**Factorising expressions**

**A LEVEL LINKS**

**Scheme of work:** 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

* Factorising an expression is the opposite of expanding the brackets.
* A quadratic expression is in the form *ax*2 + *bx* + *c*, where *a* ≠ 0.
* To factorise a quadratic equation find two numbers whose sum is *b* and whose product is *ac*.
* An expression in the form *x*2 – *y*2 is called the difference of two squares. It factorises to (*x* – *y*)(*x* + *y*).

Examples

**Example 1** Factorise 15*x*2*y*3 + 9*x*4*y*

|  |  |
| --- | --- |
| 15*x*2*y*3 + 9*x*4*y* = 3*x*2*y*(5*y*2 + 3*x*2) | The highest common factor is 3*x*2*y*. So take 3*x*2*y* outside the brackets and then divide each term by 3*x*2*y* to find the terms in the brackets |

**Example 2** Factorise 4*x*2 – 25*y*2

|  |  |
| --- | --- |
| 4*x*2 – 25*y*2 = (2*x* + 5*y*)(2*x* − 5*y*) | This is the difference of two squares as the two terms can be written as (2*x*)2and (5*y*)2 |

**Example 3** Factorise *x*2 + 3*x* – 10

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| --- | --- |
| *b* = 3, *ac* = −10  So *x*2 + 3*x* – 10 = *x*2 + 5*x* – 2*x* – 10  = *x*(*x* + 5) – 2(*x* + 5)  = (*x* + 5)(*x* – 2) | **1** Work out the two factors of *ac*= −10 which add to give *b*= 3  (5 and −2)  **2** Rewrite the *b* term (3*x*) using these two factors  **3** Factorise the first two terms and the last two terms  **4** (*x* + 5) is a factor of both terms |

**Example 4** Factorise 6*x*2 − 11*x* − 10

|  |  |
| --- | --- |
| *b* = −11, *ac* = −60  So  6*x*2 − 11*x* – 10 =6*x*2 − 15*x* + 4*x* – 10  = 3*x*(2*x* − 5) + 2(2*x* − 5)  = (2*x* – 5)(3*x* + 2) | **1** Work out the two factors of *ac*= −60 which add to give *b*= −11 (−15 and 4)  **2** Rewrite the *b* term (−11*x*) using these two factors  **3** Factorise the first two terms and the last two terms  **4** (2*x* − 5) is a factor of both terms |

**Example 5** Simplify 

|  |  |
| --- | --- |
| For the numerator:  *b* = −4, *ac* = −21  So  *x*2 − 4*x* – 21 = *x*2 − 7*x* + 3*x* – 21  = *x*(*x* − 7) + 3(*x* − 7)  = (*x* – 7)(*x* + 3)  For the denominator:  *b* = 9, *ac* = 18  So  2*x*2 + 9*x* + 9 = 2*x*2 + 6*x* + 3*x* + 9  = 2*x*(*x* + 3) + 3(*x* + 3)  = (*x* + 3)(2*x* + 3)  So    = | **1** Factorise the numerator and the denominator  **2** Work out the two factors of *ac*= −21 which add to give *b*= −4 (−7 and 3)  **3** Rewrite the *b* term (−4*x*) using these two factors  **4** Factorise the first two terms and the last two terms  **5** (*x* − 7) is a factor of both terms  **6** Work out the two factors of  *ac*= 18 which add to give *b*= 9  (6 and 3)  **7** Rewrite the *b* term (9*x*) using these two factors  **8** Factorise the first two terms and the last two terms  **9** (*x* + 3) is a factor of both terms  **10** (*x* + 3) is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1 |

Practice

**1** Factorise.

**Hint**

Take the highest common factor outside the bracket.

**a** 6*x*4*y*3 – 10*x*3*y*4 **b** 21*a*3*b*5 + 35*a*5*b*2

**c** 25*x*2*y*2 – 10*x*3*y*2 + 15*x*2*y*3

**2** Factorise

**a** *x*2 + 7*x* + 12 **b** *x*2 + 5*x* – 14

**c** *x*2 – 11*x* + 30 **d** *x*2 – 5*x* – 24

**e** *x*2 – 7*x* – 18 **f** *x*2 + *x* –20

**g** *x*2 – 3*x* – 40 **h** *x*2 + 3*x* – 28

**3** Factorise

**a** 36*x*2 – 49*y*2 **b** 4*x*2 – 81*y*2

**c** 18*a*2 – 200*b*2*c*2

**4** Factorise

**a** 2*x*2 + *x* –3 **b** 6*x*2 + 17*x* + 5

**c** 2*x*2 + 7*x* + 3 **d** 9*x*2 – 15*x* + 4

**e** 10*x*2 + 21*x* + 9 **f** 12*x*2 – 38*x* + 20

**5** Simplify the algebraic fractions.

**a**  **b** 

**c**  **d** 

**e**  **f** 

**6** Simplify

**a**  **b** 

**c**  **d** 

# **Extend**

**7** Simplify 

**8** Simplify 

Answers

**1 a** 2*x*3*y*3(3*x* – 5*y*) **b** 7*a*3*b*2(3*b*3 + 5*a*2)

**c** 5*x*2*y*2(5 – 2*x* + 3*y*)

**2 a** (*x* + 3)(*x* + 4) **b** (*x* + 7)(*x* – 2)

**c** (*x* – 5)(*x* – 6) **d** (*x* – 8)(*x* + 3)

**e** (*x* – 9)(*x* + 2) **f** (*x* + 5)(*x* – 4)

**g** (*x* – 8)(*x* + 5) **h** (*x* + 7)(*x* – 4)

**3 a** (6*x* – 7*y*)(6*x* + 7*y*) **b** (2*x* – 9*y*)(2*x* + 9*y*)

**c** 2(3*a* – 10*bc*)(3*a* + 10*bc*)

**4 a** (*x* – 1)(2*x* + 3) **b** (3*x* + 1)(2*x* + 5)

**c** (2*x* + 1)(*x* + 3) **d** (3*x* – 1)(3*x* – 4)

**e** (5*x* + 3)(2*x* +3) **f** 2(3*x* – 2)(2*x* –5)

**5 a**  **b** 

**c**  **d** 

**e**  **f** 

**6 a**  **b** 

**c**  **d** 

**7** (*x* + 5)

**8** 