

Mathematics Department Key Stage 4

Philosophy

The aims and objectives of the Mathematics scheme of work are to enable students to:

- develop fluent knowledge, skills and understanding of mathematical methods and concepts
- acquire, select and apply mathematical techniques to solve problems
- reason mathematically, make deductions and inferences, and draw conclusions
- comprehend, interpret and communicate mathematical information in a variety of forms appropriate to the information and context.

These are achieved by building on the studies in Key Stage 3; either at a foundation or higher level. At that start of year 10, students will be placed on either the foundation or higher pathway. The pathway that they follow will be chosen on the basis of what is best for each individual student, in order to maximise their learning and outcomes.

Subject content by pathway and by term (the order that topics are taught in may vary)

Foundation Year 10 Autumn

- Use and interpret algebraic manipulation
- Substitute numerical values into formulae and expressions
- Simplify and manipulate algebraic expressions (not surds) including collecting like terms, expanding and factorising
- Understand and use algebraic vocabulary
- Simplify and manipulate algebraic expressions including expanding products of two binomials, factorising quadratic expressions of the form x^2+bx+c including the difference of two squares
- Solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation); find approximate solutions using a graph
- Solve quadratic equations algebraically by factorising; find approximate solutions using a graph
- Define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; compare two quantities using percentages; work with percentages greater than 100%; solve problems using percentage change, including percentage increase/decrease and original value problems, and simple interest including financial mathematics
- Set up, solve and interpret the answers in growth and decay problems, including compound interest
- **Y10 Mock Exam 1**
- Number - applying the four operations, including formal written methods, to integers, decimals and fractions (proper and improper) and mixed numbers; understand and use place value, BIDMAS
- Use the concepts of vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, HCF, LCM, prime factorisation
- Use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5
- Calculate with roots, and with integer indices

- Simplify and manipulate algebraic expressions by: simplifying expressions involving sums, products and powers, including the laws of indices
- Calculate with and interpret standard form
- Estimate answers; check calculations using approximation & estimation, including answers obtained using technology
- Round numbers and measures to an appropriate degree of accuracy (eg to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding
- Apply and interpret limits of accuracy

Foundation Year 10 Spring

- Ratio: ratio notation; divide a quantity into a given ratio; apply to real-life problems; identify & work with fractions in ratio problems; express a multiplicative relationship between two quantities as a ratio or a fraction; understand and use proportion as equality of ratios
- Infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
- Use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing
- Interpret & construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use
- Interpret, analyse and compare the distributions of data sets from univariate empirical distributions through: appropriate graphical representation involving discrete, continuous & grouped data; appropriate measures of central tendency (median, mean, mode & modal class) and spread (range, including consideration of outliers)
- Apply statistics to describe a population
- **Y10 Mock Exam 2**
- Work with coordinates in all four quadrants
- Plot graphs of equations that correspond to straight-line graphs in the co-ordinate plane; use the form $y = mx + c$ to identify parallel and perpendicular lines; find the equation of the line through two given points, or through one point with a given gradient
- Identify and interpret gradients and intercepts of linear functions graphically and algebraically
- Interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion
- Relate ratios to fractions and to linear functions
- Recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions and the reciprocal function
- Work interchangeably with terminating decimals & their corresponding fractions; FDP conversion
- Probability: relative and expected frequency; probability scale; exhaustive probabilities; mutually exclusive events; understand that empirical unbiased samples tend towards theoretical probability distributions with increasing sample size; theoretical possibility spaces; apply systematic listing strategies

Foundation Year 10 Summer

- 4 operations with decimals
- Use standard units of measure (metric and imperial, and compound measures)
- Identify and apply circle definitions & properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector & segment
- Know the formulae: circumference of a circle; area of a circle; calculate perimeters of 2D shapes, including circles; areas of circles and composite shapes (including calculating with exact multiples of pi)
- Calculate arc lengths, angles and areas of sectors of circles
- Identify properties of the faces, surfaces, edges & vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones & spheres
- Know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)
- Surface area & volume of spheres, pyramids, cones & composite solids
- **Y10 Mock Exam 3**
- Use conventional terms & notation for points/lines/vertices etc.
- Derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite & rhombus; and triangles and other plane figures using appropriate language
- Apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angle; understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a polygon
- Solve geometrical problems on coordinate axes
- Construct and interpret plans and elevations of 3D shapes
- Measure line segments & angles in geometric figures, including interpreting maps and scale drawings and use of bearings
- Use scale factors, scale drawings and maps
- Use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)
- Apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles & sides, including Pythagoras' theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs

Foundation Year 11 Autumn

- Know the formulae for: Pythagoras' theorem and the trigonometric ratios; and apply them to find angles and lengths in right-angled triangles in two dimensional figures
- Know the exact values of $\sin \theta$ and $\cos \theta$ for $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° ; know the exact value of $\tan \theta$ for $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$
- Rearranging formulae; know the difference between an equation and an identity argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments
- Solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation); find approximate solutions using a graph
- Translate simple situations or procedures into algebraic expressions or formulae and interpret simple expressions as functions with inputs and outputs; derive an equation (or two simultaneous equations), solve the equation and interpret the solution

- Solve two simultaneous equations in two variables (linear/linear) algebraically; find approximate solutions using a graph
- Simplify and manipulate algebraic expressions (including those involving surds): collecting like terms with surds; multiplying a single term with a surd; taking out common factors over a bracket involving surds; expanding products of two brackets with surds; factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares; simplifying expressions involving sums, products and powers, including the laws of indices.
- Identify and interpret roots, intercepts, and turning points of quadratic functions graphically; deduce roots algebraically
- Plot and interpret graphs (including reciprocal graphs) and graphs of non-standard functions in real contexts to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- Solve linear inequalities in one variable; represent the solution set on a number line
- Generate terms of a sequence from either a term-to-term or position-to-term rule
- Recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci-type sequences, quadratic sequences and simple geometrical progressions (r^n where n is an integer and r is a rational number > 0)
- Deduce expressions to calculate the n th term of linear sequences
- Number: all operations with fractions
- Identify and work with fractions in ratio problems
- Calculate the probability of independent and dependent combined events, including using tree diagrams, frequency trees, Venns, tables, and know the underlying assumptions
- **Y11 Mock Exam 1**

Foundation Year 11 Spring

- Identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional scale factors)
- Apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles & sides, including Pythagoras' theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs
- Describe translations as 2D vectors
- Apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors
- Compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors
- Apply the concepts of congruence and similarity, including the relationships between lengths in similar figures; use scale factors
- Change freely between related standard units and compound units in numerical and algebraic contexts
- Use compound units such as speed, rates of pay, unit pricing, density and pressure
- Solve problems involving direct and inverse proportion, including graphical and algebraic representations
- Understand that x is inversely proportional to y is equivalent to x is proportional to $1/y$; interpret equations that describe direct and inverse proportion
- Revision: number: all operations with fractions/decimals

- Identify and work with fractions in ratio problems
- Interpret fractions and percentages as operators
- Use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from / at a given point, bisecting a given angle); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to a line
- **Mock papers, packs, examination practice and preparation**

Foundation Year 11 Summer

- **Mock papers, packs, examination practice and preparation**

Higher Year 10 Autumn

- Revision: algebraic manipulation; substitution; solving equations; expanding and factorising quadratics; rearranging formulae; know the difference between an equation & an identity
- Solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation); find approximate solutions using a graph
- Solve two simultaneous equations in two variables (linear/linear or linear/quadratic) algebraically; find approximate solutions using a graph
- Translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- Solve quadratic equations algebraically by factorising, (including those that require rearrangement); find approximate solutions using a graph
- Revision: define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; compare two quantities using percentages; work with percentages greater than 100%; solve problems using percentage change, including percentage increase/decrease and original value problems, and simple interest including financial mathematics
- Set up, solve and interpret the answers in growth and decay problems, including compound interest, and work with general iterative processes
- **Y10 Mock Exam 1**
- Revision: number - applying the four operations, including formal written methods, to integers, decimals and fractions (proper and improper) and mixed numbers; understand and use place value, BIDMAS
- Revision: use the concepts of vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, HCF, LCM, prime factorisation
- Work interchangeably with terminating decimals and their corresponding fractions; interpret fractions and percentages as operators
- Express one quantity as a fraction of another, including when the fraction is greater than 1; calculate exactly with fractions
- Change recurring decimals into their corresponding fractions and vice versa
- Use positive integer powers and associated real roots (square, cube & higher), recognise powers of 2, 3, 4, 5; estimate powers and roots of any given positive number
- Index laws
- Calculate with roots, and with integer and fractional indices
- Calculate with and interpret standard form

- Estimate answers; check calculations using approximation & estimation; round numbers to an appropriate degree of accuracy; use inequality notation to specify simple error intervals due to truncation or rounding
- Apply and interpret limits of accuracy, including upper and lower bounds; calculations with bounds

Higher Year 10 Spring

- Ratio: ratio notation; divide a quantity into a given ratio; apply to real-life problems; identify & work with fractions in ratio problems; express a multiplicative relationship between two quantities as a ratio or a fraction; understand and use proportion as equality of ratios
- Infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
- Use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing
- Interpret & construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use
- Construct and interpret diagrams for grouped discrete and continuous data, ie histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use
- Interpret, analyse and compare the distributions of data sets from univariate empirical distributions through: appropriate graphical representation, including box plots; appropriate measure of spread, including quartiles and inter-quartile range
- Apply statistics to describe a population
- **Y10 Mock Exam 2**
- Work with coordinates in all four quadrants
- Plot graphs of equations that correspond to straight-line graphs in the co-ordinate plane; use the form $y = mx + c$ to identify parallel and perpendicular lines; find the equation of the line through two given points, or through one point with a given gradient
- Identify and interpret gradients and intercepts of linear functions graphically and algebraically
- Interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion
- Relate ratios to fractions and to linear functions
- Recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions and the reciprocal function, $y = 1/x$ with $x \neq 0$, exponential functions $y = k^x$ for positive values of k
- Sketch translations and reflections of a given function
- Probability: relative and expected frequency; probability scale; exhaustive probabilities; mutually exclusive events; understand that empirical unbiased samples tend towards theoretical probability distributions with increasing sample size; theoretical possibility spaces; enumerate sets and combinations of sets systematically using Venns to calculate independent, dependent & conditional probabilities; apply systematic listing strategies

Higher Year 10 Summer

- Use standard units of measure; know the formulae: circumference of a circle; area of a circle; calculate perimeters of 2D shapes, including circles; areas of circles and composite shapes (including calculating with exact multiples of pi)
- Identify and apply circle definitions & properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector & segment
- Calculate arc lengths, angles and areas of sectors of circles
- Identify properties of the faces, surfaces, edges & vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones & spheres
- Know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)
- Surface area & volume of spheres, pyramids, cones & composite solids
- Apply and prove the standard circle theorems concerning angles, radii, tangents & chords, and use them to prove related results
- Assessment revision and half term revision guidance given
- **Y10 Mock Exam 2**
- Apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angle; understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a polygons; solve geometrical problems on coordinate axes
- Derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite & rhombus; and triangles and other plane figures using appropriate language
- Construct and interpret plans and elevations of 3D shapes
- Measure line segments & angles in geometric figures, including interpreting maps and scale drawings and use of bearings; use scale factors, scale drawings and maps
- Use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)
- Apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles & sides, including Pythagoras' theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs

Higher Year 11 Autumn

- Know the formulae for: Pythagoras' theorem and the trigonometric ratios; and apply them to find angles and lengths in right-angled triangles and, where possible, general triangles in two and three dimensional figures
- Know and apply the sine rule and cosine rule to find unknown lengths and angles
- Know and apply $\text{area} = \frac{1}{2} ab \sin C$ to calculate the area, sides or angles of any triangle
- Know the exact values of $\sin \theta$ and $\cos \theta$ for $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$ and 90° ; know the exact value of $\tan \theta$ for $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$
- Recognise, sketch and interpret graphs of trigonometric functions (with arguments in degrees) $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size
- Rearranging formulae; know the difference between an equation and an identity argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments and proofs
- Where appropriate, interpret simple expressions as functions with inputs and outputs; interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function' (the use of formal function notation is expected)

- Calculate the probability of independent and dependent combined events, including using tree diagrams, frequency trees, Venns, tables, and know the underlying assumptions
- Calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams
- Simplify and manipulate algebraic expressions (including those involving surds and algebraic fractions): collecting like terms with surds; multiplying a single term with a surd; taking out common factors over a bracket involving surds; expanding products of two brackets with surds; factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares; simplifying expressions involving sums, products and powers, including the laws of indices.
- Calculate exactly with surds; simplify surd expressions involving squares(eg $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$) and rationalise denominators
- Simplify and manipulate algebraic expressions (including those involving surds and algebraic fractions) by: expanding products and factorising of two or more binomials; factorising quadratic expressions of the form $ax^2 + bx + c$
- Generate terms of a sequence from either a term-to-term or position-to-term rule
- Recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci-type sequences, quadratic sequences and simple geometrical progressions (r^n where n is an integer and r is a rational number > 0 or a surd) and other sequences
- Deduce expressions to calculate the n th term of linear and quadratic sequences
- Solve linear inequalities in one or two variable(s), and quadratic inequalities in one variable; represent the solution set on a number line, using set notation and on a graph
- Find approximate solutions to equations numerically using iteration
- Solve quadratic equations by using the quadratic formula and completing the square; find approximate solutions using a graph
- Identify and interpret roots, intercepts, and turning points of quadratic functions graphically; deduce roots algebraically and turning points by completing the square
- Plot and interpret graphs (including reciprocal graphs and exponential graphs) and graphs of non-standard functions in real contexts to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- Recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point
- **Y11 Mock Exam 1**

Higher Year 11 Spring

- Identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional and negative scale factors)
- Describe the changes and invariance achieved by combinations of rotations, reflections and translations
- Describe translations as 2D vectors
- Apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; use vectors to construct geometric arguments and proofs

- Compare lengths, areas and volumes (using scale factors); make links to similarity (including trigonometric ratios) and scale factors
- Apply the concepts of congruence and similarity, including the relationships between lengths, areas and volumes in similar figures; use scale factors
- Change freely between related standard units and compound units in numerical and algebraic contexts
- Use compound units such as speed, rates of pay, unit pricing, density and pressure
- Solve problems involving direct and inverse proportion, including graphical and algebraic representations
- Understand that x is inversely proportional to y is equivalent to x is proportional to $1/y$; interpret equations that describe direct and inverse proportion; construct and interpret equations that describe direct and inverse proportion
- Calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts (no calculus)
- Interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of average and instantaneous rate of change (gradients of chords and tangents) in numerical, algebraic and graphical contexts
- Use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from / at a given point, bisecting a given angle); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to a line
- **Mock papers, packs, examination practice and preparation**

Higher Year 11 Summer

- **Mock papers, packs, examination practice and preparation**