The Mathematics department at the College is looking forward to welcoming you in September. Further Mathematics examines Maths topics covered at A-level in greater depth and introduces many new topics which are not covered on the ordinary A-level Mathematics syllabus, such as Complex Numbers, Network Algorithms and many more. Further Maths is excellent preparation for students who aim to study Mathematics at University, or other Maths-rich subjects such as Physics and Engineering. It will also reinforce and consolidate your standard A-level Maths work, helping you to achieve the best possible grades. We have put together a range of resources for you to explore over the next few months, which introduce you to some of the new topics you will meet.

**Overview**

Minimum Spanning Trees and Kruskal’s algorithm

This activity is designed to introduce you to Discrete Maths, which is an area of Mathematics that is only covered in the Further Maths syllabus.

Below is an example of a connected weighted graph, sometimes referred to as a network.

There are 5 vertices labelled 0 – 4, and 7 edges (connecting lines) with numerical values referred to as weights. This could, for example, represent five towns with the edges representing the roads linking them and then numerical values being the distances.

Suppose the council wished to add cycle lanes to some of these roads, so that all the towns are connected by cycle lanes but minimising the length of cycle lanes required.

Which roads would you add cycle lanes on?

Decide before reading the next paragraph.

The solution to this problem is called a **minimum spanning tree (illustrated to the left)**. It is the group of edges with the minimum total weight that connects all the vertices. A minimum spanning tree will never contain any cycle (loops) as this would always involve having unnecessary edges.

Finding minimum spanning trees on larger and more complex networks is an important skill in Discrete Maths. One common approach to this is called **Kruskal’s algorithm**.

There is a video example for Kruskal’s algorithm and a booklet to accompany it with some questions to try here: [https://www.bsfc.ac.uk/maths/furthermathsprep](https://www.bsfc.ac.uk/maths/furthermathsprep)

For more information on Kruskal’s algorithm, and how it can link to computer programming, there is an interesting overview here: [https://www.hackerearth.com/blog/developers/kruskals-minimum-spanning-tree-algorithm-example/](https://www.hackerearth.com/blog/developers/kruskals-minimum-spanning-tree-algorithm-example/)
Complex numbers and more

Complex numbers are an important element of A-level Further Mathematics. Complex numbers are used to solve equations such as $x^2 = -1$. In complex numbers, the imaginary number $i$ is defined as $\sqrt{-1}$. Therefore, the solution to $x^2 = -1$ would be $x = \pm i$.

Here are some videos which introduce the fascinating world of complex numbers:

Introduction to complex numbers - https://www.youtube.com/watch?v=hqr1DtXXHpY&list=PLmdFyQYShrjfgDBFTTXYffumKEY3mcpz1

Complex numbers are awesome, from Numberphile - https://www.youtube.com/watch?v=-LJuqR6nz_Q

This is a nice video showing how the roots of $x^2 + 1 = 0$ can be shown graphically in 3D - https://www.youtube.com/watch?v=T647CGsuOVU

This video is quite advanced - it explores one of the most famous results in Mathematics known as Euler’s Identity, which utilises complex numbers - https://www.youtube.com/watch?v=sKtloBAuP74

Here are some other interesting videos on other areas of Maths relevant to Further Maths:

Introduces game theory via the ‘Prisoner’s Dilemma’ - https://www.youtube.com/watch?v=t9Lo2fgxWHw

How curves are produced using Polar co-ordinates - https://nrich.maths.org/2755

Online Resources

The advanced mathematics support programme have a wide range of teaching videos on Further Maths topics here: https://amsp.org.uk/resource/fm-videos

These videos give you an opportunity to explore a range of topic areas you will meet on the Further Mathematics course.

Course Specifications

A Level - Download AQA Further Maths Specification