

Mathematics overview: Stage 7

Unit	Hours	KNOWLEDGE
Numbers and the number system	6	<ul style="list-style-type: none"> use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 (ALSO 6) use the concepts and vocabulary of prime numbers, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem (NEW) order positive and negative integers, decimals and fractions use the symbols $=$, \neq, $<$, $>$, \leq, \geq understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
Counting and comparing	3	<ul style="list-style-type: none"> apply the four operations, including formal written methods, to integers and decimals and simple fractions (proper and improper) and mixed numbers use conventional notation for priority of operations, including brackets (ALSO 6) use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries (ALSO 6)
Calculating	9	<ul style="list-style-type: none"> use the standard conventions for labelling and referring to the sides and angles of triangles (ALSO 6) draw diagrams from written description
Visualising and constructing	6	<ul style="list-style-type: none"> identify properties of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres the faces, surfaces, edges and vertices of (ALSO 6 and crossover with Visualising and Constructing) derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language
Investigating properties of shapes	6	<ul style="list-style-type: none"> find unknown angles in any triangles, quadrilaterals and regular polygons (NEW) understand and use the concepts and vocabulary of expressions, equations, formulae and terms (ALSO 6) use and interpret algebraic notation, including: ab in place of $a \times b$, $3y$ in place of $y + y + y$ and $3 \times y$, a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a/b in place of $a \div b$, brackets (ALSO 6) simplify and manipulate algebraic expressions by collecting like terms and multiplying a single term over a bracket (ALSO 6) where appropriate, interpret simple expressions as functions with inputs and outputs (ALSO 6)
Algebraic proficiency: tinkering	9	<ul style="list-style-type: none"> substitute numerical values into formulae and expressions (ALSO 6) rearrange formulae to change the subject (NEW) express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1 express one quantity as a percentage of another associate a fraction with division and calculate decimal fraction equivalents (for example 0.375) for a simple fraction (for example $3/8$) (NEW, ALSO 6)
Exploring fractions, decimals and percentages	6	<ul style="list-style-type: none"> recall and use equivalencies between simple fractions, decimals and percentages, including in different contexts (NEW, ALSO 6) use ratio notation, including reduction to simplest form divide a given quantity into two parts in a given part: part or part: whole ratio deduce expressions to calculate the nth term of linear sequences (NEW) use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate convert between miles and kilometres (NEW) recognise that shapes with the same areas can have different perimeters and vice versa (NEW)
Proportional reasoning	3	<ul style="list-style-type: none"> apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively compare two quantities using percentages solve problems involving percentage change, including percentage increase/decrease
Pattern sniffing	3	<ul style="list-style-type: none"> solve linear equations in one unknown algebraically



Measuring space	3	<ul style="list-style-type: none"> know and apply formulae to calculate area of triangles, parallelograms, trapezia calculate surface area of cuboids
Investigating angles	3	<ul style="list-style-type: none"> know and apply formulae to calculate volume of cuboids understand and use standard mathematical formulae round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures) estimate answers; check calculations using approximation and estimation, including answers obtained using technology
Calculating fractions, decimals and percentages	9	<ul style="list-style-type: none"> recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions) (ALSO 6) use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy (NEW) work with coordinates in all four quadrants understand and use lines parallel to the axes, $y = x$ and $y = -x$ solve geometrical problems on coordinate axes identify, describe and construct congruent shapes including on coordinate axes, by considering rotation, reflection and translation describe translations as 2D vectors
Solving equations and inequalities	6	<ul style="list-style-type: none"> plot graphs of equations that correspond to straight-line graphs in the coordinate plane. (NEW) relative expected frequencies to theoretical probability, using appropriate language and the 0 - 1 probability scale (ALSO 6) record describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees construct theoretical possibility spaces for single experiments with equally likely outcomes and use these to calculate theoretical probabilities apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments
Calculating space	9	<ul style="list-style-type: none"> interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data and know their appropriate use interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (median, mean and mode) and spread (range)
Checking, approximating and estimating	3	
Mathematical movement	6	
Understanding Risk 1	3	
Presentation of data	6	
Measuring data	3	



KNOWLEDGE	The Big Picture: Number and Place Value progression map
<ul style="list-style-type: none"> use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5 (ALSO 6) use the concepts and vocabulary of prime numbers, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem (NEW) 	

SKILLS	Suggested resources
<ul style="list-style-type: none"> Recall prime numbers up to 50 Know how to test if a number up to 150 is prime Know that there is only one set of prime factors for any integer Know the meaning of 'highest common factor' and 'lowest common multiple' Recognise when a problem involves using the highest common factor of two numbers Recognise when a problem involves using the lowest common multiple of two numbers Understand the use of notation for powers Know the meaning of the square root symbol ($\sqrt{\quad}$) Use a scientific calculator to calculate powers and roots Make the connection between squares and square roots (and cubes and cube roots) Identify the first 10 triangular numbers Recall the first 15 square numbers Recall the first 5 cube numbers Use linear number patterns to solve problems 	<ul style="list-style-type: none"> Solve problems involving prime numbers Use highest common factors to solve problems Use lowest common multiples to solve problems Explore powers and roots <p>Investigate number patterns KM: Exploring primes activities: Factors of square numbers; Mersenne primes; LCM sequence; n^2 and $(n + 1)^2$; n^2 and $n^2 + n$; $n^2 + 1$; $n! + 1$; $n! - 1$; $x^2 + x + 41$ KM: Use the method of Eratosthenes' sieve to identify prime numbers, but on a grid 6 across by 17 down instead. What do you notice? KM: Square number puzzle KM: History and Culture: Goldbach's Conjectures NRICH: Factors and multiples NRICH: Powers and roots</p> <p>Learning review www.diagnosticquestions.com</p>

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Prerequisites	Mathematical language	Agreed Common Teaching Approaches
<ul style="list-style-type: none"> Know how to find common multiples of two given numbers Know how to find common factors of two given numbers Recall multiplication facts to 12×12 and associated division facts <p>Bring on the Maths*: Moving on up! Number and Place Value: v6</p>	((Lowest) common) multiple and LCM ((Highest) common) factor and HCF Power (Square and cube) root Triangular number, Square number, Cube number, Prime number Linear sequence Notation Index notation: e.g. 5^3 is read as '5 to the power of 3' and means '3 lots of 5 multiplied together' Radical notation: e.g. $\sqrt{49}$ is generally read as 'the square root of 49' and means 'the positive square root of 49'; $\sqrt[3]{8}$ means 'the cube root of 8'	Pupils need to know how to use a scientific calculator to work out powers and roots. Note that while the square root symbol ($\sqrt{\quad}$) refers to the positive square root of a number, every positive number has a negative square root too. NCETM: Departmental workshop: Index Numbers NCETM: Glossary Common approaches <i>The following definition of a prime number should be used in order to 3entimet confusion about 1: A prime number is a number with exactly two factors.</i> Every classroom has a set of number classification posters on the wall
Reasoning opportunities and probing questions	Cross Curricular Links	Possible misconceptions
<ul style="list-style-type: none"> When using Eratosthenes sieve to identify prime numbers, why is there no need to go further than the multiples of 7? If this method was extended to test prime numbers up to 200, how far would you need to go? Convince me. Kenny says '20 is a square number because $10^2 = 20$'. Explain why Kenny is wrong. Kenny is partially correct. How could he change his statement so that it is fully correct? Always / Sometimes / Never: the lowest common multiple of two numbers is found by multiplying the two numbers together 	<ul style="list-style-type: none"> Solve problems involving prime numbers Use highest common factors to solve problems Use lowest common multiples to solve problems Explore powers and roots Investigate number patterns	<ul style="list-style-type: none"> Many pupils believe that 1 is a prime number – a misconception which can arise if the definition is taken as 'a number which is divisible by itself and 1' A common misconception is to believe that $5^3 = 5 \times 3 = 15$ See pedagogical note about the square root symbol too



KNOWLEDGE	The Big Picture: Number and Place Value progression map
<ul style="list-style-type: none"> order positive and negative integers, decimals and fractions use the symbols =, ≠, <, >, ≤, ≥ 	

SKILLS	Suggested resources	
<ul style="list-style-type: none"> Place a set of negative numbers in order Place a set of mixed positive and negative numbers in order Identify a common denominator that can be used to order a set of fractions Order fractions where the denominators are not multiples of each other Order a set of numbers including a mixture of fractions, decimals and negative numbers Use inequality symbols to compare numbers Make correct use of the symbols = and ≠ 	<p>KM: Farey Sequences KM: Decimal ordering cards 2 KM: Maths to Infinity: Fractions, decimals and percentages KM: Maths to Infinity: Directed numbers NRICH: Greater than or less than?</p> <p>Learning review</p> <ul style="list-style-type: none"> www.diagnosticquestions.com 	
Prerequisites	Mathematical language	Agreed Common Teaching Approaches
<ul style="list-style-type: none"> Understand that negative numbers are numbers less than zero Order a set of decimals with a mixed number of decimal places (up to a maximum of three) Order fractions where the denominators are multiples of each other Order fractions where the numerator is greater than 1 Know how to simplify a fraction by cancelling common factors 	Positive number Negative number Integer Numerator Denominator Notation The 'equals' sign: = The 'not equal' sign: ≠ The inequality symbols: < (less than), > (greater than), ≤ (less than or equal to), ≥ (more than or equal to)	Zero is neither positive nor negative. The set of integers includes the natural numbers {1, 2, 3, ...}, zero (0) and the 'opposite' of the natural numbers {-1, -2, -3, ...}. Pupil must use language correctly to avoid reinforcing misconceptions: for example, 0.45 should never be read as 'zero point forty-five'; $5 > 3$ should be read as 'five is greater than 3', not '5 is bigger than 3'. Ensure that pupils read information carefully and check whether the required order is smallest first or greatest first. The equals sign was designed by Robert Recorde in 1557 who also introduced the plus (+) and minus (-) symbols. NCETM: Glossary Common approaches <i>Teachers use the language 'negative number' to avoid future confusion with calculation that can result by using 'minus number'</i> <i>Every classroom has a negative number washing line on the wall</i>
Reasoning opportunities and probing questions	Cross Curricular Links	Possible misconceptions
<ul style="list-style-type: none"> Jenny writes down $0.400 > 0.58$. Kenny writes down $0.400 < 0.58$. Who do you agree with? Why? Find a fraction which is greater than $\frac{3}{5}$ and less than $\frac{7}{8}$. And another. And another ... Convince me that $-15 < -3$ 	<ul style="list-style-type: none"> Compare numbers Order numbers 	<ul style="list-style-type: none"> Some pupils may believe that 0.400 is greater than 0.58 Pupils may believe, incorrectly, that: <ul style="list-style-type: none"> A fraction with a larger denominator is a larger fraction A fraction with a larger numerator is a larger fraction A fraction involving larger numbers is a larger fraction Some pupils may believe that -6 is greater than -3. For this reason ensure pupils avoid saying 'bigger than'



KNOWLEDGE	The Big Picture: Calculation progression map
<ul style="list-style-type: none"> understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals) apply the four operations, including formal written methods, to integers and decimals and simple fractions (proper and improper) and mixed numbers use conventional notation for priority of operations, including brackets (ALSO 6) 	

SKILLS	Suggested resources	
<ul style="list-style-type: none"> Use knowledge of place value to multiply with decimals Use knowledge of place value to divide a decimal Use knowledge of place value to divide by a decimal Use knowledge of inverse operations when dividing with decimals Be fluent at multiplying a three-digit or a two-digit number by a two-digit number Be fluent when using the method of short division Know the order of operations for the four operations Use brackets in problem involving the order of operations Understand and apply the fact that addition and subtraction have equal priority Understand and apply the fact that multiplication and division have equal priority 	<p>Learning review</p> <ul style="list-style-type: none"> www.diagnosticquestions.com 	
Prerequisites	Mathematical language	Agreed Common Teaching Approaches
<ul style="list-style-type: none"> Fluently recall multiplication facts up to 12×12 Fluently apply multiplication facts when carrying out division Know the formal written method of long multiplication Know the formal written method of short division Know the formal written method of long division Convert between an improper fraction and a mixed number <p>Bring on the Maths!: Moving on up! Calculating: v2, v3, v4, v5 Fractions, decimals & percentages: v6, v7 Solving problems: v2</p>	Improper fraction Top-heavy fraction Mixed number Operation Inverse Long multiplication Short division Long division Remainder	Establish level of understanding and ability based on expectations of pupils at primary school The grid method is promoted as a method that aids numerical understanding and later progresses to multiplying algebraic statements. NCETM: Departmental workshop: Place Value NCETM: Subtraction NCETM: Multiplication and division NCETM: Glossary Common approaches <i>The use of long multiplication is to be promoted as the 'most efficient method'.</i> <i>Short division is promoted as the 'most efficient method'.</i> <i>If any acronym is promoted to help remember the order of operations, then BIDMAS is used as the I stands for indices.</i>
Reasoning opportunities and probing questions	Cross Curricular Links	Possible misconceptions
<ul style="list-style-type: none"> Jenny says that $2 + 3 \times 5 = 25$. Kenny says that $2 + 3 \times 5 = 17$. Who is correct? How do you know? Find missing digits in otherwise completed long multiplication / short division calculations Show me a calculation that is connected to $14 \times 26 = 364$. And another, and another ... 	<ul style="list-style-type: none"> Apply understanding of place value Explore written methods of calculation Calculate with decimals Know and apply the correct order of operations	<ul style="list-style-type: none"> The use of BIDMAS (or BODMAS) can imply that division takes priority over multiplication, and that addition takes priority over subtraction. This can result in incorrect calculations. Pupils may incorrectly apply place value when dividing by a decimal for example by making the answer 10 times bigger when it should be 10 times smaller.



KNOWLEDGE	The Big Picture: Properties of Shape progression map
<ul style="list-style-type: none"> use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries (ALSO 6) use the standard conventions for labelling and referring to the sides and angles of triangles (ALSO 6) draw diagrams from written description 	

SKILLS	Suggested resources	
<ul style="list-style-type: none"> Know the meaning of faces, edges and vertices Use notation for parallel lines Know the meaning of 'perpendicular' and identify perpendicular lines Know the meaning of 'regular' polygons Identify line and rotational symmetry in polygons Use AB notation for describing lengths Use $\angle ABC$ notation for describing angles Use ruler and protractor to construct triangles from written descriptions Use ruler and compasses to construct triangles when all three sides known 	KM: Shape work (selected activities) NRICH: Notes on a triangle Learning review www.diagnosticquestions.com	
Prerequisites	Mathematical language	Agreed Common Teaching Approaches
<ul style="list-style-type: none"> Use a ruler to measure and draw lengths to the nearest millimetre Use a protractor to measure and draw angles to the nearest degree 	Edge, Face, Vertex (Vertices) Plane Parallel Perpendicular Regular polygon Rotational symmetry Notation The line between two points A and B is AB The angle made by points A, B and C is $\angle ABC$ The angle at the point A is \hat{A} Arrow notation for sets of parallel lines Dash notation for sides of equal length	NCETM: Departmental workshop: Constructions The equals sign was designed (by Robert Recorde in 1557) based on two equal length lines that are equidistant NCETM: Glossary Common approaches <i>Dynamic geometry software to be used by all students to construct and explore dynamic diagrams of perpendicular and parallel lines.</i>
Reasoning opportunities and probing questions	Cross Curricular Links	Possible misconceptions
<ul style="list-style-type: none"> Given SSS, how many different triangles can be constructed? Why? Repeat for ASA, SAS, SSA, AAS, AAA. Always / Sometimes / Never: to draw a triangle you need to know the size of three angles; to draw a triangle you need to know the size of three sides. Convince me that a hexagon can have rotational symmetry with order 2. 	<ul style="list-style-type: none"> Interpret geometrical conventions and notation Apply geometrical conventions and notation Bring on the Maths!: Moving on up! Properties of shapes: v3, v4	<ul style="list-style-type: none"> Two line segments that do not touch are perpendicular if they would meet at right angles when extended Pupils may believe, incorrectly, that: <ul style="list-style-type: none"> perpendicular lines have to be horizontal / vertical only straight lines can be parallel all triangles have rotational symmetry of order 3 all polygons are regular



Investigating properties of shapes	Stage 8	6 hours
KNOWLEDGE <ul style="list-style-type: none"> • identify properties of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres the faces, surfaces, edges and vertices of (ALSO 6 and crossover with Visualising and Constructing) • derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language • find unknown angles in any triangles, quadrilaterals and regular polygons (NEW) 		The Big Picture: Properties of Shape progression map

SKILLS	SUGGESTED RESOURCES
<ul style="list-style-type: none"> • Investigate the properties of 3D shapes • Explore quadrilaterals • Explore triangles • Know the vocabulary of 3D shapes • Know the connection between faces, edges and vertices in 3D shapes • Visualise a 3D shape from its net • Recall the names and shapes of special triangles and quadrilaterals • Know the meaning of a diagonal of a polygon • Know the properties of the special quadrilaterals (including diagonals) • Apply the properties of triangles to solve problems • Apply the properties of quadrilaterals to solve problems 	<ul style="list-style-type: none"> •

Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> • Know the names of common 3D shapes • Know the meaning of face, edge, vertex • Understand the principle of a net • Know the names of special triangles • Know the names of special quadrilaterals • Know the meaning of parallel, perpendicular • Know the notation for equal sides, parallel sides, right angles <p>Bring on the Maths*: Moving on up! Properties of shapes: v1, v2</p>	Face, Edge, Vertex (Vertices) Cube, Cuboid, Prism, Cylinder, Pyramid, Cone, Sphere Quadrilateral Square, Rectangle, Parallelogram, (Isosceles) Trapezium, Kite, Rhombus Delta, Arrowhead Diagonal Perpendicular Parallel Triangle Scalene, Right-angled, Isosceles, Equilateral Notation Dash notation to represent equal lengths in shapes and geometric diagrams Right angle notation	Ensure that pupils do not use the word 'diamond' to describe a kite, or a square that is 45° to the horizontal. 'Diamond' is not the mathematical name of any shape. A cube is a special case of a cuboid and a rhombus is a special case of a parallelogram A prism must have a polygonal cross-section, and therefore a cylinder is not a prism. Similarly, a cone is not a pyramid. NCETM: Departmental workshop: 2D shapes NCETM: Glossary Common approaches <i>Every classroom has a set of triangle posters and quadrilateral posters on the wall</i> <i>Models of 3D shapes to be used by all students during this unit of work</i>

Reasoning opportunities and probing questions	Suggested resources	Possible misconceptions
<ul style="list-style-type: none"> • Show me an example of a trapezium. And another. And another ... • Always / Sometimes / Never: The number of vertices in a 3D shape is greater than the number of edges • Which quadrilaterals are special examples of other quadrilaterals? Why? Can you create a 'quadrilateral family tree'? • What is the same and what is different: Rhombus / Parallelogram? 	KM: Euler's formula KM: Visualising 3D shapes KM: Dotty activities : Shapes on dotted paper KM: What's special about quadrilaterals? Constructing quadrilaterals from diagonals and 7entimeter7 results. KM: Investigating polygons . Tasks one and two should be carried out with irregular polygons. NRICH: Property chart NRICH: Quadrilaterals game Learning review www.diagnosticquestions.com	<ul style="list-style-type: none"> • Some pupils may think that all trapezia are isosceles • Some pupils may think that a diagonal cannot be horizontal or vertical • Two line segments that do not touch are perpendicular if they would meet at right angles when extended. Therefore the diagonals of an arrowhead (delta) are perpendicular despite what some pupils may think • Some pupils may think that a square is only square if 'horizontal', and even that a 'non-horizontal' square is called a diamond • The equal angles of an isosceles triangle are not always the 'base angles' as some pupils may think



KNOWLEDGE	The Big Picture: Algebra progression map
<ul style="list-style-type: none"> understand and use the concepts and vocabulary of expressions, equations, formulae and terms (ALSO 6) use and interpret algebraic notation, including: ab in place of $a \times b$, $3y$ in place of $y + y + y$ and $3 \times y$, a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a/b in place of $a \div b$, brackets (ALSO 6) simplify and manipulate algebraic expressions by collecting like terms and multiplying a single term over a bracket (ALSO 6) where appropriate, interpret simple expressions as functions with inputs and outputs (ALSO 6) substitute numerical values into formulae and expressions (ALSO 6) rearrange formulae to change the subject (NEW) 	

SKILLS	SUGGESTED RESOURCES
<ul style="list-style-type: none"> Understand the vocabulary and notation of algebra Manipulate algebraic expressions Explore functions Evaluate algebraic statements Know the meaning of expression, term, formula, equation, function Know basic algebraic notation (the rules of algebra) Use letters to represent variables Identify like terms in an expression Simplify an expression by collecting like terms Know how to multiply a (positive) single term over a bracket (the distributive law) Substitute positive numbers into expressions and formulae Given a function, establish outputs from given inputs Given a function, establish inputs from given outputs Use a mapping diagram (function machine) to represent a function Use an expression to represent a function Use the order of operations correctly in algebraic situations 	<ul style="list-style-type: none">

Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> Use symbols (including letters) to represent missing numbers Substitute numbers into worded formulae Substitute numbers into simple algebraic formulae Know the order of operations <p>Bring on the Maths!: Moving on up! Algebra: v1</p>	<p>Algebra</p> <p>Expression, Term, Formula (formulae), Equation, Function, Variable</p> <p>Mapping diagram, Input, Output</p> <p>Represent</p> <p>Substitute</p> <p>Evaluate</p> <p>Like terms</p> <p>Simplify / Collect</p> <p>Notation</p> <p>See key concepts above</p>	<p>Pupils will have experienced some algebraic ideas previously. Ensure that there is clarity about the distinction between representing a variable and representing an unknown.</p> <p>Note that each of the statements $4x$, 42 and $4\frac{1}{2}$ involves a different operation after the 4, and this can cause problems for some pupils when working with algebra.</p> <p>NCETM: Algebra</p> <p>NCETM: Glossary</p> <p>Common approaches</p> <p><i>All pupils are expected to learn about the connection between mapping diagrams and formulae (to represent functions) in preparation for future representations of functions graphically.</i></p>
Reasoning opportunities and probing questions	Suggested resources	Possible misconceptions



- Show me an example of an expression / formula / equation
- Always / Sometimes / Never: $4(g+2) = 4g+8$, $3(d+1) = 3d+1$, $a^2 = 2a$, $ab = ba$
- [What is wrong?](#)
- Jenny writes $2a + 3b + 5a - b = 7a + 3$. Kenny writes $2a + 3b + 5a - b = 9ab$. What would you write? Why?

KM: [Pairs in squares](#). Prove the results algebraically.
 KM: [Algebra ordering cards](#)
 KM: [Spiders and snakes](#). See the 'clouding the picture' approach
 KM: Use [number patterns](#) to develop the multiplying out of brackets
 KM: [Maths to Infinity: Brackets](#)
 NRICH: [Your number is ...](#)
 NRICH: [Crossed ends](#)
 NRICH: [Number pyramids](#) and [More number pyramids](#)

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- Some pupils may think that it is always true that $a=1$, $b=2$, $c=3$, etc.
- A common misconception is to believe that $a^2 = a \times 2 = a2$ or $2a$ (which it can do on rare occasions but is not the case in general)
- When working with an expression such as $5a$, some pupils may think that if $a=2$, then $5a = 52$.
- Some pupils may think that $3(g+4) = 3g+4$
- The convention of not writing a coefficient of 1 (i.e. '1x' is written as 'x' may cause some confusion. In particular some pupils may think that $5h - h = 5$



KNOWLEDGE	The Big Picture: Fractions, decimals and percentages progression map
<ul style="list-style-type: none"> express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1 express one quantity as a percentage of another associate a fraction with division and calculate decimal fraction equivalents (for example 0.375) for a simple fraction (for example $\frac{3}{8}$) (NEW, ALSO 6) recall and use equivalencies between simple fractions, decimals and percentages, including in different contexts (NEW, ALSO 6) 	

SKILLS	SUGGESTED RESOURCES	
<ul style="list-style-type: none"> Understand and use top-heavy fractions Understand the meaning of 'percentage' Explore links between fractions and percentages Write one quantity as a fraction of another where the fraction is less than 1 Write one quantity as a fraction of another where the fraction is greater than 1 Write a fraction in its lowest terms by cancelling common factors Convert between mixed numbers and top-heavy fractions Understand that a percentage means 'number of parts per hundred' Write a percentage as a fraction Write a quantity as a percentage of another 	<ul style="list-style-type: none"> 	
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> Understand the concept of a fraction as a proportion Understand the concept of equivalent fractions Understand the concept of equivalence between fractions and percentages <p>Bring on the Maths!: Moving on up! Fractions, decimals & percentages: v1, v2</p>	Fraction Improper fraction Proper fraction Vulgar fraction Top-heavy fraction Percentage Proportion Notation Diagonal fraction bar / horizontal fraction bar	NRICH: Teaching fractions with understanding NCETM: Teaching fractions NCETM: Departmental workshop: Fractions NCETM: Glossary Common approaches <i>All pupils are made aware that 'per cent' is derived from Latin and means 'out of one hundred'</i>
Reasoning opportunities and probing questions	Suggested resources	Possible misconceptions
<ul style="list-style-type: none"> Jenny says '1/10 is the same as proportion as 10% so 1/5 is the same proportion as 5%.' What do you think? Why? What is the same and what is different: 1/10 and 10% ... 1/5 and 20%? Show this fraction as part of a square / rectangle / number line / ... 	KM: Crazy cancelling, silly simplifying NRICH: Rod fractions Learning review www.diagnosticsquestions.com	<ul style="list-style-type: none"> A fraction can be visualised as divisions of a shape (especially a circle) but some pupils may not recognise that these divisions must be equal in size, or that they can be divisions of any shape. Pupils may not make the connection that a percentage is a different way of describing a proportion Pupils may think that it is not possible to have a percentage greater than 100%



KNOWLEDGE	The Big Picture: Ratio and Proportion progression map
<ul style="list-style-type: none"> use ratio notation, including reduction to simplest form divide a given quantity into two parts in a given part: part or part: whole ratio 	

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SKILLS	SUGGESTED RESOURCES
<ul style="list-style-type: none"> Understand and use ratio notation Solve problems that involve dividing in a ratio Describe a comparison of measurements or objects using the language 'a to b' Describe a comparison of measurements or objects using ratio notation a:b Use ratio notation to describe a comparison of more than two measurements or objects Convert between different units of measurement State a ratio of measurements in the same units Simplify a ratio by cancelling common factors Identify when a ratio is written in its lowest terms Find the value of a 'unit' in a division in a ratio problem Divide a quantity in two parts in a given part:part ratio Divide a quantity in two parts in a given part:whole ratio Express correctly the solution to a division in a ratio problem 	<ul style="list-style-type: none">

Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> Find common factors of pairs of numbers Convert between standard metric units of measurement Convert between units of time Recall multiplication facts for multiplication tables up to 12×12 Recall division facts for multiplication tables up to 12×12 Solve comparison problems <p>Bring on the Maths!: Moving on up! Ratio and proportion: #1</p>	<p>Ratio Proportion Compare, comparison Part Simplify Common factor Cancel Lowest terms Unit</p> <p>Notation Ratio notation a:b for part:part or part:whole</p>	<p>Note that ratio notation is first introduced in this stage. When solving division in a ratio problems, ensure that pupils express their solution as two quantities rather than as a ratio.</p> <p>NCETM: The Bar Model NCETM: Multiplicative reasoning NCETM: Glossary</p> <p>Common approaches <i>All pupils are explicitly taught to use the bar model as a way to represent a division in a ratio problem</i></p>
Reasoning opportunities and probing questions	Suggested resources	Possible misconceptions
<ul style="list-style-type: none"> Show me a set of objects that demonstrates the ratio 3:2. And another, and another ... Convince me that the ratio 120mm:0.3m is equivalent to 2:5 Always / Sometimes / Never: the smaller number comes first when writing a ratio Using Cuisenaire rods: If the red rod is 1, explain why d (dark green) is 3. Can you say the value for all the rods? (w, r, g, p, y, d, b, t, B, o). Extend this understanding of proportion by changing the unit rod e.g. if $r = 1$, $p = ?$; $b = ?$; $o + 2B = ?$ If $B = 1$; $y = ?$ $3y = ?$; $o = ?$ $o + p = ?$ If $o + r = 6/7$; $t = ?$ 	<p>KM: Maths to Infinity: FDP RP KM: Stick on the Maths: Ratio and proportion NRICH: Toad in the hole NRICH: Mixing lemonade NRICH: Food chains NRICH: Tray bake</p> <p>Learning review www.diagnosticquestions.com</p>	<ul style="list-style-type: none"> Some pupils may think that a:b always means part:part Some pupils may try to simplify a ratio without first ensuring that the units of each part are the same Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to other quantities in order to find missing amounts



KNOWLEDGE	The Big Picture: Algebra progression map
<ul style="list-style-type: none"> deduce expressions to calculate the nth term of linear sequences (NEW) 	

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SKILLS	SUGGESTED RESOURCES	
<ul style="list-style-type: none"> Explore number sequences Explore sequences Use a term-to-term rule to generate a linear sequence Use a term-to-term rule to generate a non-linear sequence Find the term-to-term rule for a sequence Describe a number sequence Solve problems involving the term-to-term rule for a sequence Solve problems involving the term-to-term rule for a non-numerical sequence 	<ul style="list-style-type: none"> 	
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> Know the vocabulary of sequences Find the next term in a linear sequence Find a missing term in a linear sequence Generate a linear sequence from its description <p>Bring on the Maths!: Moving on up! Number and Place Value: #4, #5</p>	Pattern Sequence Linear Term Term-to-term rule Ascending Descending	'Term-to-term rule' is the only new vocabulary for this unit. Position-to-term rule, and the use of the n th term, are not developed until later stages. NRICH: Go forth and generalise NCETM: Algebra Common approaches <i>All students are taught to describe the term-to-term rule for both numerical and non-numerical sequences</i>
Reasoning opportunities and probing questions	Suggested resources	Possible misconceptions
<ul style="list-style-type: none"> Show me a (non-)linear sequence. And another. And another. What's the same, what's different: 2, 5, 8, 11, 14, ... and 4, 7, 10, 13, 16, ...? Create a (non-linear/linear) sequence with a 3rd term of '7' Always/ Sometimes /Never: The 10th term of is double the 5th term of the (linear) sequence Kenny thinks that the 20th term of the sequence 5, 9, 13, 17, 21, ... will be 105. Do you agree with Kenny? Explain your answer. 	KM: Maths to Infinity: Sequences NRICH: Shifting times tables NRICH: Odds and evens and more evens Learning review www.diagnosticquestions.com	<ul style="list-style-type: none"> When describing a number sequence some students may not appreciate the fact that the starting number is required as well as a term-to-term rule Some pupils may think that all sequences are ascending Some pupils may think the $(2n)$th term of a sequence is double the nth term of a (linear) sequence



KNOWLEDGE

The Big Picture: [Measurement and mensuration progression map](#)

- use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
- convert between miles and kilometres (NEW)
- recognise that shapes with the same areas can have different perimeters and vice versa (NEW)

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SKILLS	SUGGESTED RESOURCES	
<ul style="list-style-type: none"> • Measure accurately • Convert between measures • Solve problems involving measurement • Use a ruler to accurately measure line segments to the nearest millimetre • Use a protractor to accurately measure angles to the nearest degree • Convert fluently between metric units of length • Convert fluently between metric units of mass • Convert fluently between metric units of volume / capacity • Convert fluently between units of time • Convert fluently between units of money • Solve practical problems that involve converting between units • State conclusions clearly using the units correctly 	<ul style="list-style-type: none"> • 	
Prerequisites <ul style="list-style-type: none"> • Convert between metric units • Use decimal notation up to three decimal places when converting metric units • Convert between common Imperial units; e.g. feet and inches, pounds and ounces, pints and gallons • Convert between units of time • Use 12- and 24-hour clocks, both analogue and digital Bring on the Maths: Moving on up! Measures: #3	Mathematical language Length, distance Mass, weight Volume Capacity Metre, 13centimeter, 13centimeter Tonne, kilogram, gram, milligram Litre, 13centimeter Hour, minute, second Inch, foot, yard Pound, ounce Pint, gallon Line segment Notation Abbreviations of units in the metric system: m, cm, mm, kg, g, l, ml Abbreviations of units in the Imperial system: lb, oz	Pedagogical notes Weight and mass are distinct though they are often confused in everyday language. Weight is the force due to gravity, and is calculated as mass multiplied by the acceleration due to gravity. Therefore weight varies due to location while mass is a constant measurement. The prefix 'centi-' means one hundredth, and the prefix 'milli-' means one thousandth. These words are of Latin origin. The prefix 'kilo-' means one thousand. This is Greek in origin. Classify/Estimate angle first NCETM: Glossary Common approaches <i>Every classroom has a sack of sand (25 kg), a bag of sugar (1 kg), a cheque book (1 cheque is 1 gram), a bottle of water (1 litre, and also 1 kg of water) and a teaspoon (5 ml)</i>
Reasoning opportunities and probing questions <ul style="list-style-type: none"> • Show me another way of describing 2.5km. And another. And another. • Show me another way of describing 3.4 litres. And another. And another. • Show me another way of describing 3.7kg. And another. And another. • Kenny thinks that 14:30 is the same time as 2.30 p.m. Do you agree with Kenny? Explain your answer. • What's the same, what's different: 2 hours 30 minutes, 2.5 hours, 2½ hours and 2 hours 20 minutes? 	Suggested resources KM: Sorting units KM: Stick on the Maths: Units NRICH: Temperature Learning review www.diagnosticquestions.com	Possible misconceptions <ul style="list-style-type: none"> • Some pupils may write amounts of money incorrectly; e.g. £3.5 for £3.50, especially if a calculator is used at any point • Some pupils may apply an incorrect understanding that there are 100 minutes in a hour when solving problems • Some pupils may struggle when converting between 12- and 24-hour clock notation; e.g. thinking that 15:00 is 5 o' clock • Some pupils may use the wrong scale of a protractor. For example, they measure an obtuse angle as 60° rather than 120°.



KNOWLEDGE	The Big Picture: Position and direction progression map
<ul style="list-style-type: none"> apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles 	

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SKILLS	SUGGESTED RESOURCES
<ul style="list-style-type: none"> Investigate angles Identify fluently angles at a point, angles at a point on a line and vertically opposite angles Identify known angle facts in more complex geometrical diagrams Use knowledge of angles to calculate missing angles in geometrical diagrams Know that angles in a triangles total 180° Find missing angles in triangles Find missing angles in isosceles triangles Explain reasoning using vocabulary of angles 	<p>Bring on the Maths': Moving on up!</p> <ul style="list-style-type: none"> Properties of shapes: #5

Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> Identify angles that meet at a point Identify angles that meet at a point on a line Identify vertically opposite angles Know that vertically opposite angles are equal 	<p>Angle</p> <ul style="list-style-type: none"> Degrees Right angle Acute angle Obtuse angle Reflex angle Protractor Vertically opposite Geometry, geometrical <p>Notation</p> <ul style="list-style-type: none"> Right angle notation Arc notation for all other angles The degree symbol ($^\circ$) 	<p>It is important to make the connection between the total of the angles in a triangle and the sum of angles on a straight line by encouraging pupils to draw any triangle, rip off the corners of triangles and fitting them together on a straight line. However, this is not a proof and this needs to be revisited in Stage 8 using alternate angles to prove the sum is always 180°.</p> <p>The word 'isosceles' means 'equal legs'. What do you have at the bottom of equal legs? Equal ankles!</p> <p>NCETM: Glossary</p> <p>Common approaches</p> <p><i>Teachers convince pupils that the sum of the angles in a triangle is 180° by ripping the corners of triangles and fitting them together on a straight line.</i></p>

Reasoning opportunities and probing questions	Suggested resources	Possible misconceptions
<ul style="list-style-type: none"> Show me possible values for a and b. And another. And another. Convince me that the angles in a triangle total 180° Convince me that the angles in a quadrilateral must total 360° What's the same, what's different: Vertically opposite angles, angles at a point, angles on a straight line and angles in a triangle? Kenny thinks that a triangle cannot have two obtuse angles. Do you agree? Explain your answer. Jenny thinks that the largest angle in a triangle is a right angle? Do you agree? Explain your thinking. 	<p>KM: Maths to Infinity: Lines and angles</p> <p>KM: Stick on the Maths: Angles</p> <p>NRICH: Triangle problem</p> <p>NRICH: Square problem</p> <p>NRICH: Two triangle problem</p> <p>Learning review</p> <p>www.diagnosticquestions.com</p>	<ul style="list-style-type: none"> Some pupils may think it's the 'base' angles of an isosceles that are always equal. For example, they may think that $a = b$ rather than $a = c$.  <ul style="list-style-type: none"> Some pupils may make conceptual mistakes when adding and subtracting mentally. For example, they may see that one of two angles on a straight line is 127° and quickly respond that the other angle must be 63°.



KNOWLEDGE	The Big Picture: Fractions, decimals and percentages progression map
<ul style="list-style-type: none"> interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively compare two quantities using percentages solve problems involving percentage change, including percentage increase/decrease 	

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SKILLS	SUGGESTED RESOURCES
<ul style="list-style-type: none"> Calculate with fractions Calculate with percentages Apply addition to proper fractions, improper fractions and mixed numbers Apply subtraction to proper fractions, improper fractions and mixed numbers Multiply proper and improper fractions Multiply mixed numbers Divide a proper fraction by a proper fraction Apply division to improper fractions and mixed numbers Use calculators to find a percentage of an amount using multiplicative methods Identify the multiplier for a percentage increase or decrease Use calculators to increase (decrease) an amount by a percentage using multiplicative methods Compare two quantities using percentages Know that percentage change = actual change ÷ original amount Calculate the percentage change in a given situation, including percentage increase / decrease 	<ul style="list-style-type: none">

Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> Add and subtract fractions with different denominators Add and subtract mixed numbers with different denominators Multiply a proper fraction by a proper fraction Divide a proper fraction by a whole number Simplify the answer to a calculation when appropriate Use non-calculator methods to find a percentage of an amount Convert between fractions, decimals and percentages <p>Bring on the Maths!: Moving on up! Fractions, decimals & percentages: #3, #4, #5 Ratio and proportion: #2</p>	<p>Mixed number Equivalent fraction Simplify, cancel, lowest terms Proper fraction, improper fraction, top-heavy fraction, vulgar fraction Percent, percentage Multiplier Increase, decrease</p> <p>Notation Mixed number notation Horizontal / diagonal bar for fractions</p>	<p>It is important that pupils are clear that the methods for addition and subtraction of fractions are different to the methods for multiplication and subtraction. A fraction wall is useful to help 15centimetre and represent the calculations. NCETM: The Bar Model NCETM: Teaching fractions NCETM: Fractions videos NCETM: Glossary</p> <p>Common approaches <i>When multiplying a decimal by a whole number pupils are taught to use the corresponding whole number calculation as a general strategy</i> <i>When adding and subtracting mixed numbers pupils are taught to convert to improper fractions as a general strategy</i> <i>Teachers use the horizontal fraction bar notation at all times</i></p>

Reasoning opportunities and probing questions	Suggested resources	Possible misconceptions
<ul style="list-style-type: none"> Show me a proper (improper) fraction. And another. And another. Show me a mixed number fraction. And another. And another. Jenny thinks that you can only multiply fractions if they have the same common denominator. Do you agree with Jenny? Explain your answer. Benny thinks that you can only divide fractions if they have the same common denominator. Do you agree with Jenny? Explain. Kenny thinks that $\frac{6}{10} \div \frac{3}{2} = \frac{2}{5}$. Do you agree with Kenny? Explain. Always/Sometimes/Never: To reverse an increase of x%, you decrease by x% Lenny calculates the % increase of £6 to £8 as 25%. Do you agree with Lenny? Explain your answer. 	<p>KM: Stick on the Maths: Percentage increases and decreases KM: Maths to Infinity: FDP RP KM: Percentage methods NRICH: Would you rather? NRICH: Keep it simple NRICH: Egyptian fractions NRICH: The greedy algorithm NRICH: Fractions jigsaw NRICH: Countdown fractions</p> <p>Learning review www.diagnosticquestions.com</p>	<ul style="list-style-type: none"> Some pupils may think that you simply can simply add/subtract the whole number part of mixed numbers and add/subtract the fractional part of mixed numbers when adding/subtracting mixed numbers, e.g. $3\frac{1}{3} - 2\frac{1}{2} = 1\frac{-1}{6}$ Some pupils may make multiplying fractions over complicated by applying the same process for adding and subtracting of finding common denominators. Some pupils may think the multiplier for, say, a 20% decrease is 0.2 rather than 0.8 Some pupils may think that percentage change = actual change ÷ new amount





<p>KNOWLEDGE</p> <ul style="list-style-type: none"> solve linear equations in one unknown algebraically 	<p>The Big Picture: Algebra progression map</p>
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SKILLS	SUGGESTED RESOURCES
<ul style="list-style-type: none"> Explore way of solving equations Solve two-step equations Solve three-step equations Choose the required inverse operation when solving an equation Identify the correct order of undoing the operations in an equation Solve one-step equations when the solution is a whole number (fraction) Solve two-step equations (including the use of brackets) when the solution is a whole number Solve two-step equations (including the use of brackets) when the solution is a fraction Solve three-step equations (including the use of brackets) when the solution is a whole number Solve three-step equations (including the use of brackets) when the solution is a fraction Check the solution to an equation by substitution 	<ul style="list-style-type: none">

Prerequisites	Mathematical language	Pedagogical notes												
<ul style="list-style-type: none"> Know the basic rules of algebraic notation Express missing number problems algebraically Solve missing number problems expressed algebraically <p>Bring on the Maths!: Moving on up! Algebra: #2</p>	<p>Algebra, algebraic, algebraically</p> <p>Unknown</p> <p>Equation</p> <p>Operation</p> <p>Solve</p> <p>Solution</p> <p>Brackets</p> <p>Symbol</p> <p>Substitute</p> <p>Notation</p> <p>The lower case and upper case of a letter should not be used interchangeably when worked with algebra</p> <p>Juxtaposition is used in place of 'x'. 2a is used rather than a2.</p> <p>Division is written as a fraction</p>	<p>This unit focuses on solving linear equations with unknowns on one side. Although linear equations with the unknown on both sides are addressed in Stage 8, pupils should be encouraged to think how to solve these equations by exploring the equivalent family of equations such as if $2x = 8$ then $2x + 2 = 10$, $2x - 3 = 5$, $3x = x + 8$, $3x + 2 = x + 10$, etc.</p> <p>Encourage pupils to re-present the equations such as $2x + 8 = 23$ using the Bar Model.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>x</td><td>x</td><td>8</td></tr> <tr><td colspan="3" style="text-align: center;">23</td></tr> </table> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>x</td><td>x</td></tr> <tr><td colspan="2" style="text-align: center;">15</td></tr> </table> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>x</td></tr> <tr><td style="text-align: center;">7.5</td></tr> </table> <p>NCETM: The Bar Model</p> <p>NCETM: Algebra</p> <p>NCETM: Glossary</p> <p>Common approaches</p> <p><i>Pupils should explore solving equations by applying inverse operations, but the expectation is that all pupils should solve by balancing:</i></p> $\begin{array}{r} 2x + 8 = 23 \\ -8 \quad -8 \\ \hline 2x = 15 \\ \div 2 \quad 2 \\ \hline x = 7.5 \text{ (or } 15/2) \end{array}$ <p><i>Pupils are expected to multiply out the brackets before solving an equation involving brackets. This makes the connection with two step equations such as $2x + 6 = 22$</i></p>	x	x	8	23			x	x	15		x	7.5
x	x	8												
23														
x	x													
15														
x														
7.5														

Reasoning opportunities and probing questions	Suggested resources	Possible misconceptions		
<ul style="list-style-type: none"> Show me an (one-step, two-step) equation with a solution of 14 (positive, fractional solution). And another. And another ... Kenny thinks if $6x = 3$ then $x = 2$. Do you agree with Kenny? Explain Jenny and Lenny are solving: $3(x - 2) = 51$. Who is correct? Explain <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; vertical-align: top;"> <p>Jenny's solution</p> $\begin{array}{r} 3(x - 2) = 15 \\ \div 3 \quad \div 3 \\ x - 2 = 5 \\ \div 2 \quad \div 2 \\ x = 7 \end{array}$ </td> <td style="width: 50%; vertical-align: top;"> <p>Lenny's solution</p> $\begin{array}{r} 3(x - 2) = 15 \\ \text{Multiplying out brackets} \\ 3x - 6 = 15 \\ \div 3 \\ 3x = 21 \\ \div 3 \\ x = 7 \end{array}$ </td> </tr> </table>	<p>Jenny's solution</p> $\begin{array}{r} 3(x - 2) = 15 \\ \div 3 \quad \div 3 \\ x - 2 = 5 \\ \div 2 \quad \div 2 \\ x = 7 \end{array}$	<p>Lenny's solution</p> $\begin{array}{r} 3(x - 2) = 15 \\ \text{Multiplying out brackets} \\ 3x - 6 = 15 \\ \div 3 \\ 3x = 21 \\ \div 3 \\ x = 7 \end{array}$	<p>KM: Spiders and snakes. The example is for an unknown on both sides but the same idea can be used.</p> <p>NRICH: Inspector Remorse</p> <p>NRICH: Quince, quonce, quance</p> <p>NRICH: Weighing the baby</p> <p>Learning review</p> <p>www.diagnosticquestions.com</p>	<ul style="list-style-type: none"> Some pupils may think that equations always need to be presented in the form $ax + b = c$ rather than $c = ax + b$. Some pupils may think that the solution to an equation is always positive and/or a whole number. Some pupils may get the use the inverse operations in the wrong order, for example, to solve $2x + 18 = 38$ the pupils divide by 2 first and then subtract 18.
<p>Jenny's solution</p> $\begin{array}{r} 3(x - 2) = 15 \\ \div 3 \quad \div 3 \\ x - 2 = 5 \\ \div 2 \quad \div 2 \\ x = 7 \end{array}$	<p>Lenny's solution</p> $\begin{array}{r} 3(x - 2) = 15 \\ \text{Multiplying out brackets} \\ 3x - 6 = 15 \\ \div 3 \\ 3x = 21 \\ \div 3 \\ x = 7 \end{array}$			






KNOWLEDGE	The Big Picture: Measurement and mensuration progression map
<ul style="list-style-type: none"> know and apply formulae to calculate area of triangles, parallelograms, trapezia calculate surface area of cuboids know and apply formulae to calculate volume of cuboids understand and use standard mathematical formulae 	

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SKILLS	SUGGESTED RESOURCES
<ul style="list-style-type: none"> Develop knowledge of area Investigate surface area Explore volume Recognise that the value of the perimeter can equal the value of area Use standard formulae for area and volume Find missing lengths in 2D shapes when the area is known Know that the area of a trapezium is given by the formula $\text{area} = \frac{1}{2} \times (a + b) \times h = \frac{(a+b)h}{2}$ Calculate the area of a trapezium Understand the meaning of surface area Find the surface area of cuboids (including cubes) when lengths are known Find missing lengths in 3D shapes when the volume or surface area is known 	<ul style="list-style-type: none">

Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> Understand the meaning of area, perimeter, volume and capacity Know how to calculate areas of rectangles, parallelograms and triangles using the standard formulae Know that the area of a triangle is given by the formula $\text{area} = \frac{1}{2} \times \text{base} \times \text{height} = \text{base} \times \text{height} \div 2 = \frac{bh}{2}$ Know appropriate metric units for measuring area and volume <p>Bring on the Maths': Moving on up! Measures: #4, #5, #6</p>	Perimeter, area, volume, capacity, surface area Square, rectangle, parallelogram, triangle, trapezium (trapezia) Polygon Cube, cuboid Square 19centimeter, square 19centimeter, square metre, square 19centimetre Cubic 19centimeter, 19centimeter cube Formula, formulae Length, breadth, depth, height, width Notation Abbreviations of units in the metric system: km, m, cm, mm, mm ² , cm ² , m ² , km ² , mm ³ , cm ³ , km ³	Ensure that pupils make connections with the area and volume work in Stage 6 and below, in particular the importance of the perpendicular height. NCETM: Glossary Common approaches <i>Pupils have already derived the formula for the area of a parallelogram. They use this to derive the formula for the area of a trapezium as $\frac{(a+b)h}{2}$ by copying and rotating a trapezium as shown above.</i>  <i>Pupils use the area of a triangle as given by the formula $\text{area} = \frac{bh}{2}$.</i> Every classroom has a set of area posters on the wall.

Reasoning opportunities and probing questions	Suggested resources	Possible misconceptions
<ul style="list-style-type: none"> Always / Sometimes / Never: The value of the volume of a cuboid is greater than the value of the surface area Convince me that the area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height} = \text{base} \times \text{height} \div 2 = \frac{bh}{2}$ (Given a right-angled trapezium with base labelled 8 cm, height 5 cm, top 6 cm) Kenny uses the formula for the area of a trapezium and Benny splits the shape into a rectangle and a triangle. What would you do? Why? 	KM: Equable shapes (for both 2D and 3D shapes) KM: Triangle takeaway KM: Surface area KM: Class of rice KM: Stick on the Maths: Area and Volume KM: Maths to Infinity: Area and Volume NRICH: Can They Be Equal? Learning review www.diagnosticquestions.com	<ul style="list-style-type: none"> Some pupils may use the sloping height when finding the areas of parallelograms, triangles and trapezia Some pupils may think that the area of a triangle is found using $\text{area} = \text{base} \times \text{height}$ Some pupils may think that you multiply all the numbers to find the area of a shape Some pupils may confuse the concepts of surface area and volume Some pupils may only find the area of the three 'distinct' faces when finding surface area



Checking, approximating and estimating	Stage 8	3 hours
KNOWLEDGE <ul style="list-style-type: none"> round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures) estimate answers; check calculations using approximation and estimation, including answers obtained using technology recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions) (ALSO 6) use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy (NEW) 		The Big Picture: Number and Place Value progression map

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SKILLS	SUGGESTED RESOURCES
<ul style="list-style-type: none"> Explore ways of approximating numbers Explore ways of checking answers Approximate by rounding to any number of decimal places Know how to identify the first significant figure in any number Approximate by rounding to the first significant figure in any number Understand estimating as the process of finding a rough value of an answer or calculation Use estimation to predict the order of magnitude of the solution to a (decimal) calculation Estimate calculations by rounding numbers to one significant figure Use cancellation to simplify calculations Use inverse operations to check solutions to calculations 	<ul style="list-style-type: none">

Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> Approximate any number by rounding to the nearest 10, 100 or 1000, 10 000, 100 000 or 1 000 000 Approximate any number with one or two decimal places by rounding to the nearest whole number Approximate any number with two decimal places by rounding to the one decimal place Simplify a fraction by cancelling common factors 	Approximate (noun and verb) Round Decimal place Check Solution Answer Estimate (noun and verb) Order of magnitude Accurate, Accuracy Significant figure Cancel Inverse Operation Notation The approximately equal symbol (\approx) Significant figure is abbreviated to 's.f.' or 'sig fig'	This unit is an opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer. Pupils should be able to estimate calculations involving integers and decimals. Also see big pictures: Calculation progression map and Fractions, decimals and percentages progression map NCETM: Glossary Common approaches <i>All pupils are taught to visualise rounding through the use a number line</i>

Reasoning opportunities and probing questions	Suggested resources	Possible misconceptions
<ul style="list-style-type: none"> Convince me that 39 652 rounds to 40 000 to one significant figure Convince me that 0.6427 does <u>not</u> round to 1 to one significant figure What is wrong: $\frac{11 \times 28.2}{0.54} \approx \frac{10 \times 30}{0.5} = 150$. How can you correct it? 	KM: Approximating calculations KM: Stick on the Maths: CALC6: Checking solutions Learning review KM: 7M6 BAM Task	<ul style="list-style-type: none"> Some pupils may truncate instead of round Some pupils may round down at the half way point, rather than round up. Some pupils may think that a number between 0 and 1 rounds to 0 or 1 to one significant figure Some pupils may divide by 2 when the denominator of an estimated calculation is 0.5



Mathematical movement

Stage 8

6 hours

KNOWLEDGE	The Big Picture: Position and direction progression map
<ul style="list-style-type: none"> work with coordinates in all four quadrants understand and use lines parallel to the axes, $y = x$ and $y = -x$ solve geometrical problems on coordinate axes identify, describe and construct congruent shapes including on coordinate axes, by considering rotation, reflection and translation describe translations as 2D vectors plot graphs of equations that correspond to straight-line graphs in the coordinate plane. (NEW) 	

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SKILLS	SUGGESTED RESOURCES	
<ul style="list-style-type: none"> Explore lines on the coordinate grid Use transformations to move shapes Describe transformations Write the equation of a line parallel to the x-axis or the y-axis Draw a line parallel to the x-axis or the y-axis given its equation Identify the lines $y = x$ and $y = -x$ Draw the lines $y = x$ and $y = -x$ Carry out a reflection in a diagonal mirror line (45° from horizontal) Find and name the equation of the mirror line for a given reflection Describe a translation as a 2D vector Understand the concept and language of rotations Carry out a rotation using a given angle, direction and centre of rotation Describe a rotation using mathematical language 	<ul style="list-style-type: none"> 	
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> Work with coordinates in all four quadrants Carry out a reflection in a given vertical or horizontal mirror line Carry out a translation <p>Bring on the Maths*: Moving on up! Position and direction: #1, #2</p>	(Cartesian) coordinates Axis, axes, x-axis, y-axis Origin Quadrant Translation, Reflection, Rotation Transformation Object, Image Congruent, congruence Mirror line Vector Centre of rotation Notation Cartesian coordinates should be separated by a comma and enclosed in brackets (x, y) Vector notation $\begin{pmatrix} a \\ b \end{pmatrix}$ where a = movement right and b = movement up	Pupils should be able to use a centre of rotation that is outside, inside, or on the edge of the object Pupils should be encouraged to see the line $x = a$ as the complete (and infinite) set of points such that the x-coordinate is a. The French mathematician Rene Descartes introduced Cartesian coordinates in the 17 th century. It is said that he thought of the idea while watching a fly moving around on his bedroom ceiling. NCETM: Glossary Common approaches <i>Pupils use ICT to explore these transformations</i> Teachers do not use the phrase 'along the corridor and up the stairs' as it can encourage a mentality of only working in the first quadrant. Later, pupils will have to use coordinates in all four quadrants. A more helpful way to remember the order of coordinates is 'x is a cross, wise up!' Teachers use the language 'negative number', and not 'minus number', to avoid future confusion with calculations.
Reasoning opportunities and probing questions	Suggested resources	Possible misconceptions



- Always / Sometimes / Never: The centre of rotation is in the centre of the object
- Convince me that $y = 0$ is the x-axis
- Always / Sometimes / Never: The line $x = a$ is parallel to the x-axis

KM: [Lines](#)
 KM: [Moving house](#)
 KM: [Autograph transformations](#)
 KM: [Stick on the Maths SSM7: Transformations](#)
 NRICH: [Transformation Game](#)

Learning review
 KM: [7M11 BAM Task](#)

- Some pupils will wrestle with the idea that a line $x = a$ is parallel to the y-axis
- When describing or carrying out a translation, some pupils may count the squares between the two shapes rather than the squares that describe the movement between the two shapes.
- When reflecting a shape in a diagonal mirror line some students may draw a translation
- Some pupils may think that the centre of rotation is always in the centre of the shape
- Some pupils will confuse the order of x- and y-coordinates
- When constructing axes, some pupils may not realise the importance of equal divisions on the axes



KNOWLEDGE	The Big Picture: Probability progression map
<ul style="list-style-type: none"> relative expected frequencies to theoretical probability, using appropriate language and the 0 - 1 probability scale (ALSO 6) record describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees construct theoretical possibility spaces for single experiments with equally likely outcomes and use these to calculate theoretical probabilities apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments 	

SKILLS	SUGGESTED RESOURCES
<ul style="list-style-type: none"> Understand the meaning of probability Explore experiments and outcomes Develop understanding of probability Know that probability is a way of measuring likeliness Know and use the vocabulary of probability Understand the use of the 0-1 scale to measure probability Assess likeliness and place events on a probability scale List all the outcomes for an experiment Identify equally likely outcomes Work out theoretical probabilities for events with equally likely outcomes Know how to represent a probability Recognise when it is not possible to work out a theoretical probability for an event Know that the sum of probabilities for all outcomes is 1 Apply the fact that the sum of probabilities for all outcomes is 1 	<ul style="list-style-type: none">

Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> Understand the equivalence between fractions, decimals and percentages Compare fractions, decimals or percentages Simplify a fraction by cancelling common factors 	Probability, Theoretical probability Event Outcome Impossible, Unlikely, Evens chance, Likely, Certain Equally likely Mutually exclusive Exhaustive Possibility space Experiment Notation Probabilities are expressed as fractions, decimals or percentage. They should not be expressed as ratios (which represent odds) or as words	It is not immediately apparent how to use words to label the middle of the probability scale. 'Evens chance' is a common way to do so, although this can be misleading as it could be argued that there is an even chance of obtaining any number when rolling a fair die. NRICH: Introducing probability NRICH: Why Do People Find Probability Unintuitive and Difficult? NCETM: Glossary Common approaches <i>Every classroom has a display of a probability scale labeled with words and numbers. Pupils create events and outcomes that are placed on this scale.</i>

Reasoning opportunities and probing questions	Suggested resources	Possible misconceptions
<ul style="list-style-type: none"> Show me an example of an event and outcome with a probability of 0. And another. And another... Always / Sometimes / Never: if I pick a card from a pack of playing cards then the probability of picking a club is $\frac{1}{4}$ Label this (eight-sided) spinner so that the probability of scoring a 2 is $\frac{1}{4}$. 	KM: Probability scale and slideshow version KM: Probability loop cards NRICH: Dice and spinners interactive Learning review www.diagnosticquestions.com	<ul style="list-style-type: none"> Some pupils will initially think that, for example, the probability of it raining tomorrow is $\frac{1}{2}$ as it either will or it won't. Some students may write a probability as odds (e.g. 1:6 or '1 to 6'). There is a difference between probability and odds, and therefore probabilities must only be written as fractions, decimals or percentages. Some pupils may think that, for example, if they flip a fair coin three times and obtain three heads, then it must be more than likely they will obtain a head next.



Presentation of data	Stage 8	6 hours
KNOWLEDGE		The Big Picture: Statistics progression map
<ul style="list-style-type: none"> interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data and know their appropriate use 		

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SKILLS	SUGGESTED RESOURCES
<ul style="list-style-type: none"> Explore types of data Construct and interpret graphs Select appropriate graphs and charts Know the meaning of categorical data Know the meaning of discrete data Interpret and construct frequency tables Construct and interpret pictograms (bar charts, tables) and know their appropriate use Construct and interpret comparative bar charts Interpret pie charts and know their appropriate use Construct pie charts when the total frequency is not a factor of 360 Choose appropriate graphs or charts to represent data Construct and interpret vertical line charts 	<ul style="list-style-type: none">

Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> Construct and interpret a pictogram Construct and interpret a bar chart Construct and interpret a line graph Understand that pie charts are used to show proportions Use a template to construct a pie chart by scaling frequencies <p>Bring on the Maths': Moving on up! Statistics: #1, #2, #3</p>	Data, Categorical data, Discrete data Pictogram, Symbol, Key Frequency Table, Frequency table Tally Bar chart Time graph, Time series Bar-line graph, Vertical line chart Scale, Graph Axis, axes Line graph Pie chart Sector Angle Maximum, minimum Notation When tallying, groups of five are created by striking through each group of four	In stage 6 pupils constructed pie charts when the total of frequencies is a factor of 360. More complex cases can now be introduced. Much of the content of this unit has been covered previously in different stages. This is an opportunity to bring together the full range of skills encountered up to this point, and to develop a more refined understanding of usage and vocabulary. William Playfair, a Scottish engineer and economist, introduced the bar chart and line graph in 1786. He also introduced the pie chart in 1801. NCETM: Glossary Common approaches <i>Pie charts are constructed by calculating the angle for each section by dividing 360 by the total frequency and not using percentages. The angle for the first section is measured from a vertical radius. Subsequent sections are measured using the boundary line of the previous section.</i>

Reasoning opportunities and probing questions	Suggested resources	Possible misconceptions
<ul style="list-style-type: none"> Show me a pie chart representing the following information: Blue (30%), Red (50%), Yellow (the rest). And another. And another. Always / Sometimes / Never: Bar charts are vertical Always / Sometimes / Never: Bar charts, pie charts, pictograms and vertical line charts can be used to represent any data Kenny says 'If two pie charts have the same section then the amount of data the section represents is the same in each pie chart.' Do you agree with Kenny? Explain your answer. 	KM: Maths to Infinity: Averages, Charts and Tables NRICH: Picturing the World NRICH: Charting Success Learning review www.diagnosticquestions.com	<ul style="list-style-type: none"> Some pupils may think that a line graph is appropriate for discrete data Some pupils may think that each square on the grid used represents one unit Some pupils may confuse the fact that the sections of the pie chart total 100% and 360° Some pupils may not leave gaps between the bars of a bar chart



Measuring data

Stage 8

3 hours

KNOWLEDGE

The Big Picture: [Statistics progression map](#)

- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (median, mean and mode) and spread (range)

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SKILLS	SUGGESTED RESOURCES	
<ul style="list-style-type: none"> Investigate averages Explore ways of summarising data Analyse and compare sets of data Understand the mode and median as measures of typicality (or location) Find the mode of set of data Find the median of a set of data Find the median of a set of data when there are an even number of numbers in the data set Use the mean to find a missing number in a set of data Calculate the mean from a frequency table Find the mode from a frequency table Find the median from a frequency table Understand the range as a measure of spread (or consistency) Calculate the range of a set of data Analyse and compare sets of data Appreciate the limitations of different statistics (mean, median, mode, range) 	<ul style="list-style-type: none"> 	
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> Understand the meaning of 'average' as a typicality (or location) Calculate the mean of a set of data <p>Bring on the Maths!: Moving on up! Statistics: #4</p>	Average Spread Consistency Mean Median Mode Range Measure Data Statistic Statistics Approximate Round	<p>The word 'average' is often used synonymously with the mean, but it is only one type of average. In fact, there are several different types of mean (the one in this unit properly being named as the 'arithmetic mean'). NCETM: Glossary</p> <p>Common approaches <i>Every classroom has a set of statistics posters on the wall</i> <i>Always use brackets when writing out the calculation for a mean, e.g. $(2 + 3 + 4 + 5) \div 4 = 14 \div 4 = 3.5$</i></p>
Reasoning opportunities and probing questions	Suggested resources	Possible misconceptions
<ul style="list-style-type: none"> Show me a set of data with a mean (mode, median, range) of 5. Always / Sometimes / Never: The mean is greater than the mode for a set of data Always / Sometimes / Never: The mean is greater than the median for a set of data Convince me that a set of data could have more than one mode. What's the same and what's different: mean, mode, median, range? 	KM: Maths to Infinity: Averages KM: Maths to Infinity: Averages, Charts and Tables KM: Stick on the Maths HD4: Averages NRICH: M, M and M NRICH: The Wisdom of the Crowd Learning review www.diagnosticquestions.com	<ul style="list-style-type: none"> If using a calculator some pupils may not use the '=' symbol (or brackets) correctly; e.g. working out the mean of 2, 3, 4 and 5 as $2 + 3 + 4 + 5 \div 4 = 10.25$. Some pupils may think that the range is a type of average Some pupils may think that a set of data with an even number of items has two values for the median, e.g. 2, 4, 5, 6, 7, 8 has a median of 5 and 6 rather than 5.5 Some pupils may not write the data in order before finding the median.

