Addition

Mental Strategies: Mental recall of number bonds

6 + 4 = 10 + 3 = 10

25 + 75 = 100 19 + = 20

Use the relationship between addition and subtraction

36 + 19 = 55 19 + 36 = 55 Part + Part = Whole

55 – 19 = 36 55 – 36 = 19 Whole – Part = Part

Whole	
Part	Part

Add the nearest multiple of 10, 100 and 1000 and adjust

24 + 19 = 24 + 20 – 1 = 43

458 + 71 = 458 + 70 + 1 = 529

Counting on or back in repeated steps of 1, 10, 100, 1000

86 + 57 = 143 (by counting on in tens and then in ones)

460 - 300 = 160 (by counting back in hundreds)

Addition using partitioning and recombining:

34 + 45 = (30 + 40) + (4 + 5) = 79

Use near doubles

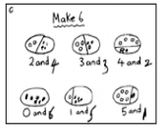
6 + 7 = double 6 + 1 = 13

6 + 8 = double 6 + 2 = 14

Written Methods:

Step 1 Practical

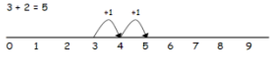
Pupils are encouraged to develop a mental picture of the number system in their heads to use for calculations. They develop ways of recording calculations using pictures/jottings.



Pupils use number lines and practical resources to support calculations and teachers *demonstrate* the use of the number line.

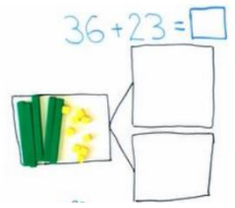
3 + 2 = 5

Pupils then begin to use number lines (up to 10) to support their own calculations to count on in ones.



Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3 for the calculation 8 + 5.

Step 2 Introducing concrete resources



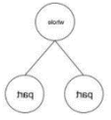
Pupils start to use part-whole models and Dienes or place value counters to add 1-digit and 2-digit numbers together.

This hands on approach allows pupils to group the tens and ones together to calculate the sum.

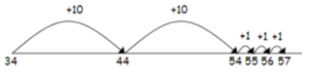
The part-whole model can be a bar model

Whole	
Part	Part

or cherry model.



Step 3 Linear Addition – this is a progressive method



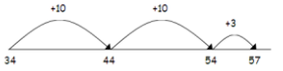
Children will begin to use 'empty number lines' themselves, starting with the larger number and counting on.

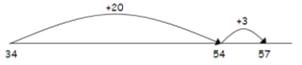
First counting on in tens and then ones.

34 + 23 = 57

Pupils become more efficient by adding the ones in one jump (by using the known fact 4 + 3 = 7).

34 + 23 = 57



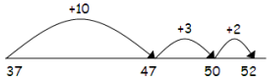


This is followed by adding the tens in one jump and the ones in one jump.

34 + 23 = 57

Bridging through ten can help children become more efficient.

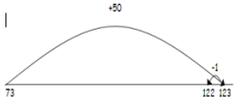
37 + 15 = 52



Pupils learn to partition the ones digit into smaller parts.

Pupils learn to always count on from the largest number irrespective of the order of the calculation.

Finally, pupils will learn to compensate by rounding and adjusting.



73 + 49 = 73 + 50 - 1

Step 4 Partitioning – Smile Addition

Pupils will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods, building on existing mental strategies.

Pupils learn to add the highest place value column first, such as the hundreds in a 3-digit number.

They add the Hundreds values, the Tens values and finally the Ones values before recombining them to find the total.

136 + 129=

H 100+100=200

T 30+20= 50

O 6+9= 15

R 200+50+15= 265

124 + 235

100 + 200 = 300

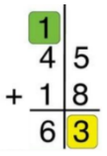
20 + 30 = 50

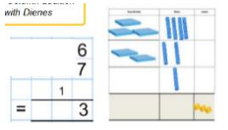
4 + 5 = 9

300 + 50 + 9 = 359

Step 5 Formal Column Addition

Pupils learn the formal written method of column addition, with increasingly larger numbers. This is modelled alongside Dienes to allow pupils to see the regrouping process.





Using column addition, children will:

Add numbers with a different numbers of digits;

783	3587
+ 42	+ 675
825	4262
1	1 1 1

Begin to add two or more decimals with a different number of decimal places;

51.37
+ 24.5

Include a place holder allows the decimals to have an equal number of decimal places, supporting the transition into decimal subtraction.

51.37
+ 24.50

Biggleswade Academy

Subtraction

Children should be encouraged to use a time line for all time calculations e.g. I arrive at the shops at 9.00 and I leave at 13.40.
How long was I shopping?

Find a small difference by counting up
82 – 79 = 3

Mental recall of addition and subtraction facts
10 – 6 = 4 17 - = 11
20 - 17 = 3 10 - = 2

Counting on or back in repeated steps of 1, 10, 100, 1000
86 - 52 = 34 (by counting back in tens and then in ones)
460 - 300 = 160 (by counting back in hundreds)

Subtract the nearest multiple of 10, 100 and 1,000 and adjust

24 - 19 → 24 - 20 + 1 = 5
458 - 71 → 458 - 70 - 1 = 387

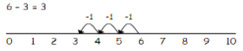
Use the relationship between addition and subtraction
36 + 19 = 55 19 + 36 = 55
55 – 19 = 36 55 – 36 = 19

Whole	
Part	Part

Written Methods

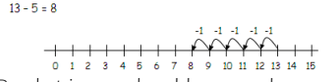
Step 1 Practical

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc.

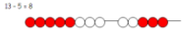


They use number lines and practical resources to support calculation. Teachers *demonstrate* the use of the number line to count backwards.

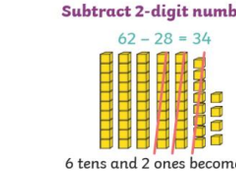
Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones.



Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

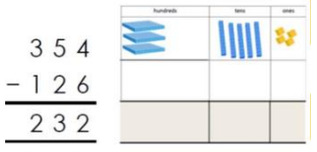


Step 2 Introducing concrete resources



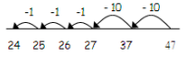
Pupils start to use part-part- whole models and Dienes or place value counters to subtract 1-digit and 2-digit numbers from numbers with more than 2 digits.

This hands on approach allows pupils to observe the number decreasing in size. Pupils can see the two parts, when recombined, make the whole.

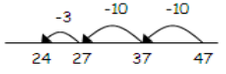


Step 3 Counting Back

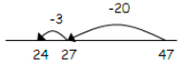
Children will begin to use empty number lines to support calculations.
First, counting back in tens and then in ones.
47 – 23 = 24



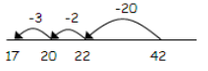
To help children become more efficient, they learn to subtract the tens in jumps of ten and the ones in one complete jump (by using the known fact 7 – 3 = 4).



Next, children learn to subtract the tens in one jump and the ones in one jump.
47 – 23 = 24



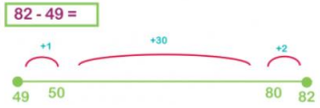
Bridging through ten can help children become more efficient.
42 – 25 = 17



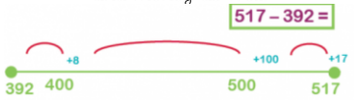
Step 4 Counting On

Counting on works for all subtraction calculations. This utilises pupils' skills with addition in order to solve subtraction equations.

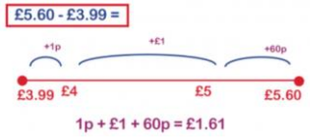
54 – 18 =



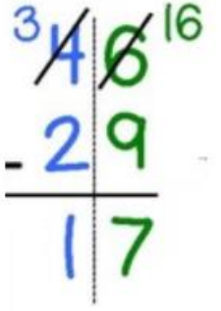
Counting on works for 3-digit numbers and money as well.



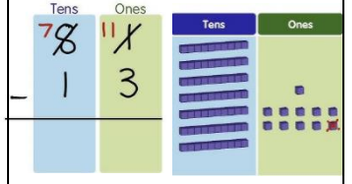
Including Money



Step 5 Introduction to Column Subtraction



Pupils are first introduced to the formal method of column subtraction and the processes of exchanging and regrouping. Pupils first practice this using Dienes and Place value counters.



In this example, pupils exchange a tens Dienes rod for ten ones in order to complete the calculation.

Step 6 Formal Decomposition

Children should:
be able to subtract numbers with different numbers of digits;
using this method, children should also begin to find the difference between two three-digit sums of money, with or without 'adjustment' from the pence to the pounds;
know that decimal points should line up under each other.

Children can and, at times, should still be encouraged to use number lines, particularly where the difference between numbers is quite small or you are close to a landmark number.
E.g. 2006-1999

BiggleswadeAcademy

Multiplication

Mental Strategies

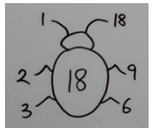
Multiplication tables:
Y2: Times Tables of x2, x5 and x10
Y3: Times tables x3, x4, x6 and x8
Y4: Times Tables x6, x7, x9, x11 and x12
Y5 and Y6: Mixed practice
Associated division facts should be taught at the same time.

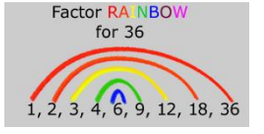
Use closely related facts already known and Partitioning:

13 x 11 = (13 x 10) + (13 x 1)
= 130 + 13
= 143

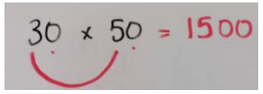
23 x 4 = (20 x 4) + (3 x 4)
= 80 + 12
= 102

Factors and Multiples





SMILE Multiplication
The process of using known facts when multiplying by powers of 10, 100 and 1000.
4 x 3 = 12, therefore 400 x 30 = 12,000



Multiplying by 10 or 100
Knowing that the effect of multiplying by 10 is a shift in the digits one place to the left.
Knowing that the effect of multiplying by 100 is a shift in the digits two places to the left.

Use of factors
8 x 12 = 8 x 4 x 3

Use of Inverse Operation
x 5 = 20 3 x ▬ = 18 x ♀ = 32

Written Methods:

Step 1 Practical

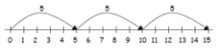
Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups.

This is supported by skip counting.

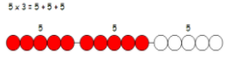
Step 2a Repeated Addition

Children will develop their understanding of multiplication and use jottings to support calculation:
3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x 3

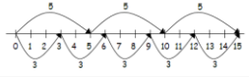
Repeated addition can be shown easily on a number line:
5 x 3 = 5 + 5 + 5



and on a bead bar/bead string:

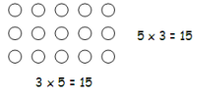


Commutativity: Children should know that 3 x 5 has the same answer as 5 x 3. This can also be shown on the number line.

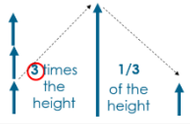


Step 2b Arrays and Scale and Bar Models

At the same time as being introduced to linear methods of multiplication, pupils should experience recording of multiplication as an array and bar model.



Children will also develop an understanding of Scaling
e.g. Find a ribbon that is 4 times as long as the blue ribbon



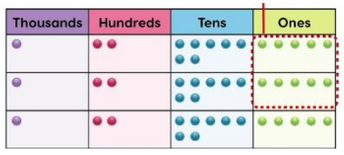
Step 3 Column Method - practical

Pupils first learn how to use column multiplication method for multiplying 2-digit and 3-digit numbers by a 1-digit number with concrete resources.

1275 x 3 =

They learn this method by first using Dines or Place value counters to represent the number. Here is 1275 represented with counters:

Pupils move onto multiplying the multiplicand by the multiplier. They have made 3 lots of 1275.

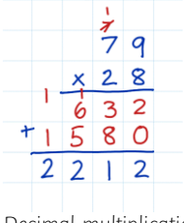


Pupils add the counters or Dienes together and learn to exchange 10 ones counters for 1 tens counter etc.
In this example, they have 15 ones and therefore exchange 10 ones for 1 ten and place that in the next column. This is modelled alongside the formal written method.

Step 5 Column Method – formal written method

Pupils continue to revise the formal written method by multilying by 1-digit numbers and progress onto multiplying larger values together.

For 2-digit x 2-digit and beyond, pupils learn the role of the place holder.



Decimal multiplication:
When multiplying decimals, for example 7.2 x 38, Pupils are taught to multiply 7.2 by 10 so they have an integer, prior to working out 72 x 38 then divide the answer by 10.

Division

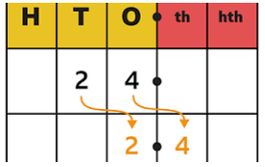
Mental Strategies

Fluency of divisional facts should be encouraged through rapid recall alongside times tables

7 x 8 = 56
56 ÷ 7 = 8
56 ÷ 8 = 7

Dividing by 10, 100 and 1,000

Knowing that the process of dividing by 10 is a shift in the digits moving one place value column to the right. Dividing by 100 require digits to move two place value columns to the right, with dividing by 1000 a movement across three place value columns.



SMILE Division

Using the inverse of SMILE multiplication, pupils learn to divide the dividend by the divisor and adjust according to place value.

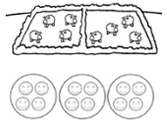
32,000 ÷ 8 = 4,000

Written Methods:

Step 1 Reception – process of sharing

Pupils will understand equal groups and share items out in practical scenarios.

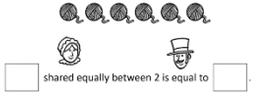
Pupils learn the difference between equal and unequal groups by looking at



images.
They learn the language associated with sharing using stem sentences:

There are ☐ groups.
There are ☐ ____ in each group.
There are ☐ ____ altogether.

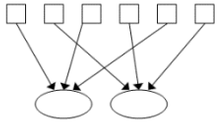
They learn to share items into groups by counting out the number of objects and sharing them out using the 'one for you, one for you' approach.



Step 2 Sharing and Grouping – practical sharing and introduction to Bar Models

Children will develop their understanding of division and use jottings to support their calculations.

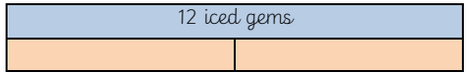
Sharing equally – partitive model (the number of parts you share between are known)
Pupils continue to learn how to divide using simple repeated subtraction such as one for you, one for you to one for each of you is...
Example: 6 sweets shared between 2 people, how many do they each get?



Pupils would first give one sweet to one person, another sweet to the other until they're all gone. This progresses to one for each of you is 2, 2 for each of you is 4, 3 for each of you is 6.

Grouping- Quotitive (the number of parts is unknown but the quantity per part is known).

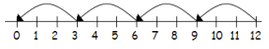
A teacher has 12 iced gems and needs to put 2 gems into each bowl. How many bowls does the teacher need?



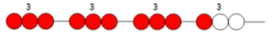
From using a bar model, pupils can see the 12 is shared between 2 equal parts and can share 12 out equally between the 2 parts.

Step 3 Repeated Subtraction

Pupils progress onto repeated subtraction using groups greater than 1. For example,
12 ÷ 3 = 4
Using a number line, pupils count back in jumps of 3 until they're at 0.



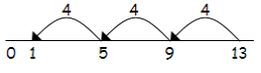
Alternatively, pupils can use bead strings to count in groups of 3 until they have used 12 beads altogether.



The bead bar will help children with interpreting division calculations such as 10 ÷ 5 as 'how many 5s make 10?'

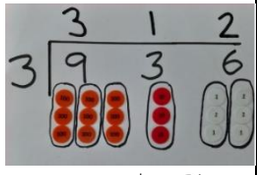
Pupils progress onto calculations involving remainders, using the same repeated subtraction method.

13 ÷ 4 = 3 r 1



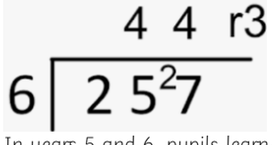
Step 4 Short Division

Short division is introduced to pupils using a 2-digit number divided by a 1-digit number, where there is an integer answer.
Pupils draw a division bracket with the divisor on the outside and the dividend inside the bracket.



Using concrete resources, such as Dienes or Place value counters, pupils practically divide the first digit (inside the bracket) by the divisor. This allows pupils to see the process of sharing.

Remainders: Pupils progress onto short division with remainders and learn how to regroup a remainder in the next place value column.



In years 5 and 6, pupils learn to express their remainders in decimal or fractional notation, or round them up or down depending on the context.

Step 5 Long Division

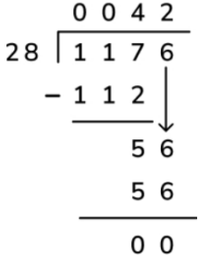
Long division is introduced in Year 6.

Pupils use long division for questions with a divisor greater than 12.

1,176 ÷ 28 =

Step 1: pupils write out the first 5 multiples of the divisor (they can record more when required).
They complete this step by partitioning the divisor into tens and ones and completing the multiples for each, before summing the parts together.
20 + 8 = 28
40 + 16 = 56
60 + 24 = 84
80 + 32 = 112
100 + 40 = 140

Once complete, pupils draw the division bracket and record the divisor on the outside and the dividend inside.



Pupils complete the same practice as with the short division method. When the divisor does not divide into the first digit, a 0 is recorded as part of the quotient. The divisor is then divided into the first and the second digit together – as a 2-digit number. As this is not possible, another 0 is recorded and the divisor is divided into the first 3 digits. The highest multiple of 28 that does not surpass the 117 is 112. A 4 is recorded in the quotient and the difference between 117 and 112 is calculated. The next digit is brought down, and the calculation continued until there is a 0 or a value less than the divisor remaining.