A-Level Further Mathematics Summer Home Work Booklet

Preparing for A-Level Further Mathematics

-Instructions

Answer ALL Questions.

-Advice

In ALL calculations, show clearly how you work out your answers. Use a Casio Class Wiz calculator or equivalent to help make some of the questions easier to answer.

-Notes

You may use other resources to help you answer these questions including e-mailing subject teachers over the Summer period to ask for help and we recommend using these YouTube videos:

Exam Solutions

Real and imaginary numbers
https://youtu.be/Vk78-7bUbzo

Addition, subtraction and multiplying complex numbers and simplifying powers of i
https://youtu.be/3Yw--3L7bCg

Complex conjugates
https://youtu.be/l5Y1KQXQ1Ys

FMSP

Solving quadratic equations with no real roots
https://youtu.be/e1XsCe0Sfr0

Adding and subtracting complex numbers
https://youtu.be/-74cX8rLqto

Multiplying complex numbers
https://youtu.be/LSUO5ck4aP4

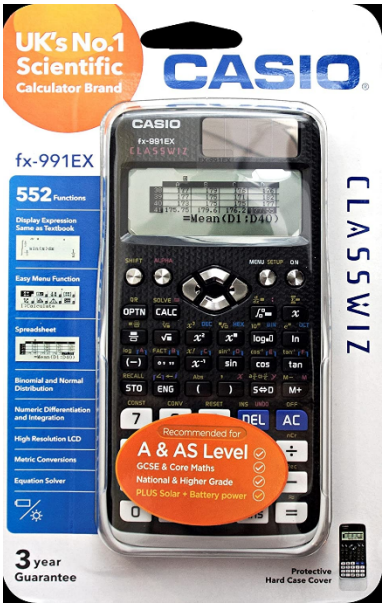
Terminology (real and imaginary part, conjugate)
https://youtu.be/A_rEiF8TUO0

-Due Date: First Further Mathematics Lesson of Year 12.

-Calculator

You will need to purchase a Casio Class Wiz FX-991ED Calculator or one that is equivalent in order to sit the final A-Level Exams. We recommend shopping around for the best price and to purchase it before the course so that you are able to use all the relevant functions for your class work. https://education.casio.co.uk/products/991ex

We have calculators to purchase from the Maths Department for a discounted cost of currently £20. These will be available on Parent Pay.



Expectations

The material in this booklet forms the building blocks for much of the A Level Further Maths course;

fluency and confidence in these topics is crucial to survival at A Level.

We expect you to work through this booklet **thoroughly** over the summer. Use the examples as a guide and complete all the exercises. We recommend you do a little work often rather than cramming it in at the end.

We also expect you to hand your booklet in to your teacher in the first lesson of Year 12. *Your teacher will mark a selection of the work and provide feedback.*

There will be an assessment on this material early in the term. Failing this assessment will result in further work being set and we may ultimately ask you to reconsider your options if we do not feel you will be successful.

Good luck and we very much look forward to seeing you in September.

Example 1

Write $\sqrt{(-36)}$ in terms of i.

$\sqrt{(-36)} = \sqrt{(36 \times -1)} = \sqrt{36}\sqrt{(-1)} = 6i$

Example 2

Write $\sqrt{(-28)}$ in terms of i.

$\sqrt{(-28)} = \sqrt{(28 \times -1)} = \sqrt{28}\sqrt{(-1)} = \sqrt{4}\sqrt{7}\sqrt{(-1)} = 2\sqrt{7}i$ or $2i\sqrt{7}$ or $(2\sqrt{7})i$

This can be written as $2i\sqrt{7}$ or $(2\sqrt{7})i$ to avoid confusion with $2\sqrt{7i}$.

Example 3

Solve the equation $x^2 + 9 = 0$.

$x^2 = -9$

$x = \pm\sqrt{(-9)} = \pm\sqrt{(9 \times -1)} = \pm\sqrt{9}\sqrt{(-1)} = \pm 3i$

$x = \pm 3i$ $\quad (x = +3i, x = -3i)$

Note that just as $x^2 = 9$ has two roots $+3$ and -3, $x^2 = -9$ also has two roots $+3i$ and $-3i$.

Example 4

Solve the equation $x^2 + 6x + 25 = 0$.

Method 1 (Completing the square)

$x^2 + 6x = (x + 3)^2 - 9$

$x^2 + 6x + 25 = (x + 3)^2 - 9 + 25 = (x + 3)^2 + 16$

$(x + 3)^2 + 16 = 0$

$(x + 3)^2 = -16$

$x + 3 = \pm\sqrt{(-16)} = \pm 4i$

$x = -3 \pm 4i$

$x = -3 + 4i, \quad x = -3 - 4i$

Because
$(x + 3)^2 = (x + 3)(x + 3)$
$= x^2 + 6x + 9$

$\sqrt{(-16)} = \sqrt{(16 \times -1)}$
$= \sqrt{16}\sqrt{(-1)} = 4i$

Method 2 (Quadratic formula)

$$x = \frac{-6 \pm \sqrt{(6^2 - 4 \times 1 \times 25)}}{2} = \frac{-6 \pm \sqrt{(-64)}}{2}$$

$\sqrt{(-64)} = \pm 8i$

$$x = \frac{-6 \pm 8i}{2} = -3 \pm 4i$$

$x = -3 + 4i, \quad x = -3 - 4i$

Using
$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$\sqrt{(-64)} = \sqrt{(64 \times -1)}$
$= \sqrt{64}\sqrt{(-1)} = 8i$

Example 5

Simplify, giving your answer in the form $a + bi$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

a $(2 + 5i) + (7 + 3i)$ **b** $(3 - 4i) + (-5 + 6i)$ **c** $2(5 - 8i)$

d $(1 + 8i) - (6 + i)$ **e** $(2 - 5i) - (5 - 11i)$ **f** $(2 + 3i) - (2 - 3i)$

a $(2 + 5i) + (7 + 3i) = (2 + 7) + i(5 + 3) = 9 + 8i$

Add real parts and add imaginary parts.

b $(3 - 4i) + (-5 + 6i) = (3 - 5) + i(-4 + 6) = -2 + 2i$

c $2(5 - 8i) = 10 - 16i$

This is the same as $(5 - 8i) + (5 - 8i)$

d $(1 + 8i) - (6 + i) = (1 - 6) + i(8 - 1) = -5 + 7i$

Subtract real parts and subtract imaginary parts.

e $(2 - 5i) - (5 - 11i) = (2 - 5) + i(-5 - (-11)) = -3 + 6i$

f $(2 + 3i) - (2 - 3i) = (2 - 2) + i(3 - (-3)) = 6i$

The answer has no real part. This is called purely imaginary.

Exercise 1A

Simplify, giving your answer in the form $a + bi$, where $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

1 $(5 + 2i) + (8 + 9i)$

2 $(4 + 10i) + (1 - 8i)$

3 $(7 + 6i) + (-3 - 5i)$

4 $(2 - i) + (11 + 2i)$

5 $(3 - 7i) + (-6 + 7i)$

6 $(20 + 12i) - (11 + 3i)$

7 $(9 + 6i) - (8 + 10i)$

8 $(2 - i) - (-5 + 3i)$

9 $(-4 - 6i) - (-8 - 8i)$

10 $(-1 + 5i) - (-1 + i)$

11 $(3 + 4i) + (4 + 5i) + (5 + 6i)$

12 $(-2 - 7i) + (1 + 3i) - (-12 + i)$

13 $(18 + 5i) - (15 - 2i) - (3 + 7i)$

14 $2(7 + 2i)$

15 $3(8 - 4i)$

16 $7(1 - 3i)$

17 $2(3 + i) + 3(2 + i)$

18 $5(4 + 3i) - 4(-1 + 2i)$

19 $\left(\frac{1}{2} + \frac{1}{3}i\right) + \left(\frac{5}{2} + \frac{5}{3}i\right)$

20 $(3\sqrt{2} + i) - (\sqrt{2} - i)$

Write in the form bi, where $b \in \mathbb{R}$.

21 $\sqrt{(-9)}$

22 $\sqrt{(-49)}$

23 $\sqrt{(-121)}$

24 $\sqrt{(-10\,000)}$

25 $\sqrt{(-225)}$

26 $\sqrt{(-5)}$

27 $\sqrt{(-12)}$

28 $\sqrt{(-45)}$

29 $\sqrt{(-200)}$

30 $\sqrt{(-147)}$

Solve these equations.

31 $x^2 + 2x + 5 = 0$

32 $x^2 - 2x + 10 = 0$

33 $x^2 + 4x + 29 = 0$

34 $x^2 + 10x + 26 = 0$

35 $x^2 - 6x + 18 = 0$

36 $x^2 + 4x + 7 = 0$

37 $x^2 - 6x + 11 = 0$

38 $x^2 - 2x + 25 = 0$

39 $x^2 + 5x + 25 = 0$

40 $x^2 + 3x + 5 = 0$

Example 6

Multiply $(2 + 3i)$ by $(4 + 5i)$

$$(2 + 3i)(4 + 5i) = 2(4 + 5i) + 3i(4 + 5i)$$
Multiply the two brackets as you would in algebra.

$$= 8 + 10i + 12i + 15i^2$$

$$= 8 + 10i + 12i - 15$$
Use the fact that $i^2 = -1$.

$$= (8 - 15) + (10i + 12i)$$

$$= -7 + 22i$$
Add real parts and add imaginary parts.

Example 7

Express $(7 - 4i)^2$ in the form $a + bi$.

$$(7 - 4i)(7 - 4i) = 7(7 - 4i) - 4i(7 - 4i)$$
Multiply the two brackets as you would in algebra.

$$= 49 - 28i - 28i + 16i^2$$

$$= 49 - 28i - 28i - 16$$
Use the fact that $i^2 = -1$.

$$= (49 - 16) + (-28i - 28i)$$

$$= 33 - 56i$$
Add real parts and add imaginary parts.

Example 8

Simplify $(2 - 3i)(4 - 5i)(1 + 3i)$

$(2 - 3i)(4 - 5i) = 2(4 - 5i) - 3i(4 - 5i)$

$= 8 - 10i - 12i + 15i^2 = 8 - 10i - 12i - 15 = -7 - 22i$

$(-7 - 22i)(1 + 3i) = -7(1 + 3i) - 22i(1 + 3i)$

$= -7 - 21i - 22i - 66i^2 = 59 - 43i$

First multiply two of the brackets.

Then multiply the result by the third bracket.

Example 9

Simplify

a i^3 **b** i^4 **c** $(2i)^5$

a $i^3 = i \times i \times i = i^2 \times i = -i$

b $i^4 = i \times i \times i \times i = i^2 \times i^2 = -1 \times -1 = 1$

c $(2i)^5 = 2i \times 2i \times 2i \times 2i \times 2i = 32(i \times i \times i \times i \times i)$

$= 32(i^2 \times i^2 \times i) = 32 \times -1 \times -1 \times i = 32i$

$i^2 = -1$

First multiply the 2s (2^5).

Exercise 1B

Simplify these, giving your answer in the form $a + bi$.

1 $(5 + i)(3 + 4i)$

2 $(6 + 3i)(7 + 2i)$

3 $(5 - 2i)(1 + 5i)$

4 $(13 - 3i)(2 - 8i)$

5 $(-3 - i)(4 + 7i)$

6 $(8 + 5i)^2$

7 $(2 - 9i)^2$

8 $(1 + i)(2 + i)(3 + i)$

9 $(3 - 2i)(5 + i)(4 - 2i)$

10 $(2 + 3i)^3$

Simplify.

11 i^6

12 $(3i)^4$

13 $i^5 + i$

14 $(4i)^3 - 4i^3$

Example 10

Write down the complex conjugate of

a $2 + 3i$ **b** $5 - 2i$ **c** $\sqrt{3} + i$ **d** $1 - i\sqrt{5}$

a $2 - 3i$ **b** $5 + 2i$

c $\sqrt{3} - i$ **d** $1 + i\sqrt{5}$

Just change the sign of the imaginary part (from + to –, or – to +).

Example 11

Find $z + z^*$ and zz^*, given that

a $z = 3 + 5i$ **b** $z = 2 - 7i$ **c** $z = 2\sqrt{2} + i\sqrt{2}$

a $z^* = 3 - 5i$

$z + z^* = (3 + 5i) + (3 - 5i) = (3 + 3) + i(5 - 5) = 6$ — Note that $z + z^*$ is real.

$zz^* = (3 + 5i)(3 - 5i) = 3(3 - 5i) + 5i(3 - 5i)$

$= 9 - 15i + 15i - 25i^2 = 9 + 25 = 34$ — Note that zz^* is real.

b $z^* = 2 + 7i$

$z + z^* = (2 - 7i) + (2 + 7i) = (2 + 2) + i(-7 + 7) = 4$ — Note that $z + z^*$ is real.

$zz^* = (2 - 7i)(2 + 7i) = 2(2 + 7i) - 7i(2 + 7i)$

$= 4 + 14i - 14i - 49i^2 = 4 + 49 = 53$ — Note that zz^* is real.

c $z^* = 2\sqrt{2} - i\sqrt{2}$

$z + z^* = (2\sqrt{2} + i\sqrt{2}) + (2\sqrt{2} - i\sqrt{2})$

$= (2\sqrt{2} + 2\sqrt{2}) + i(\sqrt{2} - \sqrt{2}) = 4\sqrt{2}$ — Note that $z + z^*$ is real.

$zz^* = (2\sqrt{2} + i\sqrt{2})(2\sqrt{2} - i\sqrt{2})$

$= 2\sqrt{2}(2\sqrt{2} - i\sqrt{2}) + i\sqrt{2}(2\sqrt{2} - i\sqrt{2})$

$= 8 - 4i + 4i - 2i^2 = 8 + 2 = 10$ — Note that zz^* is real.

Exercise 1C

1 Write down the complex conjugate z^* for

a $z = 8 + 2i$ **b** $z = 6 - 5i$

c $z = \frac{2}{3} - \frac{1}{2}i$ **d** $z = \sqrt{5} + i\sqrt{10}$

2 Find $z + z^*$ and zz^* for

a $z = 6 - 3i$ **b** $z = 10 + 5i$

c $z = \frac{3}{4} + \frac{1}{4}i$ **d** $z = \sqrt{5} - 3i\sqrt{5}$