their 'useful lists'.**
- **Depending on the context of the problem, children should also be taught how to express the remainder as a fraction ( $28 \frac{12}{15}$  or  $28 \frac{4}{5}$ ) and as a decimal (28.8).**



**Stage 7: Long Division**

$432 \div 15$

			2	8	.	8
1	5	4	3	2	.	0
		3	0	↓		
		1	3	2		
		1	2	0		
			1	2	0	
			1	2	0	
						0

**Notes:**

- **Only teach this method when children are completely secure with the previous method. The remainder is expressed as a decimal.**
- **As always, if children are making significant errors, return to the previous stage in calculation.**

**Our aim is that by the end of Year 6 children use mental methods (with jottings) when appropriate, but for calculations that they cannot do in their heads, they use an efficient formal written method accurately and with confidence.**