<u>Mathematics: Year 12 Transition Work</u>

There are ten sections for you to study. Each section covers a different skill set.

1. Substituting negatives
2. Adding and subtracting fractions
3. Laws of indices
4. Negative and fractional indices
5. Expanding single brackets
6. Factorising Expressions
7. Linear equation
8. Completing the square
9. Expanding double brackets
10. Factorising quadratic equations

The Initial Tests will be based on these skill sets.

You will work online and on paper.

<u>Online tasks</u>

On the next two pages the online tasks and instructions are given. **Read the instructions carefully, especially those underlined in the first paragraph.**

There are two categories of online task: tasks that you MUST complete and tasks we strongly recommend you to complete as a student wishing to study Maths A level. Each task requires you to complete an online homework and repeat it until you score 90%.

<u>Paper tasks</u>

The paper tasks are in this booklet. Most sections contain some notes on the skills, an Exercise A, an Exercise B and a Challenge task.

Exercise A contains questions very similar to those in the Initial Tests – you would be well advised to do these exercises.

Exercise B contains more demanding questions on the same topic – questions like these will not be in the Initial Tests, but they are good preparation for A level.

The Challenge tasks vary – most are reasonably easy to access, but more difficult to complete.

The answers to the Exercises A and B will be posted on the main school website - under the 6th form 'tab'.

You should mark any work you have done yourself.

If you complete any of the Challenge tasks, you may submit them to Mr Williams in September, but this is optional

www.mymaths.co.uk.
Username: chipping
Password: campden

After using the log in details above, use the menu system as described below to locate the Online Homework task you wish to do. **After** you have selected the Online Homework, a Welcome screen will appear**. You must now enter your personal log in details**. Failure to do this will mean that no evidence of your work will exist.

The menu is driven by clicking on **Number**, **Algebra**, **Shape** or **Data** on the left-hand side of the screen after you have logged in.

For each topic, where appropriate, the mymaths online homework has been separated into 2 sections:

- Online Homework exercises that you MUST complete – these prepare you for the initial tests
- Online Homework exercises that you SHOULD complete – these will not be tested, but are good preparation for A level

If you require support you should access the 'lessons' attached to each online homework.

You should ensure that you are getting at least 90% on each of the 'MUST' online homeworks and should complete them repeatedly until you reach this benchmark.

1. Directed Numbers:

You MUST complete:

- Number -> Counting and Place Value -> Negative Numbers 2
- Algebra -> Expressions and Formula -> Substitution 1
- Algebra -> Expressions and Formula -> Substitution 2

You SHOULD complete:

- There are no additional online homeworks for this section.

2. Adding and Subtracting Fractions:

You MUST complete:

- Number -> Fractions -> Adding Subtracting Fractions
- Number -> Fractions -> Mixed Numbers

You SHOULD complete:

- Algebra -> Algebraic Manipulation -> Cancelling Algebraic Fractions
- Algebra -> Algebraic Manipulation -> Adding Algebraic Fractions

3. Index Laws and 4. Negative and Fractional Indices:

You MUST complete:

- Number -> Powers and Roots -> Indices 1
- Number -> Powers and Roots -> Indices 2

You SHOULD complete:

- Number -> Powers and Roots -> Indices 3
- Number -> Powers and Roots -> Indices 4

5. Expanding Single Brackets and 6. Factorising Expressions:

You MUST complete:

- Algebra -> Algebraic Manipulation -> Single Brackets
- Algebra -> Algebraic Manipulation -> Factorising Linear

You SHOULD complete:

- There are no additional online homeworks for this section.

7. Solving Linear Equations:

You MUST complete:

- Algebra -> Equations – Linear -> Equations 2 – Multi-Step
- Algebra -> Equations – Linear -> Equations 3 – Both Sides
- Algebra -> Equations – Linear -> Equations 4 – Brackets

You SHOULD complete:

- Algebra -> Equations – Linear -> Equations 5 – Fractions

8. Completing the Square and the Quadratic Formula:

You MUST complete:

- Algebra –> Equations – Quadratic -> Completing the Square

You SHOULD complete:

- There are no additional online homeworks for this section.

9. Expanding Double Brackets:

You MUST complete:

- Algebra -> Algebraic Manipulation -> Brackets

You SHOULD complete:

- There are no additional online homeworks for this section.

10. Factorising Quadratic Equations:

You MUST complete:

- Algebra -> Algebraic Manipulation -> Factorising Quadratics 1
- Algebra -> Equations – Quadratic -> Quadratic Equations 1

You SHOULD complete:

- Algebra -> Algebraic Manipulation -> Factorising Quadratics 2
- Algebra -> Equations – Quadratic -> Quadratic Equations 2

1. <u>Directed Numbers</u> (positive and negative numbers)

Rules for multiplying and dividing directed numbers:

Positive x Positive = POSITIVE	Positive ÷ Positive = POSITIVE	
Negative x Negative = POSITIVE	Negative ÷ Negative = POSITIVE	
Positive x Negative = NEGATIVE	Positive ÷ Negative = NEGATIVE	
Negative x Positive = NEGATIVE	Negative ÷ Positive = NEGATIVE	

When multiplying or dividing two numbers:

if the numbers have the same sign, the answer is POSITIVE

if the numbers have different signs, the answer is NEGATIVE

We also need to remember this when adding or subtracting directed numbers.

9 + (+ 3) = 9 + 3 = 12
7 – (– 4) = 7 + 4 = 11
6 + (– 2) = 6 – 2 = 4
8 – (+ 13) = 8 – 13 = – 5

Squaring a negative will give a POSITIVE answer: $(-4)^2 = (-4) \times (-4) = 16$

Cubing a negative will give a NEGATIVE answer: $(-4)^3 = (-4)^2 \times (-4) = 16 \times (-4) = -64$

In general, when a negative has an even power, the answer is POSITIVE: $(-3)^4 = 81$

When a negative has an odd power, the answer is NEGATIVE: $(-1)^9 = -1$

Examples: 3 x (– 4) = – 12 (– 2) x (– 7) = 14 (– 20) ÷ 5 = – 4 $(-7)^2 = 49$ $(-2)^5 = -32$

$-4(-3)^2 = -4 \times 9 = -36$ 7 + (– 2) – (– 6) = 7 – 2 + 6 = 11 $(-5)^2 - (-6)^2 = 25 - 36 = -11$

Students sometimes think that the rules above should be used in straightforward addition and subtraction questions.
For example, they claim that –2 – 8 should have a positive answer "because *two negatives make a positive* ". This is wrong because we are **no**t multiplying or dividing two negatives, and we do **not** have – (– 8) for example.
The answer is –2 – 8 = – 10 (start at –2 and go down 8).

<u>Exercise 1A</u> (Do these without a calculator)

Using $a = -3$, $b = 6$, $c = -12$, find the numerical value of the following expressions.

1) $\frac{c}{a}$ 2) ab 3) $a^2 + b^2$ 4) $\frac{c^2}{2}$

5) $2a^3$ 6) $b - a$ 7) $a + c$ 8) $a(b + c)$

<u>Exercise 1B</u>
x and y are **different** numbers. x and y may be positive or negative, but neither is zero. x and y may have the same signs or opposite signs.
For each statement write ALWAYS TRUE, SOMETIMES TRUE or NEVER TRUE as appropriate.

8) xy is negative 9) x^2 is positive 10) y^3 is negative 11) $y^2 - x^2$ is negative
12) $x^2 + y^2$ is negative 13) xy and yx have the same sign 14) $x + y = 0$
15) $x - y$ and $y - x$ have opposite signs 16) $\frac{x}{y}$ and $\frac{y}{x}$ have opposite signs

<u>Challenge</u>

Given that n is a positive integer, $(-1)^n = 1$ when n is even and $(-1)^n = -1$ when n is odd.

Write an expression that equals 1 when n is odd and equals – 1 when n is even.

2. Adding and Subtracting Fractions

To add or subtract fractions:

- ✓ find the lowest common denominator of the fractions
- ✓ replace each fraction with an equivalent fraction that has the common denominator
- ✓ add or subtract
- ✓ simplify if possible

(At A-level improper fractions – top-heavy fractions – are usually preferable to mixed numbers.)

Examples:

$$\frac{2}{5}+\frac{3}{4}$$
$$=\frac{2\times 4}{5\times 4}+\frac{3\times 5}{4\times 5}$$
$$=\frac{8}{20}+\frac{15}{20}$$
$$=\frac{23}{20}$$

In arithmetic, we often do this step in our head and do not write it down. (If this was algebra, we would often write it down.)

The fractions have been replaced by equivalent fractions with the same denominator.

The first answer does not simplify, but the second does.

$$\frac{11}{15}-\frac{9}{10}$$
$$=\frac{11\times 2}{15\times 2}-\frac{9\times 3}{10\times 3}$$
$$=\frac{22}{30}-\frac{27}{30}$$
$$=-\frac{5}{30}$$
$$=-\frac{1}{6}$$

Simple algebraic examples:

$$\frac{2x}{5}+\frac{3x}{4}$$
$$=\frac{2x\times 4}{5\times 4}+\frac{3x\times 5}{4\times 5}$$
$$=\frac{8x}{20}+\frac{15x}{20}$$
$$=\frac{23x}{20}$$

Compare each algebraic example with the arithmetic example above it. You can see that exactly the same strategies are used.

$$\frac{11}{x+2}-\frac{9}{x-3}$$
$$=\frac{11(x-3)}{(x+2)(x-3)}-\frac{9(x+2)}{(x-3)(x+2)}$$
$$=\frac{11x-33}{(x+2)(x-3)}-\frac{9x+18}{(x-3)(x+2)}$$
$$=\frac{2x-51}{(x+2)(x-3)}$$

Exercise 2A (Do these without a calculator)

1) $\frac{1}{5}+\frac{1}{3}$ 2) $\frac{3}{4}+\frac{2}{7}$ 3) $\frac{4}{9}-\frac{1}{6}$ 4) $\frac{5}{12}+\frac{1}{6}-\frac{3}{4}$ 5) $\frac{5}{3}-\left(\frac{1}{4}-\frac{3}{5}\right)$

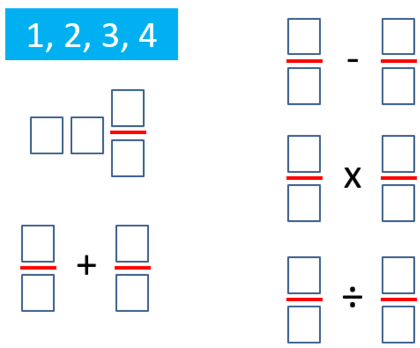
Exercise 2B

6) $\frac{x}{5}+\frac{x}{3}$ 2) $\frac{3}{x+2}+\frac{2}{x-3}$ 3) $\frac{4}{x+4}-\frac{1}{x+1}$ 4) $\frac{5}{6x}+\frac{1}{3x}-\frac{3}{2x}$ 5) $\frac{5}{x^2}-\left(\frac{x}{4}-\frac{3}{5x}\right)$

Challenge

Place one digit into each space to make the largest possible answer and the smallest possible answer. All four digits should be used once each in each of your solutions.

NOTE: you are only allowed to make proper fractions when placing the digits, i.e. the numerator must be smaller than the denominator, but the answers you create may be improper fractions.



3. <u>The rules of indices</u>

RULE	NUMERICAL EXAMPLE	ALGEBRAIC EXAMPLE
1) $a^m \times a^n = a^{m+n}$	$3^4 \times 3^5 = 3^9$	$a^2 \times a^4 = 6$
2) $a^m \div a^n = a^{m-n}$	$3^8 \div 3^6 = 3^2$	$w^{11} \div w^6 = w^5$
3) $(a^m)^n$	$(3^2)^5 = 3^{10}$	$(p^3)^5 = p^{15}$

In each of the rules a is called **the base**. In the numerical examples, **the base** is 3 each time.

In words the three rules are:

1) For a multiplication, add the powers provided the bases are the same.

2) For a division, subtract second power from the first provided the bases are the same.

3) For a power of a power, multiply the powers.

REMEMBER: These rules are used for indices (powers) only

$6a \times 2a^5 = 12a^6$ [<u>multiply</u> the numbers, use rule (1) for the powers of a – remember: a is a^1]

$32w^3 \div 8w^5 = 4w^{-2}$ [<u>divide</u> the numbers, use rule (2) for the powers of w]

$(2p^4)^3 = 8p^{12}$ [<u>cube</u> the 2, use rule (3) for the power of p]

What if there is more than one base?

$3a^4b^3 \times 2a^3b^2 = 6a^7b^5$ [<u>multiply</u> the numbers, use rule (1) for base a, use rule (1) for base b]

$20v^6w^3 \div 4v^4w^5x^3 = 5v^2w^{-2}x^{-3}$

[<u>divide</u> the numbers, use rule (2) for base v, use rule (2) for base w, use rule (2) for base x

– for base x, we have to subtract 3 so the power is –3]

<u>Exercise 3A</u>

Simplify the following:

1) $5^6 \times 5^2$	2) $d^3 \times d^4$	3) $2x^3 \times 7x^5$	4) $3y^{-5} \times 5y^6$
5) $5^6 \div 5^2$	6) $d^3 \div d^4$	7) $12x^{11} \div 3x^9$	8) $6y^7 \div 6y$
9) $(5^6)^2$	10) $(d^3)^4$	11) $(3x^5)^2$	12) $(-2x^5)^3$
13) $7a^5b^3 \times 2a^2b$	14) $24v^8w^6 \div 4v^3w^4$	15) $\frac{4c^5d^8 \times 6c^2d^{-2}}{12c^3d \times c^3d^2}$	16) $\frac{(2x^3)^4}{(4x^5)^2}$

<u>Exercise 3B</u>

Solve these two equations: 1) $\frac{20x^4y^2z^3}{7xy^5} \times \frac{14y^3}{40x^2z^3} = 5$ 2) $\frac{48x^5y^2}{12z^3} \div \frac{16x^2y^2}{z^3} = 2$

<u>Challenge</u>

1) Find values for **a** and **b** that fit these relationships.

$a^b - b^a = 1$

$a^b - b^a = 7$

$5^a - 2^b = 3^2$

$7^a - 10^b = 3^6$

$2^a - 2^b = 64$

The Challenge questions in (1) **cannot** be solved using the laws of indices. Some of the questions in (2) may be solved using these laws or in other ways.

Hint: $2^{n+1} + 2^n = 2^1 \times 2^n + 2^n = 3 \times 2^n$

2) Find a value for **n** for each of these sums

$2^{n+1} + 2^n = 96$

$2^{n+2} + 2^n = 640$

$3^n + 3^n + 3^n = 3^{12}$

$2^n + 3^n + 4^n = 7^3 + 10$

$6^n + 6^n + 6^n = 1296$

4. Negative and fractional indices

Zero index $\quad \boldsymbol{a^0 = 1}$ for any value of a except 0

Negative indices

An index (power) of –1 means the reciprocal of the base: $\boldsymbol{a^{-1} = \frac{1}{a}}$

This result can be extended to more general negative powers: $\boldsymbol{a^{-n} = \frac{1}{a^n}}$

Fractional indices

Fractional indices correspond to roots: $a^{\frac{1}{2}} = \sqrt{a} \quad a^{\frac{1}{3}} = \sqrt[3]{a} \quad a^{\frac{1}{4}} = \sqrt[4]{a}$

In general, $a^{\frac{1}{n}} = \sqrt[n]{a}$

When the denominator of the index is not 1, we may use this rule: $a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \sqrt[n]{a}^m$

Examples:

$6^0 = 1 \qquad \left(\frac{2}{5}\right)^0 = 1 \qquad (-\ 3.125)^0 = 1$

$6^{-1} = \frac{1}{6} \qquad \left(\frac{2}{5}\right)^{-1} = \frac{5}{2}$ [Find the reciprocal of a fraction by turning it upside down]

$6^{-2} = \frac{1}{6^2} = \frac{1}{36} \qquad \left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^3 = \frac{125}{8}$

$25^{\frac{1}{2}} = \sqrt{25} = 5 \qquad 8^{\frac{1}{3}} = \sqrt[3]{8} = 2 \qquad 10000^{\frac{1}{4}} = \sqrt[4]{10000} = 10$

$25^{\frac{3}{2}} = \sqrt{25}^3 = 5^3 = 125 \qquad 8^{\frac{2}{3}} = \sqrt[3]{8}^2 = 2^2 = 4$

In this last example, the index is a negative fraction:

$$1000^{-\frac{4}{3}} = (1000^{-1})^{\frac{4}{3}} = \left(\frac{1}{1000}\right)^{\frac{4}{3}} = \left(\sqrt[3]{\frac{1}{1000}}\right)^4 = \left(\frac{1}{10}\right)^4 = \frac{1}{10000}$$

Exercise 3A

Find the value of the following, expressing some answers as fractions where appropriate:

1) 2^{-1} 2) 4^0 3) 5^{-2} 4) $\left(\frac{3}{5}\right)^{-1}$ 5) $\left(\frac{4}{3}\right)^{-2}$ 6) $\left(\frac{1}{5}\right)^0$ 7) $\left(\frac{1}{2}\right)^{-5}$

8) $49^{\frac{1}{2}}$ 9) $1000^{\frac{1}{3}}$ 10) $64^{\frac{2}{3}}$ 11) $4^{\frac{5}{2}}$ 12) $36^{-\frac{3}{2}}$ 13) $\left(\frac{4}{25}\right)^{-\frac{1}{2}}$

There is no Exercise 3B.

Challenge

1) What is the units digit for the number 123^{456} ? 2) What number does this equal? $((-4^{-3})^{-2})^{-1}$

2) Without using any calculating aids, show that $7^{\frac{1}{2}} + 7^{\frac{1}{3}} + 7^{\frac{1}{4}} < 7$ [Hint: 3 + 2 + 2 = 7 and 3 = $9^{\frac{1}{2}}$]

5. Expanding Single Brackets

To expand a single bracket, we multiply each term inside the bracket by the number or the expression on the outside:

$4(2x + 5y) = 4(2x) + 4(5y) = 8x + 20y$

$-5(3x - 2) = (-5)(3x) + \ (-5)(-2) = -15\text{x} + 10$

[Remember to use the rules for multiplying directed numbers]

$3x(x - 4) = 3x(x) + 3x(-4) = 3x^2 - 12x$

$xy(x + 2y) = xy(x) + xy(2y) = x^2y + 2xy^2$

We may need to simplify by collecting like terms after we have expanded:

$10 - 2(x - 3) = 10 - 2x + 6 = 16 - 2x$

$4(2x - y) + 3(x + 2y) = 8x - 2y + 3x + 6y = 11x + 4y$

Exercise 5A

Expand the brackets and simplify where possible:

1) $3(2x + 5)$ 2) $-2(a - 5)$ 3) $4x(2x - 1)$ 4) $5b(a + b + 3)$

5) $xy(3x + 5y)$ 6) $6 - 3(a + 2)$ 7) $3(2x - y) + 4(x - 2y)$ 8) $5(a + b) - 4(3a - 2b)$

Exercise 5B

9) $5(2x - 3y) + 4x(x - 2y) - 3y(4x + 7y) - 2(y - 4x) + (y - x)$

Challenge

Expand and simplify

i) $(a + b) - 2(a - b)$

ii) $(a + b) - 2(a - b) + 3(a + b) - 4(a - b)$

iii) $(a + b) - 2(a - b) + 3(a + b) - 4(a - b) + \ 5(a + b) - 6(a - b)$

iv) $(a + b) - 2(a - b) + 3(a + b) - 4(a - b) + \ 5(a + b) - 6(a - b) + 7(a + b) - 8(a - b)$

v) Use these answers to suggest the expansion of

$$(a + b) - 2(a - b) + 3(a + b) - 4(a - b) + \dots\dots\dots\dots\dots + 19(a + b) - 20(a - b)$$

vi) Write down the expansion, in terms of *n*, of

$$(a + b) - 2(a - b) + 3(a + b) - 4(a - b) + \dots\dots\dots\dots\dots + (n - 1)(a + b) - n(a - b)$$

6. Factorising Expressions

Here are some examples. We start by finding the highest common factor.

$15x + 10 = 5(3x + 2)$ $3x^2 - 2xy = x(3x - 2y)$ $12x^3y + 8x^2y^3 = 4x^2y(3x + 2y^2)$

- 5 is a common factor of 15 and 10.
- Factorise by taking 5 outside a bracket.
- Divide $15x$ and 10 by 5 to find what remains inside the bracket.

- x is a common factor of x^2 and xy
- Factorise by taking x outside a bracket.
- Divide $3x^2$ and $-2xy$ by x to find what remains inside the bracket.

- 4 is the highest common factor of 12.
- The highest power of x found in **both** terms is x^2 and the highest power of y found in **both** terms is y.
- Factorise by taking $4x^2y$ outside a bracket.
- Divide $12x^3y$ and $8x^2y^3$ by $4x^2y$ to find what remains inside the bracket.

Make sure that you have factorised an expression fully. Look at these examples of factorising $12a^3b^2 + 18a^2b^3$.

$2a^2b^2(6a + 9b)$ is not a fully factorised answer. The terms in the bracket still contain the common factor 3.

$6ab(2a^2b + 3ab^2)$ is not a fully factorised answer. The terms in the bracket still contain the common factor ab.

$6a^2b^2(2a + 3b)$ is the **fully factorised** version because $6a^2b^2$ is the highest common factor of $12a^3b^2$ and $18a^2b^3$ – the bracket no longer contains a common factor.

If taking a negative factor outside the bracket, be careful about positive/negative signs:

$-6pq + 4q^2 = -2p(3q - 2q)$ [Remember the rules for multiplying and dividing directed numbers.]

Sometimes, when simplifying expressions, we may need to expand and collect like terms before factorising:

$7d(2c + d) - 3d^2 = 14cd + 7d^2 - 3d^2 = 14cd + 4d^2 = 2d(7c + 2d)$

Exercise 6A

Factorise fully the following expressions.

1) $28x + 21$ 2) $6 - 12x$ 3) $-10x + 15y$ 4) $a^2 - 3a$ 5) $9ab + 6b^2$ 6) $5b^3 + 2b^2$

7) $6p^3 - 15p^2$ 8) $20p^3 + 12p^4 + 8p^5$ 9) $22k^3m + 33km^3$ 10) $35k^2m - 14k^4m^2$

Simplify these expressions, giving your answer in a fully factorised form.

11) $4d(2c - d) + 6d^2$ 12) $7c(2c + d) - 3c^2$

Sometimes the common factor may actually be a bracket. For example, factorise $3x(x + 2) - 2(x + 2)$.

The bracket $(x + 2)$ is a common factor, so $3x(x + 2) - 2(x + 2) = (x + 2)(3x - 2)$ or $(3x - 2)(x + 2)$

Exercise 6B

Factorise: 1) $2y(y - 1) + 3(y - 1)$ 2) $4x(x + 5) + 3(x + 5)$ 3) $3t(2t + 1) - (2t + 1)$

Challenge

1) Prove algebraically that the sum of five consecutive integers is equal to five times the middle integer.

2) Prove algebraically that the sum of seven consecutive multiples of 3 is equal to seven times the middle integer.

7. Solving Linear Equations

When solving an equation, you must remember that whatever you do to one side must also be done to the other. You are allowed to:

- add the same amount to both sides
- subtract the same amount from each side
- multiply the whole of each side by the same amount
- divide the whole of each side by the same amount.

If the equation has unknowns on both sides, you should collect all the letters onto the same side of the equation.

If the equation contains brackets, you should start by expanding the brackets.

A linear equation is an equation that contains numbers and terms in x. A linear equation does not contain any x^2 or x^3 terms.

Examples

1) Solve the equation $6x + 7 = 5 - 2x$.

Step 1: Begin by adding $2x$ to both sides $8x + 7 = 5$

(this collects all the x terms together on the same side)

Step 2: Subtract 7 from each side: $8x = -2$

Step 3: Divide each side by 8: $x = -\frac{1}{4}$

2) Solve the equation $2(3x - 2) = 20 - 3(x + 2)$

Step 1: Multiply out the brackets: $6x - 4 = 20 - 3x - 6$

(taking care with the negative signs)

Step 2: Simplify the right-hand side: $6x - 4 = 14 - 3x$

Step 3: Add 3x to each side: $9x - 4 = 14$

Step 4: Add 4 to each side: $9x = 18$

Step 5: Divide each side by 9: $x = 2$

When an equation contains a fraction, the first step is usually to multiply through by the denominator of the fraction. This ensures that there are no fractions in the equation.

When an equation contains two fractions, you can multiply by the lowest common denominator. This will then remove both fractions.

Examples

3) Solve the equation $\frac{y}{2} + 5 = 11$

Step 1: Multiply through by 2 (the denominator in the fraction): $y + 10 = 22$

Step 2: Subtract 10 from both sides: $y = 12$

4) Solve the equation $\frac{x+1}{4} + \frac{x+2}{5} = 2$

Step 1: Find the lowest common denominator: the smallest number that both 4 and 5 divide into is 20.

Step 2: Multiply both sides by the lowest common denominator $\frac{20(x+1)}{4}+\frac{20(x+2)}{5}=40$

Step 3: Simplify the left-hand side: $\frac{\overset{5}{\cancel{20}}(x+1)}{\cancel{4}}+\frac{\overset{4}{\cancel{20}}(x+2)}{\cancel{5}}=40$

$5(x + 1) + 4(x + 2) = 40$

Step 4: Multiply out the brackets: $5x + 5 + 4x + 8 = 40$

Step 5: Simplify the equation: $9x + 13 = 40$

Step 6: Subtract 13 from both sides: $9x = 27$

Step 7: Divide both sides by 9: $x = 3$

Exercise 7A

Solve the following equations:

1) $3x - 5 = 7$ 2) $2x + 19 = 5$ 3) $11 + 3x = 8 - 2x$ 4) $7x + 2 = 4x - 5$

5) $4(2 - x) = 3(x - 9)$ 6) $14 - 3(2x + 3) = 2$ 7) $\frac{1}{2}(x+3)=5$ 8) $\frac{1-x}{4}=\frac{2x-5}{3}$

9) $\frac{y}{4}+3=5-\frac{y}{3}$ 10) $\frac{y-1}{2}+\frac{y+1}{3}=\frac{2y+5}{6}$

Exercise 7B

1) $\frac{12}{x+2}=\frac{9}{x+1}$ 2) $\frac{4}{2x+1}=\frac{3}{2x-1}$ 3) $2-\frac{5}{x}=\frac{10}{x}-1$

Challenge

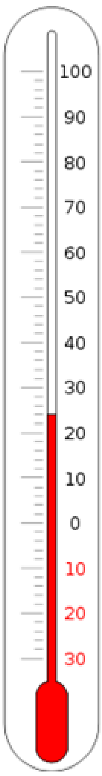
Temperature is often measured in degrees Celsius, $^\circ C$, or degrees Fahrenheit, $^\circ F$.

The freezing point of water is 0°C and 32°F.
The boiling point of water is 100°C and 212°F.

Is there a temperature at which Celsius and Fahrenheit readings are the same?

Can you describe a way of converting Fahrenheit readings into Celsius?

Can you describe a way of converting Celsius readings into Fahrenheit?



An extension challenge:

Scientists often use the Kelvin scale of temperature, where the freezing point of water is $273.15^\circ K$ and the boiling point of water is $373.15^\circ K$.

Is there a temperature at which Kelvin and Fahrenheit readings are the same?
Is there a temperature at which Kelvin and Celsius readings are the same?

Can you describe ways of converting Kelvin readings into Fahrenheit and Celsius readings?

8. Completing the Square

Sometime it is convenient to write quadratic expressions like $x^2+8x-20$ and $x^2-6x+10$ in a different form. For example, $x^2-6x+10$ may be written as $(x-3)^2+1$.
This is perfectly acceptable as $(x-3)^2+1=(x-3)(x-3)+1=x^2-3x-3x+9+1=x^2-6x+10$

$(x-3)^2+1$ is in completed square form. Completed square form can help us to work out the lowest point on the quadratic curve if we need to sketch it or to solve a quadratic equation.

In general quadratic expressions are written ax^2+bx+c. The value of a may be any number except zero**, but in this booklet we will always use $a=1$ so our quadratic expressions will be x^2+bx+c.
[** If $a=0$, we have $0x^2+bx+c=bx+c$ which is **not** quadratic.]

We can write completed square form by using this formula:

$$x^2+bx+c=\left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2+c$$

Examples:

a) Write $x^2+8x-20$ in the form $(x+q)^2+r$ (In this example b is 8 and c is –20)

b) $x^2-6x+10=(x+q)^2+r$ Find the values of q and r. (In this example b is –6 and c is 10)

$$x^2+8x-20=\left(x+\frac{8}{2}\right)^2-\left(\frac{8}{2}\right)^2-20=(x+4)^2-(4)^2-20=(x+4)^2-16-20=\boldsymbol{(x+4)^2-36}$$

Start by halving b

Write the bracket $\left(x+\frac{b}{2}\right)$ and square it

Subtract $\left(\frac{b}{2}\right)^2$ and add c

The answer should be written as a squared bracket followed by $-\left(\frac{b}{2}\right)^2+c$ simplified

$$x^2-6x+10=\left(x-\frac{6}{2}\right)^2-\left(\frac{-6}{2}\right)^2+10=(x-3)^2-(-3)^2+10=(x-3)^2-9+10=\boldsymbol{(x-3)^2+1}$$

so $\boldsymbol{q=-3}$ and $\boldsymbol{r=1}$

Take care if b is negative. The method is exactly the same, but the arithmetic is slightly harder.

$$x^2-7x+4=\left(x-\frac{7}{2}\right)^2-\left(\frac{7}{2}\right)^2+4=\left(x-\frac{7}{2}\right)^2-\frac{49}{4}+4=\left(x-\frac{7}{2}\right)^2-\frac{49}{4}+\frac{16}{4}=\boldsymbol{\left(x-\frac{7}{2}\right)^2-\frac{33}{4}}$$

<u>Exercise 7A</u>

Write the following in completed square form, $(x+p)^2+q$

1) x^2+6x+4 2) $x^2+10x-5$ 3) $x^2-8x+18$ 4) x^2-4x+1
5) x^2-2x-6 6) x^2+3x+2 7) x^2-5x-2 8) x^2+12x

If the quadratic ax^2+bx+c has a value of a that is not 1, there is an extra step required.

Example: Write $3x^2+12x+5$ in the form $p(x+q)^2+r$

$$\mathbf{3}x^2+12x+5=\mathbf{3}(x^2+4x)+5=\mathbf{3}\left(x+\frac{4}{2}\right)^2-\mathbf{3}\left(\frac{4}{2}\right)^2+5$$

Start by factorising the first **two** terms, so that we have x^2 inside the brackets	Halve the **new** coefficient of x and proceed as before, but the value subtracted must be **multiplied by the factor**	Now just simplify to get completed square form

$$=3(x+2)^2-3(2)^2+5=3(x+2)^2-12+5=\boldsymbol{3(x+2)^2-7}$$ **[Exercise on next page]**

Exercise 7B

Write the following in completed square form, $p(x+q)^2+r$:

1) $3x^2+18x+28$ 2) $5x^2+40x+70$ 3) $4x^2-8x-3$ 4) $2x^2+6x+5$

Challenge

Use the link below to find **Proof Sorter – Quadratic Equation**, then click on the link *Full screen version*.

This interactive task challenges you to show how the formula for solving a quadratic equation can be achieved by a series of steps which involves completing the square at one point.

If you succeed, either print a screenshot or take a photo on your phone.

http://nrich.maths.org/1394

9. Expanding Double Brackets

We have not provided any notes for this section. You need to be resourceful when studying A-level – not everything will be spoon fed to you. If you do not feel confident with these questions, then you could do the on-line **mymaths** lesson found as follows which starts with single brackets, but then covers double bracket expansion.

- Algebra -> Algebraic Manipulation -> Brackets

Exercise 9A

Expand the brackets:

1) $(x+2)(x+3)$	2) $(x+3)(x+4)$	3) $(x-7)(x+9)$	4) $(x+2)(x-5)$
5) $(x-4)(x+2)$	6) $(x-2)(x-3)$	7) $(x+2)(x-2)$	8) $(2x+12)(3x+4)$
9) $(2x+4)(-x-5)$	10) $(4x-2)(2x-5)$	11) $(x+2)^2$	12) $(x-4)^2$

Exercise 9B

Expand the brackets: 1) $(x-4)(x+2)(x+3)$ 2) $(x-2)(x-3)(x+5)$ 3) $(x+5)^3$

10. Factorising Quadratic Equations

Again we have not provided any notes for this section. On **mymaths** the lessons below will help you.

- Algebra -> Algebraic Manipulation -> Factorising Quadratics 1
- Algebra -> Equations – Quadratic -> Quadratic Equations 1

Exercise 10A

Solve by factorising:

1) $x^2 + 7x + 10 = 0$	2) $x^2 + 8x + 15 = 0$	3) $x^2 + 19x + 48 = 0$	4) $x^2 + 16x + 64 = 0$
5) $x^2 - 8x + 12 = 0$	6) $x^2 - 10x + 9 = 0$	7) $x^2 + 5x - 36 = 0$	8) $x^2 - 11x - 26 = 0$
9) $x^2 - 25 = 0$	10) $x^2 - 64 = 0$		

- Algebra -> Algebraic Manipulation -> Factorising Quadratics 2
- Algebra -> Equations – Quadratic -> Quadratic Equations 2

Exercise 10B

Solve by factorising:

1) $3x^2 + 13x + 4 = 0$ 2) $6x^2 + 25x + 4 = 0$

3) $3x^2 + 10x - 8 = 0$ 4) $6x^2 - 13x + 6 = 0$

Challenge

Read the statement and the question in the box.

1) Solve each equation to confirm that the statement is true.

2) Attempt the question at the bottom of the box.

These quadratics all have integer solutions

$$x^2+5x+6=0$$
$$x^2-5x+6=0$$
$$x^2+5x-6=0$$
$$x^2-5x-6=0$$

Can you find other quadratics, $x^2 \pm bx \pm c = 0$, with this property?