

Bishop Tufnell Calculation Policy

This calculation policy is adapted from the White Rose Calculation policy. It outlines the key progression in written calculations taught throughout Bishop Tufnell Primary School. Guidance is given as to the expected methods used in each year-group. In line with the approach to teaching mathematics from the 2014 National Curriculum, the emphasis in any year group is to keep students within one year-group working on the same aspect of mathematics and to deepen understanding of the key topics introduced in each year without accelerating to new content. The same philosophy applies within the teaching of written calculation. The emphasis is placed on deepening understanding of the key concepts being introduced within each year-group to prepare children for the more formal calculation methods when they are introduced.

Within this, there is an overall aim for children to progress to efficient written methods. To facilitate this progression, emphasis is placed on conceptual understanding through concrete apparatus, which should be used alongside the pictorial and abstract representations. As students move towards the more formal abstract calculations, they will be expected to manipulate numbers in the abstract. As more complex concepts are introduced (e.g. decimals) it may be necessary to revisit the concept with concrete apparatus and pictorial representations, such as place value counters, to support conceptual development.

Contents:

- Calculation Overview
- Vocabulary
- Calculation Guidance
- Exemplification of final formal written calculations

Calculation policy: Guidance

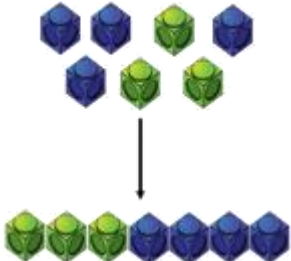
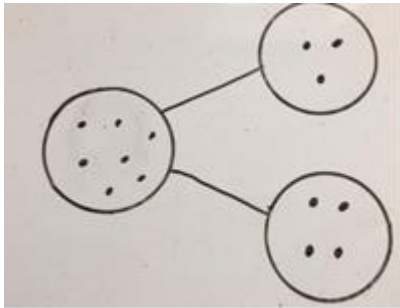
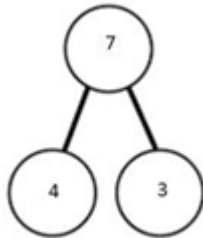
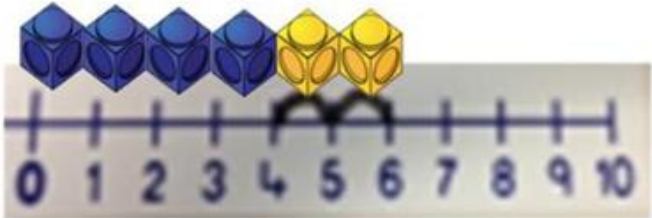
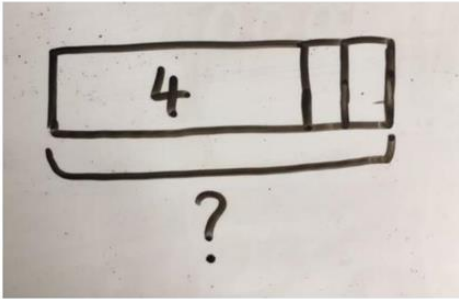
	EYFS/Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on- using cubes. Regrouping to make 10 using ten frame.	Adding three single digits. Use of base 10 to combine two numbers.	Column method- regrouping. Using place value counters (up to 3 digits).	Column method- regrouping. (up to 4 digits)	Column method- regrouping. Use of place value counters for adding decimals.	Column method- regrouping. Abstract methods. Place value counters to be used for adding decimal numbers.
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10 using the	Counting back Find the difference Part whole model Make 10 Use of base 10	Column method with regrouping. (up to 3 digits using place value counters)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. same amount of Abstract for whole numbers. Start with place value counters for decimals- with the decimal places.	Column method with regrouping. Abstract methods. Place value counters for decimals- with different amounts of decimal places.

Multiplication	Recognising and making equal groups. Doubling Counting in multiples Use cubes, Numicon and other objects in the classroom	Arrays- showing commutative multiplication	Arrays 2d × 1d using base 10	Column multiplication- introduced with place value counters. (2 and 3 digit multiplied by 1 digit)	Column multiplication (digits) Abstract only but might need a repeat of year 4 First (up to 4 digit numbers multiplied by 1 or 2 digit number)	Column multiplication Abstract methods (multi-digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups? round 3 cubes at a time.	Division as grouping Division within arrays- linking to multiplication Repeated subtraction	Division with a remainder-using lollipop sticks, times tables facts and repeated subtraction. 2d divided by 1d using base 10 or counters	Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number including remainders)	Short division Long division with place value counters (up to 4 digits by a 2 digit number) Children should exchange into the hundredths column too

Covered by:	Addition and Subtraction			
End of KS1	• add • + (addition sign) • plus • make • sum • total • altogether • combine • put together	• minus • leave • difference between • distance between • subtract • - (subtraction sign) • take away • fewer • gone	• more • less • increase • decrease • counting forwards • counting backwards • one-digit number • two-digit number • one more, two more... ten more • one less, two less, ten less...	• commutative • place value • columns • partitioning • number bonds • number line • inverse • double • near double • equals • is the same as (including equals sign)
Lower KS2	• column addition • column subtraction • calculation • calculate • estimate • regrouping • exchange • partitioning • zero as a place holder • operation		• subtraction • minuend • subtrahend • minus sign • difference • minus • less • reduce	
Upper KS2	• accuracy • rounding to check • decimal places			

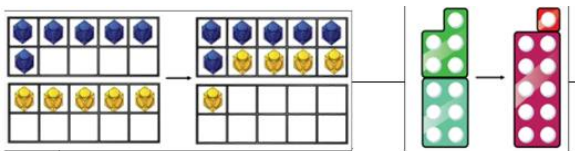
Covered by:	Multiplication and Division			
End of KS1	• odd • even • count in twos, threes, fives • count in tens (forwards from/backwards from) • lots / lots of • once, twice, three times, five times • Group in pairs, threes, etc.	• array • row • column • double • halve • share • repeated addition • place value	• multiple of • times • multiply • multiply by • lots of • groups of • x • multiple of • times as (big, long, wide and so on)	• share equally • equal groups of • divide • divided by • left • left over • grouping • sharing • ÷ • = • division • division facts • divided into
Lower KS2	• calculation • calculate • partition • place value chart • times table • multiply • multiplication • times • multiplicand • multiplier		• product • commutative • scaling • derive related facts • factor pairs • distributive law • integer • remainder	
Upper KS2	• zero as a place holder • carrying • common factors • prime numbers • prime factors • composite numbers • short division • long division • common multiple • order of operations • accuracy			

Addition:

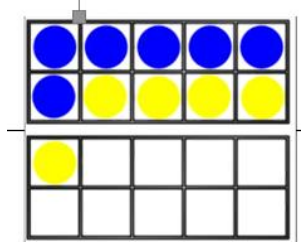
Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars) 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	$4 + 3 = 7$ Four is a part, 3 is a part and the whole is seven. 
Counting on using number lines using cubes or Numicon. 	A bar model which encourages the children to count on, rather than count all. 	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? $4 + 2$ 

Regrouping to make 10; using ten frames and counters/cubes or using Numicon.

$$6 + 5$$



Children to draw the ten frame and counters/cubes.



Children to develop an understanding of equality e.g.

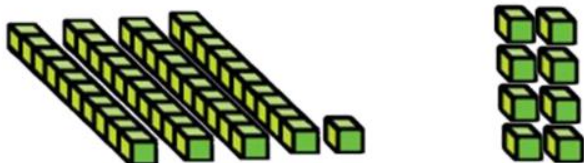
$$6 + \square = 11$$

$$6 + 5 = 5 + \square$$

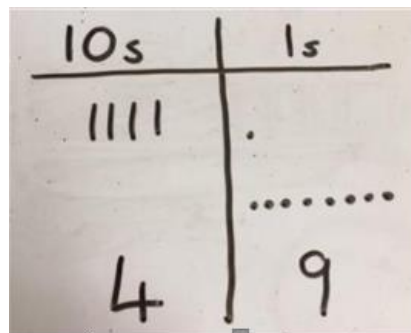
$$6 + 5 = \square + 4$$

TO + 0 using base 10. Continue to develop understanding of partitioning and place value.

$$41 + 8$$



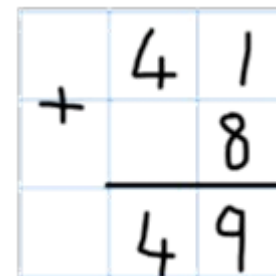
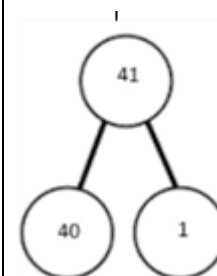
Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.



$$41 + 8$$

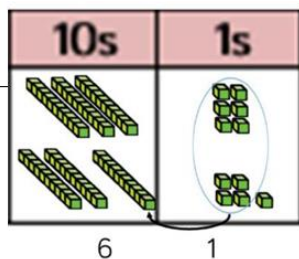
$$1 + 8 = 9$$

$$40 + 9 = 49$$

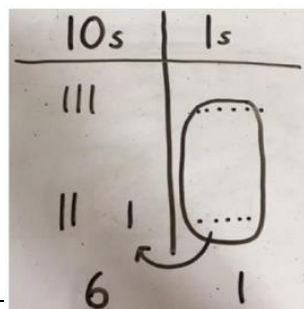


TO + TO using base 10. Continue to develop understanding of partitioning and place value.

$$36 + 25$$



Children to represent the base 10 in a place value chart.



Looking for ways to make 10.

$$36 + 25 =$$

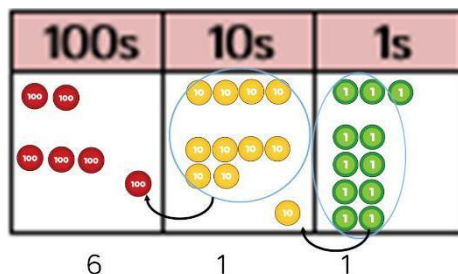
30 + 20 = 50
5 + 5 = 10
50 + 10 + 1 = 61

36

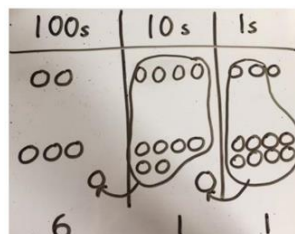
Formal method:

$$\begin{array}{r} +25 \\ 36 \\ \hline 61 \\ 1 \end{array}$$

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.



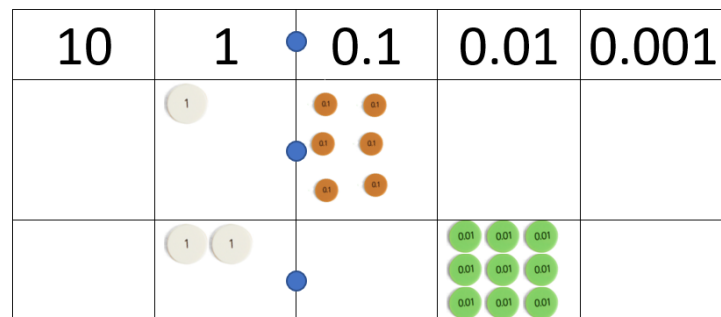
Children to represent the counters in a place value chart, circling when they make an exchange



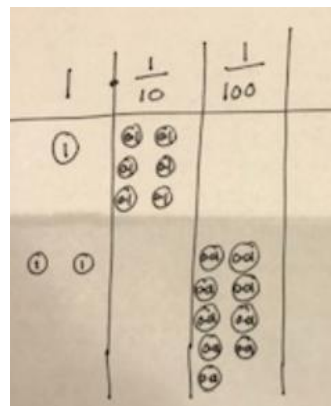
$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ 1 \quad 1 \end{array}$$

Decimal examples including those with different numbers of decimal places

$$1.6 + 2.09 = 3.09$$



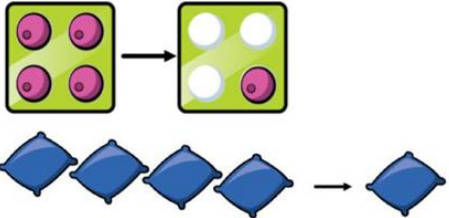
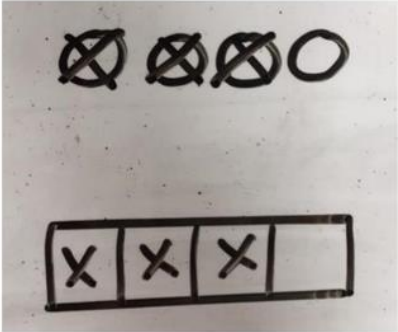
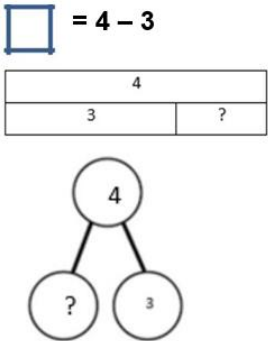
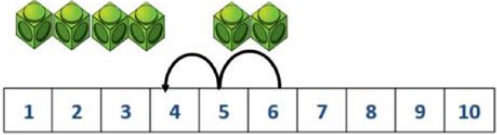
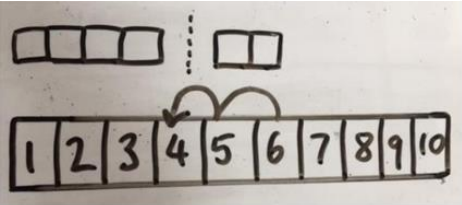
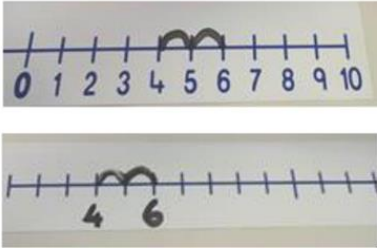
Children to represent the counters pictorially:



Use of column method with additional zero place holders as necessary:

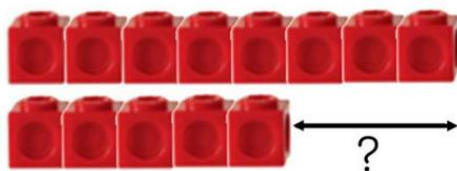
$$\begin{array}{r} 1.60 \\ + 2.09 \\ \hline 3.69 \end{array}$$

Subtraction:

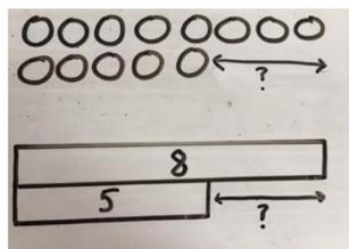
Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). $4 - 3 = 1$ 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	$4 - 3 =$ 
Counting back (using number lines or number tracks) children start with 6 and count back 2. $6 - 2 = 4$ 	Children to represent what they see pictorially e.g. 	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line 

Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).

Calculate the difference between 8 and 5.



Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.



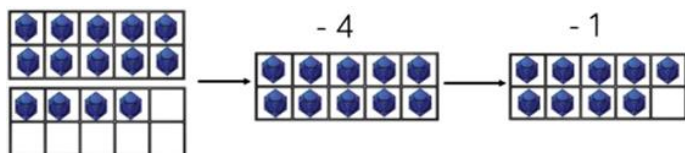
Find the difference between 8 and 5.

8 – 5, the difference is

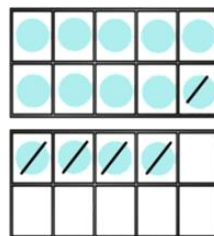
Children to explore why $9 - 6 = 8 - 5 = 7 - 4$ have the same difference.

Making 10 using ten frames.

14 – 5



Children to present the ten frame pictorially and discuss what they did to make 10.



Children to show how they can make 10 by partitioning the subtrahend.

$$14 - 5 = 9$$

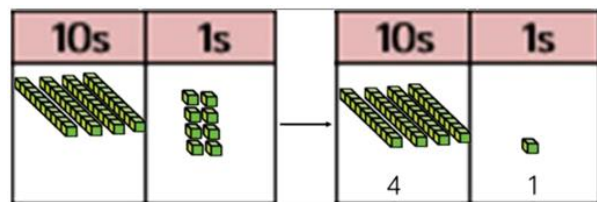
4 1

$$14 - 4 = 10$$

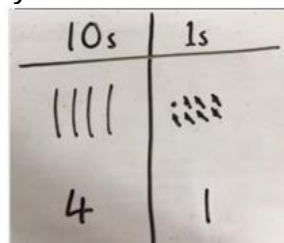
$$10 - 1 = 9$$

Column method using base 10.

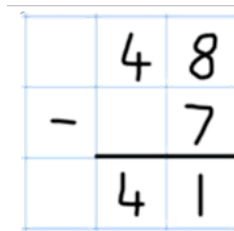
48-7



Children to represent the base 10 pictorially.

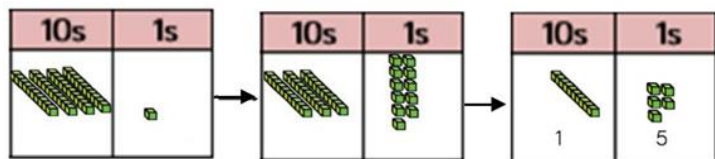


Column method or children could count back 7.

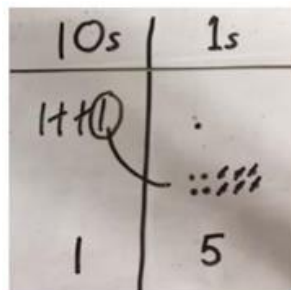


Column method using base 10 and having to exchange.

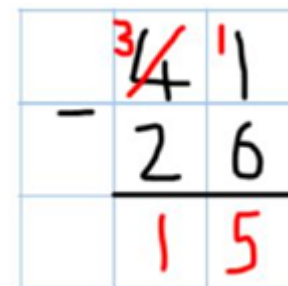
$$41 - 26$$



Represent the base 10 pictorially, remembering to show the exchange.

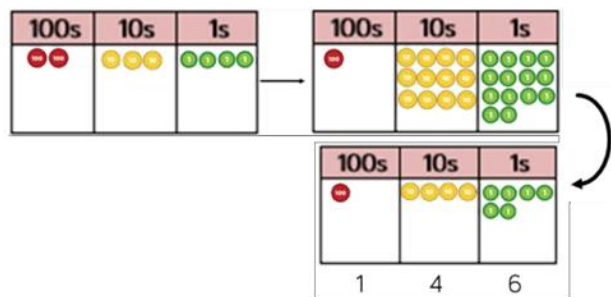


Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because $41 = 30 + 11$.

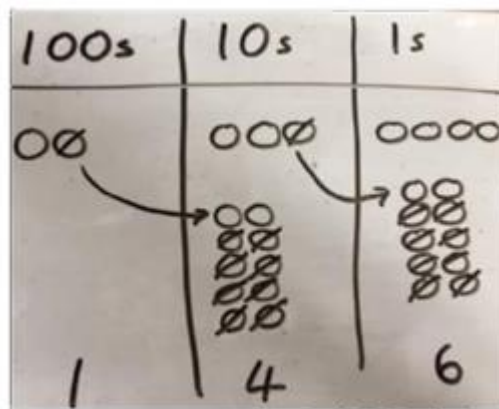


Column method using place value counters.

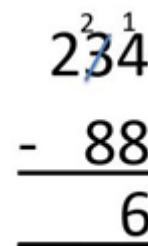
$$234 - 88$$



Represent the place value counters pictorially; remembering to show what has been exchanged.

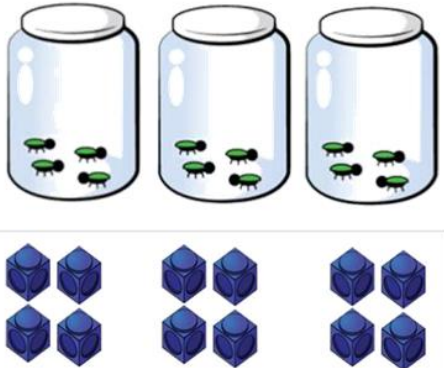
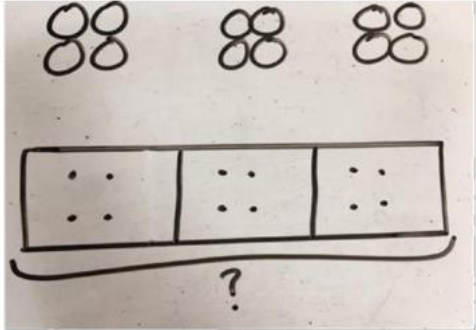
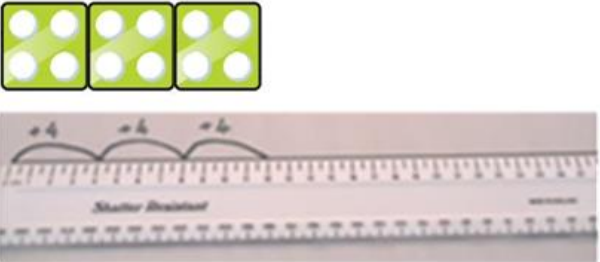
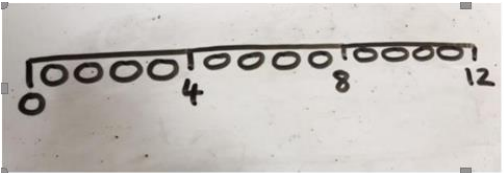
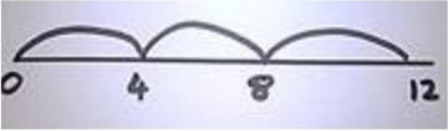


Formal column method. Children must understand what has happened when they have crossed out digits.



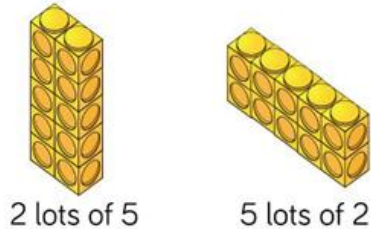
The above examples can be extended to decimal examples where formal calculation would be appropriate. However, children reaching this stage of calculation should be taught to choose appropriate methods for calculation. For example, the calculation $6 - 5.738$ (from the 2018 Y6 arithmetic paper) can be successfully solved through column subtraction, but equally can be successfully and efficiently solved through counting on, or using a number line. Children should be taught to consider methods which are reliable, effective and efficient.

Multiplication

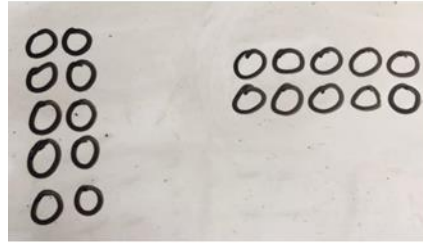
Concrete	Pictorial	Abstract
Repeated grouping/repeated addition 3×4 $4 + 4 + 4$ There are 3 equal groups, with 4 in each group. 	Children to represent the practical resources in a picture and use a bar model. 	$3 \times 4 = 12$ $4 + 4 + 4 = 12$
Number lines to show repeated groups-3×4  Cuisenaire rods can be used too.	Pictorially alongside a number line e.g.: 	Abstract number line showing three jumps of four. $3 \times 4 = 12$ 

Use arrays to illustrate commutativity counters and other objects can also be used.

$2 \times 5 = 5 \times 2$



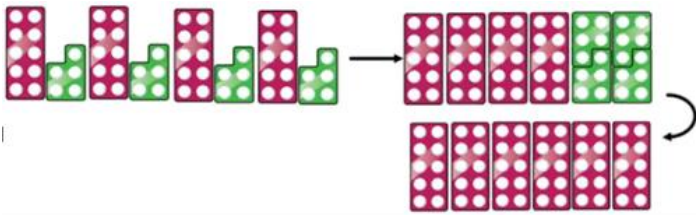
Children to represent the arrays pictorially.



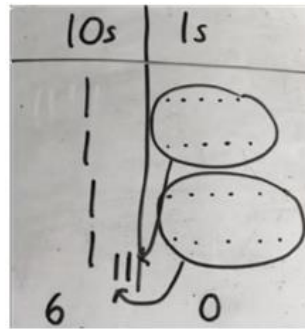
Children to be able to use an array to write a range of calculations e.g.

$$\begin{aligned} 10 &= 2 \times 5 \\ 5 \times 2 &= 10 \\ 2 + 2 + 2 + 2 + 2 &= 10 \\ 10 &= 5 + 5 \end{aligned}$$

Partition to multiply using Numicon, base 10 or Cuisenaire rods.
 4×15



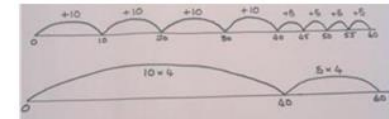
Children to represent the concrete manipulatives pictorially.



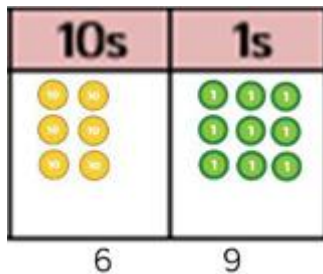
Children to be encouraged to show the steps they have taken.

$$\begin{array}{r} 4 \times 15 \\ \swarrow \searrow \\ 10 \quad 5 \end{array}$$
$$\begin{array}{l} 10 \times 4 = 40 \\ 5 \times 4 = 20 \\ 40 + 20 = 60 \end{array}$$

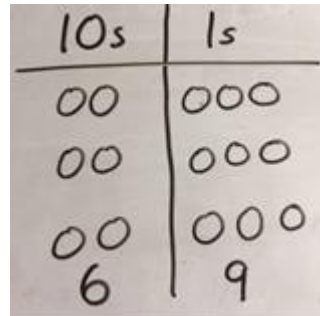
A number line can also be used



Formal column method with place value counters (base 10 can also be used.) 3×23

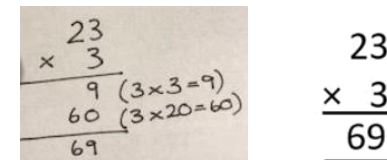


Children to represent the counters pictorially.



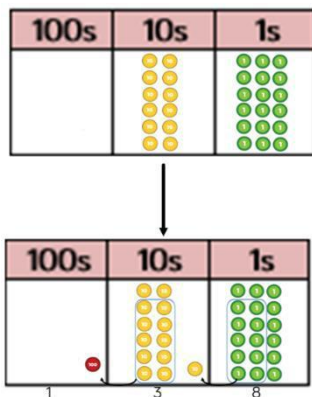
Children to record what it is they are doing to show understanding.

$$\begin{array}{r} 3 \times 23 \\ \swarrow \searrow \\ 20 \quad 3 \end{array} \quad \begin{array}{l} 3 \times 20 = 60 \\ 3 \times 3 = \underline{\underline{9}} \\ 60 + 9 = 69 \end{array}$$

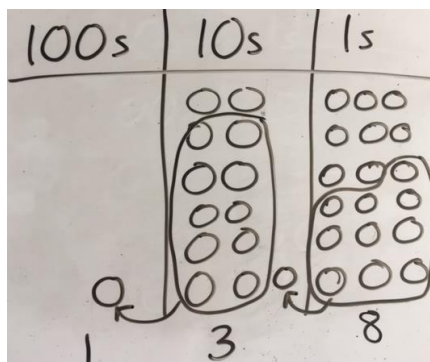


Formal column method with place value counters.

$$6 \times 23$$



Children to represent the counters/base 10, pictorially e.g. the image below.



Formal written method:

$$\begin{array}{r} 6 \times 23 = \\ 23 \\ \times 6 \\ \hline 138 \\ 11 \end{array}$$

When children start to multiply $3d \times 3d$ and $4d \times 2d$ etc., they should be confident with the abstract:

To get 744 children have solved 6×124 .

To get 2480 they have solved 20×124 .

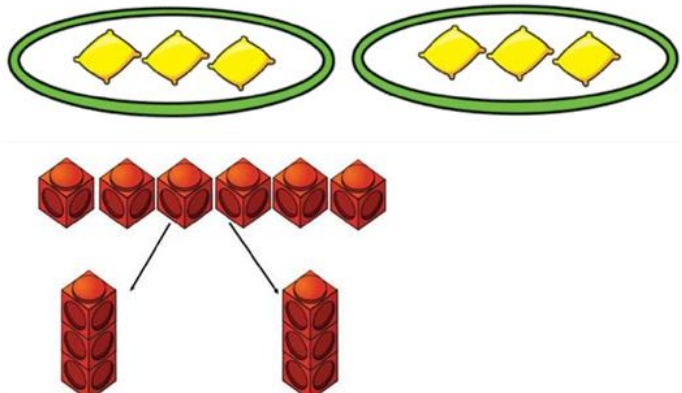
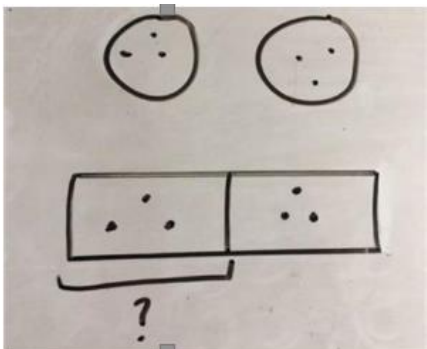
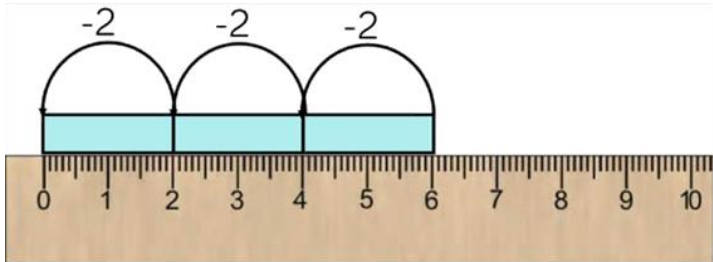
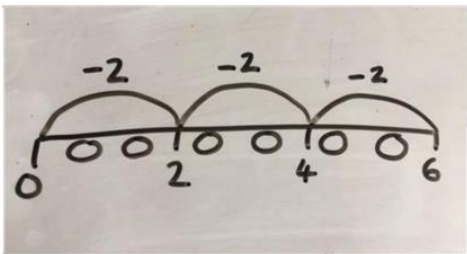
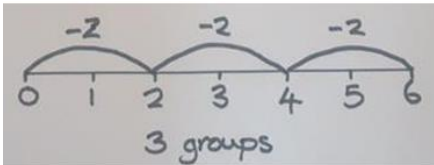
$$\begin{array}{r} 124 \\ \times 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \\ 11 \end{array}$$

Answer: 3224

Children may need to return to concrete apparatus or expanded methods when working with decimal numbers:

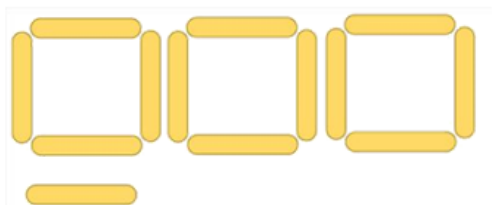
$$\begin{array}{r} 2.36 \\ \times 15 \\ \hline 11.80 \quad (5 \times 2.36) \\ 23.60 \quad (10 \times 2.36) \\ \hline 35.40 \end{array}$$

Division:

Concrete	Pictorial	Abstract
Sharing using a range of objects. $6 \div 2$  The image shows two green ovals, each containing three yellow diamonds. Below them, six red cubes are arranged in a single row. Two lines branch out from the middle of this row, pointing to two separate stacks of three red cubes each, illustrating the sharing of 6 objects into 2 equal groups of 3.	Represent the sharing pictorially  The image shows two hand-drawn circles, each containing three dots. Below them is a hand-drawn rectangle divided into two equal halves, with three dots in each half. A bracket under the first half is followed by a question mark, suggesting a problem to be solved.	$6 \div 2 = 3$  The image shows a table with two columns. Each column contains the number 3. Children should also be encouraged to use their 2 times tables facts.
Repeated subtraction using Cuisenaire rods above a ruler. $6 \div 2$  The image shows a ruler with markings from 0 to 10. Three light blue Cuisenaire rods, each representing the number 2, are placed end-to-end along the ruler from 0 to 6. Above each rod is an arc with the number -2 written above it. Below the ruler, the text '3 groups of 2' is written.	Children to represent repeated subtraction pictorially.  The image shows a hand-drawn number line from 0 to 6. There are small circles at each number. Three arcs are drawn above the line, each starting at a number and ending at the next number (0 to 2, 2 to 4, 4 to 6). Each arc is labeled -2 above it.	Abstract number line to represent the equal groups that have been subtracted.  The image shows a hand-drawn number line from 0 to 6. There are small circles at each number. Three arcs are drawn above the line, each starting at a number and ending at the next number (0 to 2, 2 to 4, 4 to 6). Each arc is labeled -2 above it. Below the number line, the text '3 groups' is written.

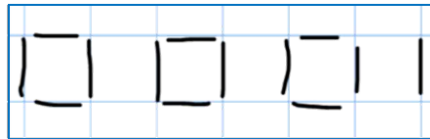
$2d \div 1d$ with remainders using lollipop sticks.
Cuisenaire rods, above a ruler can also be used.
 $13 \div 4$

Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.



There are 3 whole squares, with 1 left over.

Children to represent the lollipop sticks pictorially.



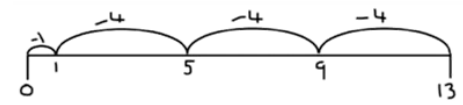
There are 3 whole squares, with 1 left over.

$13 \div 4 = 3$ remainder 1

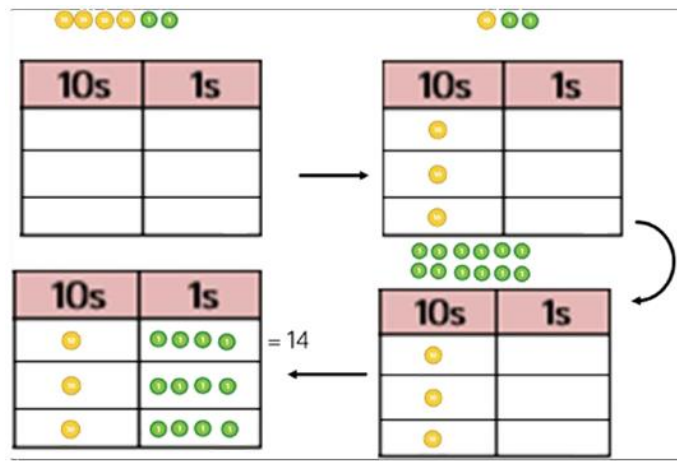
Children should be encouraged to use their

times table facts; they could also represent repeated addition on a number line.

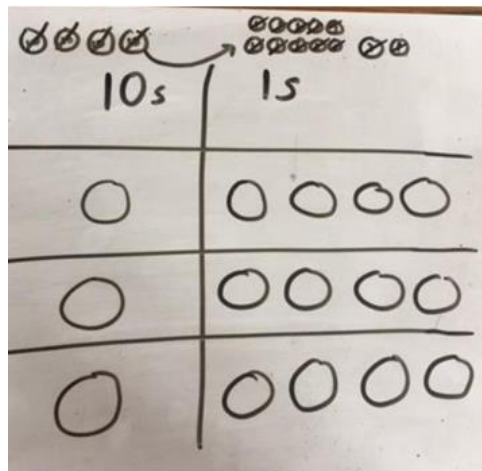
'3 groups of 4, with 1 left over'



Sharing using place value counters.
 $42 \div 3 = 14$



Children to represent the place value counters pictorially.

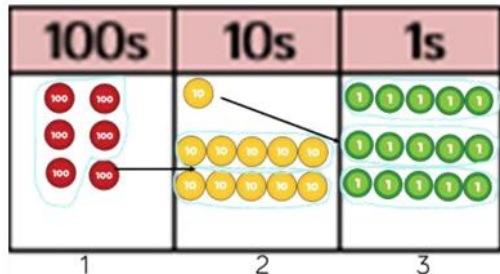


Children to be able to make sense of the place value counters and write calculations to show the process.

$42 \div 3$
 $42 = 30 + 12$
 $30 \div 3 = 10$
 $12 \div 3 = 4$
 $10 + 4 = 14$

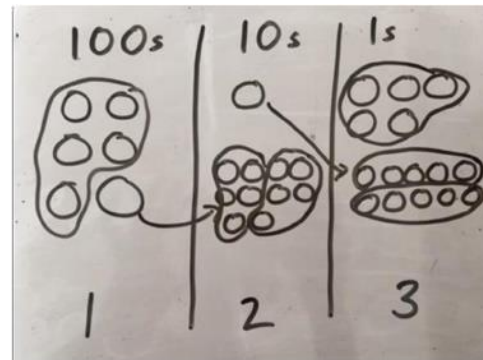
Short division using place value counters to group.

$$615 \div 5$$



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.



Children to the calculation using the short division scaffold.

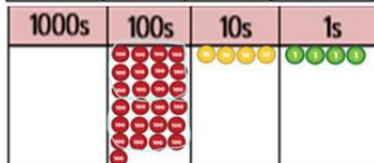
$$\begin{array}{r} 123 \\ 5 \overline{) 615} \\ \underline{5} \\ 11 \\ \underline{10} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

Long division using place value counters if necessary:

$$2544 \div 12$$

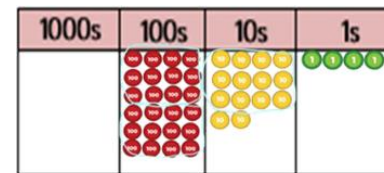


We can't group 2 thousands into groups of 12 so will exchange them.



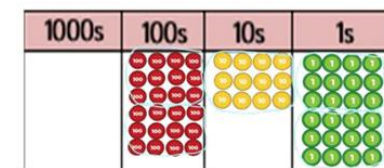
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

When meeting decimals, children may need to return to the concrete apparatus to explore the concept. However, children should be confident with abstract division before meeting decimal division. Most importantly when meeting decimals in division, students should be able to interpret the answers appropriately, knowing when a decimal, fractional or remainder answer would be appropriate.

Appendix

Examples of formal written methods for addition, subtraction, multiplication and division.

Addition (carrying)

$$\begin{array}{r} 243 \\ + 368 \\ \hline 611 \\ \hline \end{array}$$

1 1

Subtraction

$$\begin{array}{r} 234 \\ - 88 \\ \hline 146 \\ \hline \end{array}$$

Long multiplication

$$\begin{array}{r} 124 \\ \times 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \\ + + \end{array}$$

Answer = 3,224

NB: once a carried digit has been added,
cross the digit out.

Short division

$$\begin{array}{r} 123 \\ 5 \overline{) 615} \end{array}$$

Long division

$$\begin{array}{r} 28.8 \\ 15 \overline{) 432.0} \\ \underline{30} \\ 132 \\ \underline{120} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

A green arrow points down from the first remainder 30 to the second remainder 132. A blue arrow points down from the second remainder 132 to the third remainder 120.

$$\begin{array}{r} 15 \\ 30 \\ 45 \\ 60 \\ 75 \\ 90 \\ 105 \\ 120 \\ 135 \\ 150 \end{array}$$