## Clarendon Federation

## Agreed Approach to Calculation Document

Core Manipulatives to support depth learning from EYFS to Year 6.









Yr	Addition Strategies	Concrete	Pictorial/Structural	Abstract
R	Finding the sum of two numbers. Combining two parts to make a whole: part part whole model.  To know the pairs that total 5.  ELG: To add two single digit numbers and count on to find the answer.	Use cubes, objects, Numicon or 10's frames to add two numbers together verbally.  3 + 2 = 5	Use pictures to add two numbers together. These will be both pictures provided and pictures that the children draw to solve a number sentence.  3 + 2 = 5  2 + 3 = 5	Children will begin record addition number sentences. They will solve addition problems by beginning to count on from the first number.  3 + 2 = 5 Three in your head, two fingers. 3, 4, 5.  4 + 3 = 7 Four is a part, 3 is a part and the w seven.
		5 frames also used		

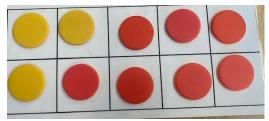
Use a range of manipulatives to add one digit and two digit numbers to 20.

Use double sided counters and 10 frames to investigate number bonds



to 10.

3 + 7 = 10



3 + 7 = 10

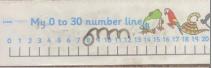
Using cubes and counting on from the biggest number.



I know that I have 8, 9, 10, 11, 12 8 + 4 = 12

Use 10 frames to count on from the biggest number.

PPW model
Missing numbers
Counting on using a
number line starting at the
largest number.



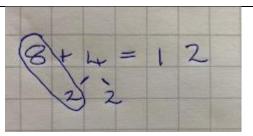
8 + 4 = 12 Counting using pictures

8 + 4 = 12

Start at the larger number on the number line and count on in ones or in jumps of one to find the answer.

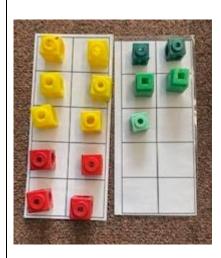
8 + 4 =8 + 2 + 2 =



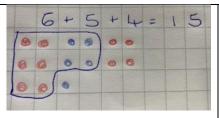


		8 + 4 = 12 Using 10 frames to bridge 10 $8 + 4 = 8 + 2 + 2$	8 + 4 = 1	
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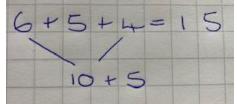
2 Adding three digits



6 + 5 + 4 = 15Adding by manipulating the apparatus to make 10 before adding the ones.

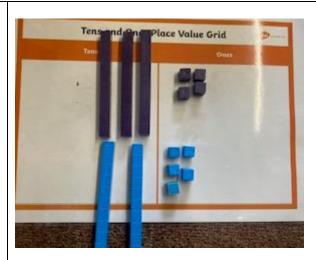


Add three digits by finding 10, partitioning the 5 and recombining. 10 + 5.

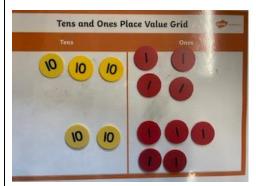


Finding number bonds to 10 before adding the ones and exploring what happens if there isn't a number bond to use.

2 Adding two 2 digit numbers up to 100

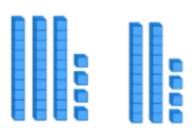


34 + 25 = 59Using base 10 to add ones first and then the tens.

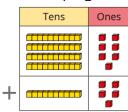


Using different representations of the same number problem.

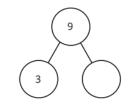
Adding with Base 10: Drawing ten sticks (lines) and ones (small circles).



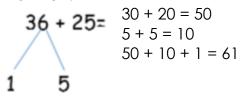
Using base ten to model the carrying of a ten:



Using the part whole model to calculate missing numbers:



Partition:



Or expanded method:

$$30 \quad 6 + 20 \quad 5 \\ 50 + 11 = 61$$

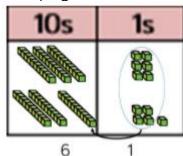
Partition numbers into T U, developing to H T U (support with arrow cards) Eg:

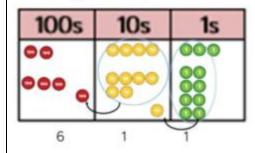
Abstract
method:

e.g £4 and 32p £3 and 24p	This informal representation is used to clarify understanding and can be used alongside number lines.	
	The bar model and part part whole model reinforces the concept of parts and wholes.	
	? 21 34 ? 21 34	
	Mentally counting on in multiples of 100, 10 and 1 386 + 57 → 386 + 50 + 7 → 436 + 7 = 443	
	386 436 443	

4 Column
method
increasing
in place
value to 4
digit
numbers

Column Addition with Carrying: Recap on use of place value counters and Base 10 to show 'carrying'.

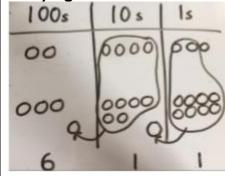


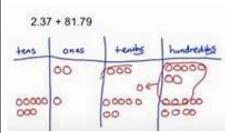


Begin in the ones column. For every ten created exchange for a 10 counter.

As children move on to decimals and money, decimal place value counters can be used to support learning.

Column Addition with Carrying:





This informal representation is used to clarity understanding and can be used alongside number lines.

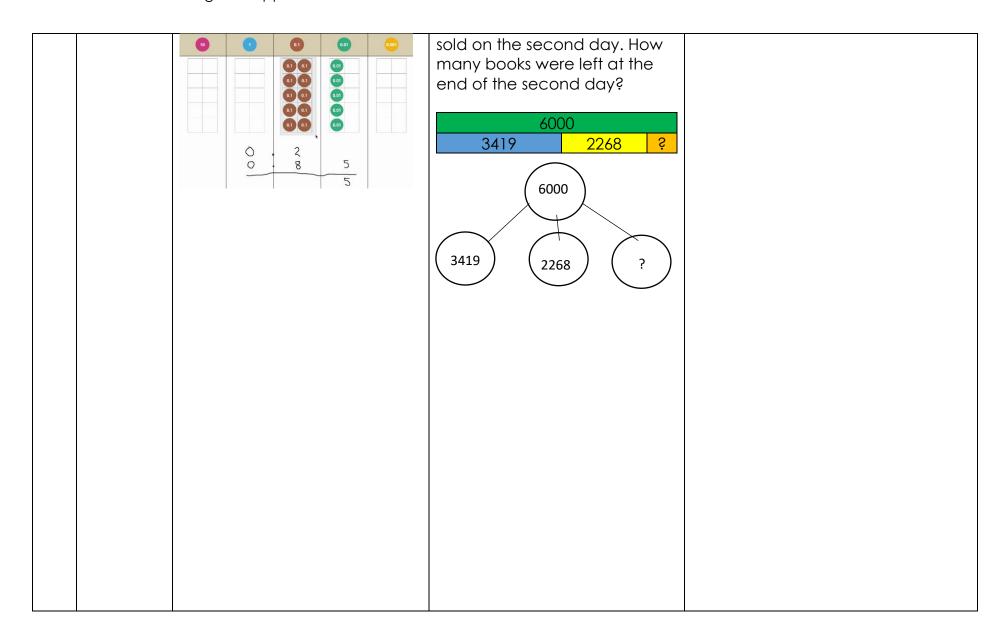
# Worded problems supported by the bar model or part part whole:

There were 6000 books for sale at a book fair. 3419 books were sold on the first day of the fair and 2268 books were **Column Addition with Carrying:** 

As the children move on, introduce decimals with the same number of decimal places and different.

Money can be used here.

Use the inverse to check answers.

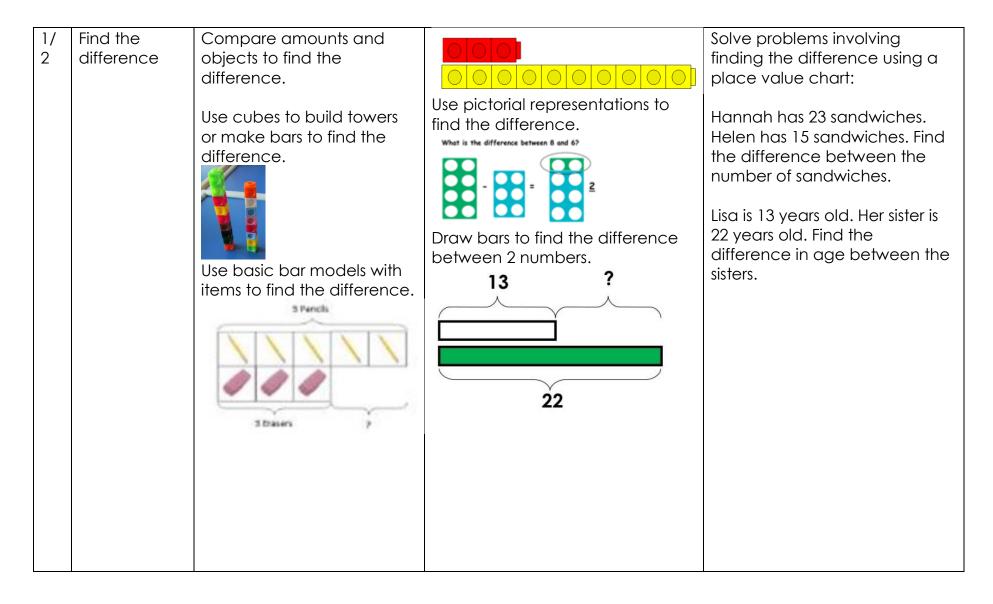


5	Column	Use written methods to support	Develop an efficient standard
	method	record and explain	method that can be applied
	increasing	calculations, achieving consistent accuracy.	generally using whole numbers and those with decimal places.
	in place value to 5	Discuss, explain and compare	mose wiin decimal places.
	and 6	methods.	For example:
	digit	memous.	To example.
	numbers		366.7
	including		+ 8 5 . 4
	those with		452.1
	up to two		111
	decimal		
	places		
	Mental		
	methods		
	for		
	addition		

6 Column method increasir in place value to		Continuation of Y5	Continue developing efficiency of written methods using carrying.  Extend methods to numbers with any number of digits.
and 6 digit numbers including those wi up to tw decimal places	th o		23.361 9.080 59.770 + 1.300 93.511 21 2
Mental methods for addition			Add several numbers with different numbers of digits. For example: Find the total of: 42 6432 786 3 and 4681
			Extend to decimals, add two or more decimal fractions with up to four digits and either one or two decimal places.  124.9 + 7.25 401.2 + 26.85 + 0.71

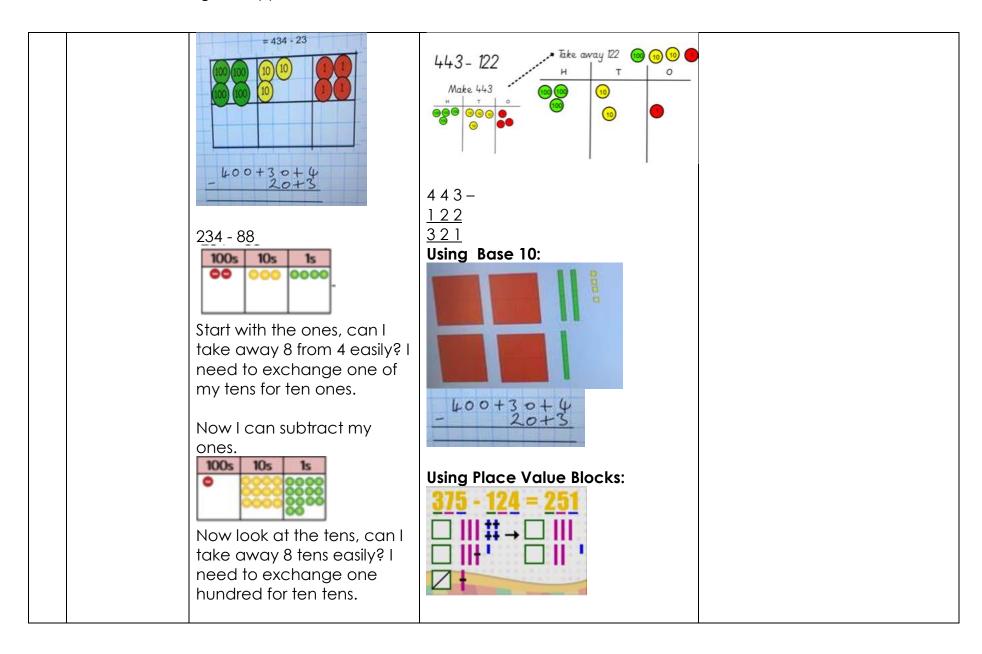
Yr	Subtraction Strategies	Concrete	Pictorial/Structural	Abstract
R/	Taking away ones  ELG: To know 1 more/1 less.  Subtract 2 single digit numbers and count back to find the answer.	Use physical objects, counters, cubes etc to show how objects can be taken away.  6-2=4  Using tens frames and counters:	Cross out drawn objects to show what has been taken away. 8 bees take away 3 bees makes 5 bees.  10-4=6  10-3=7  10-2=8	8-3=58-2=6

1/2	Counting back	Make the larger number in your subtraction. Use counters and move them away from the group as you take them away counting backwards as you go.	Count back on a number line or number track.  0 1 2 3 4 5 6 7 8 9 10  6 - 2 = 4  Start at the bigger number and count back the smaller number showing the jumps on the number line.  Using partitioning to cross boundaries:  Put 13 in your head, count back 4. What number are you at? Use your fingers to help.	Subtract mentally using number bonds.  Explore empty boxes counting back:  13 - ? = 9

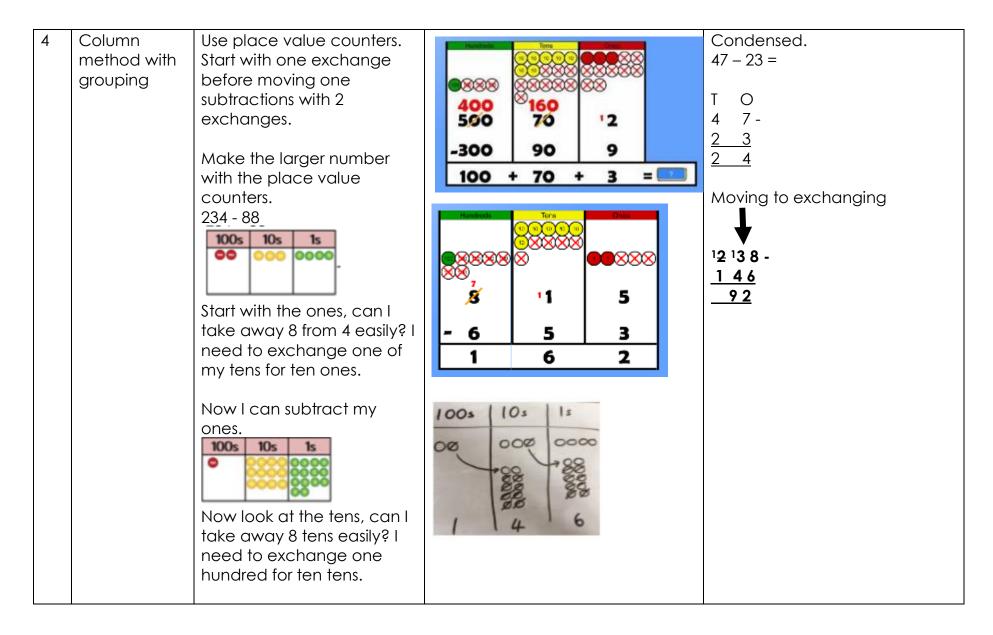


1/2	Part/Part Whole	Link to addition- use the part part whole model to help explain the inverse between addition and subtraction.  If 10 is the whole and 6 is one of the parts. What is the other part?  10 – 6 =	Use pictorial representation of objects or counters to show the part part whole model. For example (this may use counters rather than pictures)	Move to representing pictures in each part and then on to using numbers within the part whole and bar model.  9-3=
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1/2	Making 10	Make 14 on the ten frame. Take away the four first to make 10 and then take away 1 more so you have taken away 5. You are left with the answer of 9.	- 7 =  Exchange ten into ones to model crossing the boundary.	How many do we take off to reach the next 10?  How many do we have left to take off?
2/3	Column method without grouping	Use place value counters and base ten to make the bigger number then take the smaller number away.  Using Base 10:  Show how you partition numbers to subtract. Again make the larger number first.	Draw the base ten or place value counters alongside the written calculation to help to show working.  Calculations  Calculations  Calculations  Calculations  176 - 64  1176  64  112	Expanded column method to condensed.  47 - 23 =  T O TO  40 7 - 23 = 23  20 4 24  300 70 5 - 100 20 4  200 50 1 = 251  Then  3 7 5 - 12 4  2 5 1



Now I can take away tens and complete my subtraction.	
Show children how the concrete method links the written method alongside your working Cross out the numbers	to g. with
exchanging and show where we write our near amount.	



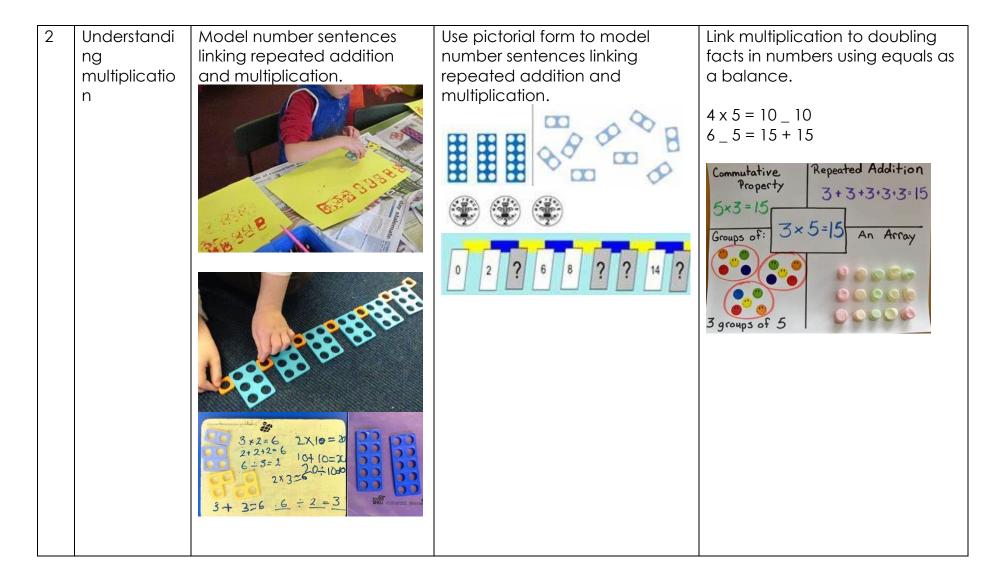
Now I can take away eight Using Base 10: tens and complete my Tens subtraction. 100s 10s subtracting, the I borrowed top number is ten ones from the tens. the most important. Show children how the concrete method links to the written method alongside your working. Cross out the numbers with exchanging and show where we write our new amount.

5	To use decompositio n with efficiency when exchanging		exch	angir	g of	amou	nters to show nts including imal places	Revise exchanging of amounts in formal written form including numbers up to 2 decimal places. Use variation of measures and money.
								3,904 - 1,583 8,000 - 5,648 14,800 - 7,822 3 8 0 4 8 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
6		To recap Year 5 to ensure eff	ficienc	y with	n any	size c	of digit.	

Yr	Multiplicatio n Strategies	Concrete	Pictorial/Structural	Abstract
R	Doubling and finding lots of  ELG: solve problems involving doubling and halving  Tables: 2 and 10	Use practical activities to show how to double a number.  Start off with a tower of 8 cubes  Build a tower the same as the one you have already.  How many do you have altogether now? What is double 8?	Draw pictures to show how to double a number.  Double 4 is 8	Count in multiples of a number aloud using actions.  Using Mastering Number approach- I can see two twos/ two fours.
		Real life problems: One teddy has two buttons, how many buttons will we need for 3 teddies?		

1	Counting in multiples  Tables: 2, 5 and 10	Count in multiples supported by concrete objects in equal groups. Adding equal groups.	Use a number line or pictures to continue to support in counting in multiples.	Write in sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 15, 30  Write addition sentences to describe objects and pictures.
1	Repeated addition	Use different objects to add equal groups.  Using Cuisenaire, multilink, number tracks and bead strings:	Use pictures and number lines to show repeated addition:  There are 5 pairs of socks on the washing line. 5 pairs of 2 = 10 5 2s = 10 2 + 2 + 2 + 2 + 2 = 10 2 x 5 = 10 5 lots of 2 = 10 2 multiplied by 5 5 hops of 2 There are 10 altogether.	Record in number sentences: 5 + 5 + 5 + 5 = 20 2 + 2 + 2 + 2 = 8

1/	Partitioning	Use objects to model	Use place value cards and
2	to double	doubling ten and ones	Numicon to show:
	(pre-grid)	practically.	Tromison to sheve.
	(pro grid)	practically.	10 x 2 =
			10 x 2 = 4 x 2 =
		double 12 = 24	
		12+12=24	□ + □ = □
			+ = =
			+ = =
		12+12=24	
		double 12=24	
		2000 to 12 - 27	
			Double 5 is Double 6 is

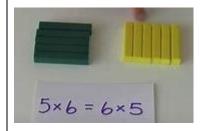


3 Using arrays to show commutati ve multiplicati on

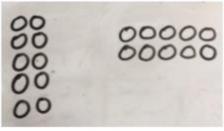
Using arrays to lead to grid method Create arrays using counters/cubes to show multiplication sentences.







Draw arrays to represent multiplication sentences.



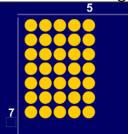
Use arrays in different orientations to find commutative multiplication sentences.



3 x 6 6 x 3

#### **Grid Method:**

ITP array to support the introduction of grid TU x U



Children can draw the counters, using different colours for different

Use an array to write multiplication sentences and reinforce repeated addition.

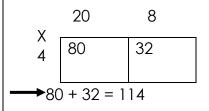


$$3 \times 4 = 12$$
  
 $4 \times 3 = 12$   
 $3 + 3 + 3 + 3 = 12$   
 $4 + 4 + 4 = 12$ 

Empty box sentences using known facts.

#### **Grid Method:**

Introduce the grid method TU x U using multiplication tables that they know to match related facts.





#### Grid Method:

Show the link with arrays first to introduce the grid method.

Use counters to show arrays:

10	8

Move on to Base 10 to move towards a more compact method.



Use place value counters to show how to find groups of a number.

For example:

26 x 4

"We are multiplying by 4 so we need 4 rows."

amounts or just circles within the place value columns to show their thinking as shown below:

#### Use the bar model:

			35			
5	5	5	5	5	5	5

$$7 \times 5 = 35$$

$$5 \times 7 = 35$$

$$35 \div 5 = 7$$

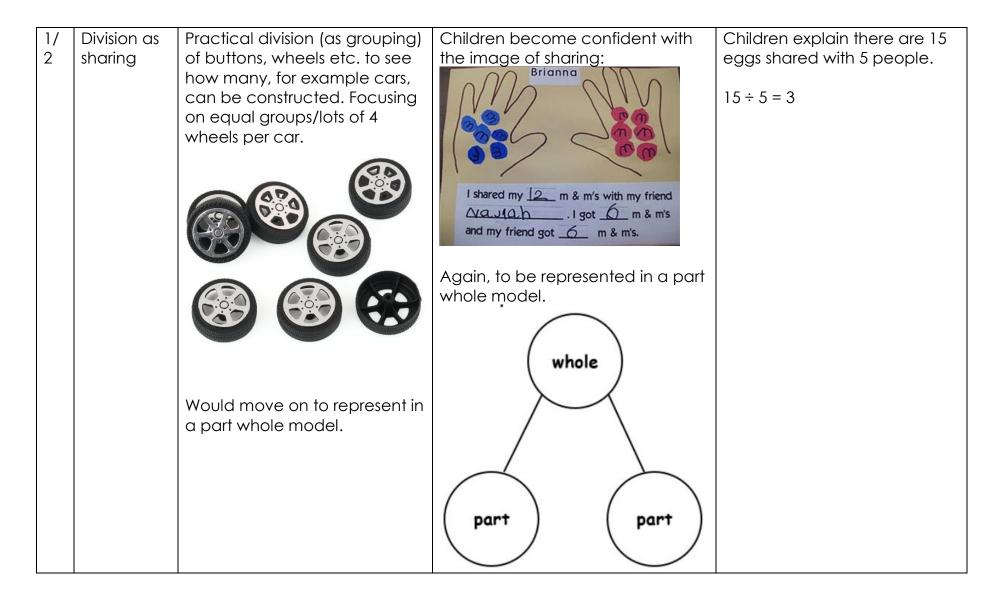
$$35 \div 7 = 5$$

	Τ	T	T	
4	Column	Children continue to use place	Develop fluency through	Start with long multiplication,
	Multiplicati	value counters at this stage of	variation:	reminding the children about
	on	calculation.		lining up their numbers clearly
		6 x 23	2 x 3 =	in columns.
		100s 10s 1s	2 x 30 =	
		000	2 x 300 =	Children to write out the
		88 888	20 x 3 =	calculation that they are
		000	200 x 3 =	solving next to their answer.
				23 x 6
		100s 10s 1s		
		88 800		23
		80 800		X 6
		9 88 9 888		1 8 (3 x 6)
		It is important that they always		<u>1 2 0</u> (20 x 6)
		multiply the ones first, note the		138
		answers and follow with the		
		tens that they note below.		Move to short multiplication
		Toris mar mey here belew.		(to include TU x U and HTU x U)
				6 x 23 =
				0 X 23 -
				23
				v 6
				<del>x 0</del>
				<u>× 6</u> 138
				1 1

5	Develop efficiency in short multiplicati on  Long multiplicati on for TU x TU	Recap any Year 4 methods/visuals as necessary.	Recap short multiplication up to 4 digits by 1 digit.  Introduce long multiplication when using TU x TU and increase to HTU and ThHTU by U.  TO 3 2 X 2 4 8 (2 x 4) 1 2 0 (30 x 4) 4 0 (2 x 20) 6 0 0 (20 x 30) 7 6 8
6	Develop efficiency in written methods		Short multiplication and long multiplication methods revised and developed in terms of accuracy and efficiency.  Extend to multiplying with decimals up to two decimal places. Know that decimal points should line up under each other.

Yr	Division Strategies	Concrete	Pictorial/Structural	Abstract
R/ 1	Sharing objects into groups	Sharing objects equally into groups.  10 shared into 2 groups.	Sharing pictures equally into groups. Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	Sharing amounts in numbers.  9 buns shared between three people:  9 ÷ 3 = 3

R/	ELG: To	Children understand sharing	Children understand sharing and	Verbal recall in stem
1	solve	and halving in <u>equal</u> groups	halving in <u>equal</u> groups in pictures.	sentences
	problems using halving, doubling	practically.	Half of 2 is	
	and sharing.		Half of 4 is	
		8		



1 /	D:	B: : 1	1 1 1 1	00.7
1/	Division as	Divide quantities into equal	Use a number line to show jumps	28 ÷ 7 = 4
2	grouping	groups. Use cubes, counters,	in groups. The number of jumps	
		objects, Numicon, place value	equals the number of groups.	Divide 28 into 7 groups. How
		counters or bead strings to aid	0 1 2 3 4 5 6 7 8 9 10 11 12	many are in each group
		understanding.	<del>k                                      </del>	
		onderstanding.	3 3 3	
			Think of the bar as a whole. Split it	
		2222	into the number of groups you are	
			dividing by and work out how	
		D 9 10 10 20 20 90 M	many would be within each	
		122 1 2 122	group.	
		96 + 3 = 32	20	
		o_ o_ o_		
			?	
		A A A	20 ÷ 5 = ?	
		CHE CHE CHE	5 x ? = 20	
		GEO GEO GEO		
			( whole )	
			( )	
			\ \ \	
			/	
			/	
			_/	
			( part ) ( part )	
			)	

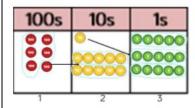
Division as Use physical objects to show Children continue to use drawn No abstract method covered division. Use tens and ones diagrams with dots or circles to as the concept of division is grouping and arrays counters to show division help them divide numbers into explored verbally and equal groups. physically.  $62 \div 2$ Encourage moving to counting multiples to divide more efficiently. Use place value counters to divide Division into groups: Divide objects into groups and with a Problem Solving
Cameron uses place value counters to help him calculate 66 + 3 see how much is left over. remainder (I) (II) (II) (II) Division with @ (D) (II) (II) (D) (D) (D) He gets the answer of 13. Is he correct?
Use place value counters to explain how you kno Problem Solving 10 (0) (B) (CI 0 0 0 (D) (II) 10 10 (B) (CI He gets the answer of 22 There are 3 whole squares, with 1 left over.

Short division

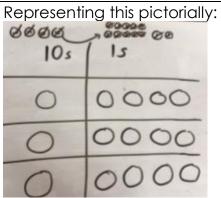
Sharing using place value counters:

Short division:

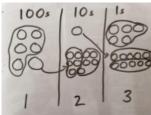
615 ÷ 5



- 1. Make 615 with place value counters.
- 2. How many groups of 5 hundreds can you make with 6 hundred counters?
- 3. Exchange 1 hundred for 10 tens.
- 4. How many groups of 5 tens can you make with 11 ten
- 5. Exchange 1 ten for 10 ones.
- 6. How many groups of 5 ones can you make with 15 one



Short division:

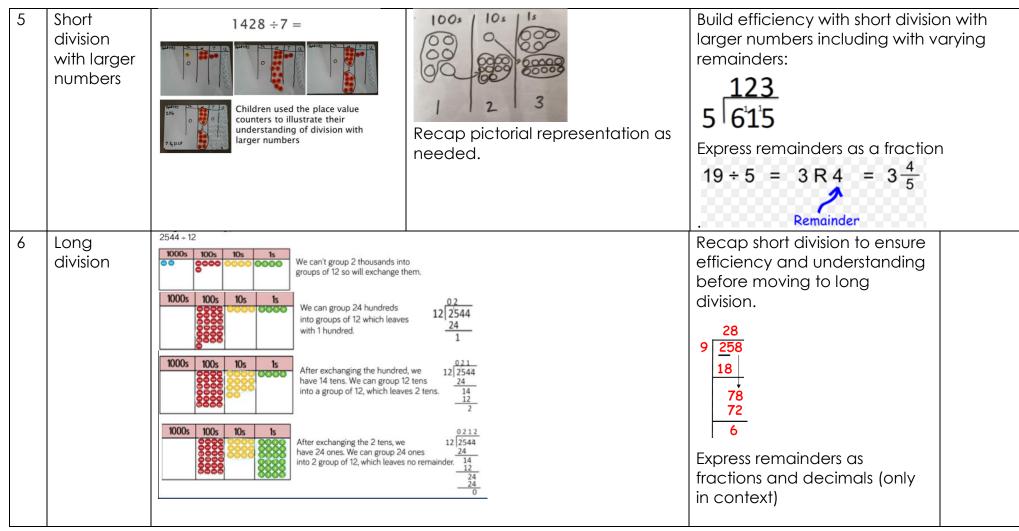


Expressing what is happening using short numerical expressions:

$$42 \div 3$$
  
 $42 = 30 + 12$   
 $30 \div 3 = 10$ 

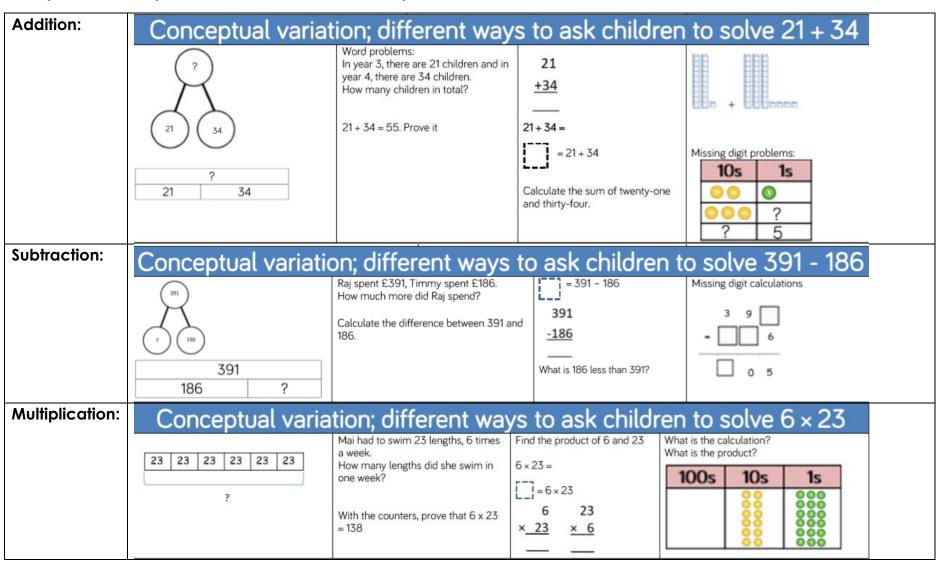
$$12 + 3 = 4$$
  
 $10 + 4 = 14$ 

Introduce short division: Begin with divisors that divide equally with no remainder



To ensure the children gain fluency and competence with each operation, we will provide them with a wide range of procedural and conceptual variation questions that will strengthen and develop the depth of each child's understanding of each method and process.

#### Examples of Conceptual Variation Ideas for Each Operation:



Division:	Conceptual varia	tion; different way	s to ask chil	dren to s	olve 61	15 ÷ 5	
	Using the part whole model below, how can you divide 615 by 5 without using short division?	I have £615 and share it equally between 5 bank accounts. How much will be in each account?	5 615	What is the calc			
	615	615 pupils need to be put into 5	615 + 5 =	100s	10s	1s	
	500 100	groups. How many will be in each group?	= 615 ÷ 5	000	00000	00000	