## Calculating Probabilities

1) A box contains 3 grey counters and 2 white counters.

A counter is taken from the box at random.
What is the probability of choosing a white counter?


$$
\frac{2}{5}
$$

2) There are 3 blue counters, 5 red counters and 7 green counters in abag.

A counter is taken from the bag at random.
a) What is the probability that a green counter will be chosen? $\frac{7}{15}$
b) What is the probability that a blue or red counter will be chosen? $\frac{8}{15}$
3) In a class there are 10 boys and 15 girls.

A teacher chooses a student at random from the class.
Eric says that the probability a boy will be chosen is 0.5 because a student can be either a boy or a girl.

Jenny says that Eric is wrong.
Decide who is correct - Eric or Jenny - giving reasons for your answer.
Jenny is correct because there are more girls than boys, so the probability of choosing a girl will be greater than that of choosing a boy.
4) Spinner A has numbers 1 to 4 on it.

Spinner B has numbers 1 to 3 on it.
Both spinners are spun and the numbers on each are added together to give a score.
What is the probability that the score will be
a) $7 ? \frac{1}{12}$
b) 3 or 4 ? $\frac{5}{12}$

## Mutually Exclusive Events

1) If the probability of passing a driving test is 0.54 , what is the probability of failing it? 0.46
2) The probability that a football team will win their next game is $\frac{2}{11}$.

The probability they will lose is $\frac{3}{11}$.
What is the probability the game will be a draw? $\frac{6}{11}$
3) On the school dinner menu there is only ever one of four options.

Some of the options are more likely to be on the menu than others.
The table shows the options available on any day, together with three of the probabilities.

| Food | Curry | Sausages | Fish | Casserole |
| :--- | :---: | :---: | :---: | :---: |
| Probability | 0.36 | 0.41 | 0.14 | 0.09 |

a) Work out the probability of the dinner option being Fish. 0.14
b) Which option is most likely? Sausages
c) Work out the probability that it is a Curry or Sausages on any particular day.
d) Work out the probability that it is not Casserole. 0.91
4) Julie buys a book every week.

Her favourite types are Novel, Drama, Biography and Romance.
The table shows the probability that Julie chooses a particular type of book.

| Type of book | Novel | Drama | Biography | Romance |
| :--- | :---: | :---: | :---: | :---: |
| Probability | 0.24 | 0.16 | $x$ | $x$ |

a) Work out the probability that she will choose a Novel or a Drama. 0.4
b) Work out the probability that she will choose a Biography or a Romance. 0.6

The probability that she will choose a Biography is the same as the probability she will choose a Romance.
c) Work out the probability that she will choose a Biography. 0.3

## Two-Way Tables

1) Billy has been carrying out a survey.

He asked 100 people the type of water they like to drink (still, sparkling or both).
Here are part of his results:

|  | Still | Sparkling | Both | Total |
| :---: | :---: | :---: | :---: | :---: |
| Male | 26 | 21 | 6 | 53 |
| Female | 17 | 20 | 10 | 47 |
| Total | 43 | 41 | 16 | 100 |

a) Complete the two-way table.
b) How many males were in the survey? 53
c) How many females drink only still water? 17
d) How many people drink only sparkling water? 41
2) 90 students each study one of three languages.

The two-way table shows some information about these students.

|  | French | German | Spanish | Total |
| :---: | :---: | :---: | :---: | :---: |
| Female | 6 | 11 | 23 | 40 |
| Male | 14 | 7 | 29 | 50 |
| Total | 20 | 18 | 52 | 90 |

50 of the 90 students are male.
29 of the 50 male students study Spanish.
a) Complete the two-way table.
b) How many females study French? 6
c) How many people study Spanish? 52
3) Karen asks 100 students if they like milk, plain or white chocolates best.

36 of the students are girls.
19 of these girls like milk chocolates best.
16 boys like white chocolates best.
8 out of the 24 students who like plain chocolates best are girls.
Work out the number of students who like milk chocolates the best. 51

## Averages and the Range

1) Kaya made a list of his homework marks.

$$
\begin{array}{llllllll}
3 & 2 & 3 & 4 & 1 & 4 & 5 & 4
\end{array}
$$

a) Write down the mode of Kaya's marks. 4
b) Work out his mean homework mark. 3.25
2) Lydia rolled an 8 -sided dice ten times.

Here are her scores.

## $\begin{array}{llllllllll}5 & 1 & 2 & 5 & 3 & 8 & 6 & 6 & 3 & 2\end{array}$

a) Work out Lydia's median score. 4
b) Work out the mean of her scores. 4.1
3) In a two-week period, a train was this many minutes late each day:

$$
\begin{array}{cccccccccccccc}
3 & 0 & 0 & 0 & 7 & 4 & 5 & 2 & 0 & 1 & 14 & 0 & 5 & 1
\end{array}
$$

a) What was the mean average number of minutes late? 3 minutes late
b) What was the median average number of minutes late? 1.5 minutes late
4) Two small Year 10 classes, Set A and Set B, sat the same Science test.

Set A had these scores for the test:
$63 \%, 71 \%, 48 \%, 95 \%, 46 \%, 82 \%, 77 \%, 36 \%, 73 \%$
Set B had these scores:
$58 \%, 63 \%, 85 \%, 61 \%, 59 \% 38 \%, 90 \%, 84 \%, 75 \%, 48 \%$
How much bigger was Set B's mean average score than Set A's mean average score?
Give your answer correct to 1 decimal place. $0.4 \%$
5) A rugby team played six games.

The mean score for the six games is 15
The rugby team played one more game.
The mean score for all seven games is 16
Work out the number of points the team scored in the seventh game. 22

## Experimental Probabilities

1) Ahmad does a statistical experiment.

He throws a dice 600 times.
He scores one, 200 times.
Is the dice fair? Explain your answer Two possible answers:
No, you would expect to score 1 about 100 times.
Yes, although you would expect 1 about 100 times, you could still get it 200 times.
2) Chris has a biased coin.

The probability that the biased coin will land on a tail is 0.3
Chris is going to flip the coin 150 times.
Work out an estimate for the number of times the coin will land on a tail. 45 times
3) On a biased dice, the probability of getting a six is $\frac{2}{3}$.

The dice is rolled 300 times.
Work out an estimate for the number of times the dice will land on a six. 200 times
4) On a biased dice, the probability of getting a three is 0.5

The dice is rolled 350 times.
Work out an estimate for the number of times the dice will land on a three. 175 times
5) Jenny throws a biased dice 100 times.

The table shows her results.

| Score | Frequency |
| :---: | :---: |
| 1 | 15 |
| 2 | 17 |
| 3 | 10 |
| 4 | 24 |
| 5 | 18 |
| 6 | 16 |

a) She throws the dice once more.

Find an estimate for the probability that she will get a four.

$$
\frac{24}{100} \text { or } 0.24
$$

b) If the dice is rolled 250 times, how many times would you expect to get a five? 45 times

## Possibility Spaces

1) Two dice are rolled and their scores are multiplied together.
a) Complete the possibility space to show all the possible results.

## First dice

|  |  | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 1 | 2 | 3 | 4 | 5 | 6 |
|  | 2 | 2 | 4 | 6 | 8 | 10 | 12 |
|  | 3 | 3 | 6 | 9 | 12 | 15 | 18 |
|  | 4 | 4 | 8 | 12 | 16 | 20 | 24 |
|  | 5 | 5 | 10 | 15 | 20 | 25 | 30 |
|  | 6 | 6 | 12 | 18 | 24 | 30 | 36 |

b) What is the probability of getting a result that is an even number? $\frac{27}{36}$
2) Suppose there are three cards:

A black card that is black on both sides,
A white card that is white on both sides,
A mixed card that is black on one side and white on the other side.
All the cards are placed into a hat and one is taken out at random.
It is placed on a table and the side facing up is black.
What is the probability that the other side of the card is also black? $\frac{1}{2}$

## Averages From a Table

1) The number of pens in each pupil's pencil case in a classroom has been counted. The results are displayed in a table.

| Number of pens | Number of pupils |
| :---: | :---: |
| 0 | 4 |
| 1 | 6 |
| 2 | 7 |
| 3 | 5 |
| 4 | 3 |
| 5 | 1 |

a) Work out the total number of pens in the classroom. 52
b) Write down the modal number of pens in a pencil case. 2
c) Work out the mean number of pens in a pencil case. 2
d) Work out the range of the number of pens in a pencil case. 5
2) Thomas is analysing the local football team. He records the number of goals scored in each football match in the past twelve months.

Thomas said that the mode is 7
Thomas is wrong.
a) Explain why.

Thomas gave the highest frequency instead of giving the number of 'goals scored' associated with it.
b) Calculate the mean number of goals scored.

| Goals scored | Frequency |
| :---: | :---: |
| 0 | 7 |
| 1 | 5 |
| 2 | 3 |
| 3 | 6 |
| 4 | 2 |
| 5 | 1 |
| 6 | 1 |

1.92
3) Sindy recorded the time, in minutes, that her train was late over 100 days.

Information about these times is shown in the table.

| Time $(t$ minutes | Frequency | Midpoint | MP $\times$ Frequency |
| :---: | :---: | :---: | :---: |
| $0<t \leqslant 6$ | 15 | 3 | 45 |
| $6<t<12$ | 23 | 9 | 207 |
| $12<t \leqslant 18$ | 28 | 15 | 420 |
| $18<t \leqslant 24$ | 19 | 21 | 399 |
| $24<t \leqslant 30$ | 15 | 27 | 405 |

Calculate an estimate for the mean time that her train was late.
Give your answer to 1 decimal place. 14.8 minutes

## Simple Tree Diagrams

1) Jim and Sue each take a driving test.

The probability that Jim will pass the driving test is 0.7
The probability that Sue will pass the driving test is 0.6
a) Complete the probability tree diagram.

b) Work out the probability that both Jim and Sue will pass the driving test. 0.42
c) Work out the probability that only one of them will pass the driving test. $0.28+0.18=0.46$
2) Terri has 7 pens in a box.

2 of the pens are blue.
5 of the pens are red.
Terri takes at random a pen from the box and writes down its colour.
Terri puts the pen back in the box.
Then Terri takes at random a second pen from the box and writes down its colour.
a) Complete the probability tree diagram.


$$
\frac{10}{49}+\frac{10}{49}=\frac{20}{49}
$$

b) Work out the probability that Terri takes exactly one pen of each colour from the box.

## Simple Tree Diagrams

1) Tim puts 3 red counters and 4 blue counters in a bag.

He takes at random a counter from the bag.
He writes down the colour of the counter.
He puts the counter in the bag again.
He then takes at random a second counter from the bag.
a) Complete the probability tree diagram.

1st counter
2nd counter

b) Work out the probability that Tim takes two red counters. $\frac{9}{49}$
2) In a game of chess, a player can either win, lose or draw.

The probability that Jane wins any game of chess is 0.2
The probability that Jane draws any game of chess is 0.3
Jane plays 2 games of chess.
1st game
2nd game
a) Complete the probability tree diagram.
b) Work out the probability that Jane will

 win both games. 0.04

1) Lucy throws a biased dice twice.

Complete the probability tree diagram to show the outcomes.
Label clearly the branches of the tree diagram.

2) A bag contains 10 coloured balls.

7 of the balls are blue and 3 of the balls are green.
Nathan is going to take a ball, replace it, and then take a second ball.
a) Complete the tree diagram.

b) Work out the probability that Nathan will take two blue balls. $\frac{49}{100}$
c) Work out the probability that Nathan will take one of each coloured balls.
$\frac{42}{100}$
d) Work out the probability that Nathan will take two balls of the same colour.
$\frac{58}{100}$

## Harder Tree Diagrams

1) There are 5 red pens, 3 blue pens and 2 green pens in a box.

Jerry takes at random a pen from the box and gives the pen to his friend.
Jerry then takes at random another pen from the box.
Work out the probability that both pens are the same colour. $\frac{28}{90}$
2) There are 3 red sweets, 2 blue sweets and 4 green sweets in a bag. Jack takes a sweet at random.
He eats the sweet.
He then takes another sweet at random.

Work out the probability that both sweets are the same colour. $\quad \frac{20}{72}$
3) There are 13 buttons in a bag.

9 buttons are white.
4 buttons are black.
Carol takes a button at random from the bag, and keeps it.
She now takes another button from the bag.
Work out the probability that Carol takes a button of each colour. $\frac{72}{156}$

## Harder Tree Diagrams

1) A bag contains 7 green and 3 yellow balls.

A ball is taken from the bag at random and not replaced. Another ball is taken from the bag at random.
a) Draw a tree diagram to show all the possibilities.
b) What is the probability that both balls are different colours? $\frac{42}{90}$
a)


2) A box contains 5 red counters and 3 blue counters.

A counter is taken from the box at random and not replaced.
Another counter is taken at random.
a) Draw a tree diagram to show all the possibilities.
b) What is the probability of choosing at least one blue counter? $\frac{36}{56}$
c) What is the probability of choosing two counters of the same colour? $\frac{26}{56}$
d) What is the probability of choosing two counters of different colours? $\frac{30}{56}$

3) A box contains 4 red counters and 3 blue counters.

A counter is taken from the box at random and not replaced.
A second counter is taken from the box at random and not replaced.
A third counter is taken from the box.
a) Draw a tree diagram to show all the possibilities.
b) What is the probability that all three counters are the same colour? $\frac{30}{210}$
c) What is the probability that exactly two of the counters are red? $\frac{108}{210}$
a)


## Harder Tree Diagrams

1) Sara has two boxes.

There are 6 black and 4 white counters in box A.
There are 7 black and 3 white counters in box B.

Sara takes at random a counter from box $A$ and puts it in box B.
She then takes at random a counter from box B and puts it in box A.
a) Complete the probability tree diagram.

b) Find the probability that after Sara has put the counters from box B into box A there will still be 6 black counters and 4 white counters in box $A$.

## Venn Diagrams

1) The Venn diagram shows the number of students studying one or more of the sciences Chemistry, Biology and Physics.

a) How many students are represented in this Venn diagram? 36
b) How many students are studying exactly two sciences? 5
c) What is the probability that a student chosen at random is not studying Physics? $\frac{24}{36}$
2) There are 31 student who study languages.

All 31 study at least one of Spanish, French or German.
4 study all three languages.
9 study Spanish and French.
7 study French and German.
6 study Spanish and German.
7 study only French and 5 study only German.
Complete the Venn diagram to show this information.


## Probability Using Venn Diagrams

1) In a class of 30 students, all of them have brothers or sisters or both.

19 have a brother.
$P(B$ or $S)=30$
16 have a sister.
$P(B)=19$
$P(S)=16$
a) Complete the Venn diagram.
$19+16-30=5$
5 have both

b) Find the probability that a student in the class has a brother and a sister. $\frac{5}{30}=1 / 6$
c) If it is known that a student has a sister, what is the probability that they also have a brother?

$$
\frac{5}{16}
$$

2) A cafeteria serves only main courses and desserts. Some people just have a main, some have just a dessert and some have both.
One day, $65 \%$ of the customers had a main course and $90 \%$ had a dessert.
a) Show this information on a Venn diagram.
U.6S-0.55


$$
\begin{aligned}
& P(M)=0.65 \\
& P(D)=0.9 \\
& P(m \text { or } D)=1 \quad 55 \% \text { have } \\
& 0.6 s+0.9-1=0.55
\end{aligned}
$$

b) What is the probability that a customer had a main course and a dessert?

$$
0.55
$$

c) If it is known that a customer had a dessert, what is the probability that they also had a main course?

$$
\frac{0.55}{0.9}=11 / 18
$$

## Box Plots

1) The ages of 20 teachers are listed below.

a) On the grid below, draw a boxplot to show the information about the teachers.

2) A warehouse has 60 employees working in it.

The age of the youngest employee is 16 years.
The age of the oldest employee is 55 years.
The median age is 37 years.
The lower quartile age is 29 years.
The upper quartile age is 43 years.
On the grid below, draw a boxplot to show information about the ages of the employees.


## Boxplots

Terry drew a line of length 60 cm .
He asked some children to estimate the length of the line he had drawn.
He recorded their estimates.
The box plot gives some information about these estimates.

## Children's estimates



Adults' estimates

a) Write down the median of the children's estimates.

$$
55 \mathrm{~cm}
$$

b) Write down the interquartile range of the children's estimates.


Terry then asked some adults to estimate the length of the line he had drawn. The table gives some information about the adults' estimates.

|  | Length |
| :--- | :---: |
| Lowest estimate | 20 cm |
| Lower quartile | 45 cm |
| Median | 62 cm |
| Upper quartile | 75 cm |
| Highest estimate | 95 cm |

c) On the grid above, draw a box plot to show this information.

## And \& Or Questions

1) Jordan designs a game for a school fair.

He has two 8-sided spinners.
The spinners are equally likely to land on each of their sides.

One spinner has 3 blue sides, 2 yellow sides and 3 white sides.
The other spinner has 2 blue sides, 2 green sides and 4 white sides.
Calculate the probability that the two spinners will land on the same colour. $\frac{18}{64}$ or $\frac{9}{32}$
Blue and Blue: $\frac{3}{8} \times \frac{2}{8}=\frac{6}{64}$
Blue and Blue OR White and White

$$
\frac{6}{64}+\frac{12}{64}=\frac{18}{64}
$$

White and White: $\frac{3}{8} \times \frac{4}{8}=\frac{12}{64}$
ela

Blue and Blue OR White and White
2) The probability that it will snow in Paris on Christmas day is 0.06 .
a) Work out the probability that it will snow in Paris on both Christmas day 2008 and Christmas day 2009. $0.00360 .06 \times 0.06$
b) Work out the probability that it will snow in Paris on either Christmas Day 2008 or Christmas Day 2009, but not on both. 0.1128
$0.06 \times 0.94+0.94 \times 0.06$
snow not snow not snow $\begin{aligned} & \text { snow }\end{aligned}$
3) A bag contains 2 black beads, 5 yellow beads and 3 red beads.

Natalie takes a bead at random from the bag, records its colour and replaces it.
She does this two more times.

Work out the probability that, of the three beads Natalie takes, exactly two are the same colour. $\frac{660}{1000}$ (or any equivalent fraction)

$$
\begin{aligned}
P(\text { two same colour })=1-P(B B B)-P(Y Y Y)-P(R R R) & -P(B Y R)-P(B R Y)-P(Y B R)-P(Y R B)-P(R Y B)-P(R B Y) \\
& \frac{8}{1000} \\
& \frac{125}{1000} \\
& \frac{27}{1000} \\
& \frac{30}{1000} \\
& \frac{30}{1000} \\
& 1-\frac{340}{1000}
\end{aligned}
$$

## Sampling Populations

1) In a school there were 800 students who regularly had a school dinner.

The Headteacher of the school wanted to know whether the students liked the dinners.
a) What is the main advantage of asking a sample of the students whether they like school dinners rather than asking all of them? It is quicker.
b) The Headteacher asked 100 KS 3 students whether they liked the dinners and 40 of them said they did.

Use this information to estimate how many of the 800 students liked school dinners. 320
c) In finding your answer to part b), what assumption have you made? KS4 students are as likely
d) What could be done to make your estimate more accurate? KS3 students. Include KS4 students in the sample, as well as KS3 students.
2) A park-keeper wanted to know how many fish there were in the park pond.

He went to the pond early one morning and used his fishing rod to catch 20 fish. The bait he used was maggots.

Then, he marked each of the fish with a white dot on their tail and released them.
A week later, he used his fishing rod and maggots to catch another 20 fish.
He found that 4 of these 20 fish had the white dot on their tails.
a) Use this information to estimate how many fish there are in the pond. 100
b) In finding your estimate, what assumption have you made? All of the fish are attracted to the specific bait that he is using.

## Stratified Sampling

1) Ellen wants to do a survey with Years 9, 10 and 11 at her school.

The table shows the number of students in each of these year groups.

| Year 11 | Year 10 | Year 9 |
| :---: | :---: | :---: |
| 750 | 700 | 900 |

Ellen takes a sample of 50 students stratified by year group.
Work out the number of students from Year 10 in the sample. 15
2) The table shows information about the year groups of 1000 students in a school.

| Year group | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number in year | 157 | 180 | 166 | 140 | 132 | 114 | 111 |

Tony takes a sample of 50 of these students, stratified by year group.
Calculate the number of Year 8 students he should have in his sample. 9
3) The table shows information about Ben's collection of 652 coins.

| Country | France | Spain | Germany | Italy | Total |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Number of coins | 240 | 182 | 133 | 97 | 652 |

Ben takes a sample of 50 coins stratified by country.
Work out the number of coins from Italy in this sample. 7
4)

|  | Male | Female |
| :--- | :---: | :---: |
| Lower sixth | 399 | 602 |
| Upper sixth | 252 | 198 |

The table gives information about the number of students in the two years of a sixth form.
Amy wants to interview some of these students.
She takes a random sample of 70 students stratified by year and by gender.
Work out the number of students in the sample who are male and in the lower sixth. 19

# Stratified Sampling 

$\frac{\text { Sample size }}{\text { Population size }} \times$ Stratum size

1) The table below shows the number of employees in each section of a company.

| Department | Managerial | Sales | Technical | Production |
| :--- | :---: | :---: | :---: | :---: |
| Number of employees | 18 | 45 | 288 | 549 |$=900$

A survey on job satisfaction is to be carried out.
a) Explain why a simple random sample of employees is unsuitable.

The numbers in each department are very different - it might not be fair.
b) A stratified random sample of 100 is used. Complete the table below to show how many employees from each department will be included.

| Department | Managerial | Sales | Technical | Production |
| :--- | :---: | :---: | :---: | :---: |
| Number of employees <br> in sample | 2 | 5 | 32 | 61 |
| $\frac{100}{900} \times 18$ |  |  |  |  |$\frac{100}{900} \times 45 \quad \frac{100}{900} \times 288 \quad \frac{100}{900} \times 549$

2) MathsWatch High-School has 798 pupils.

The size of each year group is shown below.

| Year Group | Boys | Girls |
| :---: | :---: | :---: |
| $\mathbf{7}$ | 77 | 72 |
| $\mathbf{8}$ | 74 | 79 |
| $\mathbf{9}$ | 72 | 74 |
| $\mathbf{1 0}$ | 93 | 107 |
| $\mathbf{1 1}$ | 85 | 65 |

The headteacher wants to find out the opinions of the pupils on changing the timing of the school day. A stratified sample of 80 pupils is taken.
a) Complete the table below to show the numbers of pupils to be sampled.

| Year Group | Boys in Sample |  | Girls in Sample |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{7}$ | $\frac{80}{78} \times 77$ | 8 | $\frac{80}{189} \times 72$ | 7 |
| $\mathbf{8}$ | $\frac{80}{798} \times 74$ | 7 | $\frac{80}{78} \times 79$ | 8 |
| $\mathbf{9}$ | $\frac{80}{798} \times 72$ | 7 | $\frac{80}{798} \times 74$ | 7 |
| $\mathbf{1 0}$ | $\frac{80}{78} \times 93$ | 9 | $\frac{80}{788} \times 107$ | 11 |
| $\mathbf{1 1}$ | $\frac{80}{798} \times 85$ | 9 | $\frac{80}{798} \times 65$ | 7 |

The table below shows the number of pupils in the sample who answered YES to a change in the timing of the school day.

| Year Group | Boys in Sample who answered YES | Girls in Sample who answered YES | $\begin{aligned} & 8+14=22 \\ & \text { answered YES } \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| 7 | 2 | 3 |  |
| 8 | 3 | 5 |  |
| 9 | 2 | 1 | $\frac{22}{80} \times 100=27.5$ |
| 10 | 1 | 4 |  |
| 11 | 0 | 1 |  |
|  | 8 | 14 |  |

b) Use the table to estimate the percentage of pupils in the school who would answer YES to the question. 27.5\%

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
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1) A class of pupils is asked to solve a puzzle.

The frequency table below shows the times taken by the pupils to solve the puzzle.

| Time $(t)$ in min | Frequency |
| :---: | :---: |
| $0<t \leqslant 5$ | 3 |
| $5<t \leqslant 10$ | 4 |
| $10<t \leqslant 15$ | 5 |
| $15<t \leqslant 20$ | 7 |
| $20<t \leqslant 25$ | 5 |

Draw a frequency diagram to show this information.


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| :--- | :--- |

Grade 3 answers

## Scatter Diagrams

1) The scatter graph shows some information about the marks of six students.
It shows each student's marks in Maths and Science.

The table below shows the marks for four more students.

| Maths | 22 | 8 | 17 | 26 |
| :--- | :---: | :---: | :---: | :---: |
| Science | 30 | 12 | 24 | 24 |

a) On the scatter graph, plot the information from the table.
b) Draw a line of best fit.
c) Describe the correlation between the marks in Maths and the marks in Science. There is a positive correlation

Another student has a mark of 18 in Science.
d) Use the line of best fit to estimate the mark in Maths of this student.

2) The table below shows the average daily number of hours sleep of 10 children.

| Age (years) | 4 | 2 | 5 | 1 | 9 | 6 | 8 | 7 | 10 | 1.5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of hours sleep | 14 | 13 | 12.5 | 15 | 10 | 12.5 | 10.8 | 12 | 11 | 14 |

The first five results have been plotted on the scatter diagram.
a) Plot the next five points.
b) Draw a line of best fit.
c) Describe the relationship between the age of the children and their number of hours sleep per day. A negative correlation
d) Use your scatter graph to estimate the number of hours sleep for a 3 year old child.
My answers is 13.6 Yours will depend on your line of best fit.

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Grade 3 answers

## Scatter Diagrams

1) Sue did an experiment to study the times, in minutes, it took 1 cm ice cubes to melt at different temperatures.
Some information about her results is given in the scatter graph.


The table shows the results from two more experiments.

| Temperature $\left({ }^{\circ} \mathbf{C}\right)$ | 15 | 55 |
| :--- | :---: | :---: |
| Time (Minutes) | 21 | 15 |

a) On the scatter graph, plot the results from the table.
b) Describe the relationship between the temperature and the time it takes a 1 cm ice cube to melt. Negative correlation
c) Find an estimate for the time it takes a 1 cm ice cube to melt when the temperature is $25^{\circ} \mathrm{C}$. 19 minutes (your answer will depend on your line of best fit)

Sue's data cannot be used to predict how long it will take a 1 cm ice cube to melt when the temperature is $100^{\circ} \mathrm{C}$.
d) Explain why.

Line of best fit would give a negative time
or
You cannot draw and use a line of best fit which goes beyond the values.

## Scatter Diagrams

1) Henry reads eight books.

For each book he recorded the number of pages and the time he took to read it.
The scatter graph shows information about his results.

a) Describe the relationship between the number of pages in a book and the time Henry takes to read it. Positive correlation

Henry reads another book.
The book has 150 pages.
b) Estimate the time it takes Henry to read it. 7 hours (your answer will depend on your line of best fit)

## Scatter Diagrams

1) Mr Jones sells umbrellas.

The scatter graph shows some information about the number of umbrellas he sold and the rainfall, in cm, each month last year.

Number of umbrellas


In January of this year, the rainfall was 6.2 cm .
During January, Mr Jones sold 32 umbrellas.
a) Show this information on the scatter graph.
b) What type of correlation does this scatter graph show? Positive correlation

In February of this year, Mr Jones sold 40 umbrellas.
c) Estimate the rainfall for February. 6.7 cm (your answer will depend on your line of best fit)

## Cumulative Frequency

The heights of 80 plants were measured and can be seen in the table, below.

a) Complete the cumulative frequency table for the plants.

| Height $(\mathrm{cm})$ | Cumulative Frequency |
| :---: | :---: |
| $0<\mathrm{h} \leqslant 10$ | 2 |
| $0<\mathrm{h} \leqslant 20$ | 7 |
| $0<\mathrm{h} \leqslant 30$ | 26 |
| $0<\mathrm{h} \leqslant 40$ | 64 |
| $0<\mathrm{h} \leqslant 50$ | 77 |
| $0<\mathrm{h} \leqslant 60$ | 80 |

b) Draw a cumulative frequency graph for your table.
c) Use your graph to find an estimate for
(i) the median height of a plant. 34 cm
(ii) the interquartile range of the heights of the plants.

$$
39-27.7=11.3 \mathrm{~cm}
$$

d) Use your graph to estimate how many plants had a height that was greater than 45 cm . $80-72=8$ plants

## Cumulative Frequency

The table shows information about the amount spent by 100 customers in a supermarket.

| Amount spent $(£ n)$ | Frequency |
| :---: | :---: |
| $0<n \leqslant 20$ | 17 |
| $20<n \leqslant 40$ | 23 |
| $40<n \leqslant 60$ | 36 |
| $60<n \leqslant 80$ | 14 |
| $80<n \leqslant 100$ | 8 |
| $100<n \leqslant 120$ | 2 |

a) Complete the cumulative frequency table for this information.

| Amount spent (£n) | Cumulative frequency |
| :---: | :---: |
| $0<n \leqslant 20$ | 17 |
| $0<n \leqslant 40$ | 40 |
| $0<n \leqslant 60$ | 76 |
| $0<n \leqslant 80$ | 90 |
| $0<n \leqslant 100$ | 98 |
| $0<n \leqslant 120$ | 100 |

b) On the grid, draw a cumulative frequency graph for your table.
 money spent.

$$
6_{0}-27=f_{33}
$$

Histograms

The incomplete table and histogram give some information about the heights (in cm ) of some plants.

| Height $(h \mathrm{~cm})$ | Frequency |
| :---: | :---: |
| $100<h \leqslant 130$ | 30 |
| $130<h \leqslant 150$ | 64 |
| $150<h \leqslant 160$ | 50 |
| $160<h \leqslant 180$ | 40 |
| $180<h \leqslant 210$ | 18 |


a) Use the histogram to complete the table.
b) Use the table to complete the histogram.

## Histograms

1. The table gives information about the heights, in centimetres, of some 18 year old students.

Use the table to draw a histogram.

| Height $(h \mathrm{~cm})$ | Frequency |
| :---: | :---: |
| $135<h \leqslant 145$ | 12 |
| $145<h \leqslant 165$ | 46 |
| $165<h \leqslant 180$ | 45 |
| $180<h \leqslant 190$ | 25 |
| $190<h \leqslant 195$ | 4 |

Frequency density

2. The histogram shows the amount of time, in hours, that students spend on their homework per week.

Frequency density $=\frac{\text { Frequency }}{\text { Class width }}$
From the numbers in the table:
Frequency density $=\frac{27}{1}=27$


Use the histogram to complete the table.

| Time $(t$ hours $)$ | Frequency |
| :---: | :---: |
| $0<t \leqslant 1 / 2$ | 5 |
| $1 / 2<t \leqslant 1$ | 12 |
| $1<t \leqslant 2$ | 34 |
| $2<t \leqslant 3$ | 27 |
| $3<t \leqslant 5$ | 8 |

1) $y=5 x$
a) Work out the value of $y$ when $x=3$15
b) Work out the value of $y$ when $x=-2 \quad-10$
2) $y=2 x+7$
a) Work out the value of $y$ when $x=4$
b) Work out the value of $y$ when $x=-3 \quad 1$
3) $y=2 x+4 t$
$x=6$
$t=1$
Work out the value of $y$.16
4) $y=2 a-3 b$
$a=4$
$b=-2$
Work out the value of $y$. 14
5) $v=3 a+5 b$
$a=6$
$b=-3$
Work out the value of $v$. 3
6) $y=x^{2}$
a) Work out the value of $y$ when $x=6 \quad 36$
b) Work out the value of $y$ when $x=-4$
7) $y=2 x^{2}$
a) Work out the value of $y$ when $x=5$
b) Work out the value of $y$ when $x=-3 \quad 18$
8) $y=3 x^{2}+2 x$
a) Work out the value of $y$ when $x=2 \quad 16$
b) Work out the value of $y$ when $x=-4$
9) $v=u^{2}+5 a s$
$u=6$
$a=2.5$
$s=9$
Work out the value of $v .148 .5$
10) $y=p-2 q x^{2}$
$p=-10$
$q=2$
$x=-5$
Work out the value of $y .-110$
11) $v^{2}=u^{2}+2 a s$
$u=6$
$a=2.5$
$s=9$
Work out the value of $v .9$
12) $v^{2}=u^{2}+2 a s$
$u=3$
$a=9.8$
$s=12$
Work out the value of $v .15 .6$
Give your answer correct to 1 decimal place
13) $s=u t+0.5 a t^{2}$
$a=9.8$
$t=5$
$u=7$
Work out the value of $s$. 157.5

## Straight Line Graphs

1) a) Complete the table of values for $y=4 x-2$

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -10 | -6 | -2 | 2 | 6 | 10 |

b) On the grid, draw the graph of $y=4 x-2$, for values of $x$ from -2 to 3 .
c) Use the graph to find the value of $y$ when $x=2.5$ $y=8$
d) Use the graph to find the value of $x$ when $y=-8$ $x=-1.5$

2) a) Complete the table of values for $y=2 x+2$

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -2 | 0 | 2 | 4 | 6 | 8 |

b) On the grid, draw the graph of $y=2 x+2$.


## Straight Line Graphs

1) On the grid, draw the graph of $y=2 x-4$

2) a) Complete the table of values for $3 x+2 y=6$

| $x$ | -2 | -1 | 0 | 1 | 2 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 6 | 4.5 | 3 | 1.5 | 0 | -1.5 |

b) On the grid, draw the graph of $3 x+2 y=6$

c) Find the gradient of the graph of $3 x+2 y=6$. Gradient is -1.5

## Straight Line Graphs

1) a) Complete the table of values for $y=2 x-3$

| $x$ | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | -5 | -3 | -1 | 1 | 3 | 5 |

b) Using the axes on the right draw the graph of $y=2 x-3$
c) Use your graph to work out the value of $y$ when $x=2.5 \quad y=2$
d) Use your graph to work out the value of $x$ when $y=4.5 \quad x=3.75$
2) a) Complete the table of values for $y=2-x$

| $x$ | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 3 | 2 | 1 | 0 | -1 | -2 |

b) Using the axes on the right, again, draw the graph of $y=2-x$

3) a) Complete the table of values for $y=1 / 2 x-1$
b) Draw the graph of $y=1 / 2 x-1$

| $x$ | -1 | 0 | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | $-1 \frac{1}{2}$ | -1 | $-\frac{1}{2}$ | 0 | $\frac{1}{2}$ | 1 |


c) Use your graph to find the value of $y$ when $x=3.5 \quad x=0.75$

1) Solve the following equations.
a) $2 x-7=11$

$$
x=9
$$

b) $5 x+3=43$

$$
x=8
$$

2) Solve the following equations.
a) $\frac{x}{5}+1=7$

$$
x=30
$$

b) $\frac{x}{2}-6=2.5$

$$
x=17
$$

3) Solve the following equations.
a) $2(4 x-1)=46$

$$
x=6
$$

b) $6(2 x+7)=48$

$$
x=0.5
$$

4) Solve the following equations.
a) $3\left(\frac{x}{7}+2\right)=6$

$$
x=0
$$

b) $2\left(\frac{5 x}{3}-1\right)=8$

$$
x=3
$$

## Subject of a Formula

1) Make $x$ the subject of the formula $w=5 x+2$

$$
x=\frac{w-2}{5}
$$

2) Make $x$ the subject of the formula $y=2 x-6$

$$
x=\frac{y+6}{2}
$$

3) Make $x$ the subject of the formula $2 w=3 y+\frac{x}{5}$

$$
x=5(2 w-3 y)
$$

4) Make $t$ the subject of the formula $a=2(b+3 t)+1$

$$
t=\frac{\frac{a-1}{2}+b}{3}
$$

5) Make $x$ the subject of the formula $y=5+\sqrt{x}$

$$
x=(y-5)^{2}
$$

6) Make $t$ the subject of the formula $w=x^{2}+t$

$$
t=w-x^{2}
$$

7) Make $n$ the subject of the formula $m=3 n^{2}-p$

$$
n=\sqrt{\frac{m+p}{3}}
$$

8) Make $q$ the subject of the formula $2\left(5 q^{2}+1\right)=c$
$q=\sqrt{\frac{\frac{c}{2}-1}{5}}$

## Rearranging Formulae

1) Make $c$ the subject of the formula.

$$
\begin{gathered}
a=b+c d \\
c=\frac{a-b}{d}
\end{gathered}
$$

2) Make $t$ the subject of the formula.

$$
\begin{aligned}
& u=v+2 t \\
& t=\frac{u-v}{2}
\end{aligned}
$$

3) Make $n$ the subject of the formula.
$M=3 n+5$

$$
n=\frac{M-5}{3}
$$

4) Make $z$ the subject of the formula.

$$
\begin{aligned}
& x=3 y+z \\
& z=x-3 y
\end{aligned}
$$

5) $r=5 s+3 t$
a) Make $t$ the subject of the formula. $\quad t=\frac{r-5 s}{3}$
b) Make $s$ the subject of the formula. $s=\frac{r-3 t}{5}$
6) Rearrange $y=3 x+1$ to make $x$ the subject.

$$
x=\frac{y-1}{3}
$$

7) Rearrange $y=\frac{1}{2} x+2$ to make $x$ the subject.

$$
x=2(y-2)
$$

8) Rearrange $y=\frac{1}{3} x+1$ to make $x$ the subject.
$x=3(y-1)$

## Rearranging Formulae

1) Make $c$ the subject of the formula.
$v=2 a+3 b+c \quad c=v-2 a-3 b$
2) Make $t$ the subject of the formula.
$A=\pi t+5 t \quad \dagger=\frac{A}{\pi+5} \quad A=\dagger(\pi+5)$
3) Make $s$ the subject of the formula.

$$
R-2 t=3 s+\pi s
$$

$R=3 s+\pi s+2 t s=\frac{\mathrm{R}-2 \dagger}{3+\pi}$

$$
R-2 t=s(3+\pi)
$$

4) $k=\frac{l}{m-l}$
a) Make $l$ the subject of the formula. $I=\frac{\mathrm{km}}{1+\mathrm{k}}$

$$
\begin{aligned}
k(m-I) & =1 \\
k m-k I & =1 \\
k m & =I+k I \\
k m & =I(1+k)
\end{aligned}
$$

b) Make $m$ the subject of the formula. $m=\frac{|+k|}{k}$
5) $A=\frac{k(x+5)}{3}$

Make $x$ the subject of the formula. $x=\frac{3 A-5 k}{k}$

$$
\begin{aligned}
3 A & =k(x+5) \\
3 A & =k x+5 k \\
3 A-5 k & =k x
\end{aligned}
$$

6) $R=\frac{u+v^{2}}{u+v}$

Make $u$ the subject of the formula. $u=\frac{v^{2}-R v}{R-1}$

$$
\begin{aligned}
R(u+v) & =u+v^{2} \\
R u+R v & =u+v^{2} \\
R u-u & =v^{2}-R v \\
u(R-1) & =v^{2}-R v
\end{aligned}
$$

7) $\frac{3 x+2}{5}=\frac{y}{10+y}$

Make $y$ the subject of the formula. $y=\frac{30 x+20}{3-3 x}$

$$
\begin{aligned}
& \quad(3 x+2)(10+y)=5 y \\
& 30 x+3 x y+20+2 y=5 y \\
& 30 x+20=5 y-3 x y-2 y \\
& 30 x+20=3 y-3 x y \\
& 30 x+20=y(3-3 x)
\end{aligned}
$$

8) $\sqrt{\frac{a-3}{5}}=4 b$

$$
\begin{aligned}
& \frac{a-3}{5}=16 b^{2} \\
& a-3=80 b^{2}
\end{aligned}
$$

Rearrange this formula to give $a$ in terms of $b, \quad a=80 b^{2}+3$
9) $S=2 \pi d \sqrt{h^{2}+d^{2}}$

Rearrange this formula to make $h$ the subject. $\quad h=\sqrt{\frac{S^{2}}{4 \pi^{2} d^{2}}-d^{2}}$

$$
\begin{aligned}
\frac{S}{2 \pi d} & =\sqrt{h^{2}+d^{2}} \\
\frac{S^{2}}{4 \pi^{2} d^{2}} & =h^{2}+d^{2}
\end{aligned}
$$

$$
\frac{S^{2}}{4 \pi^{2} d^{2}}-d^{2}=h^{2}
$$

## Simultaneous Equations

1) Solve

$$
\begin{aligned}
& 4 x+3 y=6 \\
& 5 x-3 y=71
\end{aligned} \quad x=3 \text { and } y=-2
$$

2) Solve

$$
\begin{aligned}
& 4 x+3 y=19 \\
& 3 x-5 y=7
\end{aligned} \quad x=4 \text { and } y=1
$$

3) Solve

$$
3 x+5 y=13
$$

$$
2 x+3 y=8
$$

$$
x=1 \text { and } y=2
$$

4) Solve

$$
\begin{aligned}
x+4 y & =5 \\
4 x-2 y & =11 \quad x=3 \text { and } y=0.5
\end{aligned}
$$

5) Solve

$$
\begin{aligned}
& 2 a+b=3 \\
& 4 a-5 b=20
\end{aligned} \quad a=2.5 \text { and } b=-2
$$

6) Solve

$$
\begin{aligned}
& 5 x+3 y=4 \\
& 3 x+4 y=9
\end{aligned} \quad x=-1 \text { and } y=3
$$

7) Solve

$$
\begin{aligned}
& 6 x-2 y=13 \\
& 2 x+3 y=-3
\end{aligned} \quad x=1.5 \text { and } y=-2
$$

8) Solve
$3 a-2 b=14$
$4 a+3 b=13$
$a=4$ and $b=-1$
9) Solve

$$
\begin{aligned}
& 5 x+4 y=5 \\
& 2 x+7 y=29
\end{aligned} \quad x=-3 \text { and } y=5
$$

10) Solve

$$
\begin{aligned}
& 6 x-4 y=39 \\
& 2 x+y=6
\end{aligned} \quad x=4.5 \text { and } y=-3
$$

