

Elm Tree Mathematical Progression

Pre- Formal Teaching

At Elm Tree we have high expectations of all our children and strive for all of our children to be accessing formal Maths teaching following our Maths overview and the below calculation policy. It is however important to note that this calculation policy should be read in conjunction with the “Elm Tree Early Maths Overview”. For some of our younger cohorts at Elm Tree their developmental and cognitive delay poses challenges in their ability to access and engage with formal Maths lessons. In light of this, it is crucial for us to support the individual needs of every child and provide appropriate and tailored mathematical learning opportunities.

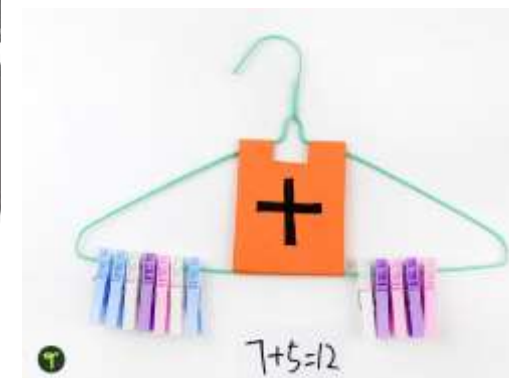
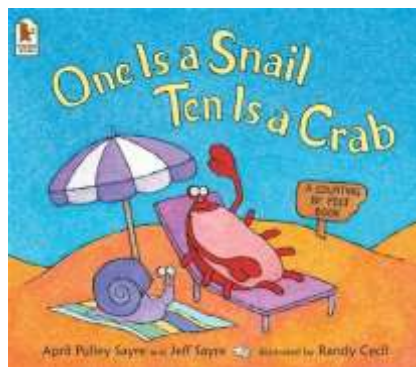
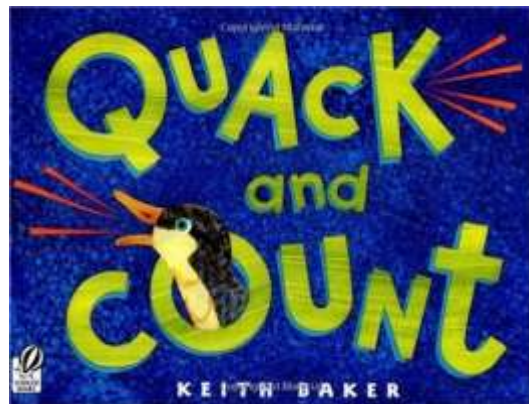
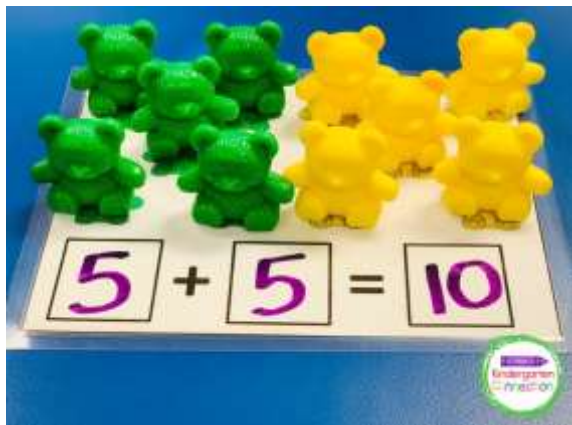
Children working below Range 5 will be taught Maths through continuous provision and pre-formal learning strategies. This approach allows for a more hands-on, experiential, and concrete learning experience, which can better align with their current developmental abilities. During continuous provision, these children are provided with a range of carefully planned and structured activities that encourage the acquisition of mathematical skills and concepts. These activities are embedded within their daily routines and learning experiences and take into account their unique learning styles and needs building a strong foundation for future formal learning.

Whilst these children are not currently engaging in formal Maths lessons, it is essential to note that they are still progressing and deepening their mathematical understanding through accessing continuous provision and pre-formal learning approaches. We are documenting their progress and regularly assessing their development so that we have a comprehensive overview of their learning journey. When children reach Range 5 they will be introduced to short teacher inputs and they will start a transition into formal Maths lessons at which point the Calculation Policy below is followed in conjunction with the Maths overview. It is important to note that for many children they will still continue to develop their Mathematical understanding through formal teaching but also through continuous provision throughout their time at Elm Tree.

Elm Tree Mathematical Progression

Transition into Formal Teaching

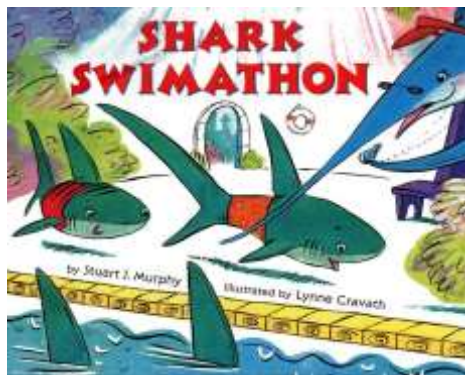
'Experience it' Addition



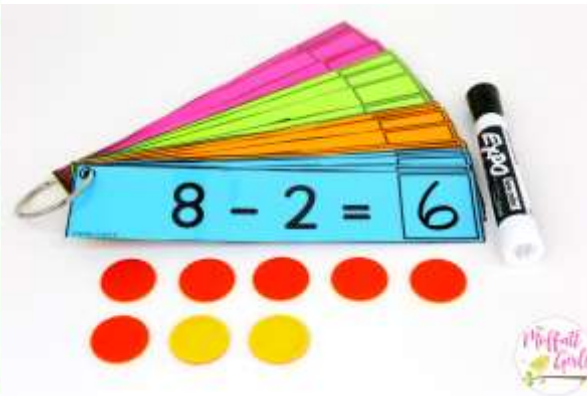
Elm Tree Mathematical Progression

Transition into Formal Teaching

'Experience it' Subtraction



Whack-a-Ball Subtraction Game



SUBTRACTION COVER UP

UP TO 12

COVER UP ANY SUBTRACTION FACT WHICH EQUALS THE NUMBER YOU ROLL ON THE DICE.

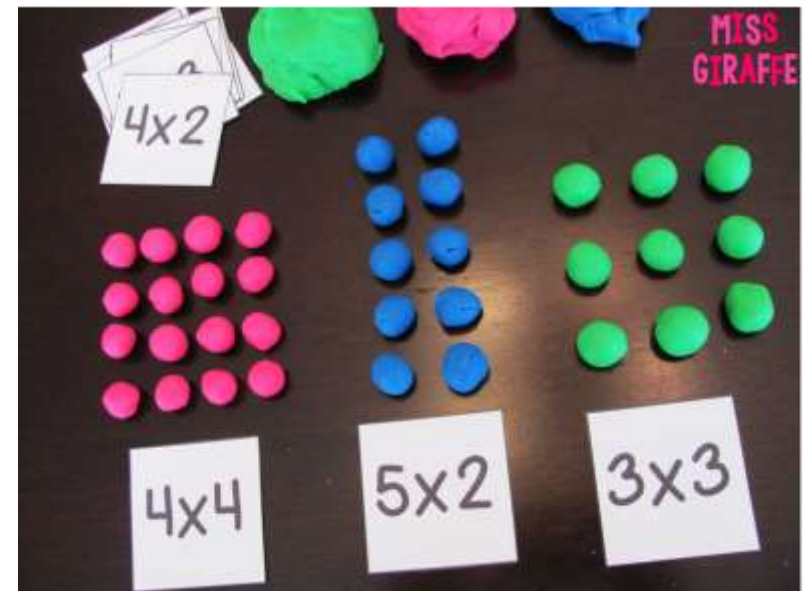
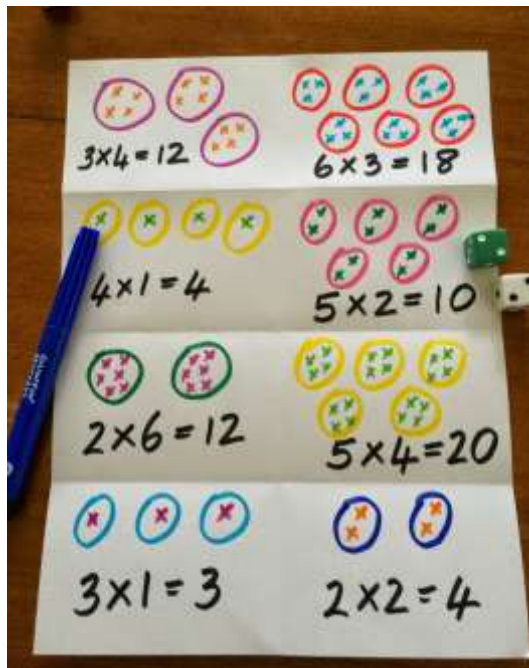
5-3	9-6	11-5	8-7	4-2	9-5
4-1	6-2	8-3	8-2	6-1	8-7
10-5	6-4	9-8	10-7	9-3	12-8
11-7	12-9	10-4	6-5	9-4	7-1
9-4	11-9	7-6	5-1	12-6	8-5
12-9	11-5	8-6	10-5	10-9	7-3

Hungry Monster Subtraction Game



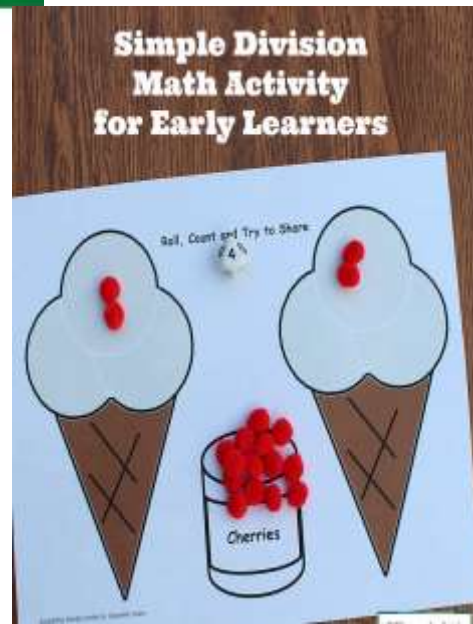
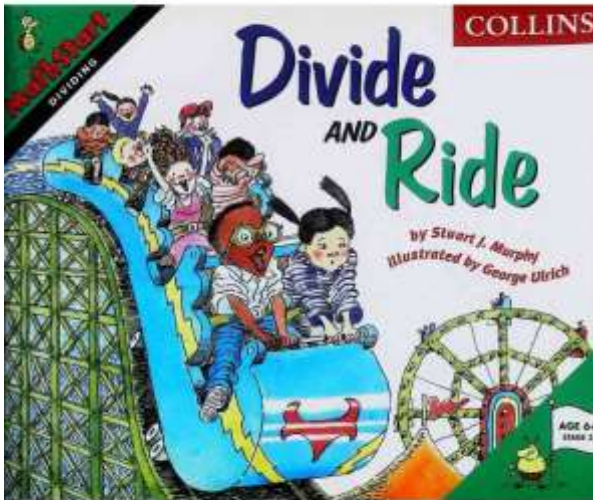
Elm Tree Mathematical Progression

Transition into Formal Teaching 'Experience it' Multiplication



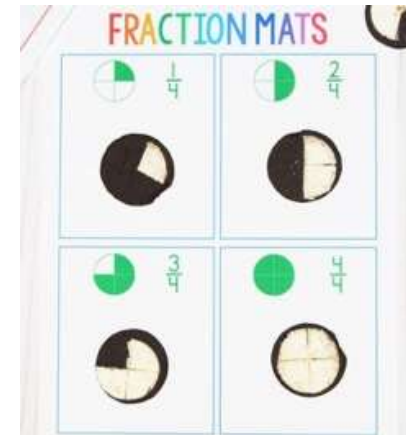
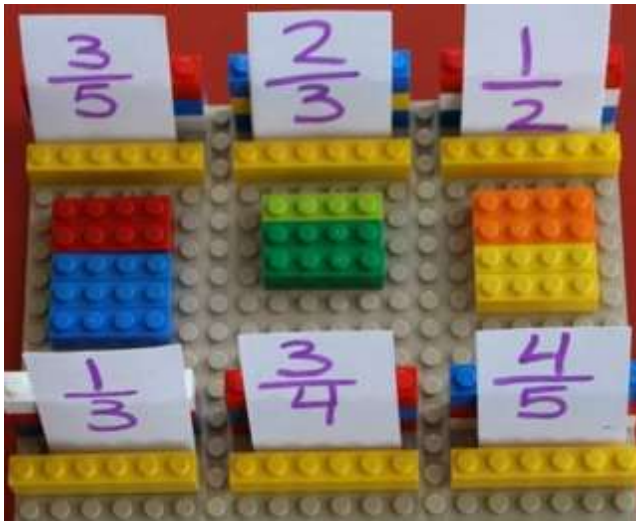
Elm Tree Mathematical Progression

Transition into Formal Teaching 'Experience it' Division



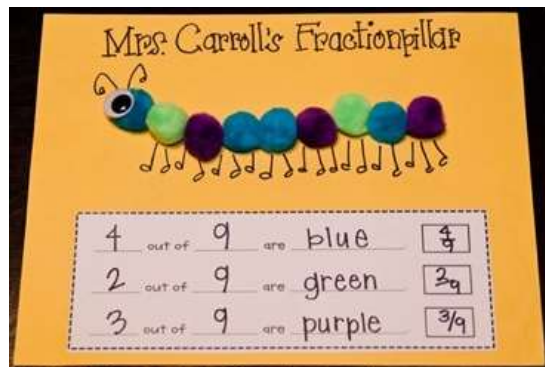
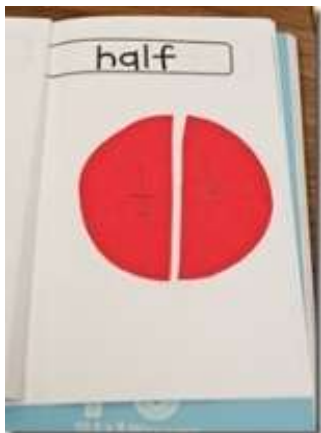
Elm Tree Mathematical Progression

Transition into Formal Teaching 'Experience it' Fractions

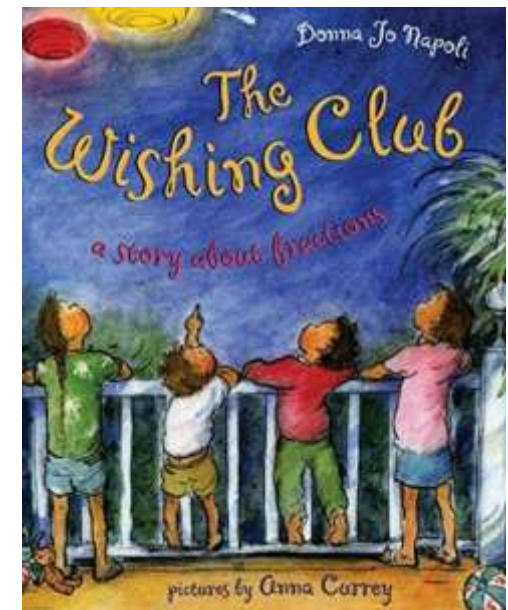


Shape fraction cards

	$\frac{1}{2}$		$\frac{1}{2}$
	$\frac{1}{2}$		$\frac{1}{2}$
	$\frac{1}{4}$		$\frac{1}{4}$
	$\frac{1}{4}$		$\frac{1}{4}$
	$\frac{3}{4}$		$\frac{3}{4}$
	$\frac{3}{4}$		$\frac{3}{4}$



1											
$\frac{1}{3}$				$\frac{1}{3}$				$\frac{1}{3}$			
$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$



Elm Tree Mathematical Progression

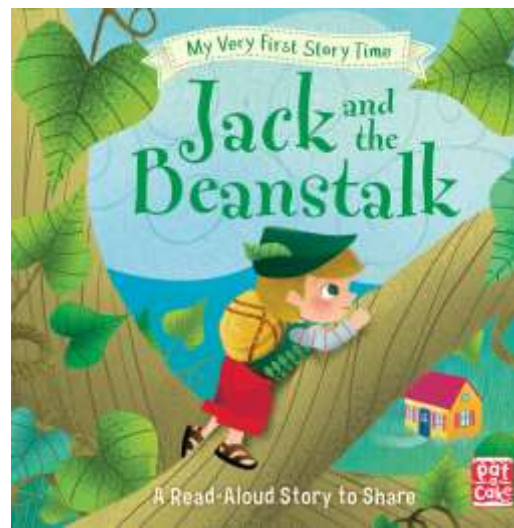
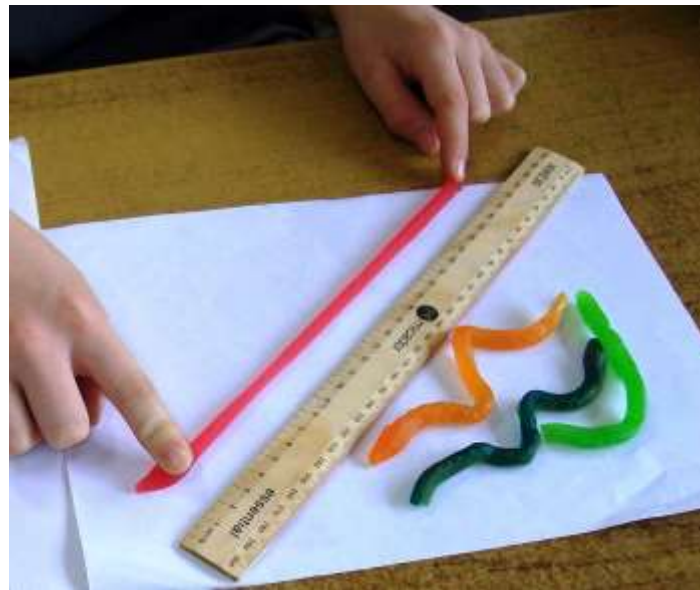
Transition into Formal Teaching 'Experience it' Time



Elm Tree Mathematical Progression

Transition into Formal Teaching

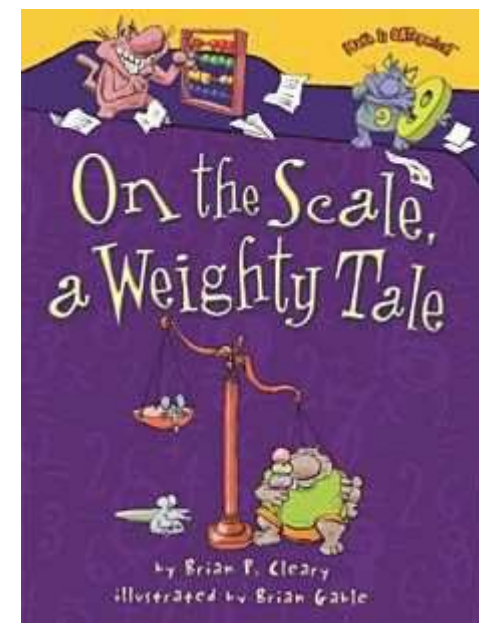
'Experience it' Length



Elm Tree Mathematical Progression

Transition into Formal Teaching

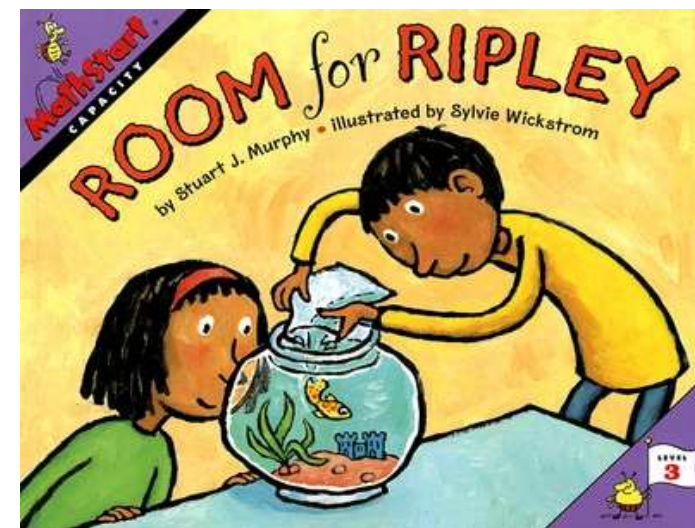
'Experience it' Weight



Elm Tree Mathematical Progression

Transition into Formal Teaching

'Experience it' Capacity



Elm Tree Mathematical Progression
Transition into Formal Teaching
'Experience it' Temperature

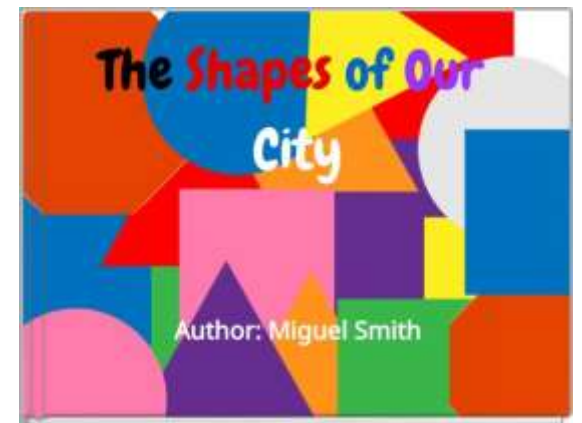
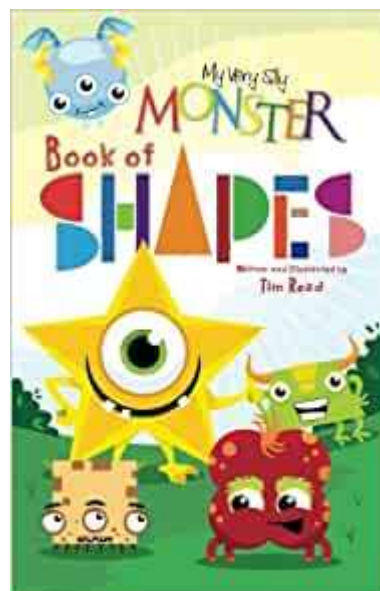
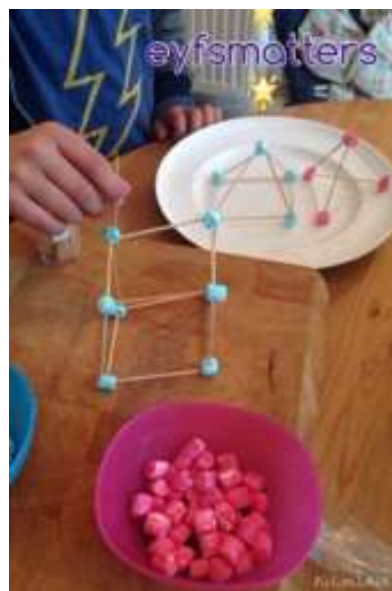
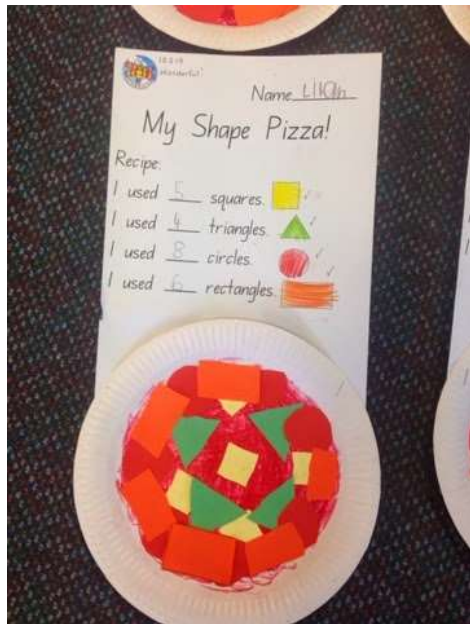


Transition into Formal Teaching

Elm Tree Mathematical Progression

Transition into Formal Teaching

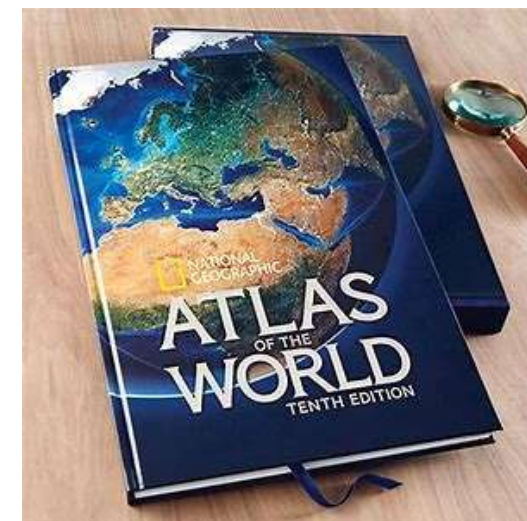
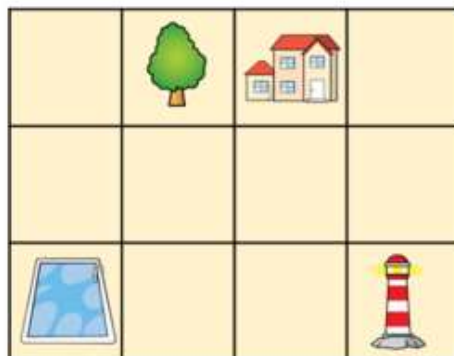
'Experience it' Shape



Elm Tree Mathematical Progression

Transition into Formal Teaching

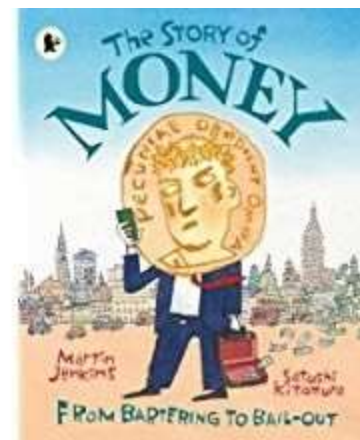
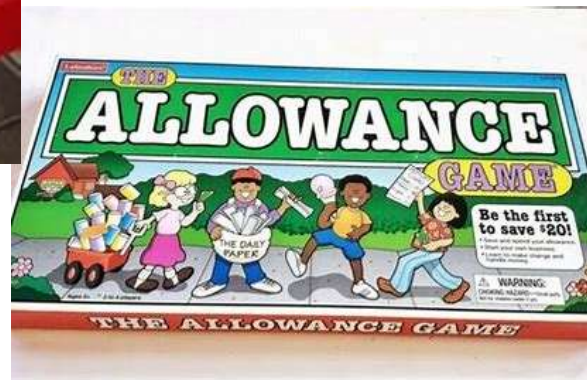
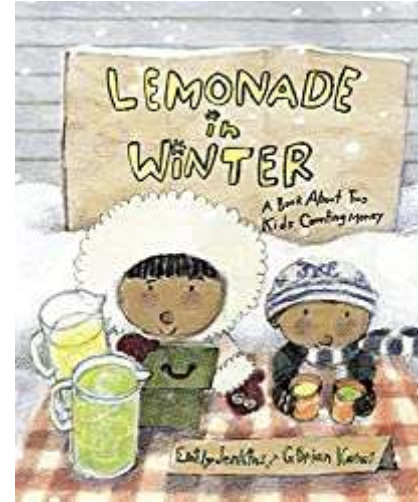
'Experience it' Position and Direction



Elm Tree Mathematical Progression

Transition into Formal Teaching

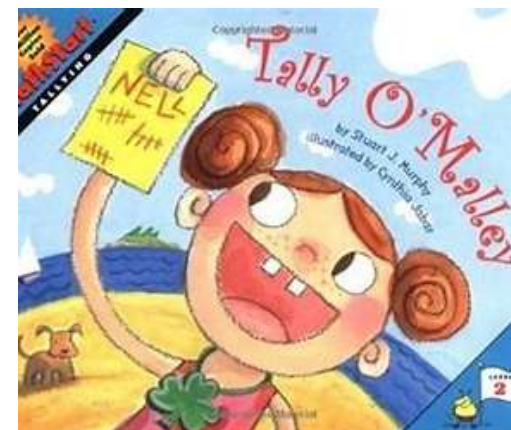
'Experience it' Money



Elm Tree Mathematical Progression

Transition into Formal Teaching

'Experience it' Statistics



Elm Tree Mathematical Progression

'Experience it' vocabulary

Step	<u>Number and Place Value</u>
1	Number names to 3
2	Number names to five,
3	number zero, one, two...to twenty and beyond none, first, second, order, more, less
4	numbers to 100 and beyond units, ones, tens digit 'teens' numbers, tens digit 'teens' numbers
5	exchange fewer most least first...second...eleventh...twentieth half-way between roughly count on in twos...fives...tens odd, even forwards, backwards repeating pattern number line number square number track, hundreds sequence continue predict partition sequence consecutive value rule one-, two-, three-digit number place, place value stands for represents twenty-first, twenty-second... exact, exactly round, nearest > and < numeral
6	Thousand, zero... thousand+ relationship one hundred more one hundred less approximate approximately round up round down place holder estimate near double, thousands, numeral, one thousand more/less, round to the nearest 10...100...1,000, negative numbers,
7	ten thousand, hundred thousand, integer positive integer, positive number negative integer, above zero, below zero, minus next consecutive sort, classify, property Roman numerals to 100 (I to C), greater than or equal to \geq less than or equal to \leq ascending order descending order round to the nearest 10,000 and 100,000 \approx is approximately equal to approximation square/d number cube/d number to the power of... prime number composite number one squared, two squared etc million term-to-term rule Roman numerals to 1,000 (M)
8	factorise recurring numbers to ten million+ (10,000,000)

Step	<u>Addition and subtraction</u>
1	one, two, three, count
2	lots, how many...? Groups
3	number zero, one, two...to twenty and beyond none, count, count (up) to count on, count on in ones, one more, more, how many more...? All together
4	Add, altogether, add, make, makes, put together, plus, total, plus, ones, more than, number bond, count on in ones, forwards,

Elm Tree Mathematical Progression

5	Count on in twos, fives and tens, number bonds, bond, number bonds to twenty, commutative, addition, sum, hundreds boundary equation column(ar) addition column(ar) subtraction inverse operation exchange
6	One hundred more, one, two, three...digit, combine, facts, number facts, increase, decrease
7	units/ones boundary tenths boundary
8	Common multiple

Step	<u>Multiplication and division</u>
1	Match, not enough
2	Share, groups
3	evens and odds, double facts
4	equal to, multiple, sharing, grouping, array
5	Times table, multiply, multiplied by multiplication, multiple of remainder equation inverse operation product once, twice, three times...ten times as big... long... wide... as... repeated addition array row column, one each, two each, three each... group in threes...tens equal groups, divided by, divided into division inverse, lots of, groups of, as big... long... wide... as... repeated addition array row column share equally one each, two each, three each... group in pairs, groups of, ÷, divide
6	Factor, factor pair, divisible by, remainder, equation,
7	divisibility common factor prime factor divisor factorise
8	long division long multiplication brackets order of operations - BODMAS / BIDMAS quotient recurring decimal

Step	<u>Fractions, Ratio, Proportions</u>
1	Matches
2	Shares, splits
3	Part of, whole, half

Elm Tree Mathematical Progression

4	Fraction, halve, quarter
5	part equal parts fraction one whole one half, two halves one quarter, two... three... four... quarters one third
6	two thirds three thirds one tenth tenths denominator numerator eighth sixth fifth twentieth hundredth proportion in every for every decimal decimal fraction decimal point decimal place simplify
7	common fraction, simple fraction, vulgar fraction proper fraction improper fraction mixed number, mixed fraction reduced to cancel ninth twelfth thousandth percentage per cent %
8	Thousandth common denominator rational number quantities comparison scale scale factor proportion ratio

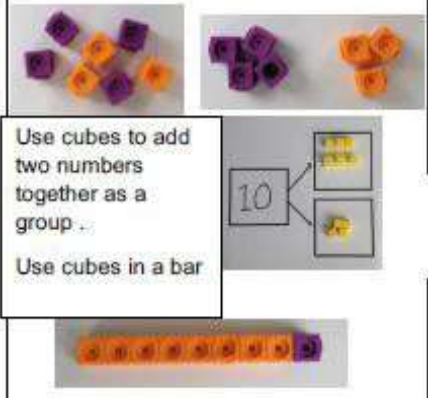
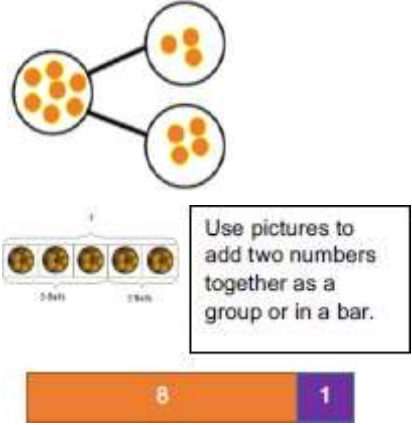
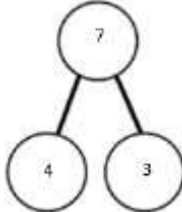
Step	<u>Time</u>
1	Finished, now, next
2	First, last, early, late, old, new
3	time Monday, Tuesday etc day week, birthday, holiday morning afternoon evening night bedtime dinnertime playtime today yesterday tomorrow, hour o'clock clock watch hands
4	Spring, summer, autumn, winter, month, year, weekend, midnight, half past How long ago...? How long will it be to...? How long will it take to...? How often...? always, never, sometimes, usually
5	January, February...December fortnight minute second quarter to, quarter past digital clock, analogue clock
6	century leap year calendar date am, pm, noon earliest, latest 12 hour clock, 24 hour clock
7	Millennium, date of birth,
8	Miles per hour

Elm Tree Mathematical Progression

Formal Teaching of Maths

'Use it' and 'Develop it': Calculation and Thinking

Addition

Objective and strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole (Part-part whole model)	 <p>Use cubes to add two numbers together as a group . . .</p> <p>Use cubes in a bar</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	<p>$4 + 3 = 7$ (four is a part, 3 is a part and the whole is seven)</p> 

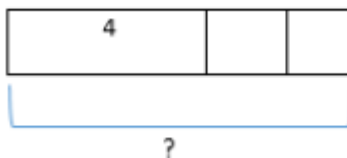
Elm Tree Mathematical Progression

Starting at the bigger number and counting on

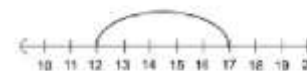


Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.

A bar model which encourages the children to count on

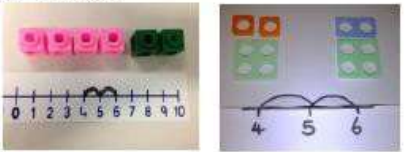

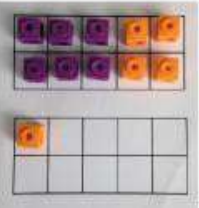


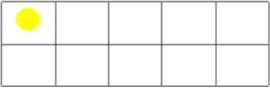

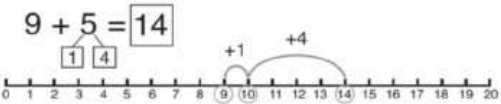


$$12 + 5 = 17$$



Start at the larger number on the number line and count on in ones or in one jump to find the answer.

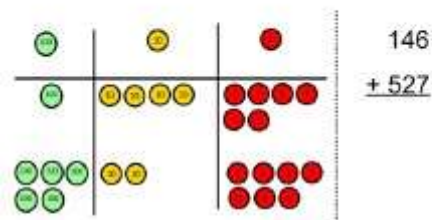
Elm Tree Mathematical Progression

	<p>Counting on using number lines by using cubes or numicon</p> 		
<p>Regrouping to make 10.</p>	 <p>$6 + 5 = 11$</p>  <p>Start with the bigger number and use the smaller number to make 10.</p> 	<p>Children to draw the ten frame and counters/cubes</p>    <p>$3 + 9 =$</p> <p>Use pictures or a number line. Regroup or partition the smaller number to make 10.</p>	<p>$9 + 5 = 14$</p>  <p>$7 + 4 = 11$</p> <p>If I am at seven, how many more do I need to make 10. How many more do I add on now?</p>

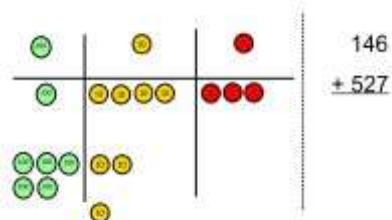
Elm Tree Mathematical Progression

Column Method regrouping

Make both numbers on a place value grid.



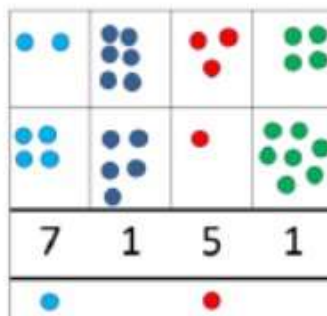
Add up the units and exchange 10 ones for one 10.



Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.

Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.



Start by partitioning the numbers before moving on to clearly show the exchange below the addition.

$$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$$

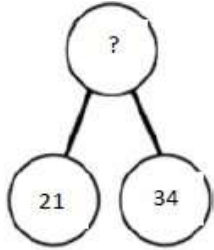
$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$

As the children move on, introduce decimals with the same number of decimal places and different. Money can be

$$\begin{array}{r} 23.361 \\ 9.080 \\ 59.770 \\ \hline + 1.000 \\ \hline 93.111 \\ + 1.000 \\ \hline \pounds 94.111 \end{array}$$

Elm Tree Mathematical Progression

Fluency variation, different ways to ask children to solve 21+34:



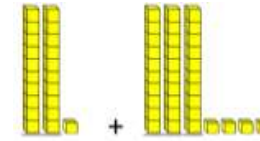
Sam saved £21 one week and £34 another.
How much did he save in total?
 $21+34=55$. Prove it! (reasoning but the children need to be fluent in representing this)

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

$$21 + 34 =$$

$$\square = 21 + 34$$

What's the sum of twenty one and thirty four?


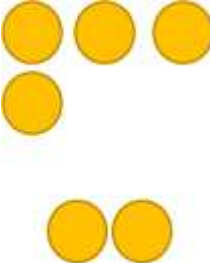
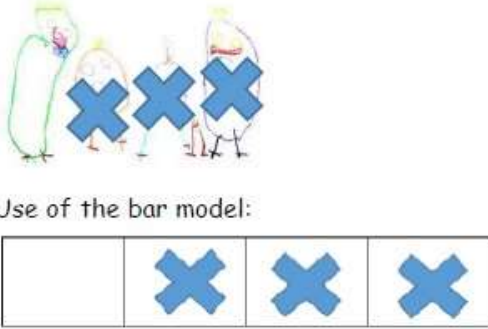


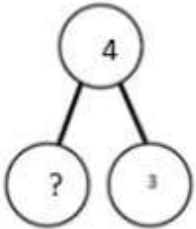


Always use missing digit problems too:

Tens	Ones
● ●	●
● ● ●	?
?	4

Elm Tree Mathematical Progression

Subtraction

Objective and strategies	Concrete	Pictorial	Abstract				
<p>Physically taking away and removing objects from a whole (use various objects too) rather than crossing out- children will physically remove the objects</p>	<p>$4 - 3 = 1$</p>  <p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p> <p>$6 - 2 = 4$</p> 	<p>Children to draw the concrete resources they are using and cross out.</p>  <p>Use of the bar model:</p>  	<p>$4 - 3 =$</p> <p><input type="text"/> = $4 - 3$</p> <table border="1" data-bbox="1615 639 1935 719"> <tr> <td colspan="2" style="text-align: center;">4</td> </tr> <tr> <td style="text-align: center;">3</td> <td style="text-align: center;">?</td> </tr> </table> 	4		3	?
4							
3	?						

Elm Tree Mathematical Progression

Counting back

Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.

$$13 - 4$$



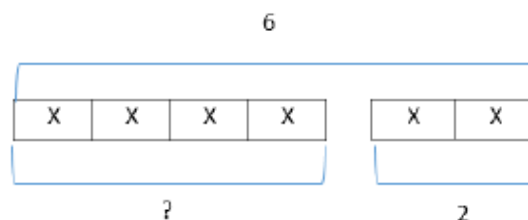
Use counters and move them away from the group as you take them away counting backwards as you go.



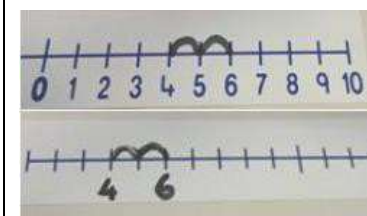
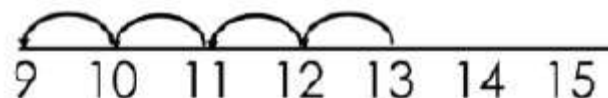
Counting back (using number lines or number tracks)



Children to represent what they see pictorially
e.g.



Count back on a number line or number track




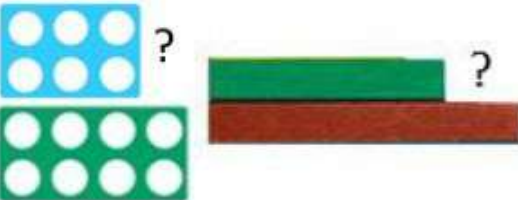
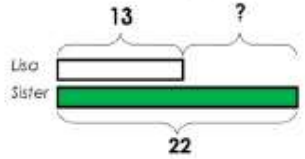
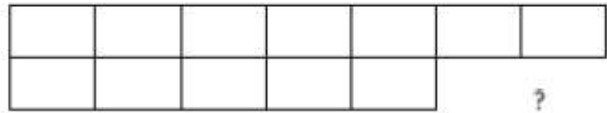
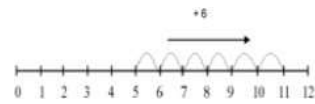
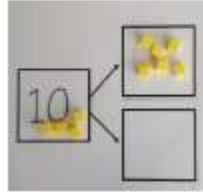
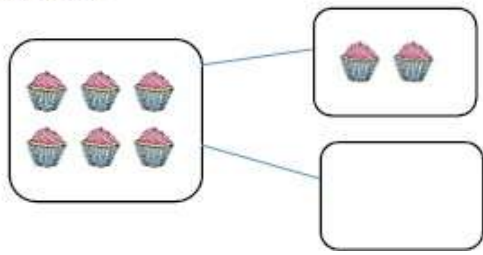
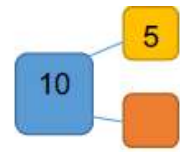
Start at the bigger number and count back the smaller number showing the jumps on the number line.




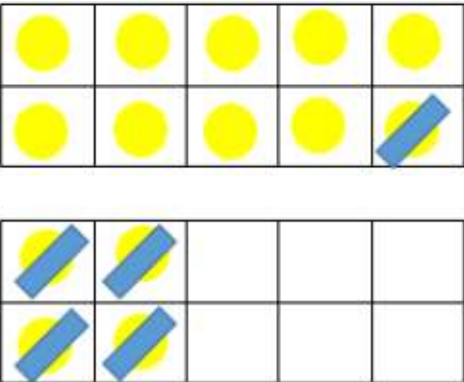
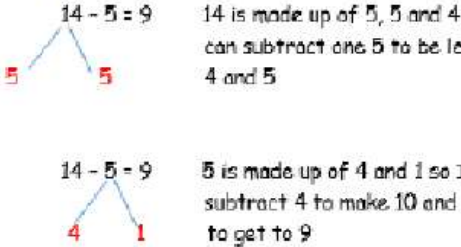
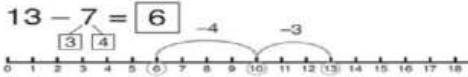
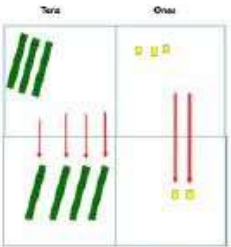
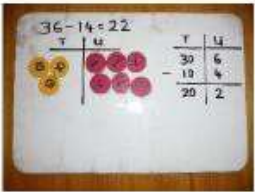
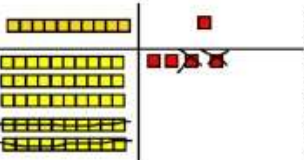
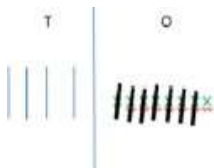

This can progress all the way to counting back using two 2 digit numbers.

Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

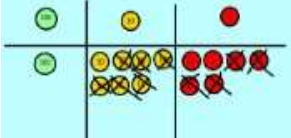
Elm Tree Mathematical Progression

<p>Find the difference</p>	<p>Finding the difference (using cubes, numicon or Cuisenaire rods, other objects can also be used)</p>  	<p>Comparison Bar Models</p> <p>Draw bars to find the difference between 2 numbers.</p> <p>Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.</p>  <p>Children to draw the cubes/other concrete objects which they have used</p> <p>XXXXXXXXX XXXXXX</p> <p>Use of the bar model</p> 	 <p>Count on to find the difference.</p> <p>Find the difference between 8 and 6. 8 - 6, the difference is ?</p> <p>Children to also explore why 9 - 7 = 8 - 6 (the difference, of each digit, has changed by 1 do the difference is the same- this will help when solving 10000-9987)</p>
<p>Part -Part Whole Model – linked back to addition.</p>	 <p>Link to addition- use the part whole model to help explain the inverse between addition and subtraction.</p> <p>If 10 is the whole and 6 is one of the parts. What is the other part?</p> <p>$10 - 6 =$</p>	<p>Use a pictorial representation of objects to show the part whole model.</p> 	 <p>Move to using numbers within the part whole model.</p>

Elm Tree Mathematical Progression

<p>Make 10</p>	<p>$14 - 9 =$</p>  <p>Make 14 on the ten frame. Take away the four first to make 10 and then take away one more so you have taken away 5. You are left with the answer of 9.</p>	<p>Children to present the ten frame pictorially</p> 	<p>$14 - 5 = 9$ You also want children related facts e.g. $15 - 9 = 5$</p> <p>Children to represent how they have solved it e.g.</p>  <p>$13 - 7 = 6$</p> 
<p>Column method without regrouping</p>	<p>Use Base 10 to make the bigger number then take the smaller number away.</p>  <p>Show how you partition numbers to subtract. Again make the larger</p> 	 <p>Calculations</p> $\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$ <p>Draw the Base 10 or place value counters alongside the written calculation to help to show working.</p> 	<p>$47 - 24 = 23$</p> $\begin{array}{r} 47 \\ - 24 \\ \hline 20 + 3 \end{array}$ <p>This will lead to a clear written column subtraction.</p> 

Elm Tree Mathematical Progression

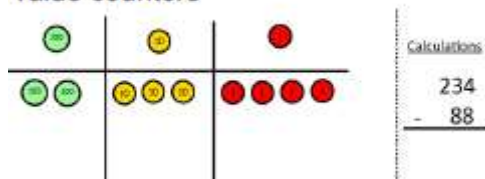
		<p>Calculations</p> $\begin{array}{r} 176 - 64 = \\ 176 \\ - 64 \\ \hline 112 \end{array}$			
--	---	--	--	--	--

Elm Tree Mathematical Progression

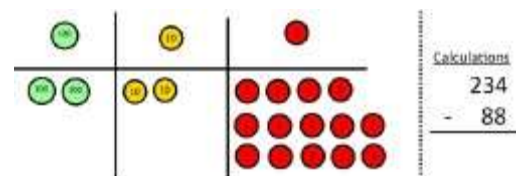
Column method with regrouping and replacing.

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value counters



Start with the ones, can I take away 8 from 4 easily? I need to exchange and replace one of my tens for ten ones.



Now I can subtract my ones.

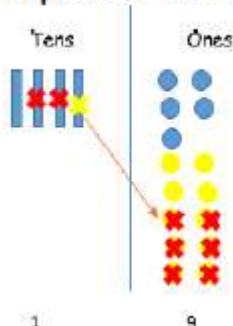
Now look at the tens, can I take away 8 tens easily? I need to exchange and replace one hundred for ten tens.



When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

Represent the base 10 pictorially



Children can start their formal written method by partitioning the number into clear place value columns.

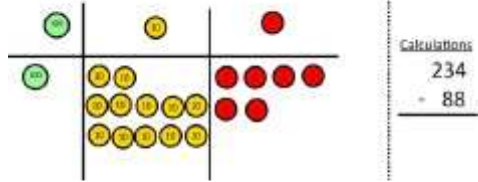
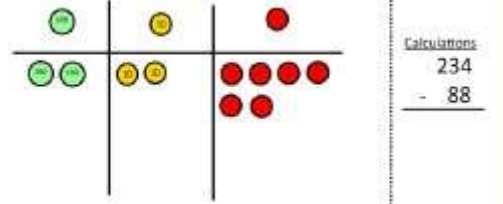


Moving forward the children use a more compact method.

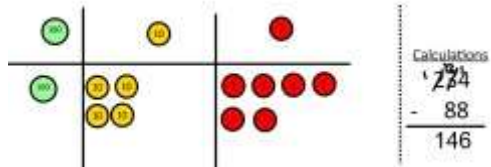
This will lead to an understanding of subtracting any number including decimals.

$$\begin{array}{r} 5 \quad 12 \quad 1 \\ 2 \quad \cancel{6} \quad \cancel{3} \quad . \quad 0 \\ - \quad 2 \quad 6 \quad . \quad 5 \\ \hline 2 \quad 3 \quad 6 \quad . \quad 5 \end{array}$$

Elm Tree Mathematical Progression



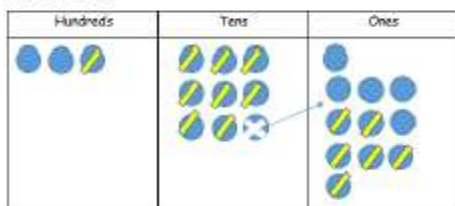
Now I can take away eight tens and complete my subtraction



Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and replacing and show where we write our new amount.

Elm Tree Mathematical Progression

What's the calculation? What's the answer?



$$\begin{array}{r}
 3 \quad 9 \quad \square \\
 - \quad \square \quad \square \quad 6 \\
 \hline
 \square \quad 0 \quad 5
 \end{array}$$

Column method (using base 10 and having to exchange)

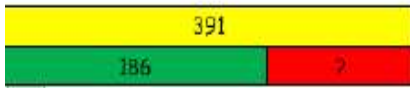
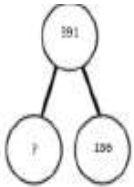
45-26



- 1) Start by partitioning 45
- 2) Exchange one ten for ten more ones
- 3) Subtract the ones, then the tens.

Elm Tree Mathematical Progression

Fluency variation, different ways to ask children to solve 391-186:



Raj spent £391, Timmy spent £186. How much more did Raj spend?
I had 391 metres to run. After 186 I stopped. How many metres do I have left to run?

$$391 - 186$$

$$\square = 391 - 186$$

$$391$$

$$\underline{-186}$$

Find the difference between 391 and 186

Subtract 186 from 391.

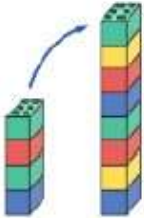

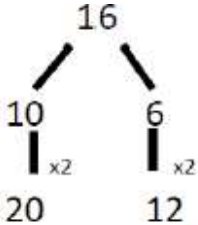
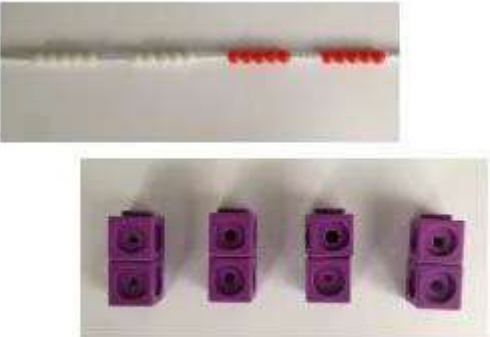
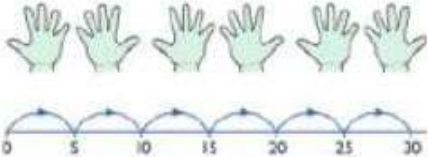
What is 186 less than 391?

Missing digit calculations

$$\begin{array}{r} 39\square \\ - \square\square 6 \\ \hline \square 0 5 \end{array}$$

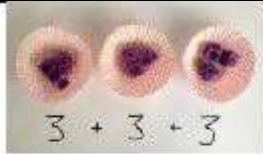
Elm Tree Mathematical Progression

Multiplication

Objective and strategies	Concrete	Pictorial	Abstract
Doubling	<p>Use practical activities to show how to double a number.</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double a number.</p> <p>Double 4 is 8</p> 	 <p>Partition a number and then double each part before recombining it back together.</p>
Counting in multiples	 <p>Count in multiples supported by concrete objects in equal groups.</p>	 <p>Use a number line or pictures to continue support in counting in multiples.</p>	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30</p> <p>Games used to practice.</p>

Elm Tree Mathematical Progression

Repeated addition



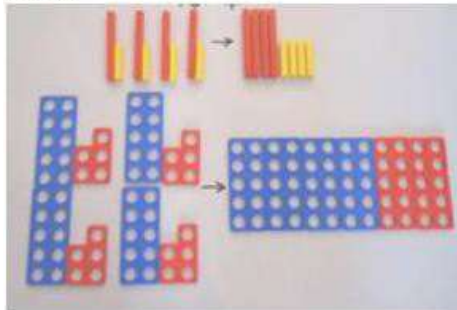
Use different objects to add equal groups.

Repeated grouping/repeated addition
(does not have to be restricted to cubes)

3×4 or 3 lots of 4



4×15



There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?



2 add 2 add 2 equals 6



$2 + 2 + 2 + 2 + 2 = 10$

Use of a bar model for a more structured method



Write addition sentences to describe objects and pictures.

3×4

$4 + 4 + 4$

12		
4	4	4

Elm Tree Mathematical Progression

Use arrays to illustrate commutativity

(counters and other objects can also be used)

$$2 \times 5 = 5 \times 2$$



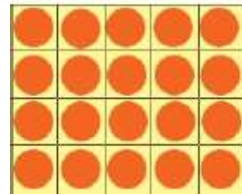
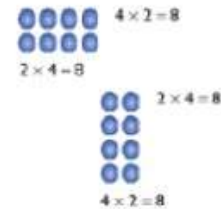
Create arrays using counters/ cubes to show multiplication sentences.



Children to draw the arrays



Draw arrays in different rotations to find **commutative** multiplication sentences.



Link arrays to area of rectangles.

Children to be able to use an array to write a range of calculations e.g.

$$2 \times 5 = 10$$

$$5 \times 2 = 10$$

$$2 + 2 + 2 + 2 + 2 = 10$$

$$5 + 5 = 10$$

Use an array to write multiplication sentences and reinforce repeated addition.



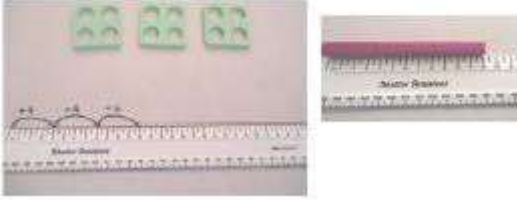
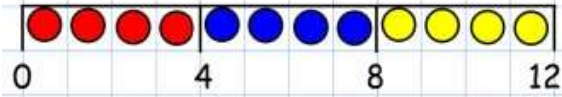
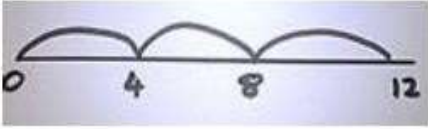
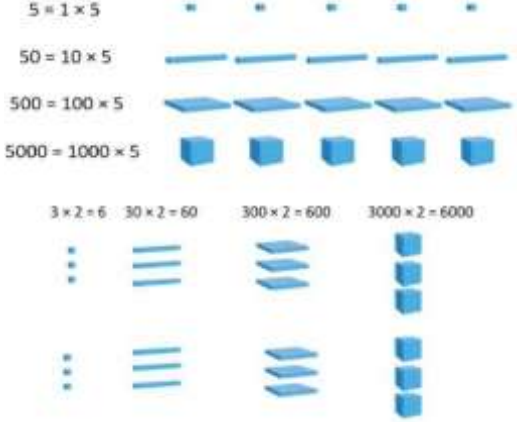
$$5 + 5 + 5 = 15$$

$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

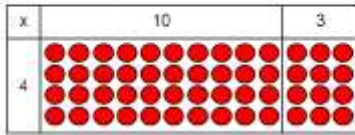
Elm Tree Mathematical Progression

<p>Use number lines to show repeated groups.</p>	<p>Use number lines to show repeated groups - 3×4</p> 	<p>Represent this pictorially alongside a number line e.g:</p> 	<p>Abstract number line: $3 \times 4 = 12$</p> 
<p>Using known facts for multiplying by multiples of 10, 100 and 1000 <i>Pupils' growing understanding of place value, allows them to make use of known facts to derive multiplications using powers of 10. It is important to use tables with which they are already familiar (i.e. not 7 or 9 tables in Year 3)</i></p>		<p>32×100</p> <p>Take 32 'ones' and replace them with 32 'one hundreds'. What happens when we times by 100?</p> <p>We are trying to avoid answers like add two zeros with limited understanding. They should see and be able to explain that each part we have multiplied by 100 is 100 times larger.</p>	<p>$360 \div 10 = 36$</p>

Elm Tree Mathematical Progression

Grid Method

Show the link with arrays to first introduce the grid method.



4 rows of 10
4 rows of 3

Move on to using Base 10 to move towards a more compact method.

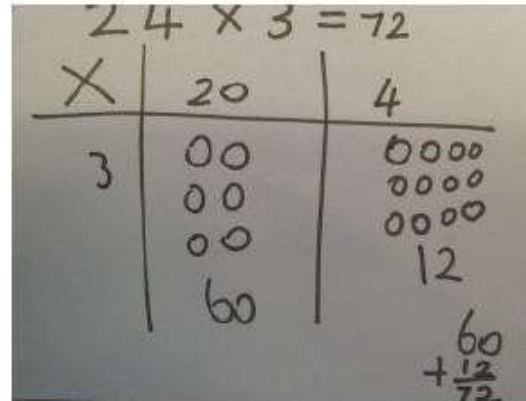


4 rows of 13

Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

x	30	5
7	210	35

$$210 + 35 = 245$$

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

	10	8
10	100	80
3	30	24

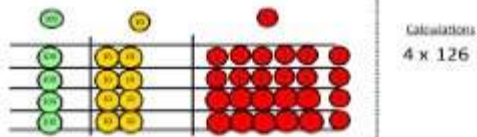
x	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

Elm Tree Mathematical Progression

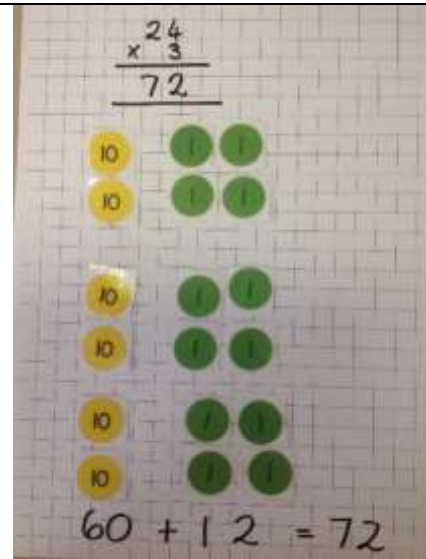
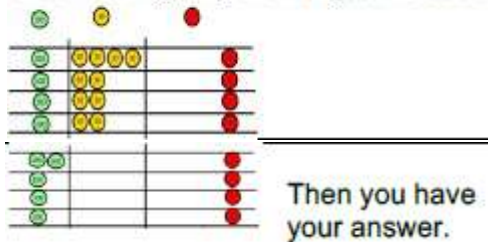
Short
Column
Multiplicati
on



Fill each row with 126.



Add up each column, starting with the ones making any exchanges needed.



Children to record what it is they are
doing to show understanding

$$3 \times 23 \quad 3 \times 20 = 60$$

$$\quad \quad \quad 3 \times 3 = 9$$

$$20 \quad 3 \quad 60 + 9 = 69$$

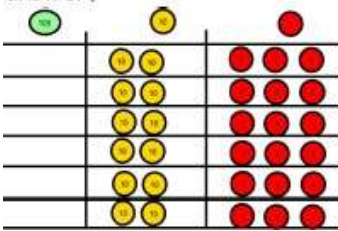
$$23$$

$$\times 3$$

$$69$$

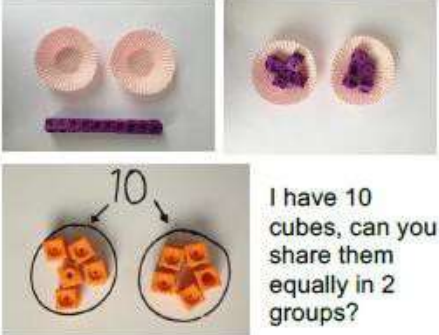
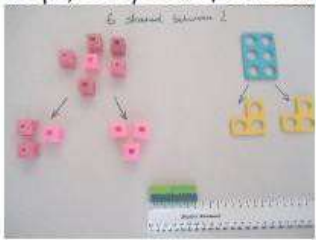
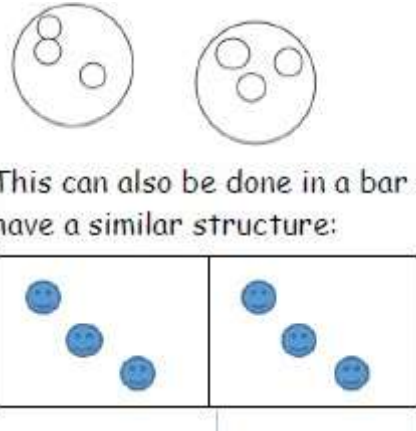
Fluency variation, different ways to ask children to solve 6×23 :

Elm Tree Mathematical Progression

<table border="1" style="margin-left: auto; margin-right: auto; text-align: center;"> <tr> <td style="padding: 2px 10px;">23</td> <td style="padding: 2px 10px;">23</td> <td style="padding: 2px 10px;">23</td> <td style="padding: 2px 10px;">23</td> <td style="padding: 2px 10px;">23</td> <td style="padding: 2px 10px;">23</td> </tr> </table> <div style="border: 1px solid blue; width: 100%; height: 20px; margin: 10px auto;"></div> <p style="text-align: center; margin-top: 10px;">?</p> <p>With the counters, prove that $6 \times 23 = 138$</p> <p>Why is $6 \times 23 = 32 \times 6$?</p>	23	23	23	23	23	23	<p>Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?</p> <p>Tom saved 23p three days a week. How much did he save in 2 weeks?</p>	<p>Find the product of 6 and 23</p> <p>$6 \times 23 =$</p> <div style="display: flex; align-items: center; margin-top: 10px;"> <div style="border: 1px solid black; width: 20px; height: 20px; margin-right: 5px;"></div> = 6×23 </div> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="text-align: right;"> $\begin{array}{r} 6 \\ \times 23 \\ \hline \end{array}$ </div> <div style="text-align: left;"> $\begin{array}{r} 23 \\ \times 6 \\ \hline \end{array}$ </div> </div>	<p>What's the calculation? What's the answer?</p> 
23	23	23	23	23	23				

Elm Tree Mathematical Progression

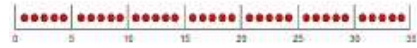
Division

Objective and strategies	Concrete	Pictorial	Abstract		
Sharing objects into groups	 <p>I have 10 cubes, can you share them equally in 2 groups?</p> <p>6 shared between 2 (other concrete objects can also be used e.g. children and hoops, teddy bears, cakes and plates)</p> 	 <p>This can also be done in a bar so all 4 operations have a similar structure:</p>	$6 \div 2 = 3$ <p>What's the calculation?</p> <table border="1" data-bbox="1662 686 2132 758"> <tr> <td style="text-align: center;">3</td> <td style="text-align: center;">3</td> </tr> </table>	3	3
3	3				

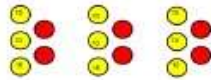
Elm Tree Mathematical Progression

Division as grouping and repeated subtraction.

Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.

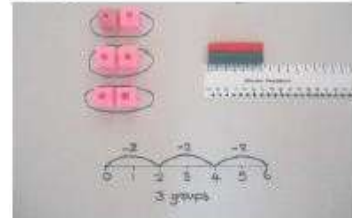


$$96 \div 3 = 32$$

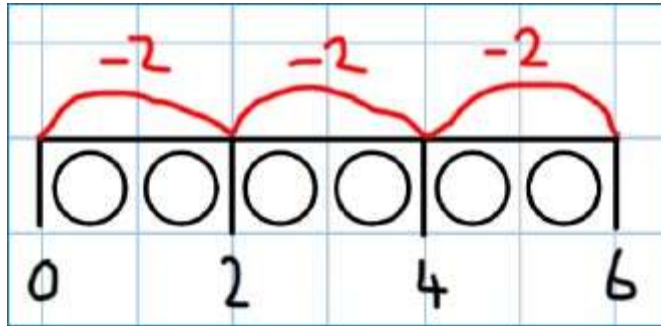
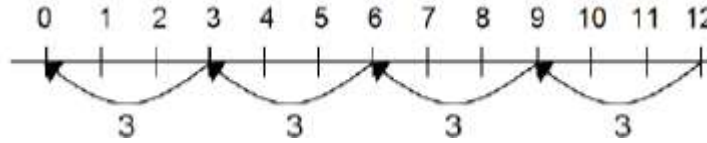


Understand division as repeated grouping and subtracting

$$6 \div 2$$



Use a number line to show jumps in groups. The number of jumps equals the number of groups.



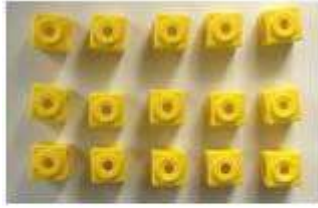
$$28 \div 7 = 4$$

Divide 28 into groups of 7. How many are in each group?

28			
7	7	7	7

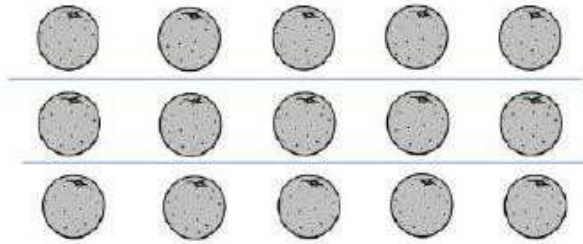
Elm Tree Mathematical Progression

Division with arrays including fact families and inverse



Link division to multiplication by creating an array and thinking about the number sentences that can be created.

Eg $15 \div 3 = 5$ $5 \times 3 = 15$
 $15 \div 5 = 3$ $3 \times 5 = 15$



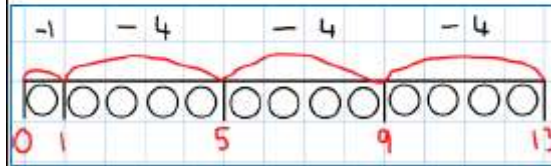
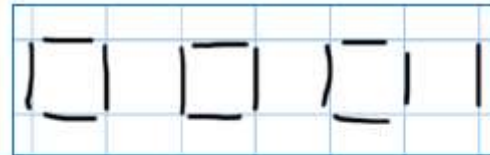
Draw an array and use lines to split the array into groups to make multiplication and division sentences.

Find the inverse of multiplication and division sentences by creating four linking number sentences.

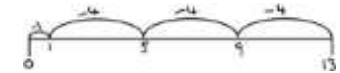
$7 \times 4 = 28$
 $4 \times 7 = 28$
 $28 \div 7 = 4$
 $28 \div 4 = 7$

Division with a remainder

Use of lollipop sticks to form wholes



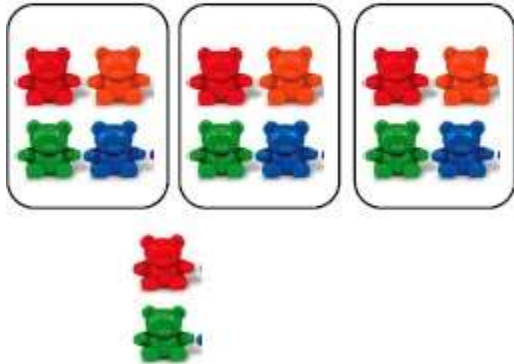
$13 \div 4 = 3r1$



Elm Tree Mathematical Progression

$14 \div 3 =$

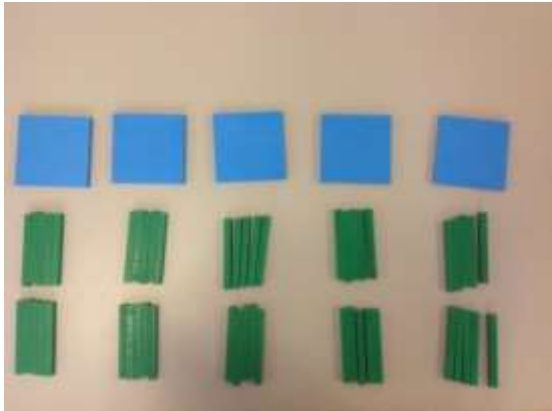
Divide objects between groups and see how much is left over



Using known facts for dividing by multiples of 10, 100 and 1000

$500 \div 10 =$

Show using base 10 - showing 50 tens.



$400 \div 10 =$

Show sharing PV counters into groups



$27 \div 10 =$

Show with PV counters sharing into groups.

Elm Tree Mathematical Progression

Short
division

Use of the 'bus stop method' using grouping and counters. Key language for grouping- how many groups of X can we make with X hundreds'- *this can also be done using sharing!*

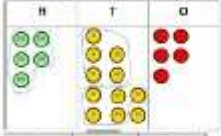
$$615 \div 5$$



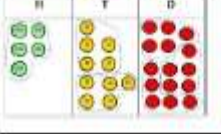
Step 1: make 615



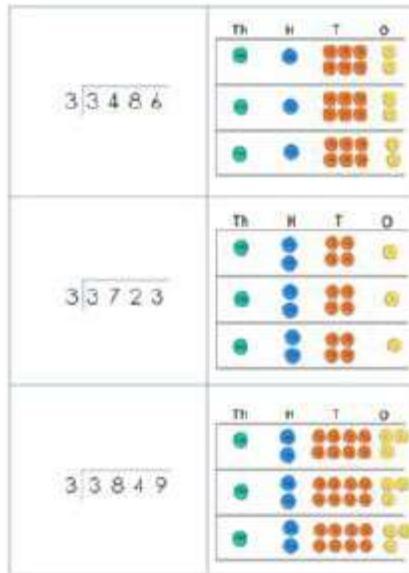
Step 2: Circle your groups of 5



Step 3: Exchange 1H for 10T and circle groups of 5



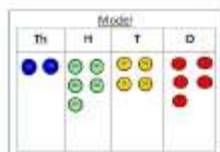
Step 4: exchange 1T for 10ones and circles groups of 5



$$5 \overline{) 615} \begin{matrix} 123 \\ \hline \end{matrix}$$

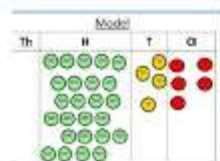
Elm Tree Mathematical Progression

Long division

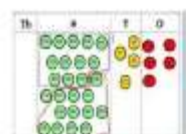


$$\begin{array}{r}
 0212 \\
 12 \overline{) 2544} \\
 \underline{24} \\
 14 \\
 \underline{12} \\
 24 \\
 \underline{24} \\
 0
 \end{array}$$

2544 ÷ 12
How many groups of 12 thousands do we have? None



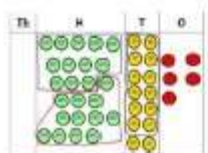
Exchange 2 thousand for 20 hundreds.



$$\begin{array}{r}
 02 \\
 12 \overline{) 2544} \\
 \underline{24} \\
 1
 \end{array}$$

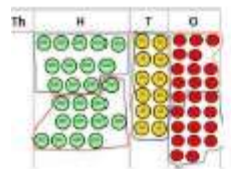
How many groups of 12 are in 25 hundreds? 2 groups. Circle them.

We have grouped 24 hundreds so can take them off and we are left with one.



$$\begin{array}{r}
 021 \\
 12 \overline{) 2544} \\
 \underline{24} \\
 14 \\
 \underline{12} \\
 2
 \end{array}$$

Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.



Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2

Children to represent the counters, pictorially and record the subtractions beneath.

$$\begin{array}{r}
 0 \\
 12 \overline{) 2544}
 \end{array}$$

Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds.

$$\begin{array}{r}
 02 \\
 12 \overline{) 2544} \\
 \underline{24} \\
 1
 \end{array}$$

Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.

$$\begin{array}{r}
 021 \\
 12 \overline{) 2544} \\
 \underline{24} \\
 14 \\
 \underline{12} \\
 2
 \end{array}$$

Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens? The 14 shows how many tens I have, the 12 is how many I

grouped and the 2 is how many tens I have left.

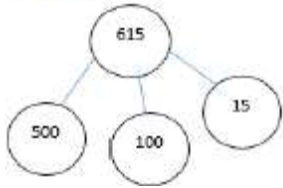
$$\begin{array}{r}
 0212 \\
 12 \overline{) 2544} \\
 \underline{24} \\
 14 \\
 \underline{12} \\
 24 \\
 \underline{24} \\
 0
 \end{array}$$

Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.

Elm Tree Mathematical Progression

Fluency variation, different ways to ask children to solve $615 \div 5$:

Using the part whole model below, how can you divide 615 by 5 without using the 'bus stop' method?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?
615 pupils need to be put into 5 groups. How many will be in each group?

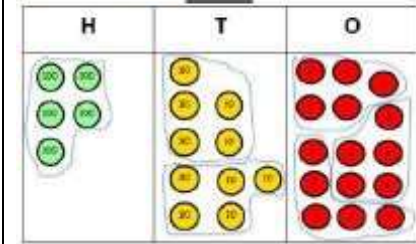
$$5 \overline{)615}$$

$$615 \div 5 =$$

$$\square = 615 \div 5$$



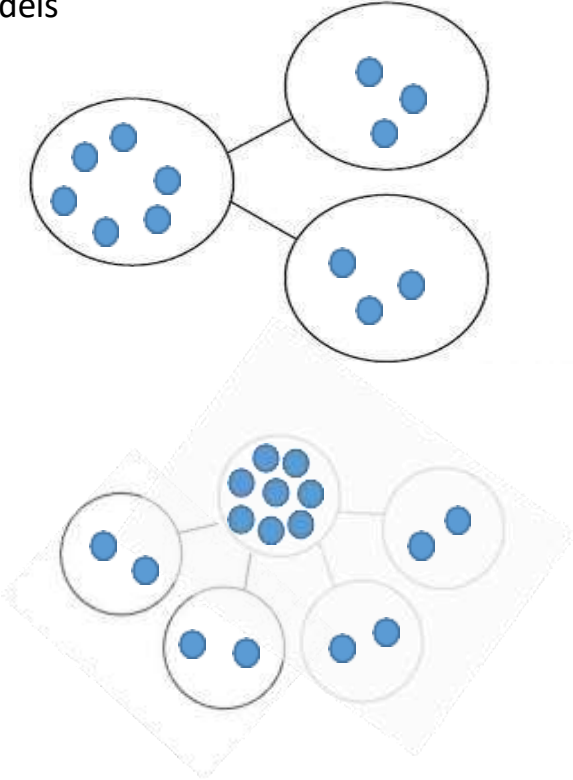
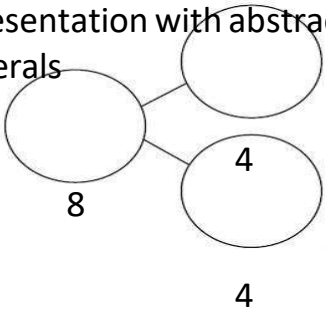
How many 5's go into 615?

What's the calculation? What's the answer?

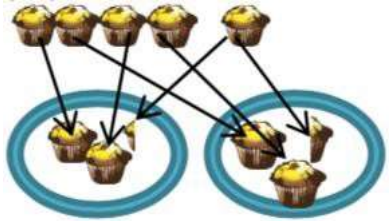
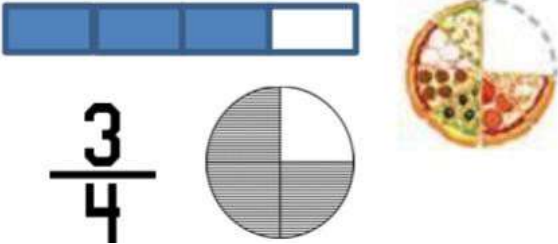
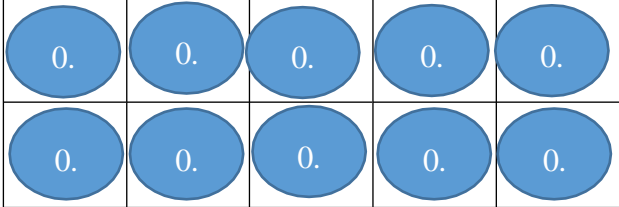
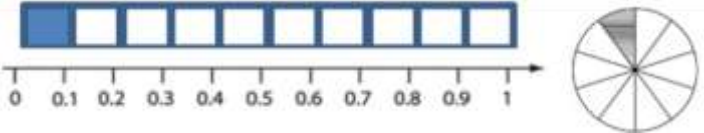


Elm Tree Mathematical Progression

Fractions

Objective and strategies	Concrete	Pictorial	Abstract
<p>Identifying half as 2 equal parts or quarter as 4 equal parts of a quantity</p>	<p>Using physical resources, children manipulate skittles, sweets, cubes to link back to parts having to be equal.</p>  <p>One half of 12 is 6. "There are two equal parts, 6 is one equal part of the whole."</p>  <p>One quarter of 12 is 3. "There are four equal parts, 3 is one equal part of the whole."</p>	<p>Representations are chosen to support pupils' understanding of fractions, e.g part-part-whole models</p> 	<p>Pupils are confident to use a range of abstract written versions:</p> <p>$12 \div 4 = 3$</p> <p>or</p> <p>Half of 16 is 8</p> <p>Or they use the pictorial representation with abstract numerals</p> 

Elm Tree Mathematical Progression

<p>write simple fractions for example, $\frac{1}{2}$ of $6 = 3$</p>	<p>Pupils will make links between division and fractions using denominators</p> 	<p>A range of pictorial representations can be used to demonstrate fractions, always linking back to equal parts in a whole.</p> 	<p>$\frac{1}{3}$ of 12 is 4</p> <p>3</p> <p>To find three quarters of a number, you can find the four equal parts and calculate the total of three of those four parts</p>
<p>recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10</p>	<p>Use place value knowledge and a tens frame to identify that 1 split into 10 equal parts → each equal part is 1 tenth</p>  <p>Equally, 10 equal parts that are tenths combined is the same as 1 whole</p>	 <p>These pictorial representations demonstrate what happens on a number line, with blocks or cubes or with circles when sharing into 10 equal parts.</p> <p>N.B Consider how these represent the decimals between 0 and 1, i.e 6 of the 10ths is the same as 0.6. 10 of the 10ths is the same as 1.</p>	<p>Divide an object into 10 equal parts and you have ten tenths.</p> <p>If you divide whole numbers into 10th, each whole is divided into 10 equal parts. Each part is 1 tenth. If you divide 3 into tenths, each whole provides 1/10 which combined total 3/10.</p>

Elm Tree Mathematical Progression

Add and subtract fractions with the same denominator (within and to beyond 1)

Using different shaped fraction tools allows pupils to manipulate and compare fractions



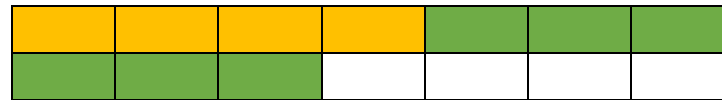
Use a range of representations to demonstrate two fractions being like fractions, i.e each part is always the same.



$$4/7 + 2/7 = 6/7$$

I have split the whole into seven equal parts. I take 4 of those 7 equal parts and combine it with 2 more of those equal parts – how many of the 7 equal parts do I have in total?

You can also represent this:



$$4/7 + 6/7 = 10/7 \text{ or } 1 \text{ and } 3/7$$

$5/8$ and $1/8$ are like fractions as they have the same denominator.

How much of a pizza / chocolate bar would this be all together?

$$5/8 + 1/8 = 6/8$$

Elm Tree Mathematical Progression

Subtraction works with the same model:

$9/10 - 4/10$ is:

I have split the whole into ten equal parts. I take 9 of those 10 equal parts and subtract 4 of those equal parts – how many of the 10 equal parts do I have left?

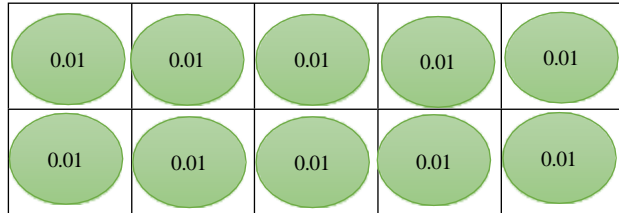


5 of the 10 equal parts remain (or I have $5/10$ left over).

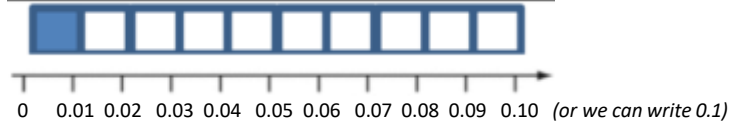
Elm Tree Mathematical Progression

recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.

Use place value knowledge and a tens frame to identify that 0.1 split into 10 equal parts → each equal part is 1 hundredth

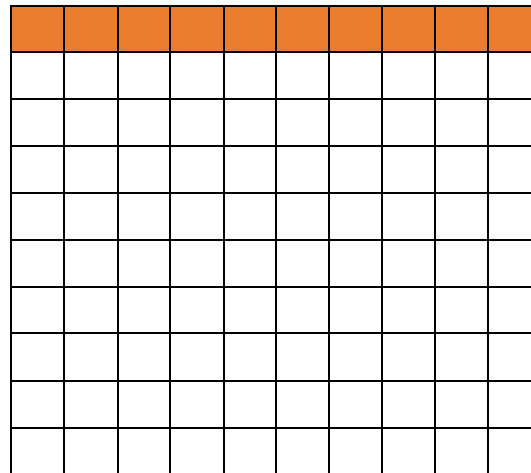


Equally, 10 equal parts that are hundredths combined is the same as 1 whole



Similar to looking at tenths, pictorial representations linked to number lines can help us to get a sense of the size of hundredths.

Similarly, you can use empty hundred squares on lined paper to support them with the idea of size and proportion of these fractions.

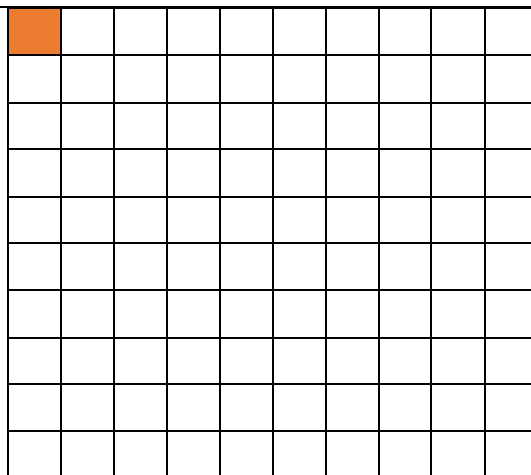


This is one tenth of the whole (10 out of 100)

Divide an object into 100 equal parts and you have one hundred hundredths.

If you divide whole numbers into 100ths, each whole is divided into 100 equal parts. Each part is 1 hundredth. If you divide 7 into hundredth, each whole provides 1/100 which combined total 7/100.

Elm Tree Mathematical Progression



This is one hundredth of the whole (1 out of 100)

You can also use fraction bars to show the difference in size.



This represents one tenth.



This represents one hundredth.

Elm Tree Mathematical Progression

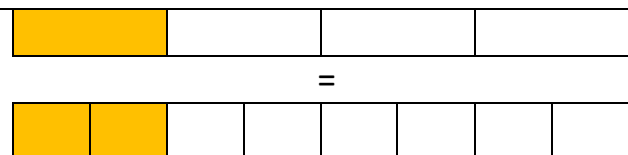
Add and subtract fractions with denominators that are multiples of the same number.

When demonstrating this to pupils, using paper strips is useful.

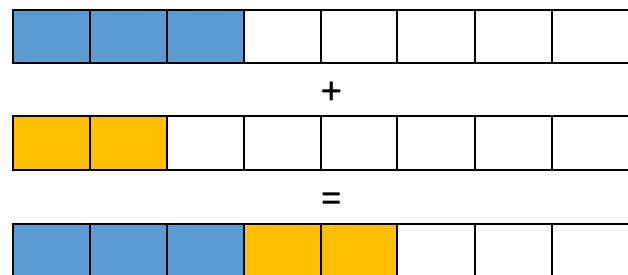
$$3/8 + 1/4 = ?$$

You can show that the lowest common multiple is linked to their knowledge of equivalent fractions.

$1/4$ is the same as $2/8$:



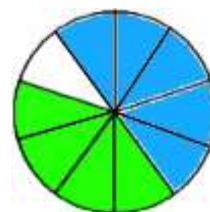
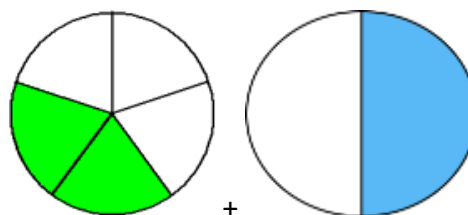
So now we can add the two fractions:



The total is $5/8$

Asking the pupils to draw a representation either on a bar or a circle can be useful:


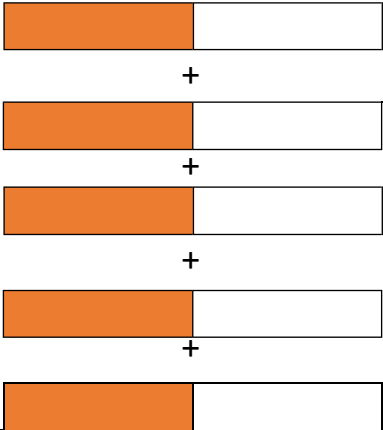


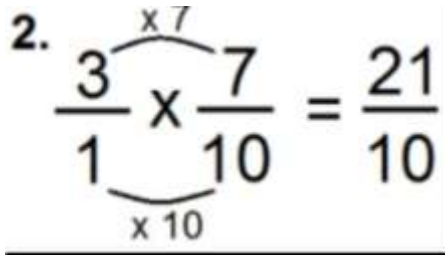
$$2/5 + 1/2$$



When adding two fractions with different denominators, we need to look for the lowest common multiple and convert both numbers (if necessary) to this fraction.

For example, 5 and 2 have the lowest common multiple of 10 – so to add these together, we need to ensure we keep to rules of equivalence (same rule applied to numerator and denominator).

Elm Tree Mathematical Progression

<p>Multiply proper fractions and mixed numbers by whole numbers</p>	<p>$\frac{1}{2} \times 5$</p> <p>Using strips of paper is an excellent practical resource to support understanding of this concept.</p>  <p>This is $\frac{1}{2}$ of the whole.</p> <p>$5 \times n$ (<i>n meaning any number</i>) is the same as 5 groups of n.</p> <p>Therefore $5 \times \frac{1}{2}$ is the same as 5 groups, with half in each group.</p> 	<p>1.</p> $3 \times \frac{7}{10} = \frac{21}{10}$ <hr/> <p>If this is $\frac{7}{10}$ (7 tenths):</p>  <p>Then this would be 3 groups of $\frac{7}{10}$ (7 tenths):</p>  <p>The product is clear 21 tenths (because each whole is split in to 10 equal parts and the denominator represents this information).</p>	<p>1.</p> $3 \times \frac{7}{10} = \frac{21}{10}$ <hr/> <p>The reason why this works:</p> <p>2.</p>  <hr/> <p>...is because you do have '3 wholes' (or $\frac{3}{1}$) – if they are getting too focused on this method alone and start to forget why it works, move them back into concrete or abstract methods.</p>
---	---	---	---

Elm Tree Mathematical Progression

Combined, it would look like this:



The product must be 2 and a half or $2\frac{1}{2}$

Multiplying a mixed number is the same:

$$2\frac{1}{2} \times 2$$



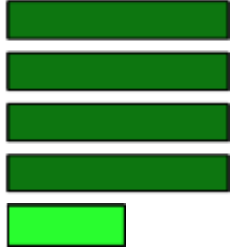






+



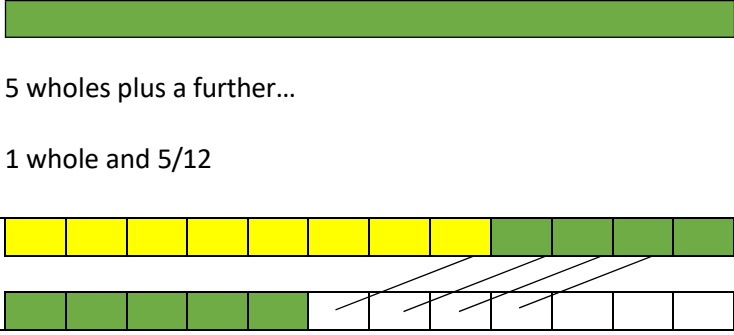


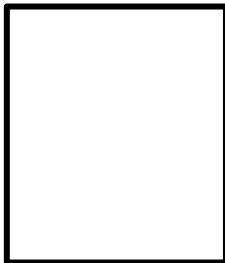
=



Elm Tree Mathematical Progression

<p>To add and subtract fractions with different denominators and mixed numbers (using equivalent fractions)</p>	<p>Practical resources such as cuisinaire allow you to see mixed fractions:</p>  <p>If Dark Green is 1 whole then...</p> <p>What is light green?</p> <p>What is the total of this number?</p> <p>What if I had 2 more wholes? What would I have in total now?</p>	<p>If we take the example of adding 2 and $\frac{2}{3}$ with 3 and $\frac{3}{4}$ then we can consider that the wholes and parts can be combined separately before being combined as two new totals.</p>   <p style="text-align: center;">+</p>   <p>So we have:</p>  	<p>2 and $\frac{2}{3}$ plus 3 $\frac{3}{4}$</p> <p>2 wholes plus 3 wholes is 5 wholes</p> <p>This leaves us with $\frac{2}{3}$ plus $\frac{3}{4}$</p> <p>Now we are simply adding two fractions with different denominators.</p> <p>The rules of equivalence inform us that the lowest common multiple is 12 and so we convert both fractions.</p> <p>$\frac{2}{3} \times 4 = \frac{8}{12}$ and $\frac{3}{4} \times 3 = \frac{9}{12}$</p> <p>Now we can add them together which will total $\frac{17}{12}$ or 1 whole and $\frac{5}{12}$.</p> <p>If we combine this with 5 wholes, we would have 6 wholes and $\frac{5}{12}$.</p>
---	--	---	--

Elm Tree Mathematical Progression

		 <p>5 wholes plus a further...</p> <p>1 whole and 5/12</p> <p>Combining both of these physically, pictorially or abstract will allow us to see that there are 6 wholes and 5/12</p>	
<p>To multiply simple pairs of proper fractions (writing the answer in its simplest form)</p>	<p>Start with what they do know:</p>  <p>Each pink block is worth $\frac{1}{4}$</p> <p>so $\frac{1}{4} \times 3 = 3$ groups of $\frac{1}{4}$</p> <p>We can write this as:</p> $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$ 	 <p>If this is one whole.</p> $1 \times 1 = 1$	$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ $\frac{2}{3} \times \frac{1}{4} = \frac{2}{12} = \frac{1}{6}$ <p>They should be expected to draw it to continue demonstrating, and developing, their conceptual understanding.</p> <p>This will support 'genuine' fluency and therefore application.</p>

Elm Tree Mathematical Progression

Each pink block is worth $\frac{1}{4}$

so $\frac{1}{4} \times 1 = 1$ group of $\frac{1}{4}$ is $\frac{1}{4}$

We can write this as:

$$\frac{1}{4} = \frac{1}{4}$$



Each pink block is worth $\frac{1}{4}$

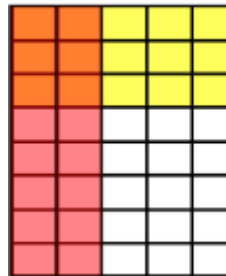
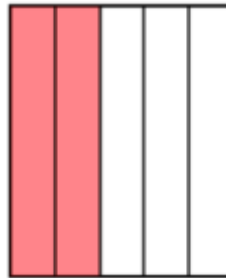
So $\frac{1}{4} \times \frac{1}{3} = \frac{1}{3}$ of 1 group of $\frac{1}{4}$

We can write this as:

1 group of $\frac{1}{4}$ is $\frac{1}{4}$

$\frac{1}{3}$ of 1 group of $\frac{1}{4}$ is $\frac{1}{12}$.

First, you split into three equal sections as we need to find $\frac{1}{3}$ of one group of $\frac{1}{4}$.



Then this is the whole shared equally into 8 equal parts.

This is 3 of the 8 equal parts.
1 group of $\frac{3}{8} = \frac{3}{8}$

Then this is the whole shared equally into 5 equal parts.

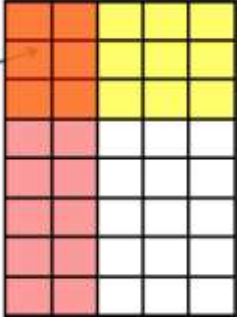

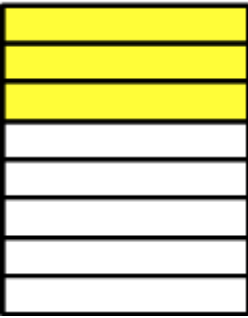
This is 2 of the 5 equal parts.
1 group of $\frac{2}{5} = \frac{2}{5}$

$$\frac{3}{8} \times \frac{2}{5}$$

Relate to what pupils know already.

When we had 1 group of $\frac{2}{5}$, the product was $\frac{2}{5}$. Now we are calculating what $\frac{3}{8}$ of 1 group of $\frac{2}{5}$ would be.

Elm Tree Mathematical Progression

		$\frac{3}{8} \times \frac{2}{5} = \frac{6}{40}$  <p>6 and 40 have common factors of 1 and 2. Dividing it by the Highest Common Factor (HCF) will provide this fraction in its simplest form: $\frac{3}{20}$</p>	
<p>To divide proper fractions by whole numbers</p>	<p>Always start with a familiar context.</p> <p>$\frac{1}{2}$ of a pizza is to be shared between four friends. How much do they each get?</p> 	$\frac{3}{8}$  <p>What is the generalisation?</p>	$\frac{1}{3} \div 2 = \frac{1}{6}$ <p>Pupils understand the strategy and can calculate mentally.</p> <p>If necessary, they can use simple drawn models if they confuse their own understanding.</p>

Elm Tree Mathematical Progression

$\frac{3}{8}$ of the pizza remains. Two friends are going to share it equally.

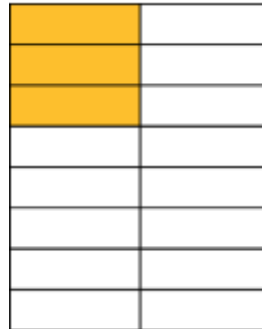
Each friend will get half each.

How can we represent this?

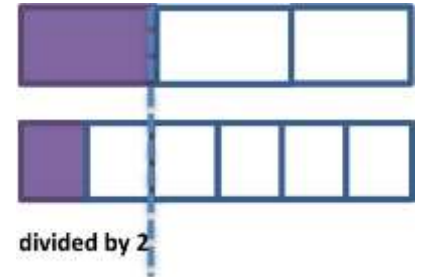


Multiply the denominator by the whole number. The numerator stays the same. Why does this work?

$$\frac{3}{8} \div 2$$

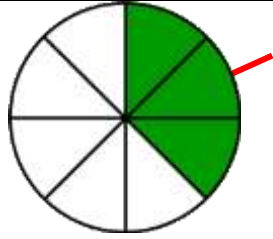


This is $\frac{3}{16}$ because you have shared $\frac{3}{8}$ equally between two people meaning they get half each. Half of $\frac{3}{8}$ is $\frac{3}{16}$.



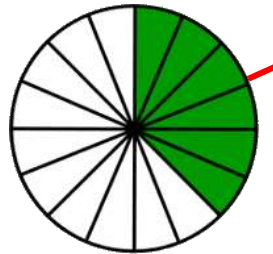
divided by 2

Elm Tree Mathematical Progression

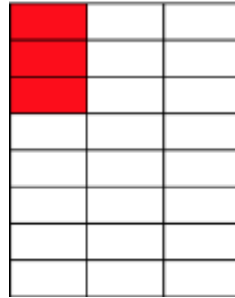


Find half of the $\frac{3}{8}$.

Now to ensure you are representing it fairly, you have to make sure all the parts are equal (e.g sixteenths, so draw the rest of the grid out to support in counting that each child has three sixteenths – see below).

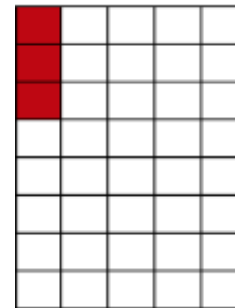


$$\frac{3}{8} \div 3$$



This pattern continues. $\frac{3}{8}$ shared equally between three people will mean they get one third each. One Third of $\frac{3}{8}$ is $\frac{3}{24}$.

$$\frac{3}{8} \div 5$$



Elm Tree Mathematical Progression