



#### Concrete/Pictorial/Abstract

#### Maths Calculation Policy 24/25

This policy has been largely adapted from White Rose Maths Calculation Policy, NCETM Guidance with further material added. It is a working document and will be revised and amended as necessary. Many variations have been included to provide teachers with a range of tools to support pupils in their grasp of number and calculation. To ensure consistency for pupils, it is important that the mathematical language used in maths lessons reflects the vocabulary used throughout this policy.

#### Recommended practice delivering a mastery approach

True mastery aims to develop all children's mathematical understanding at the same pace. As much as possible, children should be accessing the same learning. Differentiation should primarily be through support, scaffolding and deepening, not through task.

Consistency in language is essential for pupils to understand the concepts presented in mathematics. If other, 'child-friendly' terminology is used, this must be alongside the current terminology recommended by maths specialists. Using this will support children with their examinations and throughout secondary school.

Concrete, pictorial, abstract (CPA) concepts should not be confused as differentiation for lower, middle, higher attaining children. CPA is an approach to be used with the whole class and teachers should promote each area as equally valid. Manipulatives must not be presented as a resource to support the less confident or lower attaining pupils. The abstract should run alongside the concrete and pictorial stage as this enables pupils to better understand mathematical statements and concepts. Real things and structured images enable children to understand the abstract. The concrete and the images are a means for children to understand the symbolic so it's important to move between all modes to allow children to make connections. Morgan, D. (2016)

Used well, manipulatives can enable pupils to inquire themselves- becoming independent learners and thinkers. They can also provide a common language with which to communicate cognitive models for abstract ideas. Drury, H. (2015)

Children aged seven to ten years old work in primarily concrete ways and that the abstract notions of mathematics may only be accessible to them through embodiment in practical resources. Jean Piaget's (1951)

Real things and structured images enable children to understand the abstract. The concrete and the images are a means for children to understand the symbolic so it's important to move between all modes to allow children to make connections. Morgan, D. (2016)

The abstract should run alongside the concrete and pictorial stage as this enables pupils to better understand mathematical statements and concepts.

Here at Devonshire, we follow the 5 Big Ideas from mastery using representation and structure, mathematical thinking, fluency variation and learning is broken down into small steps so the children are making clear connections throughout. This in turn allows the children to know more, remember more.



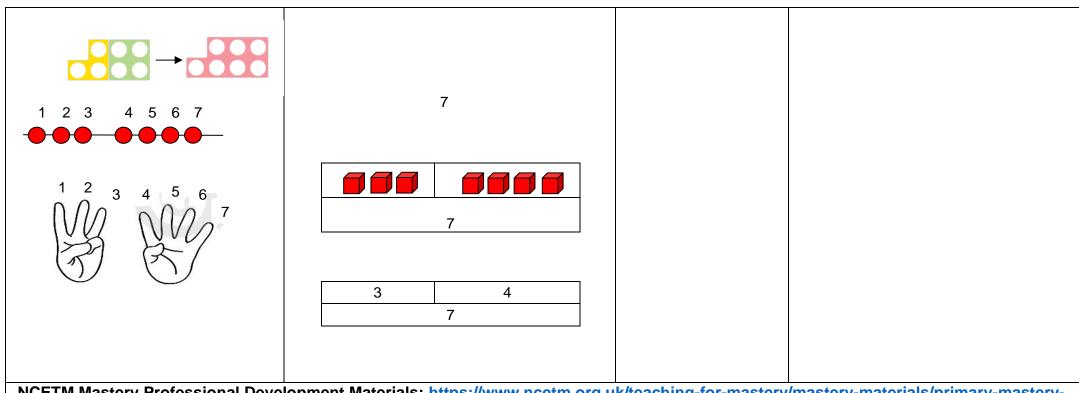
Our overall aim is that when the children leave Devonshire Primary Academy, they:

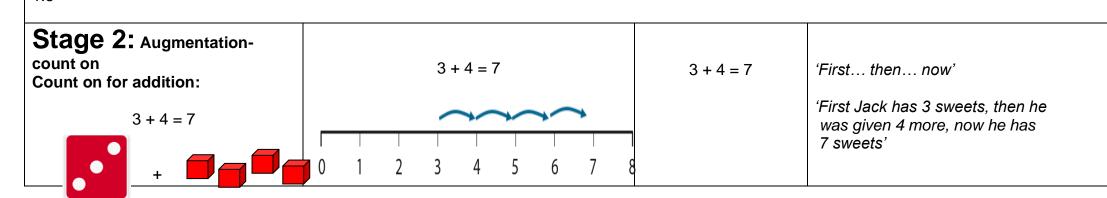
- Have a secure knowledge of number facts and a good understanding of the four operations.
- Are able to use this knowledge and understanding to carry out calculations mentally and to apply general strategies when using one-digit and two-digit numbers and particular strategies to special cases involving bigger numbers.
- Make use of diagrams and informal notes to help record stages and part answers when using mental methods that generate more information than can be kept in their heads.

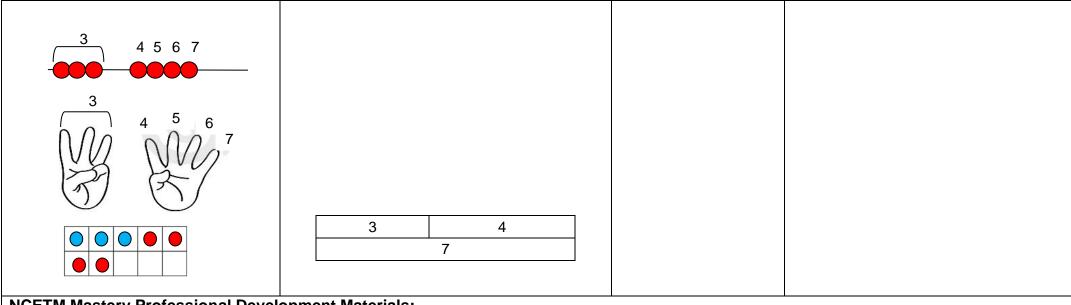
## **Calculation Policy for Addition**

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as', increase. Key representations: Base 10, bead strings, multilink, counters, Numicon, bar model, number lines, ten frames, fingers, place value counters, 100 square

Concrete	Pictorial	Abstract	Stem Sentences
Stage 1: Aggregation-count all Adding groups to make a whole:	3 + 4 = 7	3 + 4 = 7	' plus is equal to'
3 + 4 = 7			' is equal to plus'
			' is the sum of and'

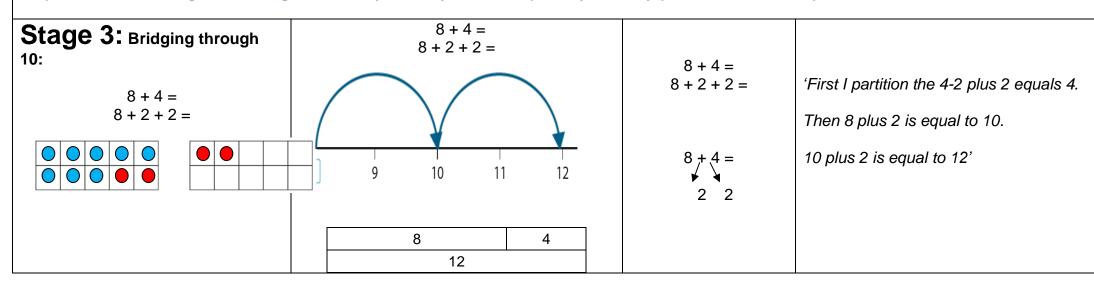






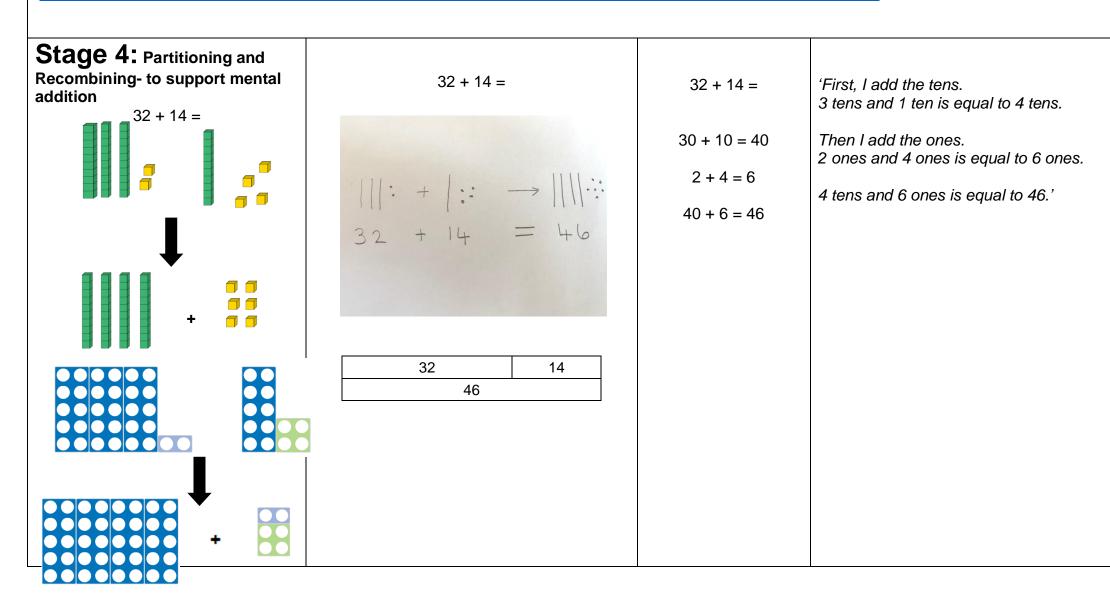
## **NCETM Mastery Professional Development Materials:**

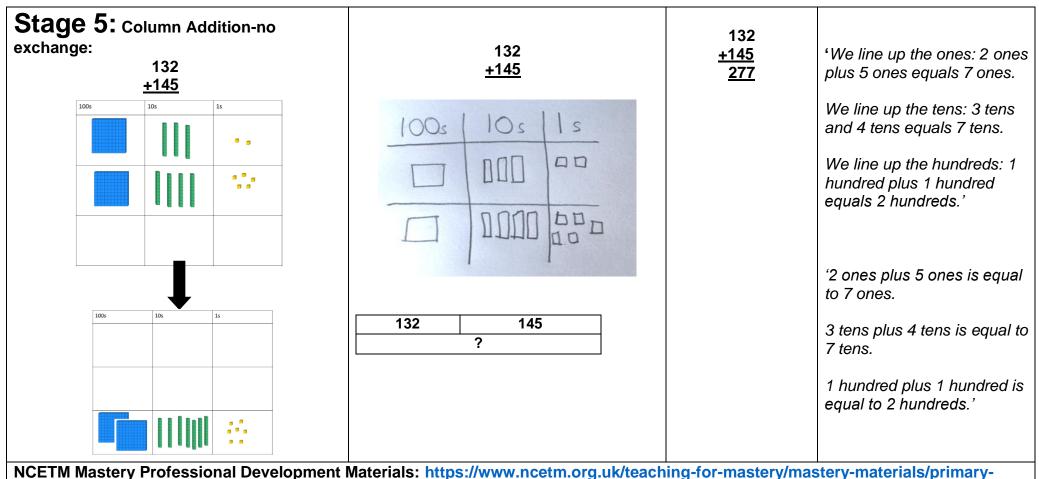
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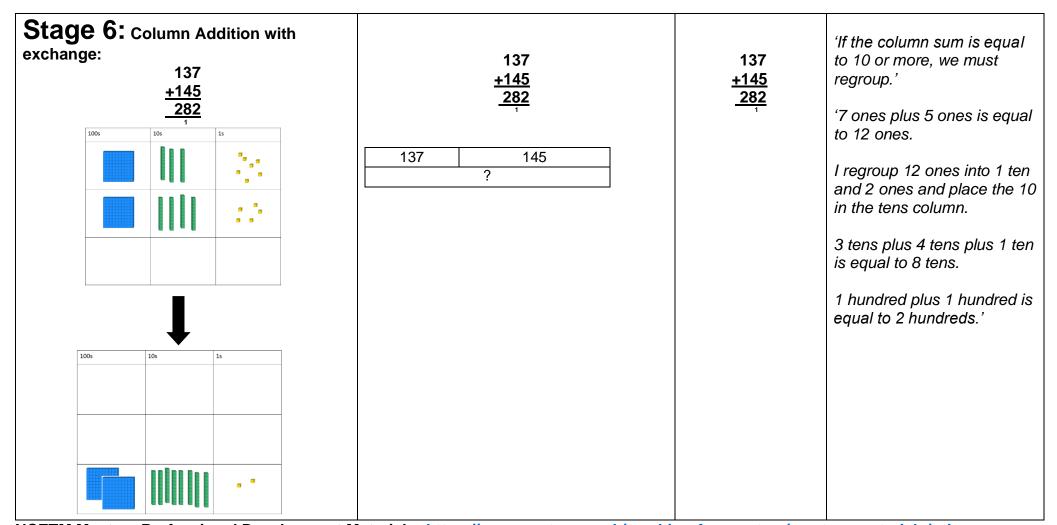


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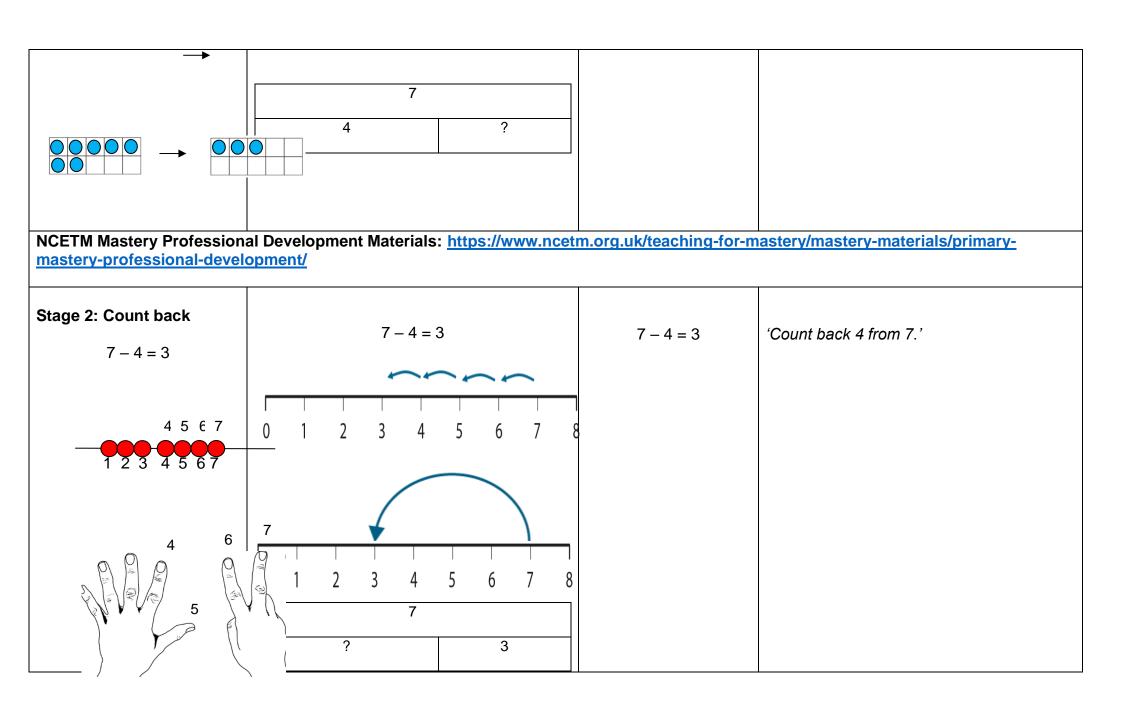


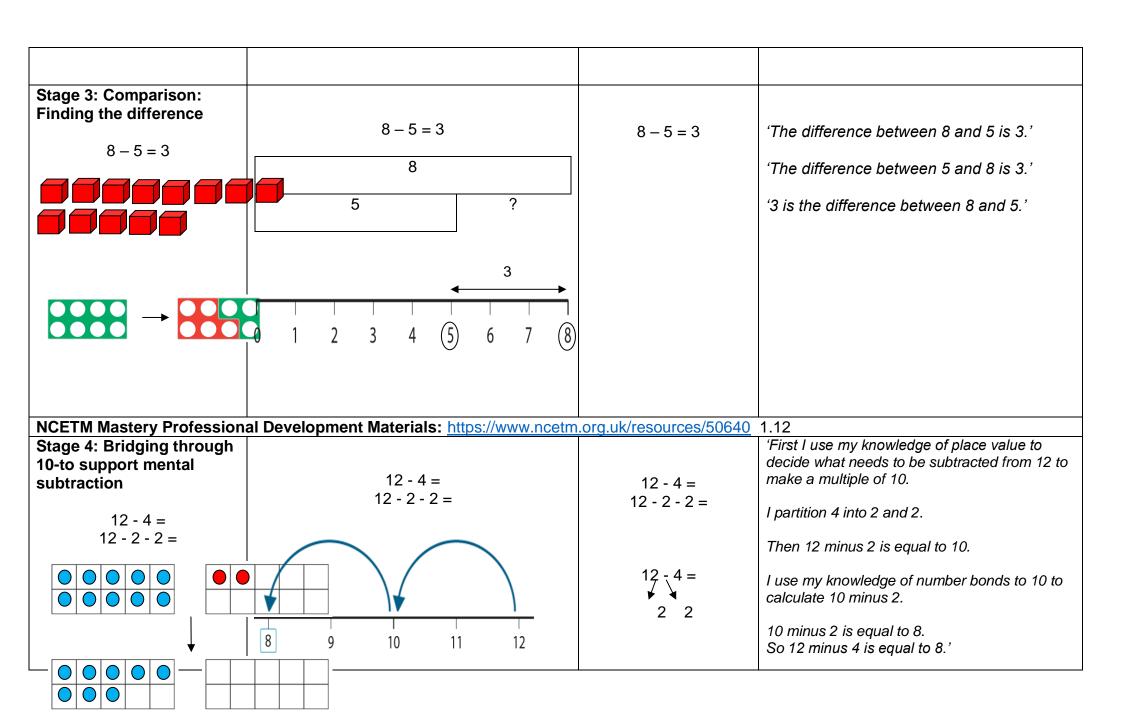
## **Calculation Policy for Subtraction**

## Key language: subtract, less, fewer, takeaway, difference, leave, minus, decrease

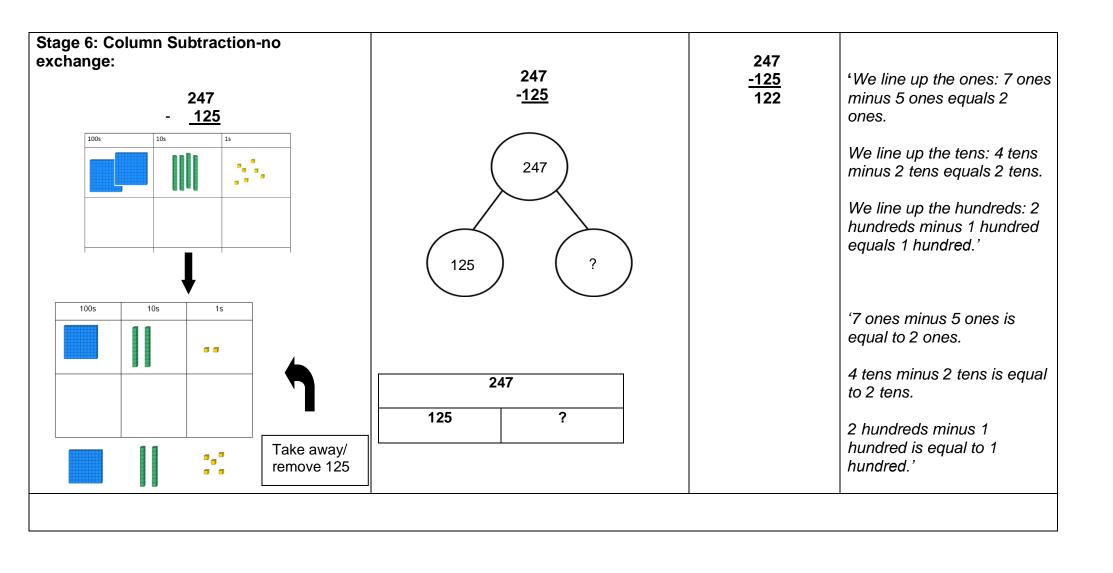
Key representations: Base 10, bead strings, multilink, counters, Numicon, bar model, number lines, ten frames, fingers, place value counters, 100 square

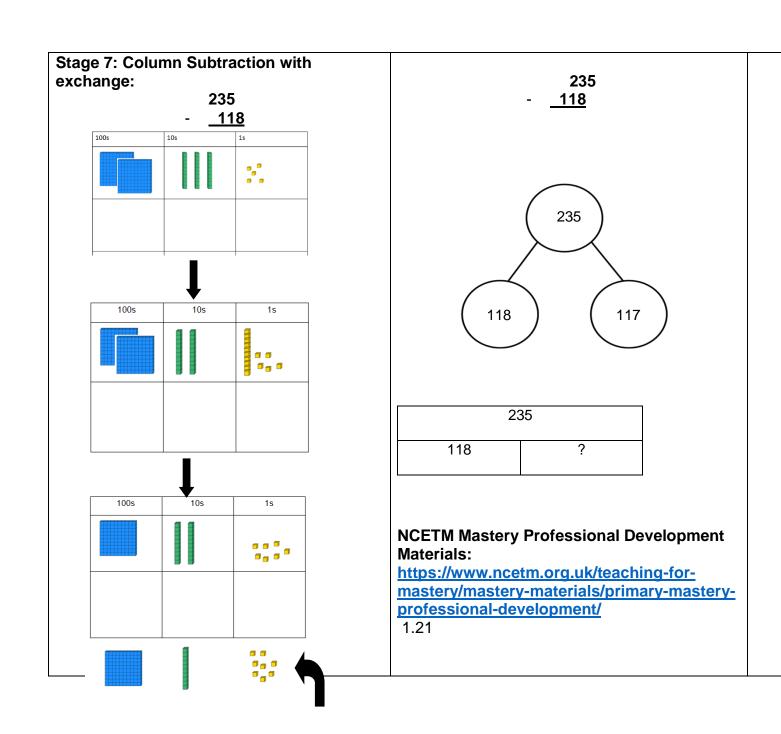
Concrete	Pictorial	Abstract	Stem Sentences
Stage 1: Reduction: Take away			
	7 – 4 = 3	7 - 4 = 3	'First then now'
7 - 4 = 3	0000444		'First Jack had 7 sweets. Then he ate 4. Now he has 3 sweets left. 7 minus 4 equals 3.'
	7		
	4 ?		





	12		
	4 ?		
	?		
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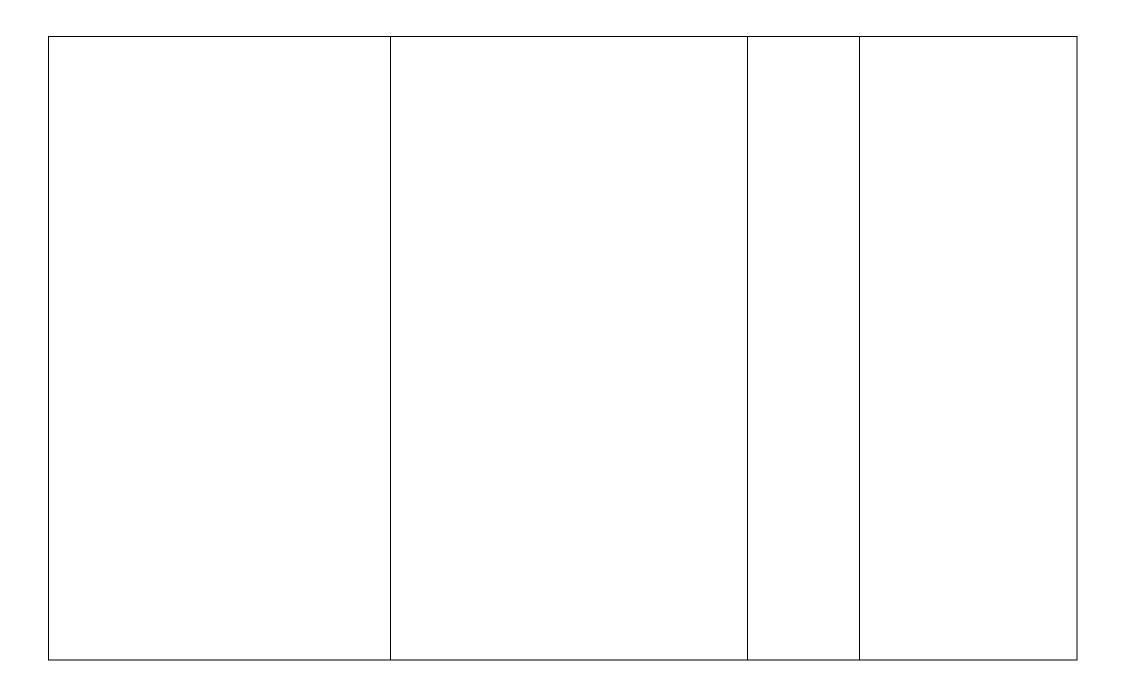
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'5 ones minus 8 ones...we need to exchange 1 ten for 10 ones.

15 ones minus 8 ones is equal to 7 ones.

2 tens minus 1 ten is equal to 1 ten.

2 hundreds minus 1 hundred is equal to 1 hundred.'

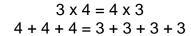


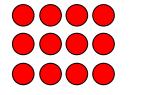
# **Calculation Policy for Multiplication**

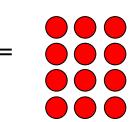
## Key language: multiply, times, product, lots of, groups of, multiple, commutative

Key representations: place value counters, base 10, bead strings, multilink, counters, Numicon, bar model, number line

### **Stage 2: Arrays to show commutativity:**







3 groups of 4

4 groups of 3

$3 \times 4 = 4 \times 3$ 4 + 4 + 4 = 3 + 3 + 3 + 3								
3	3 3 3 3							
?								
4		4	4					
12								

$$3 \times 4 = 4 \times 3$$
  
 $4 + 4 + 4 = 3 + 3 + 3 + 3$ 

'I can see 3 groups of 4 and I can see 4 groups of 3.'

'3 times 4 can represent 3 groups of 4.

It can also represent 4 times 3.'

'If there are \_\_\_ equal groups, we can use the \_\_\_ times table.'

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## Stage 3: Partition to multiply:

$$3 \times 15 =$$





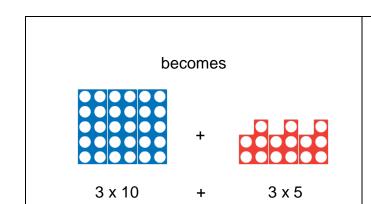


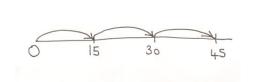
15	15	15
	?	

$$3 \times 10 = 30$$
  
 $3 \times 5 = 15$ 

'15 is equal to 10 plus 5.

So 3 times 15 is equal to 3 times 10 plus 3 times 5.

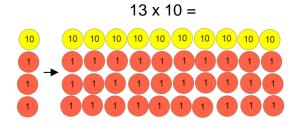




$$30 + 15 = 45$$

X	10	5
3	30	15

Stage 4: Multiply by 10:



100s	10s	1s
		000
, <u> </u>		

1	3	Χ	1	0	=
---	---	---	---	---	---

100s	10s	1s
	1	3
1	3	0

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

13	13	13	13	13	13	13	13	13	13
				•	?				

'\_\_ multiplied by 10 is equal to \_\_.'

'\_\_ is ten times the size of\_\_.'

**Stage 5: Short multiplication:** 2 digit by 1 digit with no exchange

$$23 \times 3 =$$

10s 1s

60 + 9

23 x 3 =

23 23 23

23 x 3 =

23 <u>x 3</u> 69 'First we multiply the ones and then we multiply the tens. We add those products together.'

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Stage 6: Short multiplication: 2 digit by 1 digit with exchange

$$34 \times 4 =$$

 $34 \times 4 =$ 

34	34	34	34
	(	<b>&gt;</b>	

34 x 4 =

34 <u>x 4</u> 16 +<u>120</u> <u>136</u> <u>I</u> 'First we multiply the ones and then we multiply the tens. We add those products together.'

'If there are 10 or more ones, we must regroup the ones into 10s and 1s.'

120 + 16  NCETM Mastery Professional Development  mastery-professional-development/	Materials: https://www.ncetm.org.	34 <u>x 4</u> <u>136</u> uk/teaching-for-mastery/ma	astery-materials/primary-
Stage 7: Long multiplication: 3 digit by 2 digit	124 x 26 = 124 x 26 124 x 6	124 x 26 =  1 2 4  x 2.6  7 4 4  2 4 8 0  3 2 2 4	'If there are 10 or more ones, we must regroup the ones into 10s and 1s.'  'If there are 10 or more hundreds, we must regroup then tens into 100s and 10s.'  'If there are 10 or more hundreds, we must regroup the hundreds into 1000s and 100s.'  'To multiply a three digit number by a two digit number, first multiply by the ones, then multiply by the tens and then add them together.'

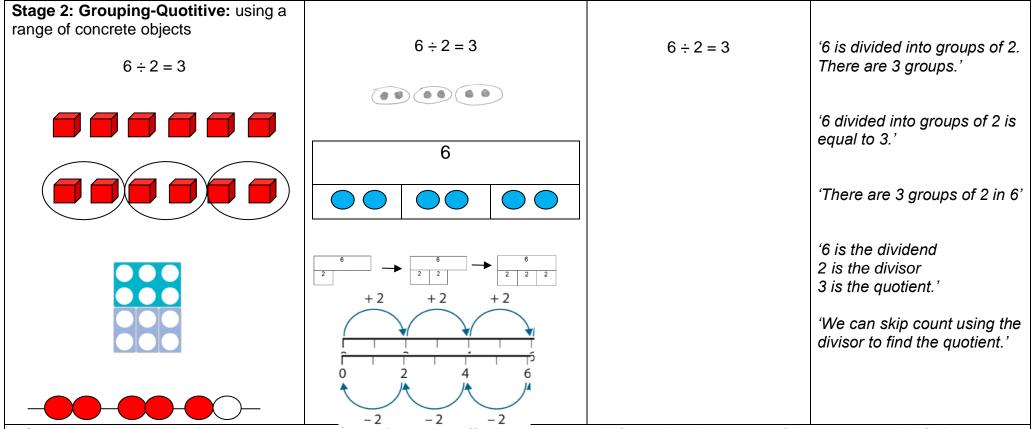
## **Calculation Policy for Division**

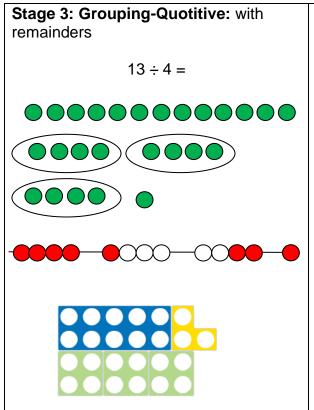
## - important to make links with Fractions

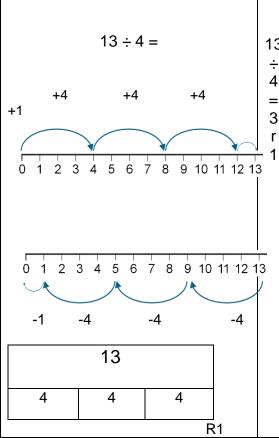
## Key language: divide, halve, share, division, factor, remainder, dividend, equal groups of, quotient, divisor

Key representations: place value counters, base 10, bead strings, multilink, counters, Numicon, bar model, number line

Concrete	Picto		Abstract	Stem Sentences
Stage 1: Sharing-Partitive: using a range of discrete concrete objects				'6 divided between 2 is equal to 3 each.'
6 ÷ 2 = 3	6 ÷ 2	2 = 3	$6 \div 2 = 3$	
	• •	•		'6 shared into 2 equal groups, there are 3 in each group.'
	6	3		'6 is the dividend 2 is the divisor
				3 is the quotient.'
	6	3		
	3	3		
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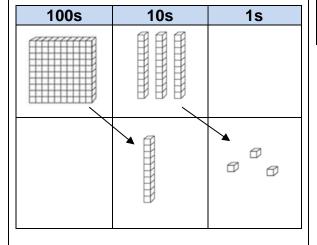
'3 is divided into groups of 4. There are 3 groups and a remainder of 1.'

'13 divided into groups of 4 is equal to 3 remainder 1.'

'The remainder is always less than the divisor.'

'If the dividend is not a multiple of the divisor, there is a remainder.'

# Stage 4: Divide by 10:



100s	10s	1s
1	3	0
	1	3

1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	130	40	50	60	70	80	90
1	2	<b>♦</b> 3	4	5	6	7	8	9

				13	30				
?	?	?	?	?	?	?	?	?	?

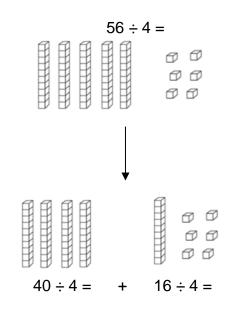
 $130 \div 10 = 13$ 

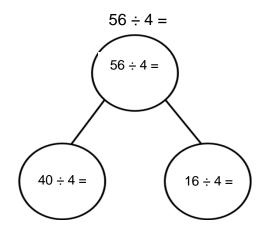
\_\_\_ divided by 10 is equal to

'\_\_ is ten times smaller than\_\_.'

**Stage 5: Partitioning:** to support mental calculation

(Only use when the dividend is beyond the twelfth multiple. Before the twelfth multiple, times tables facts should be used).





 $56 \div 4 =$ 

Partitive:

'56 is partitioned into 40 and 16.

'40 divided between 4 is equal to 10 each.

16 divided between 4 is equal to 4 each.

56 divided between 4 is equal to 1 ten and 4 ones.

#### Quotitive:

'56 is partitioned into 40 and 16.

40 is divided into groups of 4. There are 10 groups.'

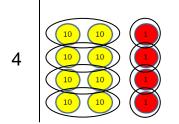
'16 divided into groups of 4 is equal to 4.'

'There are 14 groups of 4 in 56'

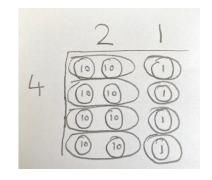
Stage 6: Short division: partitive: 2 digit by 1 digit (with no exchange) (Only use when the dividend is beyond the twelfth multiple. Before the twelfth multiple, times tables facts should be used).

10s 1s

2



10s 1s



84					
?		?	?	?	

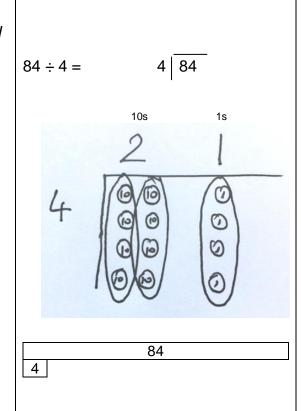
$$84 \div 4 = 21$$

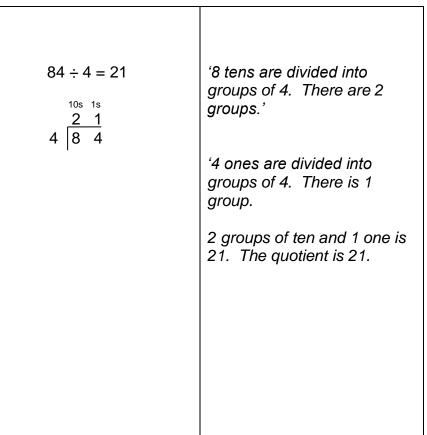
10s 1s 2 1 4 8 4 '8 tens divided between 4 is equal to 2 tens each.

4 ones divided between 4 is equal to 1 one each.

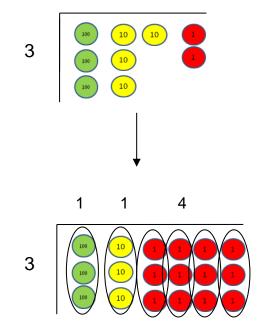
8 tens and 4 ones divided between 4 is equal to 2 tens and 1 one, which is 21.

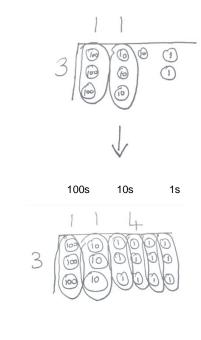
Stage 7: Short division: quotitive 2 digit by 1 digit (with no exchange) (Only use when the dividend is beyond the twelfth multiple. Before the twelfth multiple, times tables facts should be used).





Stage 8: Short division: quotitive 2 digit by 1 digit (with exchange) (Only use when the dividend is beyond the twelfth multiple. Before the twelfth multiple, times tables facts should be used).





$$342 \div 3 = 114$$

'3 hundreds divided by 3 is 1 hundred.

4 tens divided by 3 is 1 group remainder 1 ten.

Exchange 1 ten for 10 ones.

12 ones divided by 3 is equal to 4.'

Stage 9: Formal division: with 2-digit divisors	Short Division	Long Division
uivisors	465 ÷ 31 =	465 ÷ 31 =
	31 4 4 6 15 5 (It may be helpful to write a list of multiples of 31 e.g. 31, 62, 93, 124, 155)	0 1 5 31 4 6 5 3 1 1 5 5 1 5 5 0
		Get children to make up their own acronyms to help them remember the process.

# Calculation Policy for Reasoning, Greater Depth and Mastery Key language:

#### What do we mean by mastery?

The essential idea behind mastery is that ALL children need a deep understanding of the mathematics they are learning so that:

- Future mathematical learning is built on solid foundations which do not need to be re-taught;
- There is no need to for separate catch-up programmes due to some children falling behind;
- Children who, under other teaching approaches, can often fall a long way behind, are better able to keep up with their peers, so that gaps in attainment are narrowed whilst the attainment of all is raised.

There are generally four ways in which the term mastery is being used in the current debate around raising standards in mathematics.

**1.A** mastery approach: a set of principles and beliefs. This includes a belief that all pupils are capable of understanding and doing mathematics, given sufficient time.

Pupils are neither 'born with the maths gene' nor 'just no good at maths'. With good teaching, appropriate resources, effort and a 'can do' attitude all children can achieve in and enjoy mathematics.

- **2.A mastery curriculum:** one set of mathematical concepts and big ideas for all. All pupils need access to these concepts and idea and to the rich connections between them. Mathematics is mathematics and the key ideas and building blocks are important to everyone.
- **3.Teaching for mastery:** a set of pedagogic practices that keep the class working together on the same topic, whilst at the same time addressing the need for all pupils to master the curriculum and for some to gain greater depth of proficiency and understanding. Challenge is provided by going depth and sufficient practice to embed learning.

**4.Achieving mastery of particular topics and areas of mathematics.** Mastery is not just being able to memorise key facts and procedures and answer test questions accurately and quickly. It involves knowing 'why' as well as knowing 'that' and knowing 'how'. It means being able to use apply it in new and unfamiliar situations.

#### Mastery of the curriculum requires that all pupils:

- Use mathematical concepts, facts and procedures appropriately, flexibly and fluently;
- Recall key number facts with speed and accuracy and use them to calculate and work out unknown facts;
- Have sufficient depth of knowledge and understanding to reason and explain mathematical concepts and procedures and use them to solve a variety of problems.

#### **Key questions:**

- -Odd one out
- -True or false?

#### -Would you rather have...

- -Here is the..
- -Tell me about this ...
- -prove/disprove this statement

#### -find the mistake

- -always, sometimes, never
- -convince me that
- -how are these linked?

#### What is the same and what is different?

Give me a silly answer to this problem. What makes it silly?

What if...?

If you know this fact, what else do you know? E.g. if you know:

4 + 6 = 10

- Give me a hard and easy	-challenge	You know:
example of a five- digit	-agree	40+60=100
calculation.	-Build on	100-40=60
-Give me a hard and easy	-what do you know	The sum of 6 and 4 is 10.
example of a question		4000+600=10,000
you could ask about this		100,000-60,000=40,000
graph/pie chart etc.		If it is 6'o'clock now, in 4 hours it will be 10'o'clock.