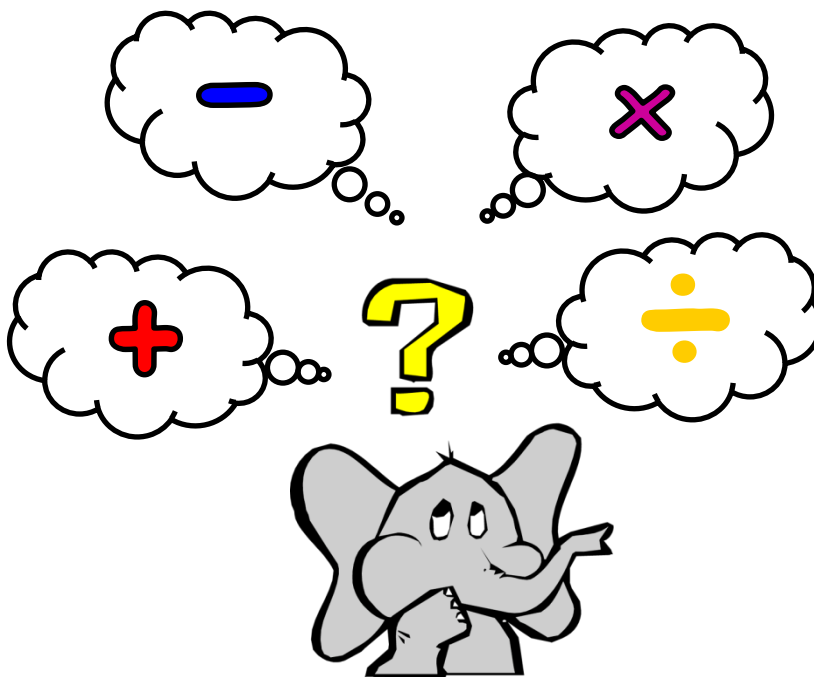
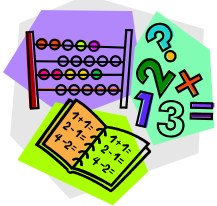


# Progression in Calculations

## Information for Parents



Spring 2013



# Introduction

The maths work your child is doing at school may look very different to the kind of 'sums' you remember.

This is because children are encouraged to work mentally, where possible, using personal jottings to help support their thinking. Number lines are one example of this.

Even when children are taught more formal written methods they are only encouraged to use these methods for calculations they cannot solve in their heads.

It will be a great help to your child, and to their teachers, if you could encourage them to use methods which they have learnt at school rather than teaching them different methods at home.

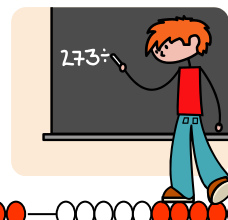
This booklet is designed to inform you about the progression in calculation methods that we use at Dowson for addition, subtraction, multiplication and division.

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation when the children are ready for them. For many children this will be in the later years of primary school or into secondary school.

Strategies for calculation need to be supported by familiar models and images to reinforce understanding. When teaching a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the concept.

The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.



By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved. Discussing the efficiency and suitability of different strategies is important.

Remember that the expanded methods are perfectly good ways of working out an answer if the children feel more comfortable and therefore find it easier. They give the same answer and it can often be quicker if they are confident about what they are doing.

These methods are very useful when children are extending their work, for example to numbers involving decimals.

Children should not be made to go onto the next stage if:

- they are not ready.
- they are not confident.



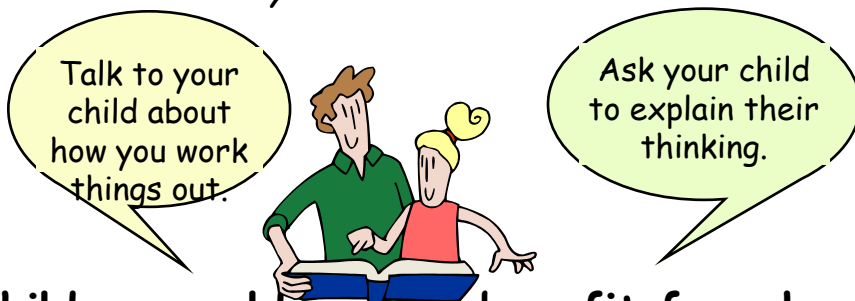
# Mental calculation

Developing confidence and efficiency in mental calculations is a vital part of Maths teaching throughout Key Stage 2.

Regular practice of number facts is important both at school and at home. Any opportunities to practise are very useful, for example through **real life situations** such as shopping as well as activities such as games.

*See 'Fun Activities to do at home' booklets for ideas. Many card and board games involve mental calculations as well.*

*A variety of Maths games will soon be available to borrow from the school library.*



**The children would greatly benefit from knowing key number facts by heart and recalling them instantly (e.g. number bonds to 20, tables).**

There are many useful games on the internet which give children chance to practise number facts and mental calculations. Links to some that we know are particularly good can be found through the children's links on the school website: [www.dowson.tameside.sch.uk](http://www.dowson.tameside.sch.uk)

For example, Hit the Button is good for practising number bonds and tables facts.

For those who have subscribed to Education City a selection of basic number facts games will be able all year.

# Multiplication Facts

Remember that truly **knowing** tables is not the same as just being able to count up in steps of a given number or being able to recite the table.

Really knowing a table means that the children can instantly tell you any fact up to  $10x$ . It also means knowing the corresponding division facts.

*For example, a child who knows the  $3x$  table well would be able to answer questions like these with very little hesitation:*

*$9 \times 3$ , 7 lots of 3,  $3 \times 4$ ,  $18 \div 3$ , how many 3s in 24?*

As the children get more confident they should also have strategies for using known facts to help them work out other facts and also to work with larger numbers or decimals.

*e.g. I know  $5 \times 3$  is 15, so I can work out  $50 \times 3$ ,  $5 \times 30$ ,  $150 \div 5$ ,  $500 \times 3$ ,  $50 \times 30$ ,  $5 \times 0.3$ ,  $150 \div 30$ ...*

A suggested order for learning tables:

$2x$ ,  $10x$ ,  $5x$ ,  $4x$  (double  $2x$ ),  $3x$ ,  $6x$  (double  $3x$ ),  $9x$ ,  $8x$ ,  $7x$

**Just a few minutes a day  
could make a real  
difference to your child's  
confidence with number.**





# ICT links

There are many useful games on the internet which give children chance to practise number facts and mental calculations. Links to some that we know are particularly good can be found through the children's links on the **school website**:

[www.dowson.tameside.sch.uk](http://www.dowson.tameside.sch.uk)

For example, **Hit the Button** is good for practising number bonds and tables facts.

For those who have subscribed to **Education City** a selection of basic number facts games will be available all year.

**Other useful sites include:**

[www.topmarks.co.uk](http://www.topmarks.co.uk) select Games, 7-11 then category e.g. addition and subtraction

[www.woodlands-junior.kent.sch.uk/maths](http://www.woodlands-junior.kent.sch.uk/maths)

[www.bbc.co.uk/schools/ks2bitesize/maths](http://www.bbc.co.uk/schools/ks2bitesize/maths) (particularly useful for Y6)

[www.bbc.co.uk/schools/digger](http://www.bbc.co.uk/schools/digger) (select 7-9 or 9-11)

[www.channel4learning.com/sites/puzzlemaths](http://www.channel4learning.com/sites/puzzlemaths)

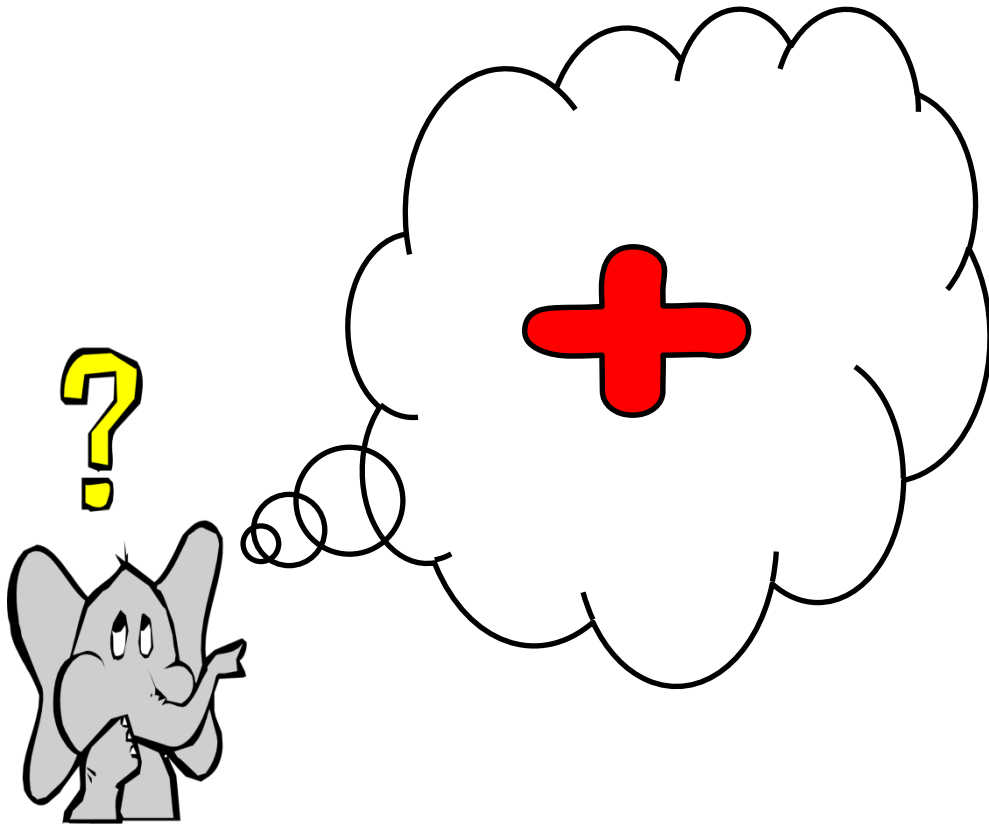
[www.ictgames.com](http://www.ictgames.com) (select numeracy - designed for infants but some useful games to practise basic number facts)

[www.mad4maths.com](http://www.mad4maths.com)

[www.counton.org/games](http://www.counton.org/games)

[www.primaryinteractive.co.uk/maths](http://www.primaryinteractive.co.uk/maths) (includes an easy link to Moon Maths for tables practice)

# Addition



add and count on  
addition plus  
more sum total  
altogether increase



Recognise numbers 0 to 10

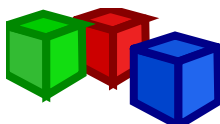
0 1 2 3 4 5 6 7 8 9 10



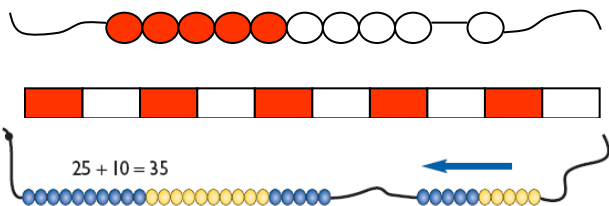
1, 2, 3, 4, 5, 6  
... there are 6  
teddies

Count reliably up to 10 everyday objects

Find one more than a number



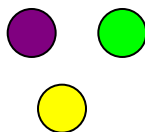
One more than  
three is four



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Count in ones and tens

Begin to relate addition to  
combining two groups of objects



and makes 5

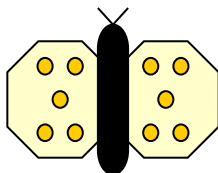
$$3 + 2 = 5$$



Count along a number line to  
add numbers together

Begin to use the + and = signs to record  
mental calculations in a number sentence

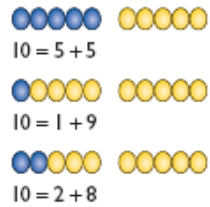
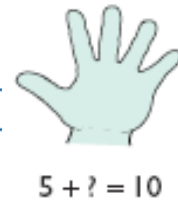
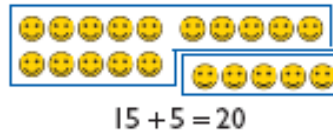
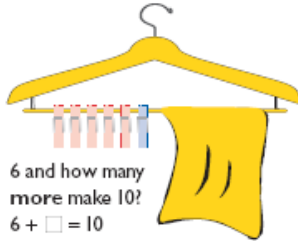
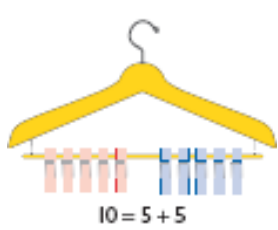
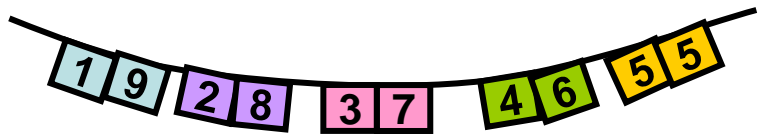
$$6 + 4 = 10$$



$$5 + 5 = 10$$

Know doubles of numbers

Know by heart all pairs of numbers with a total of 10 and 20



$$1 + 2 = 3$$



$$2 + 1 = 3$$



$$2 + 5 = 7$$

2 count on 5



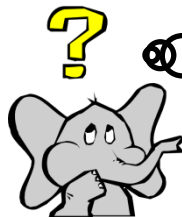
$$5 + 2 = 7$$

5 count on 2

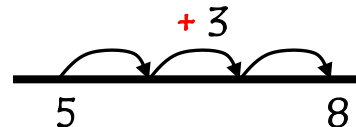


Know that addition can be done in any order

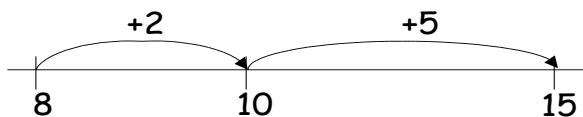
Put the biggest number first and count on



$$3 + 5$$

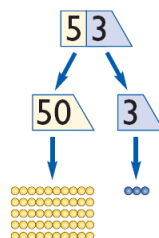


$$8 + 7 = 15$$



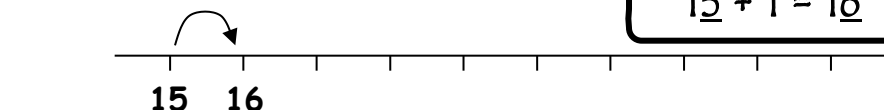
Add two single-digit numbers that bridge 10

Begin to partition numbers in order to add

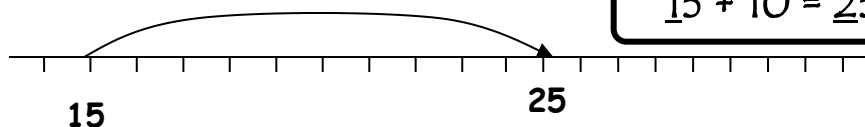


$$30p + 4p = 34p$$

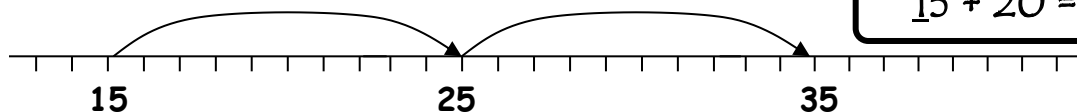
Know which digit changes when adding 1s or 10s to any number



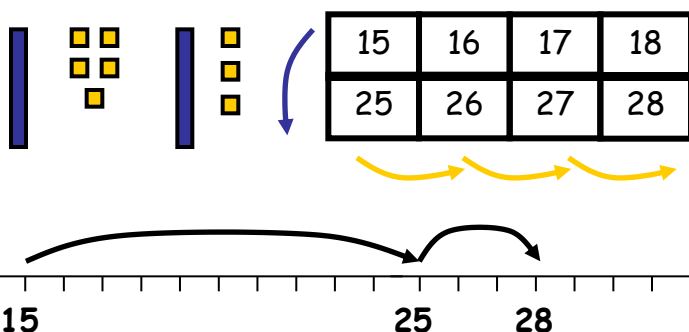
$$15 + 1 = 16$$



$$15 + 10 = 25$$



$$15 + 20 = 35$$



Adding two two-digit numbers (without bridging)

Counting in tens and ones

Partitioning and recombining

$$15 + 13 = 28$$

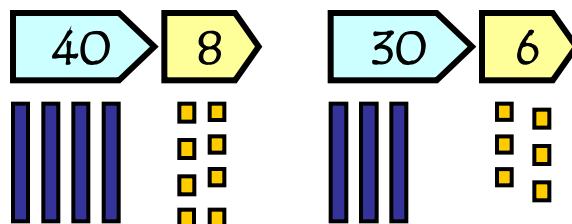
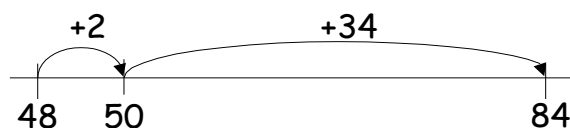
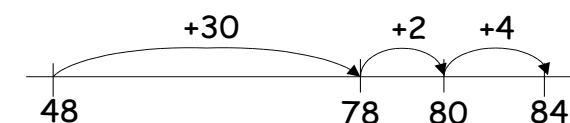
Adding two two-digit numbers (bridging through tens boundary)

Using a number line

OR

Using place value cards and place value apparatus to partition numbers and recombine

$$48 + 36 = 84$$



$$40 + 30 + 8 + 6$$

$$40 + 30 = 70$$

$$8 + 6 = 14$$

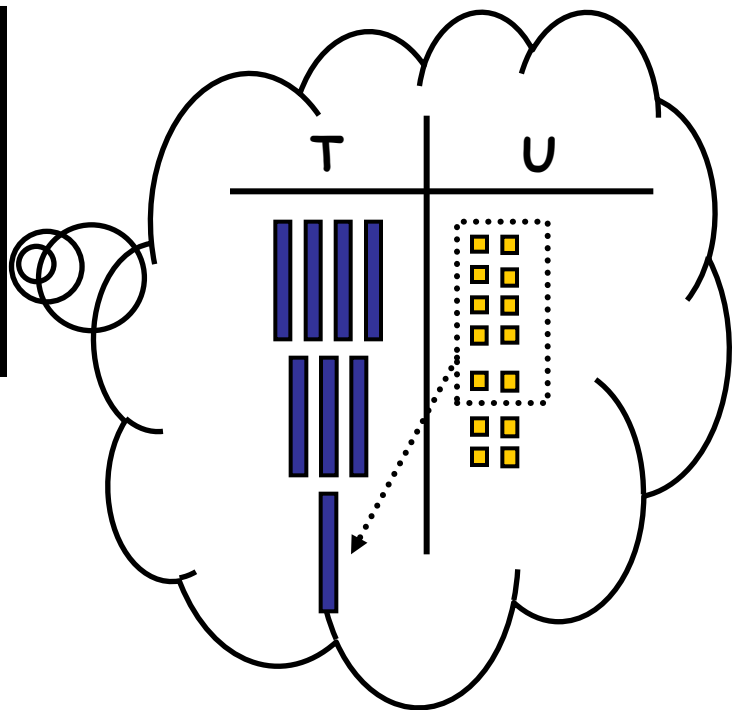
$$70 + 14 = 84$$

### Expanded method

It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support calculations. The expanded method enables children to see what happens to numbers in the standard written method.

$$48 + 36$$

$$\begin{array}{r} 48 \\ + 36 \\ \hline \end{array}$$



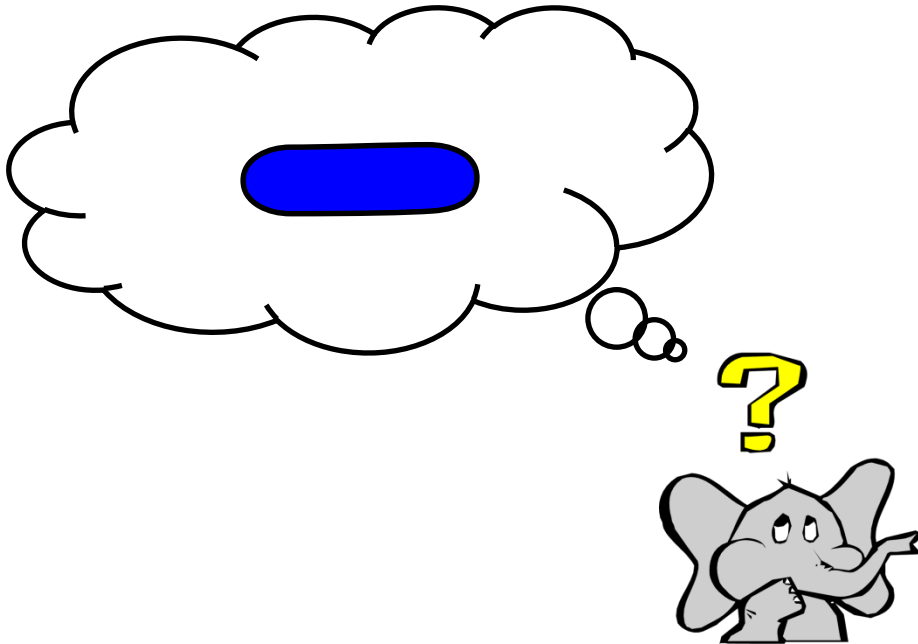
T	U
40	+ 8
30	+ 6
80	+ 4
<hr/>	
10	

$$\begin{array}{r} 48 \\ + 36 \\ \hline 84 \\ 1 \end{array}$$

### Standard written method

The previous stages reinforce what happens to the numbers when they are added together using more formal written methods.

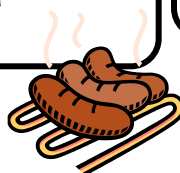
# Subtraction



count back    take away  
fewer    subtract  
minus    less  
difference between

Begin to count backwards in familiar contexts such as number rhymes or stories

Five fat sausages frying in a pan ...



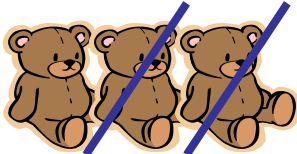
Ten green bottles hanging on the wall ...



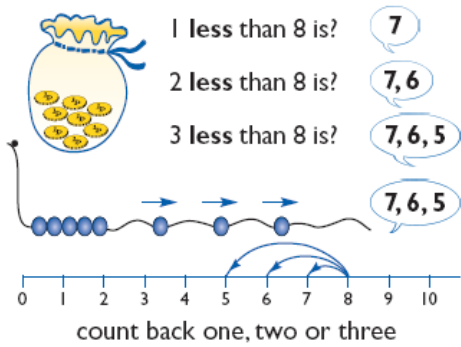
10, 9, 8, 7, ...

Continue the count back in ones from any given number

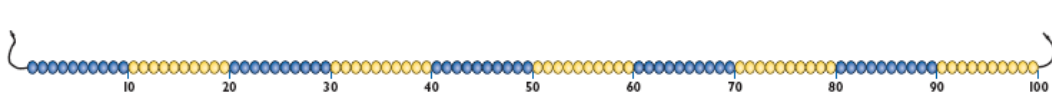
Begin to relate subtraction to 'taking away'



Three teddies take away two teddies leaves one teddy



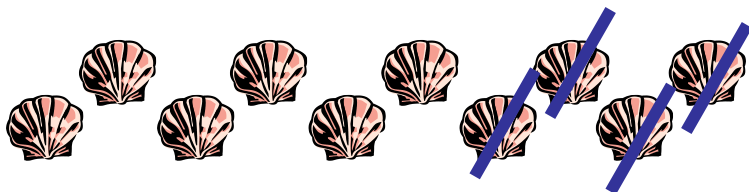
Find one less than a number



Count back in tens



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



If I take away four shells there are six left

Count backwards along a number line to 'take away'



Begin to use the - and = signs to record mental calculations in a number sentence

Maria had six sweets and she ate four. How many did she have left?



?



$$6 - 4 = 2$$



$$6 + ? = 10$$

$$10 - 6 = ?$$

$$? + 6 = 10$$

$$10 - 4 = 6$$



$$20 = 12 + 8$$

$$8 + 12 = 20$$

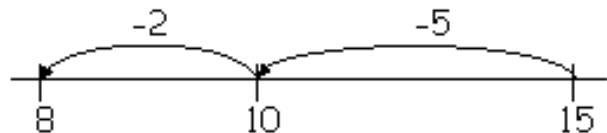
$$20 - 8 = 12$$

$$20 - 12 = 8$$

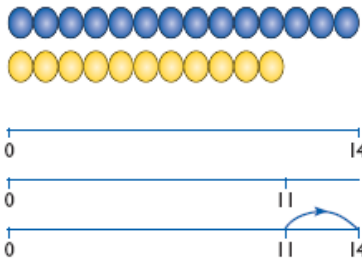
Know by heart subtraction facts for numbers up to 10 and 20

Subtract single digit numbers often bridging through 10

$$15 - 7 = 8$$



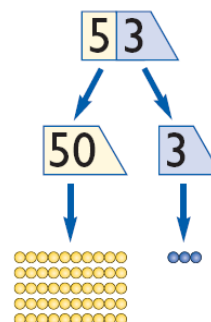
The difference is?



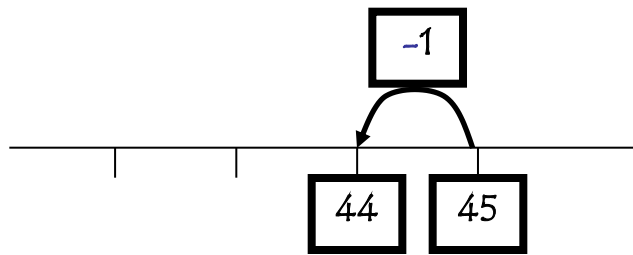
The difference between 11 and 14 is 3.  
 $14 - 11 = 3$   
 $11 + \square = 14$

Begin to find the difference by counting up from the smallest number

Begin to partition numbers in order to take away



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



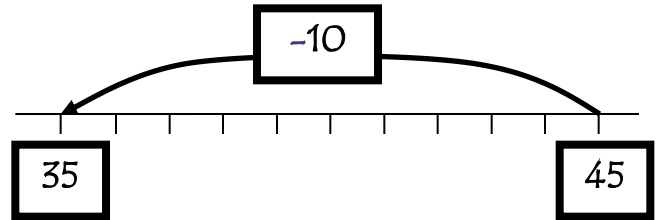
Subtract 1 from a two-digit number

$$45 - 1$$

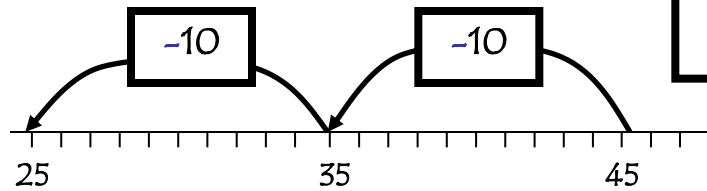
Subtract 10 from a two-digit number

$$45 - 10$$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



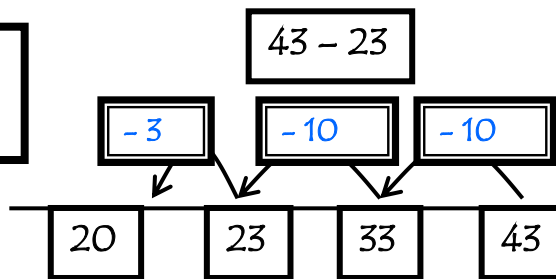
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Subtract multiples of 10 from any number

$$45 - 20$$

Partition the number to be subtracted (no exchanging)



$$43 - 20 = 23$$

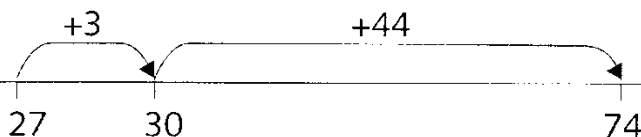
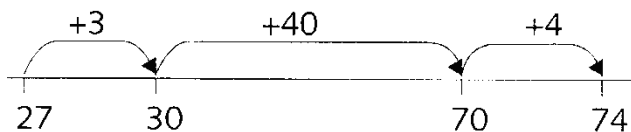
$$23 - 3 = 20$$

$$23 - 3 = 20$$

Decide whether to count on or count back

$$74 - 27 = 47$$

Now what's the answer?



Partitioning number to be subtracted - with exchanging (links to counting back on number line)

$$\begin{array}{|c|c|} \hline 4 & 3 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 20 & 7 \\ \hline \end{array}$$

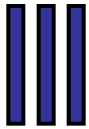
$$43 - 27 = 16$$

$$43 - \begin{array}{|c|c|} \hline 20 & 7 \\ \hline \end{array}$$

$$43 - 20 = 23$$

$$23 - 7 = 16$$

$$43 - 27 = 16$$



to subtract 7 units  
we need to exchange  
a ten for ten units



NOTE: the correct language is 'exchange' not 'borrow'

T	U
- 2	7
<hr/>	<hr/>

### Expanded method

It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support calculations. The expanded method enables children to see what happens to numbers in the standard written method.

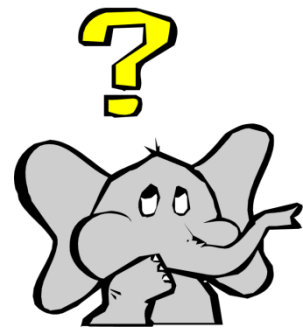
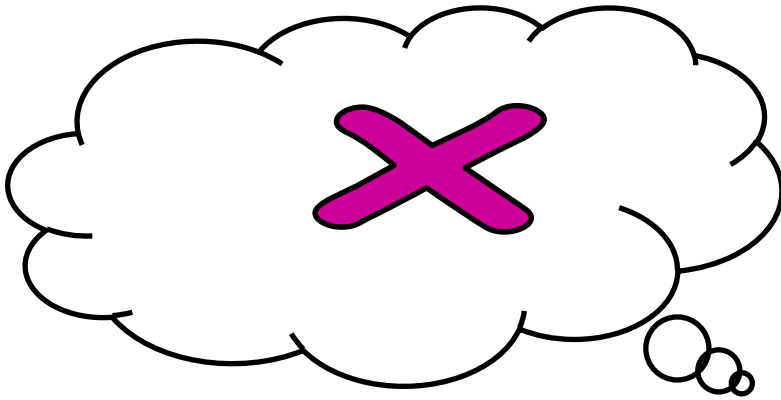
$$\begin{array}{r} 30 \quad \cancel{40} \quad + \quad 10 + 3 \\ - \quad 20 \quad + \quad 7 \\ \hline 10 \quad + \quad 6 \end{array}$$

### Standard written method

The previous stages reinforce what happens to numbers when they are subtracted using more formal written methods. It is important that the children have a good understanding of place value and partitioning.

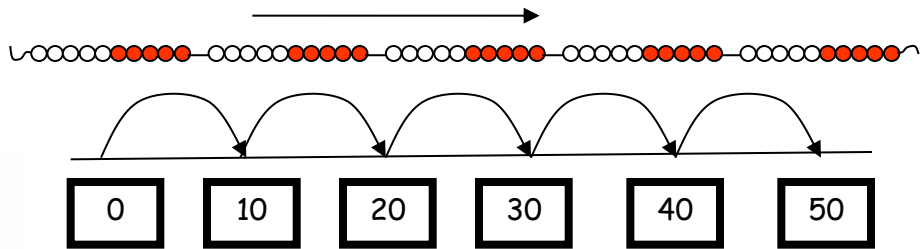
$$\begin{array}{r} 3 \quad \cancel{4} \quad 13 \\ - \quad 27 \\ \hline 16 \end{array}$$

# Multiplication

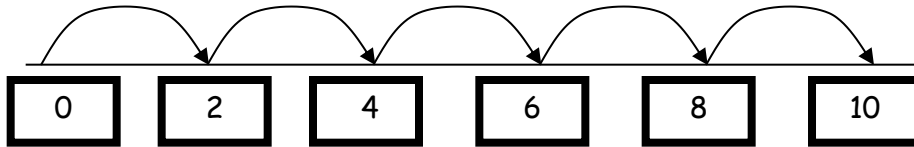


multiplication	product
once, twice, three times	
double	groups of
repeated addition	lots of
array, row, column	multiply
times	multiple

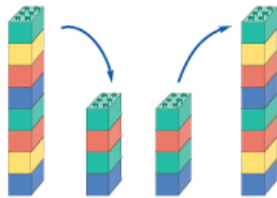
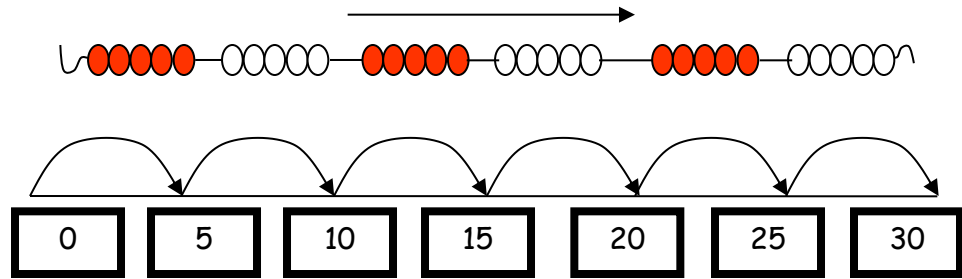
Count in tens  
from zero



Count in twos  
from zero



Count in fives  
from zero



half of 8 is 4  
 $8 \div 2 = 4$

double 4 is 8  
 $4 \times 2 = 8$

Know doubles and  
corresponding halves

Know multiplication tables to  $10 \times 10$

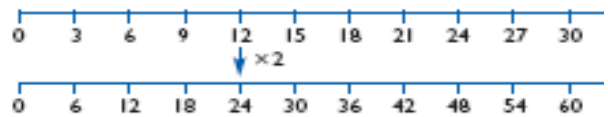
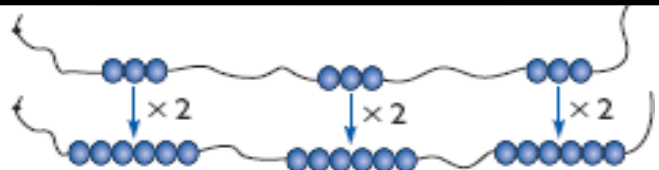
$$2 \times 5 = 10$$

$$\times 5$$

$$6 \times 5 = 30$$

$$3 \times 5 = 15$$

$$8 \times 5 = 40$$



$$12 \times 2 = 24$$

Twice as  
many

Use known facts to  
work out new ones


Understand that ...

$$24 \times 20 = 24 \times 2 \times 10$$

$$24 \times 50 = 24 \times 5 \times 10$$

Use factors to multiply

Understand multiplication  
as repeated addition



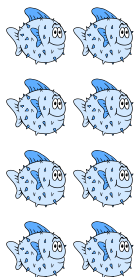
$$2 + 2 + 2 + 2$$

$$2 + 2 + 2 + 2 = 8$$

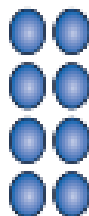
$$4 \times 2 = 8$$

2 multiplied by 4

4 lots of 2

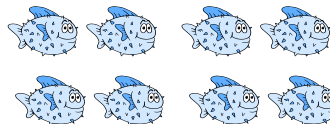


$$2 \times 4$$

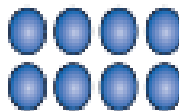


$$4 \times 2 = 8$$

$$2 \times 4 = 8$$



$$4 \times 2$$

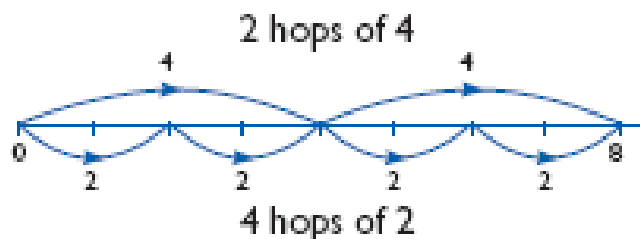


$$2 \times 4 = 8$$

$$4 \times 2 = 8$$

Understand  
multiplication  
as an array

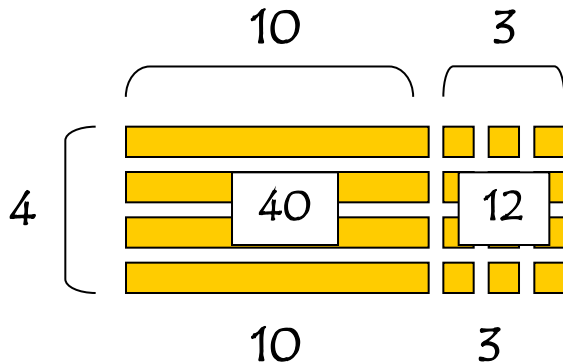
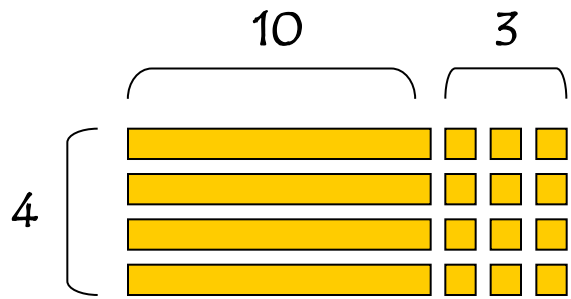
Understand how to  
represent arrays  
on a number line





Use place value apparatus to support the multiplication of  $U \times TU$

$$4 \times 13$$



Use place value apparatus to support the multiplication of  $U \times TU$  alongside the grid method

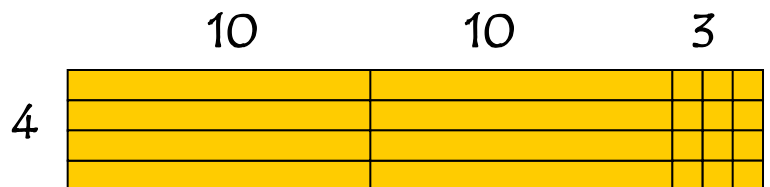
4	40	12
---	----	----

$$4 \times 13$$

$$40 + 12 = 52$$

Use place value apparatus to represent the multiplication of  $U \times TU$  alongside the grid method

$$4 \times 23$$



4	40	40	12
---	----	----	----

4	80	12
---	----	----

$$80 + 12 = 92$$

Multiplying TU x TU

$$14 \times 33$$

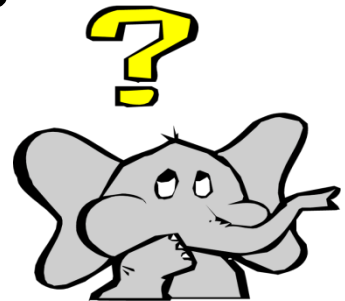
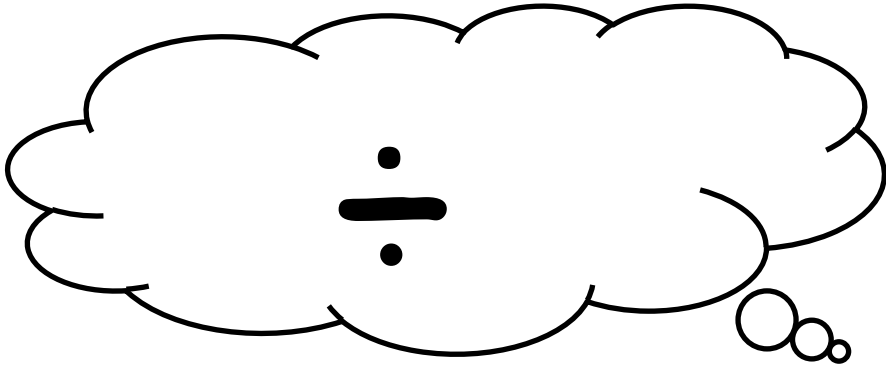
	30	3	
10	300	30	= 330 +
4	120	12	= 132
			<u>462</u>

300
120
30
+ 12
<u>462</u>

56	
$\times$ 27	
1120	(56 $\times$ 20)
<u>392</u>	(56 $\times$ 7)
1512	
1	

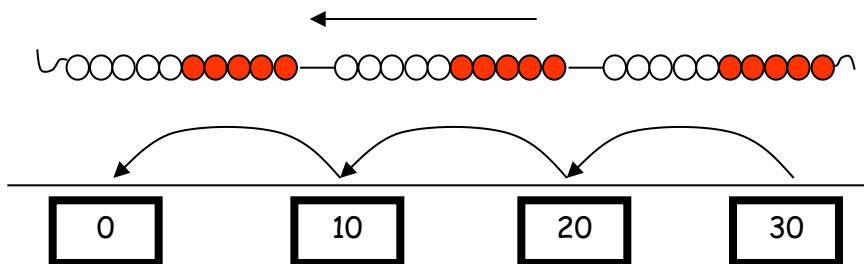
Standard written method

# Division

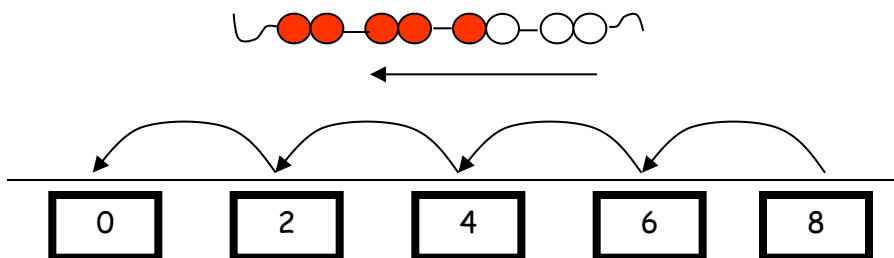


group                      groups of  
                                 lots of                      divide  
divided by                      quotient  
division                      factor  
                                 remainder                      divisible  
half                      halve                      share

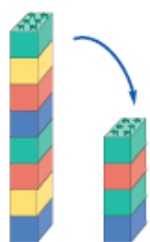
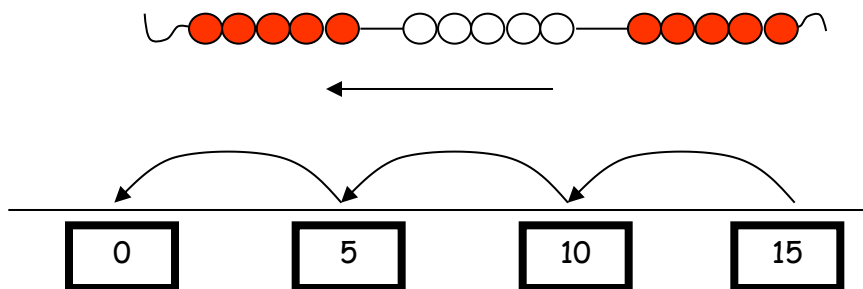
Count back in tens



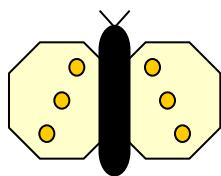
Count back in twos



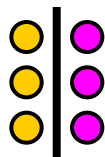
Count back in fives



half of 8 is 4  
 $8 \div 2 = 4$



Half of 6 is 3  
 $\frac{1}{2}$  of 6 = 3



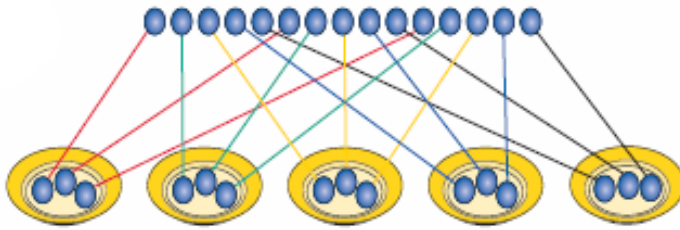
Know halves

Use known multiplication facts to work out corresponding division facts

If  $2 \times 10 = 20$   
then  
 $20 \div 10 = 2$   
 $20 \div 2 = 10$

$$15 \div 3 = 5$$

15 shared between 5



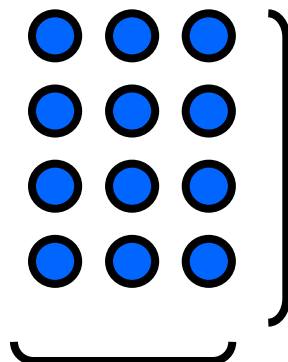
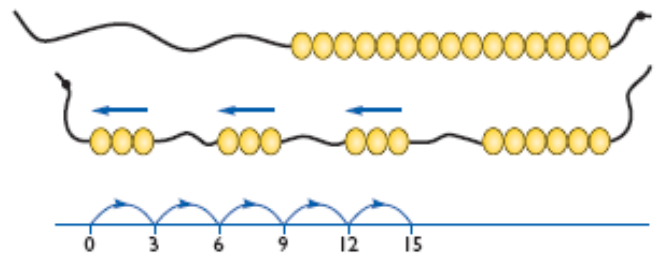
Understand division  
as sharing

Understand division  
as grouping

How many 3s  
in 15?



$$15 \div 3 = 5$$



12 divided into groups  
of 3 gives 4 groups

$$12 \div 3 = 4$$

12 divided into groups  
of 4 gives 3 groups

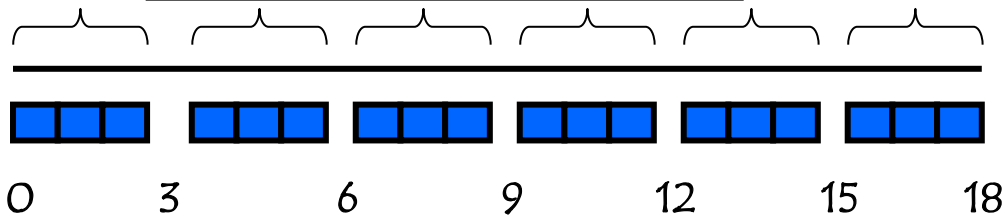
$$12 \div 4 = 3$$

Reinforce division as  
grouping through the  
use of arrays

Represent 'groups' for division on a number line using apparatus alongside the line

18 divided into groups of 3

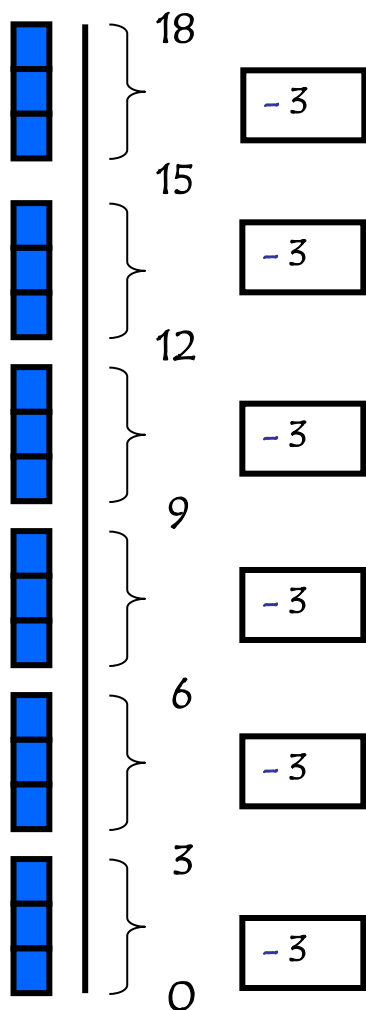
$$18 \div 3 = 6$$



$$18 \div 3 = 6$$



$$18 \div 6 = 3$$



$$18 \div 3 = 6$$

$$\begin{array}{r} 18 \\ - 3 (1 \times 3) \\ \hline 15 \\ - 3 (1 \times 3) \\ \hline 12 \\ - 3 (1 \times 3) \\ \hline 9 \\ - 3 (1 \times 3) \\ \hline 6 \\ - 3 (1 \times 3) \\ \hline 3 \\ - 3 (1 \times 3) \\ \hline 0 \end{array}$$

Understand division as repeated subtraction using a vertical line and apparatus to make the links



Children need to see that as the numbers get larger, large chunk subtraction is the more efficient method. Multiples of the divisor (large chunks) are taken away. Multiplication facts are needed to see the size of the 'chunk'.

What facts do I know about the 7 times-table?

$$100 \div 7 = \underline{14} \text{ r } 2$$

$$\begin{array}{r} 100 \\ - 70 \quad (10 \times 7) \\ \hline 30 \\ - 28 \quad (4 \times 7) \\ \hline 2 \end{array}$$

$$518 \div 7 = \underline{74}$$

$$\begin{array}{r} 518 \\ - 350 \quad (50 \times 7) \\ \hline 168 \\ - 140 \quad (20 \times 7) \\ \hline 28 \\ - 28 \quad (4 \times 7) \\ \hline 0 \end{array}$$

#### Fact Box

$$1 \times 7 = 7$$

$$2 \times 7 = 14$$

$$5 \times 7 = 35$$

$$10 \times 7 = 70$$

$$20 \times 7 = 140$$

$$50 \times 7 = 350$$

$$100 \times 7 = 700$$

$$560 \div 24$$

$$\begin{array}{r} 23 \text{ r } 8 \\ 24 \overline{) 560} \\ - 480 \\ \hline 80 \\ - 72 \\ \hline 8 \end{array}$$

Standard written method

Links directly to large chunk subtraction

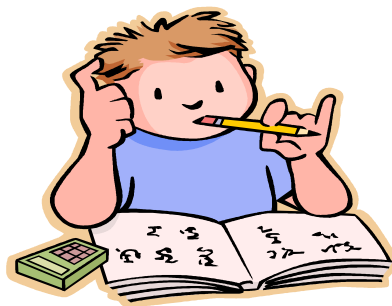
When faced with a calculation problem,  
encourage your child to ask...

★ Can I do this in my head?

★ Could I do this in my head using  
drawings or jottings to help me?

★ Do I need to use a written method?

★ Should I use a calculator? *(only if is  
necessary with the numbers involved)*



Also help your child to estimate and then check  
the answer.

Encourage them to ask...

Is the answer sensible?