8.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The sum S_n of the first n terms of an infinite series is called a(n) _____.

2. **WRITING** Explain how to tell whether the series $\sum_{i=1}^{\infty} a_1 r^{i-1}$ has a sum.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–6, consider the infinite geometric series. Find and graph the partial sums S_n for $n = 1, 2, 3, 4,$ and 5. Then describe what happens to S_n as n increases. *(See Example 1.)*

3. $\frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \frac{1}{54} + \frac{1}{162} + \cdots$

4. $\frac{2}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{24} + \cdots$

5. $4 + \frac{12}{5} + \frac{36}{25} + \frac{108}{125} + \frac{324}{625} + \cdots$

6. $2 + \frac{2}{6} + \frac{2}{36} + \frac{2}{216} + \frac{2}{1296} + \cdots$

In Exercises 7–14, find the sum of the infinite geometric series, if it exists. *(See Example 2.)*

7. $\sum_{n=1}^{\infty} 8\left(\frac{1}{5}\right)^{n-1}$

8. $\sum_{k=1}^{\infty} -6\left(\frac{3}{2}\right)^{k-1}$

9. $\sum_{k=1}^{\infty} \frac{11}{3}\left(\frac{3}{8}\right)^{k-1}$

10. $\sum_{i=1}^{\infty} \frac{2}{5}\left(\frac{5}{3}\right)^{i-1}$

11. $2 + \frac{6}{4} + \frac{18}{16} + \frac{54}{64} + \cdots$

12. $-5 - 2 - \frac{4}{5} - \frac{8}{25} - \cdots$

13. $3 + \frac{5}{2} + \frac{25}{12} + \frac{125}{72} + \cdots$

14. $\frac{1}{2} - \frac{5}{3} + \frac{50}{9} - \frac{500}{27} + \cdots$

ERROR ANALYSIS **In Exercises 15 and 16, describe and correct the error in finding the sum of the infinite geometric series.**

15. $\sum_{n=1}^{\infty} \left(\frac{7}{2}\right)^{n-1}$

✗ For this series, $a_1 = 1$ and $r = \frac{7}{2}$.

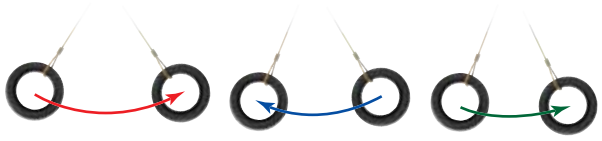
$S = \frac{a_1}{1 - r} = \frac{1}{1 - \frac{7}{2}} = \frac{1}{-\frac{5}{2}} = -\frac{2}{5}$

16. $4 + \frac{8}{3} + \frac{16}{9} + \frac{32}{27} + \cdots$

✗ For this series, $a_1 = 4$ and $r = \frac{4}{\frac{8}{3}} = \frac{3}{2}$.

Because $\left|\frac{3}{2}\right| > 1$, the series has no sum.

17. **MODELING WITH MATHEMATICS** You push your younger cousin on a tire swing one time and then allow your cousin to swing freely. On the first swing, your cousin travels a distance of 14 feet. On each successive swing, your cousin travels 75% of the distance of the previous swing. What is the total distance your cousin swings? *(See Example 3.)*



18. **MODELING WITH MATHEMATICS** A company had a profit of $350,000 in its first year. Since then, the company's profit has decreased by 12% per year. Assuming this trend continues, what is the total profit the company can make over the course of its lifetime? Justify your answer.

In Exercises 19–24, write the repeating decimal as a fraction in simplest form. *(See Example 4.)*

19. $0.222\ldots$

20. $0.444\ldots$

21. $0.161616\ldots$

22. $0.625625625\ldots$

23. $32.323232\ldots$

24. $130.130130130\ldots$

25. **PROBLEM SOLVING** Find two infinite geometric series whose sums are each 6. Justify your answers.