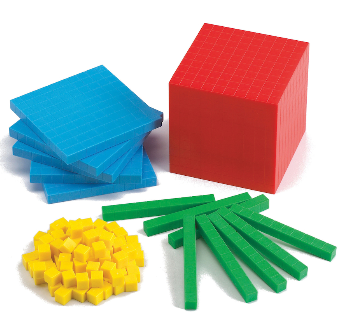
Clarendon Junior School

Agreed Approach to Calculation Document

Core Manipulatives to support depth learning from Year 3 to Year 6.



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| **Yr** | **Addition Strategies** | **Concrete** | **Pictorial/Structural** | **Abstract** |
| 3 | Column method including regrouping up to 3 digit numbers including tenths | **Using regrouping:**    **Using the expanded column method:**    Begin in the ones column and move to the left.  **Using Tenths:**  As children move on to decimals and money, decimal place value counters can be used to support learning. | **Using regrouping:**      **Using the expanded column method:**    This informal representation is used to clarify understanding and can be used alongside number lines.  The bar model and part part whole model reinforces the concept of parts and wholes.      **Mentally counting on in multiples of 100, 10 and 1**  386 + 57 386 + 50 + 7 436 + 7 = 443  +50 +7  386 436 443  It will also aid fluency in mental calculations. | **Using the expanded column method:**  Expanded vertical method using the least significant numbers first to add up to 3 digit numbers bridging 10 and 100.  **H T O**  2 5 6  + 1 3 7   1. 3 (6+7) Ones   8 0 (50 + 30) Tens  3 0 0 (200 + 100) Hundreds  3 9 3  Children must always show place value headings. |
| 4 | Column method increasing in place value to 4 digit numbers | **Column Addition with Carrying:**  Recap on use of place value counters and Base 10 to show ‘carrying’.      Begin in the ones column. For every ten created exchange for a 10 counter.  As children move on to decimals and money, decimal place value counters can be used to support learning. | **Column Addition with Carrying:**      This informal representation is used to clarity understanding and can be used alongside number lines.  **Worded problems supported by the bar model or part part whole:**  There were 6000 books for sale at a book fair. 3419 books were sold on the first day of the fair and 2268 books were sold on the second day. How many books were left at the end of the second day?   |  |  |  | | --- | --- | --- | | 6000 | | | | 3419 | 2268 | ? |   6000  2268  3419  ?  To promote fluency, number lines can be used for addition of decimals.  +7 +0.4 +0.09  46.34 53.34 53.74 53.83 | **Column Addition with Carrying:**  Start by using the expanded method before moving on to clearly show carrying below the addition.  **H T O H T O**  2 5 6 2 5 6  + 1 3 7 + 1 3 7   1. 3 3 9 3   8 0 1  3 0 0  3 9 3  As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.  Use the inverse to check answers. |
| 5 | Column method increasing in place value to 5 and 6 digit numbers including those with up to two decimal places  Mental methods for addition |  | Use written methods to support record and explain calculations, achieving consistent accuracy.  Discuss, explain and compare methods.  **Mental Methods**  Add numbers mentally up to 1000 and beyond.  e.g : 12462 + 2300  Compensation:  Add too much and take some off:  +300  -14  654 940 954  Develop expanded horizontal method (least significant numbers first)  7587 + 675  7000 + 500 + 80 + 7  600 + 70 + 5  7000 + 1100 + 150 + 12 | Develop an efficient standard method that can be applied generally using whole numbers and those with decimal places.  For example:  3 6 6 . 7  + 8 5 . 4  4 5 2 . 1  1 1 1 |
| 6 | Column method increasing in place value to 5 and 6 digit numbers including those with up to two decimal places  Mental methods for addition |  | **Mental Methods:**  Blank number line to support with larger numbers:  e.g: 309,997 + 24  + 4 +20  309,997 310,001 310,021  Continue using compensation (add too much and take off)  +3000  -6  6467 9461 9467 | Continue developing efficiency of written methods using carrying.  Extend methods to numbers with any number of digits.  2 3 . 3 6 1  9 . 0 8 0  5 9 . 7 7 0  + 1 . 3 0 0  9 3 . 5 1 1  2 1 2  Add several numbers with different numbers of digits.  For example:  Find the total of:  42 6432 786 3 and 4681  Extend to decimals, add two or more decimal fractions with up to four digits and either one or two decimal places.  124.9 + 7.25  401.2 + 26.85 + 0.71  Add fractions with different denominations and mixed numbers, using the concept of equivalent fractions. |

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| **Yr** | **Subtraction Strategies** | **Concrete** | **Pictorial/Structural** | **Abstract** |
| 3 | Column method without grouping | **Using Base 10:**    234 - 88    Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.  Now I can subtract my ones.    Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Now I can take away eight tens and complete my subtraction.    Show children how the concrete method links to the written method alongside your working. Cross out the numbers with exchanging and show where we write our new amount. | Draw the Base 10 or place value counters alongside the written calculation to help to show working.    400 40 3 –  100 20 2  300 20 1 = 321  **Using Place Value Blocks:** | Expanded column method to condensed.  300 70 5 –  100 20 4  200 50 1  = 251  Then…  3 7 5 -  1 2 4  2 5 1 |
| 4 | Column method with grouping | Use Cuisenaire to start with before moving on to place value counters. Start with one exchange before moving one subtractions with 2 exchanges.  Make the larger number with the place value counters.  234 - 88    Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.  Now I can subtract my ones.    Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.  Now I can take away eight tens and complete my subtraction.    Show children how the concrete method links to the written method alongside your working. Cross out the numbers with exchanging and show where we write our new amount. | **Using Base 10:** | Recap expanded to condensed.  47 – 23 =  T O TO  40 7 - 47 -  20 3 23  20 4 24  **Example:**      **1~~2~~ 13 8 -**  **1 4 6**  **9 2** |
| 5 | To use decomposition with efficiency when exchanging |  | Use place value counters to show exchanging of amounts including numbers up to 2 decimal places.. | Revise exchanging of amounts in formal written form including numbers up to 2 decimal places.  Use variation of measures and money. |
| 6 |  | To recap Year 5 to ensure efficiency with any size of digit. | | |

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| **Yr** | **Multiplication Strategies** | **Concrete** | | **Pictorial/Structural** | | **Abstract** |
| 3 | Using arrays to show commutative multiplication  Using arrays to lead to grid method | Create arrays using counters/cubes to show multiplication sentences.          **Grid Method:**  Show the link with arrays first to introduce the grid method.  Use counters to show arrays:    Move on to Base 10 to move towards a more compact method.    Use place value counters to show how to find groups of a number.  For example:  126 x 4  “We are multiplying by 4 so we need 4 rows.”   |  |  |  | | --- | --- | --- | | H | T | O | |  |  |  | |  |  |  | |  |  |  | |  |  |  |   Fill each row with 126.  Add up each column, starting with the ones, carrying where needed. Then you have your answer. | | Draw arrays to represent multiplication sentences.  Use arrays in different orientations to find commutative multiplication sentences.    3 x 6 6 x 3  Link arrays to the area of rectangles.    **Grid Method:**  ITP array to support the introduction of grid TU x U    Children can draw the counters, using different colours for different amounts or just circles within the place value columns to show their thinking as shown below:  **Use the bar model:**   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | 35 | | | | | | | | 5 | 5 | 5 | 5 | 5 | 5 | 5 |   7 x 5 = 35  5 x 7 = 35  35 ÷ 5 = 7  35 ÷ 7 = 5 | | Use an array to write multiplication sentences and reinforce repeated addition.    3 x 4 = 12  4 x 3 = 12  3 + 3 + 3 + 3 = 12  4 + 4 + 4 = 12  Empty box sentences using known facts.  3 x \_\_ = 12  **Grid Method:**  Introduce the grid method TU x U using multiplication tables that they know:  20 8  X   |  |  | | --- | --- | | 80 | 32 |   4  80 + 32 = 114  Or broken down further:  X 10 10 8   |  |  |  | | --- | --- | --- | | 40 | 40 | 32 |     4  40 + 40 + 32 = 114  Move forward, multiply TU x TU showing different rows within the grid method.  18 x 13   |  |  |  | | --- | --- | --- | | **X** | **10** | **8** | | **10** | 100 | 80 | | **3** | 30 | 24 |   100 + 80 + 30 + 24 = 234 |
| 4 | Column Multiplication | Children continue to use place value counters at this stage of calculation.    It is important that they always multiply the ones first, note the answers and follow with the tens that they note below. | | Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.   |  |  |  |  | | --- | --- | --- | --- | | ? | | | | | 23 | 23 | 23 | 23 |   Develop fluency through variation:  2 x 3 =  2 x 30 =  2 x 300 =  20 x 3 =  200 x 3 = | | Start with long multiplication, reminding the children about lining up their numbers clearly in columns.  Children to write out the calculation that they are solving next to their answer.  23 x 4    2 3  X 4  1 2 (3 x 4)  8 0 (20 x 4)  9 2  Move to short multiplication (to include TU x U and HTU x U) |
| 5 | Develop efficiency in short multiplication  Long multiplication for TU x TU |  | | Recap any Year 4 methods/visuals as necessary. | | Recap short multiplication up to 4 digits by 1 digit.  Introduce long multiplication when using TU x TU and increase to HTU and ThHTU by U.  T O  3 2  X 2 4  8 (2 x 4)  1 2 0 (30 x 4)  4 0 (2 x 20)  6 0 0 (20 x 30)  7 6 8 |
| 6 | Develop efficiency in written methods |  |  | | Short multiplication and long multiplication methods revised and developed in terms of accuracy and efficiency.  Extend to multiplying with decimals up to two decimal places. Know that decimal points should line up under each other. | |

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| **Yr** | **Division Strategies** | **Concrete** | **Pictorial/Structural** | **Abstract** |
| 3 | Division as grouping and arrays  Division with a remainder | Use physical objects to show division:    Use tens and ones counters to show division  62 ÷ 2    Use Cuisenaire rods to show repeated groups with an amount:  24 ÷ 3    24 ÷ 8  Divide objects into groups and see how much is left over. | Children continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.    Encourage moving to counting multiples to divide more efficiently.    Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.    Draw dots or shapes and group them to divide an amount and clearly show a remainder.      Jump back on a number and show remainders. | Children become fluent with representing division stories and images with number sentences they can explain:  96 ÷ 3 = 32  “96 sweets were shared between 3 friends. They had 32 sweets each.”    Complete written divisions showing the remainder as “r”. |
| 4 | Short division | Sharing using place value counters:  Short division: | Representing this pictorially:    Short division: | Expressing what is happening using short numerical expressions:    Introduce short division:  Begin with divisors that divide equally with no remainder |
| 5 | Short division with larger numbers |  | Recap pictorial representation as needed. | Build efficiency with short division with larger numbers including with varying remainders:    Express remainders as a fraction. |
| 6 | Long division |  | | Recap short division to ensure efficiency and understanding before moving to long division.  Express remainders as fractions and decimals (only in context) |

To ensure the children gain fluency and competence with each operation, we will provide them with a wide range of procedural and conceptual variation questions that will strengthen and develop the depth of each child’s understanding of each method and process.

**Examples of Conceptual Variation Ideas for Each Operation:**

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| **Addition:** |  |
| **Subtraction:** |  |
| **Multiplication:** |  |
| **Division:** |  |