

# Calculation Policy



This Calculation Policy has been written with the aims of the National Curriculum at the heart of it.

Each of the four operations build on a solid understanding of place value, the connections between the four number operations and number sense, such as: whether they are odd or even, whether they are close to multiples of ten or if they are close together.

- Children need to use correct mathematical terminology in context and be able to verbalise their calculation strategies.
- Children need to make considered decisions as to the most appropriate methods to make mathematics more functional. They need to choose the most appropriate, fluent, efficient and accurate method to solve a particular calculation.
- Children need to use concrete resources before they progress to pictorial and abstract representations. This CPA (concrete, pictorial and abstract) approach needs to be available to children throughout school, as and when necessary.

Use of manipulatives (Numicon, Cuisenaire, Dienes, Base Ten, HTO counters etc.) helps reinforce understanding and provides support when calculating mentally, mentally with jottings, using expanded methods and formal written methods. Use of the bar model, number lines and part-part-whole diagrams are recommended.

## The National Curriculum Aims

*The national curriculum for mathematics aims to ensure that all pupils:*

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

Children should progress between the stages working towards formal written methods (where appropriate), once they have mastered each stage. However, they should not be hurried and, after the method has been taught, children should still be able to make their preferred choice of the most appropriate, efficient and accurate method for them. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.



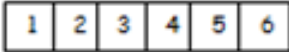

As new methods of calculations are introduced, children should have the opportunity to examine them, alongside the method they have consolidated, to make connections between the methods and establish the similarities and differences between them.

This policy includes sections on: Addition, Subtraction, Multiplication and Division. It outlines progression in teaching, from mental through to formal written methods.

# EYFS ADDITION







Maths for young children should be meaningful. Where possible, concepts should be taught in the context of real life.

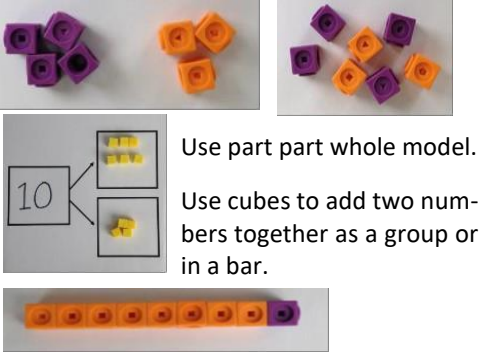
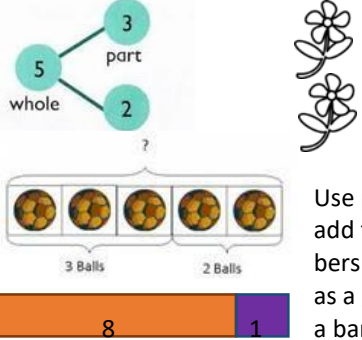


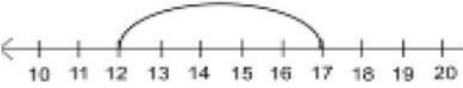
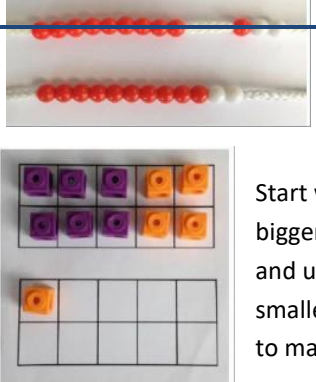
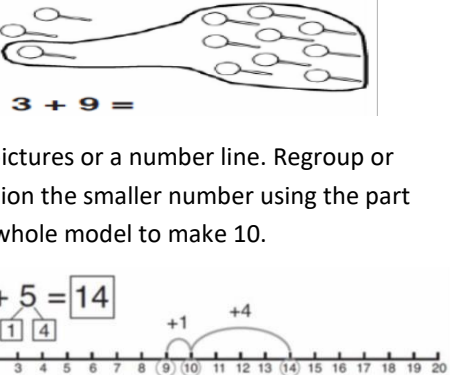

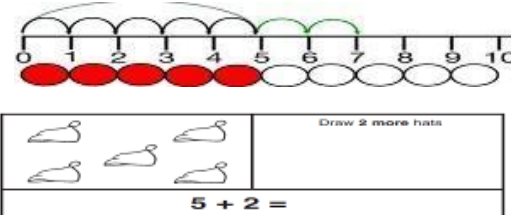
GUIDANCE/ MODELS AND IMAGES	KEY VOCABULARY
<p>Children are encouraged to use a variety of resources to:</p> <ul style="list-style-type: none"> <li>• Identify 1 more 1 less</li> <li>• Combine pieces to add</li> <li>• Find number bonds</li> <li>• Add without counting</li> <li>• Children can record this by printing or drawing around shapes or objects</li> </ul> <p>Children can begin to combine groups of objects using concrete apparatus</p>  <p>Construct number sentences verbally or using cards to go with practical activities.</p> <p>Children are encouraged to read number sentences aloud in different ways "Three add two equals 5" "5 is equal to three and two"</p> <p>Children make a record in pictures, words or symbols of addition activities already carried out.</p> <p>Solve simple problems using fingers</p>  <p style="text-align: center;"><math>5 + 1 = 6</math></p> <p>Number tracks can be introduced to count up on and to find one more:</p>  <p>What is one more than 4? 1 more than 13?</p> <p><u>Numberlines</u> can be used alongside number tracks and practical apparatus to solve addition calculations and word problems.</p>  <p><b>Children will need opportunities to look at and talk about different models and images as they move between representations.</b></p>	<p>Games and songs can be a useful way to begin using vocabulary involved in addition e.g. Alice the Camel Number Fun</p> <p>add</p> <p>more</p> <p>and</p> <p>make</p> <p>sum</p> <p>total</p> <p>altogether</p> <p>score</p> <p>double</p> <p>one more, two more, ten more ...</p> <p>how many more to make...?</p> <p>how many more is...than...?</p>

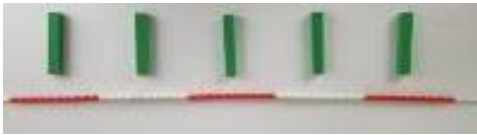
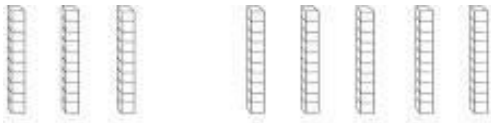
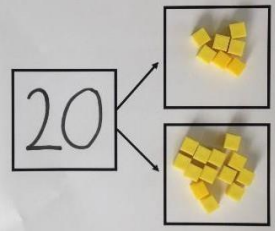
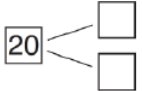
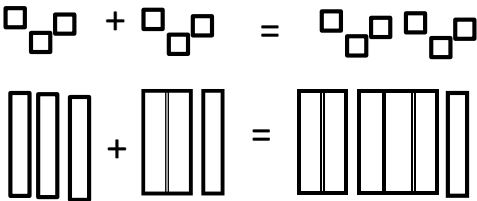
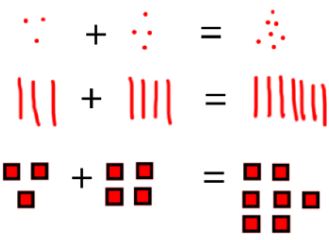


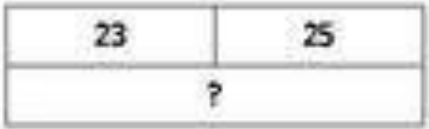
# EYFS SUBTRACTION



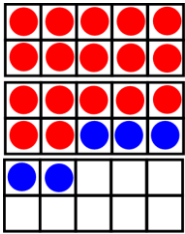
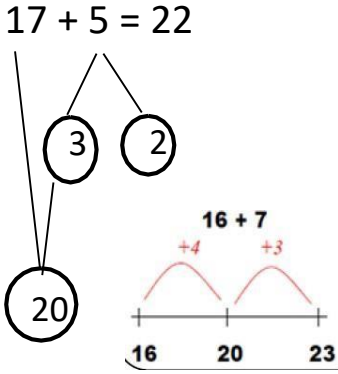
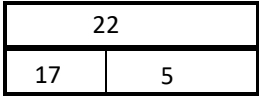
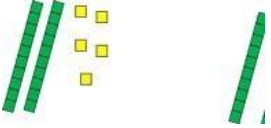
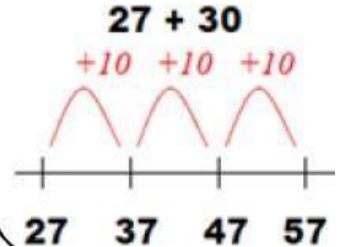
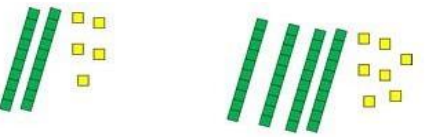
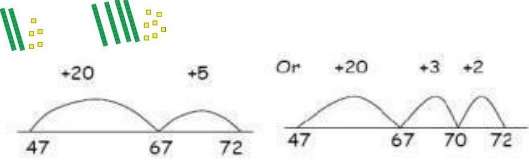
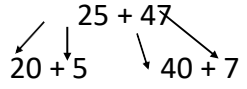
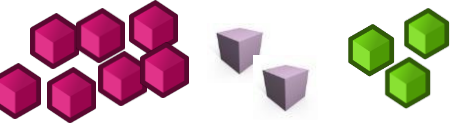
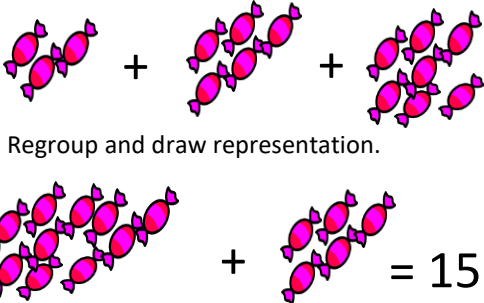
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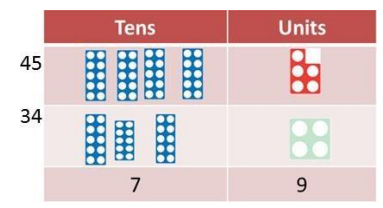
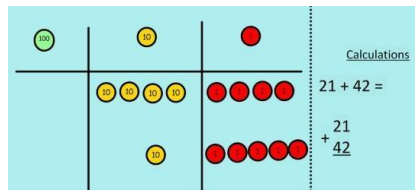
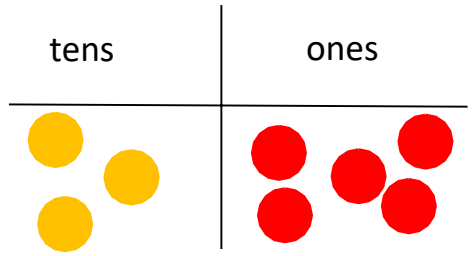
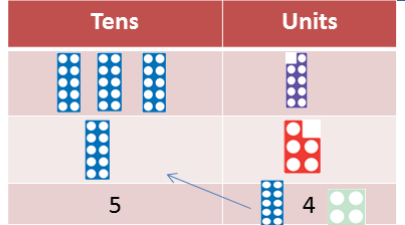
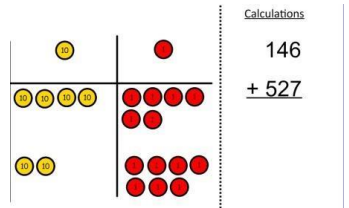
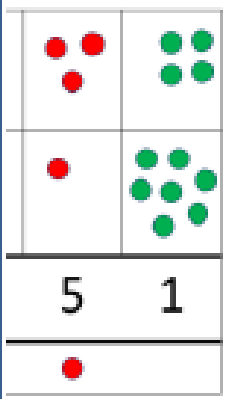
GUIDANCE/ MODELS AND IMAGES	KEY VOCABULARY
<p>Children begin with mostly pictorial representations</p> <p></p> <p>Concrete apparatus is used to relate subtraction to taking away and counting how many objects are left.</p> <p>Concrete apparatus models the subtraction of 2 objects from a set of 5.</p> <p>Construct number sentences verbally or using cards to go with practical activities</p> <p>Children are encouraged to read number sentences aloud in different ways "five subtract one leaves four" "four is equal to five subtract one"</p> <p>Children have a record in pictures, words or symbols of subtraction activities already carried out.</p> <p>Solve simple problems using fingers </p> <p>Number tracks can be introduced to count back and to find one less: </p> <p>What is 1 less than 9? And 1 less than 20?</p> <p>Number lines can then be used alongside number tracks and practical Apparatus to solve subtraction calculations and word problems. Children Count back under the number line.</p> <p></p> <p><b>Children will need opportunities to look at and talk about different models and images as they move between representations.</b></p>	<p>Games and songs can be a useful way to begin using vocabulary involved in subtraction e.g. Five little men in a flying saucer.</p> <p>take (away)</p> <p>leave</p> <p>how many are left/left over?</p> <p>how many have gone?</p> <p>one less, two less... ten less...</p> <p>how many fewer is... than ...?</p> <p>difference between</p> <p>is the same as</p>

Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	 <p>Use part part whole model. Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	$4 + 3 = 7$  $10 = 6 + 4$ Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	$12 + 5 = 17$  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	$5 + 12 = 17$ Place the larger number in your head and count on the smaller number to find your answer.  Use Place Value headings to record number sentence.
Regrouping to make 10.  <i>This is an essential skill for column addition later.</i>	 <p>Start with the bigger number and use the smaller number to make 10. Use ten frames.</p>	 <p>Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10.</p>	$7 + 4 = 11$  If I am at seven, how many more do I need to make 10. How many more do I add on now? Use Place Value headings to record number sentence.
Represent & use number bonds and related subtraction facts within 20	 <p>2 more than 5.</p>	 <p><math>5 + 2 =</math></p>	Emphasis should be on the language '1 more than 5 is equal to 6.' '2 more than 5 is 7.' '8 is 3 more than 5.'

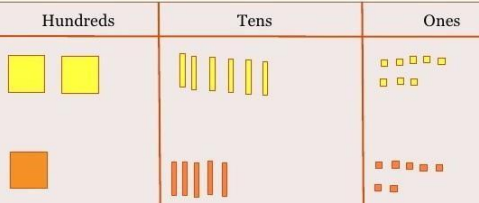
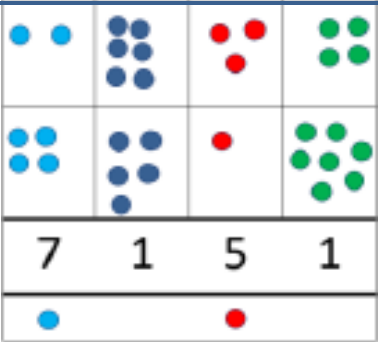
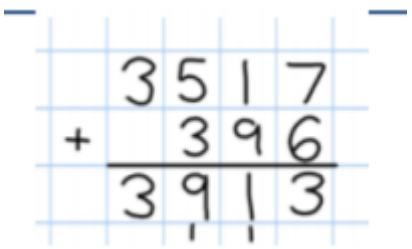
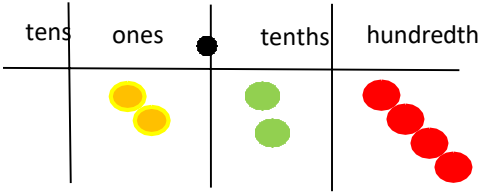
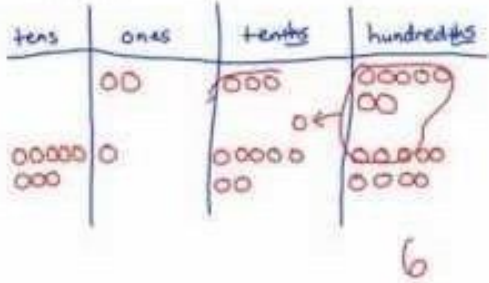
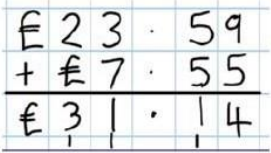
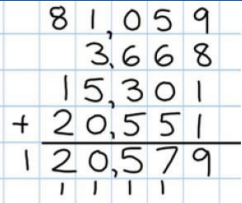
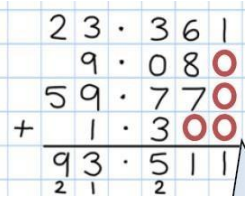
Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	$50 = 30 + 20$  Model using dienes and bead strings	 $3 \text{ tens} + 5 \text{ tens} = \underline{\quad} \text{ tens}$ $30 + 50 = \underline{\quad}$ Use representations for base ten.	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$
Use known number facts Part part whole	 Children explore ways of making numbers within 20	 $\square + \square = 20$ $20 - \square = \square$ $\square + \square = 20$ $20 - \square = \square$	$\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$
Using known facts		 Children draw representations of H,T and O	$3 + 4 = 7$ <i>leads to</i> $30 + 40 = 70$ <i>leads to</i> $300 + 400 = 700$
Bar model	 $3 + 4 = 7$	 $7 + 3 = 10$	 $23 + 25 = 48$



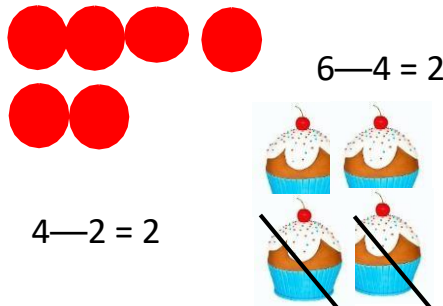
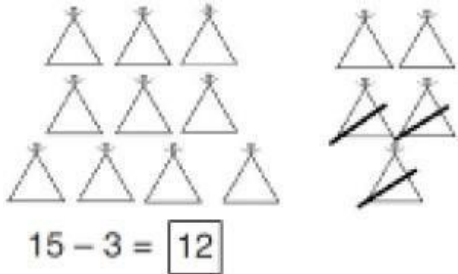
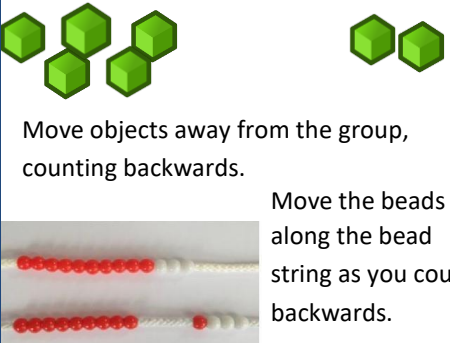
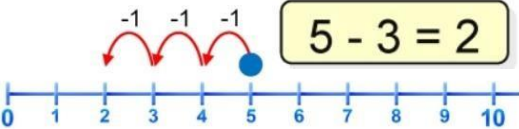
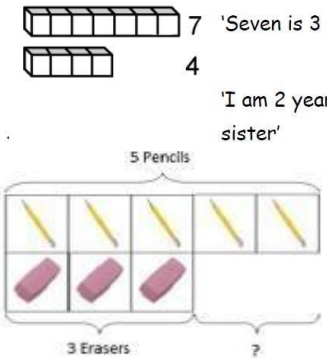
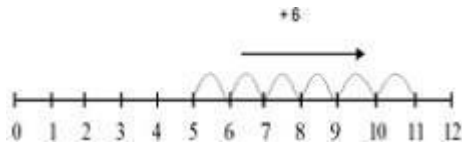
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Add a two digit number and ones</p>	 <p><math>17 + 5 = 22</math></p> <p>Use ten frame to make 'magic ten'</p> <p>Children explore the pattern.</p> <p><math>17 + 5 = 22</math></p> <p><math>27 + 5 = 32</math></p>	<p><math>17 + 5 = 22</math></p> <p>Use part part whole and number line to model.</p> 	<p><math>17 + 5 = 22</math></p> <p>Explore related facts</p> <p><math>17 + 5 = 22</math></p> <p><math>5 + 17 = 22</math></p> <p><math>22 - 17 = 5</math></p> <p><math>22 - 5 = 17</math></p> 
<p>Add a 2 digit number and tens</p>	 <p><math>25 + 10 = 35</math></p> <p>Explore that the ones digit does not change</p>	<p><math>27 + 30</math></p> 	<p><math>27 + 10 = 37</math></p> <p><math>27 + 20 = 47</math></p> <p><math>27 + \square = 57</math></p>
<p>Add two 2-digit numbers</p>	 <p>Model using dienes, place value counters and numicon</p>	 <p>Use number line and bridge ten using part whole if necessary.</p>	<p><math>25 + 47</math></p>  <p><math>20 + 40 = 60</math></p> <p><math>5 + 7 = 12</math></p> <p><math>60 + 12 = 72</math></p>
<p>Add three 1-digit numbers</p>	 <p>Combine to make 10 first if possible, or bridge 10 then add third digit</p>	 <p>Regroup and draw representation.</p> <p><math>= 15</math></p>	<p><math>4 + 7 + 6 = 10 + 7</math></p> <p><math>= 17</math></p> <p>Combine the two numbers that make/ bridge ten then add on the third.</p>

Objective & Strategy	Concrete	Pictorial	Abstract																																	
<p>Column Addition—no regrouping (friendly numbers)</p> <p>Add two or three 2 or 3-digit numbers.</p>	<p>Model using Dienes or Numicon</p>  <p>Add together the ones first, then the tens.</p>   <p>Move to using place value counters</p>	<p>Children move to drawing the counters using a tens and one frame.</p> 	<table border="0"> <tr><td>H</td><td>T</td><td>O</td></tr> <tr><td>200</td><td>20</td><td>3</td></tr> <tr><td>100</td><td>10</td><td>4</td></tr> <tr><td>300</td><td>30</td><td>7</td></tr> <tr><td colspan="3">= 337</td></tr> </table> <table border="0"> <tr><td>H</td><td>T</td><td>O</td></tr> <tr><td>2</td><td>2</td><td>3</td></tr> <tr><td>+</td><td>1</td><td>1</td><td>4</td></tr> <tr><td colspan="4"><hr/></td></tr> <tr><td>3</td><td>3</td><td>7</td><td></td></tr> </table> <p>Add the ones first, then the tens, then the hundreds.</p>	H	T	O	200	20	3	100	10	4	300	30	7	= 337			H	T	O	2	2	3	+	1	1	4	<hr/>				3	3	7	
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<p>Column Addition with regrouping.</p>	 <p>Exchange ten ones for a ten. Model using Numicon, Base Ten and PV counters.</p> 	 <p>Children can draw a representation of the grid to further support their understanding, carrying the ten <u>underneath</u> the line</p>	<table border="0"> <tr><td>20</td><td>+</td><td>5</td><td></td></tr> <tr><td>40</td><td>+</td><td>8</td><td></td></tr> <tr><td colspan="4"><hr/></td></tr> <tr><td>60</td><td>+</td><td>13</td><td>= 73</td></tr> </table> <p>Start by partitioning the numbers before formal column to show the exchange.</p> <table border="0"> <tr><td>536</td></tr> <tr><td>+ 85</td></tr> <tr><td><hr/></td></tr> <tr><td>621</td></tr> <tr><td>11</td></tr> </table>	20	+	5		40	+	8		<hr/>				60	+	13	= 73	536	+ 85	<hr/>	621	11												
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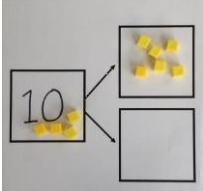
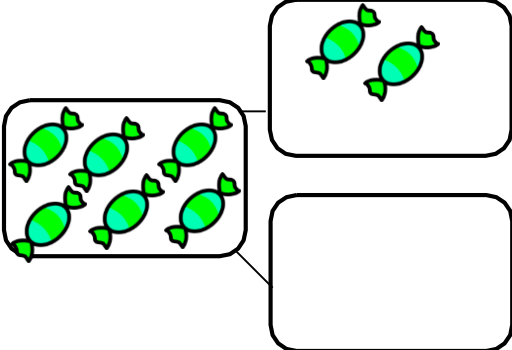
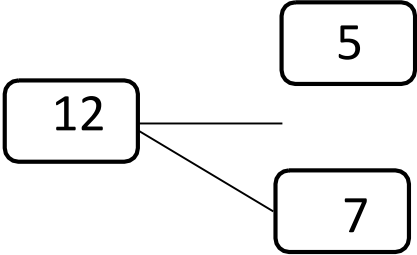
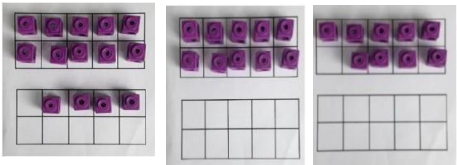
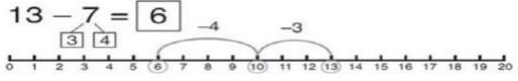
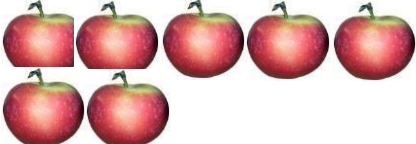

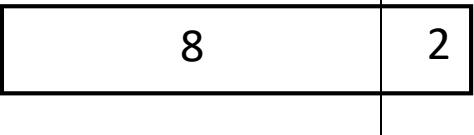
# Y4-6 ADDITION +

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Y4—add numbers with up to 4 digits</p>	<p>Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.</p> 	 <p>Draw representations using pv grid.</p>	 <p>Continue from previous work to carry hundreds as well as tens.</p> <p>Relate to money and measures.</p>
<p>Y5—add numbers with more than 4 digits.</p> <p>Add decimals with 2 decimal places, including money.</p>	<p>As year 4</p>  <p>Introduce decimal place value counters and model exchange for addition.</p>	<p>2.37 + 81.79</p> 	<p>72.8</p> <p>+ 54.6</p> <p><u>127.4</u></p> <p>11</p> 
<p>Y6—add several numbers of increasing complexity</p> <p>Including adding money, measure and decimals with different numbers of decimal points.</p>	<p>As Y5</p>	<p>As Y5</p>	 <p>Insert zeros for place holders.</p> 

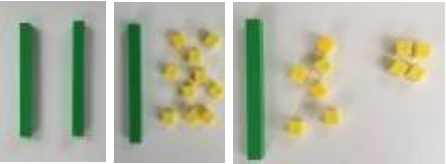
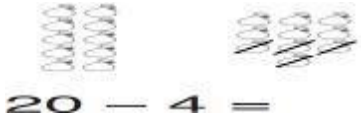

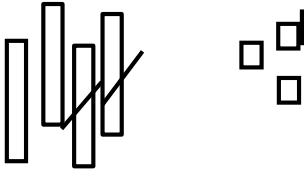
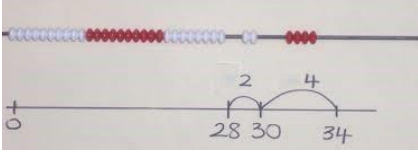
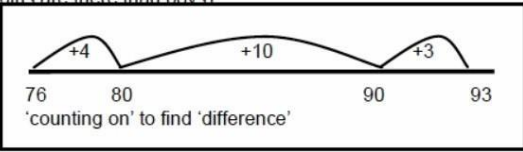
# Y1 SUBTRACTION -

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Taking away ones.</p>	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  <p><math>6 - 4 = 2</math></p> <p><math>4 - 2 = 2</math></p>	 <p><math>15 - 3 = 12</math></p> <p>Cross out drawn objects to show what has been taken away.</p>	<p><math>7 - 4 = 3</math></p> <p><math>16 - 9 = 7</math></p>
<p>Counting back</p>	 <p>Move objects away from the group, counting backwards.</p> <p>Move the beads along the bead string as you count backwards.</p>	 <p><math>5 - 3 = 2</math></p> <p>Count back in ones using a number line.</p>	<p>Put 13 in your head, count back 4. What number are you at?</p>
<p>Find the Difference</p>	<p>Compare objects and amounts</p>  <p>'Seven is 3 more than four'</p> <p>4</p> <p>'I am 2 years older than my sister'</p> <p>5 Pencils</p> <p>3 Erasers</p> <p>?</p> <p>Lay objects to represent bar model.</p>	<p>Count on using a number line to find the difference.</p>  <p><math>+6</math></p>	<p>Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister.?</p>

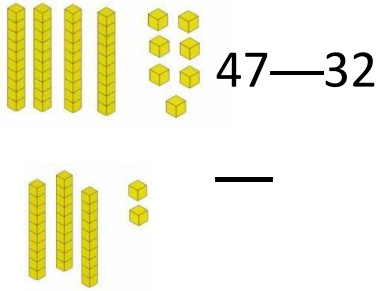
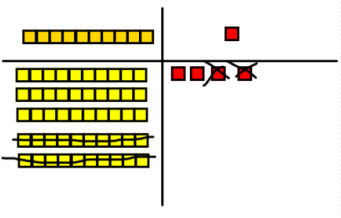
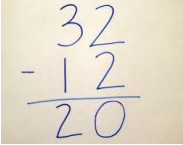
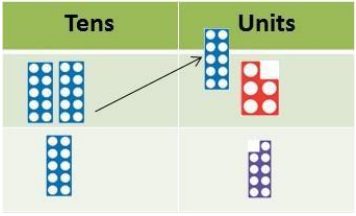
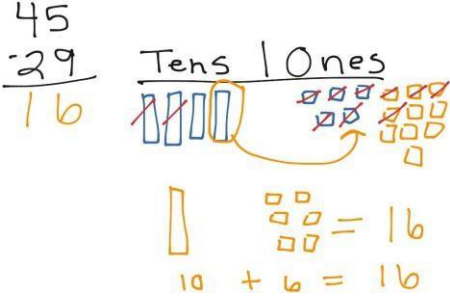
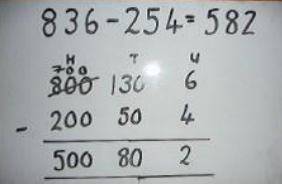
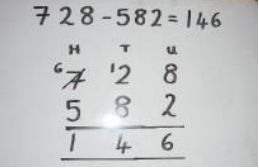
# Y1 SUBTRACTION -

Objective & Strategy	Concrete	Pictorial	Abstract
<p><b>Represent and use number bonds and related subtraction facts within 20</b></p> <p>Part Part Whole model</p>	 <p>Link to addition. Use PPW model to model the inverse.</p> <p>If 10 is the whole and 6 is one of the parts, what is the other part?</p> $10 - 6 = 4$	 <p>Use pictorial representations to show the part.</p>	<p>Move to using numbers within the part whole model.</p> 
<p><b>Make 10</b></p>	<p><b>14—9</b></p>  <p>Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.</p>	<p><b>13—7</b></p>  <p>Jump back 3 first, then another 4. Use ten as the stopping point.</p>	<p><b>16—8</b></p> <p>How many do we take off first to get to 10? How many left to take off?</p>
<p><b>Bar model</b></p>	 $5 - 2 = 3$		 $10 = 8 + 2$ $10 = 2 + 8$ $10 - 2 = 8$ $10 - 8 = 2$

# SUBTRACTION -

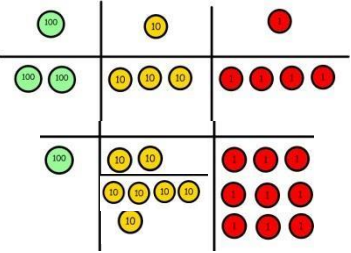
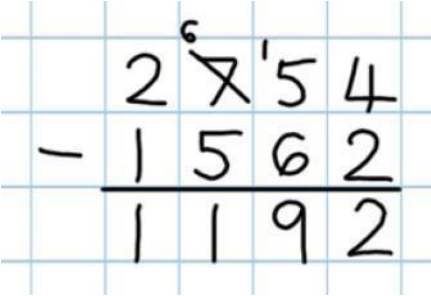
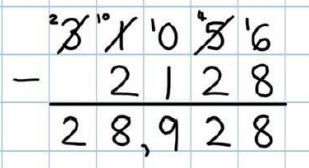
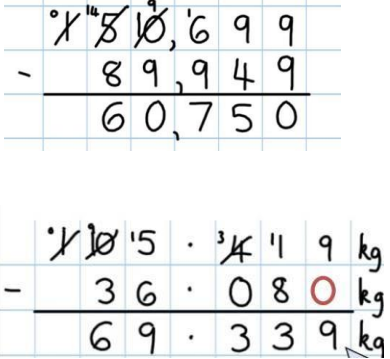
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Regroup a ten into ten ones</p>	 <p>Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'</p>	 <p><math>20 - 4 =</math></p>	<p><math>20 - 4 = 16</math></p>
<p>Partitioning to subtract without regrouping.</p> <p><i>'Friendly numbers'</i></p>	<p><math>34 - 13 = 21</math></p>  <p>Use Dienes to show how to partition the number when subtracting without regrouping.</p>	<p>Children draw representations of Dienes and cross off.</p>  <p><math>43 - 21 = 22</math></p>	<p>T O</p> <p><math>40 + 3</math></p> <p><math>- 20 + 1</math></p> <p><math>= 20 + 2 = 22</math></p> <p><math>43 - 21 = 22</math></p>
<p>Make ten strategy</p> <p><i>Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</i></p>	 <p><math>34 - 28</math></p> <p>Use a bead bar or bead strings to model counting to next ten and the rest.</p>	 <p>Use a number line to count on to next ten and then the rest.</p>	<p><math>93 - 76 = 17</math></p>

# SUBTRACTION -

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Column subtraction without regrouping (friendly numbers)</p>	 <p>47—32</p> <p>Use base 10 or Numicon to model</p>	 <p>Calculations</p> $\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$ <p>Draw representations to support understanding</p>	$47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$ <p>Intermediate step may be needed to lead to clear subtraction understanding.</p> 
<p>Column subtraction with regrouping</p>	 <p>Begin with base 10 or Numicon. Move to PV counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.</p>	 <p>Children may draw base ten or PV counters and cross off.</p>	$836 - 254 = 582$  <p>Begin by partitioning into pv columns</p> $728 - 582 = 146$  <p>Then move to formal method.</p>

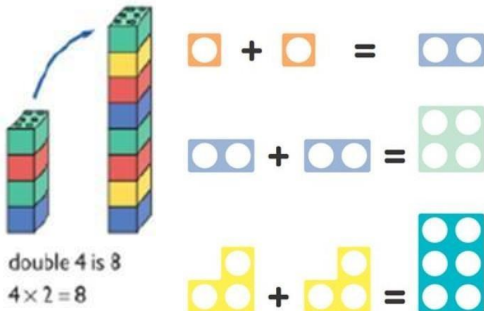

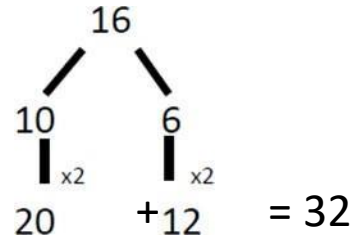
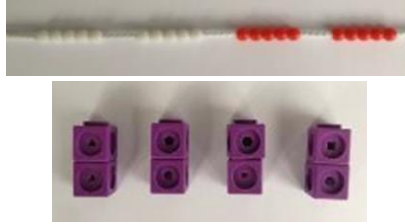
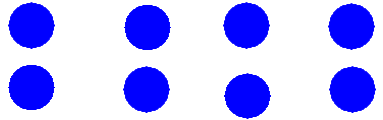
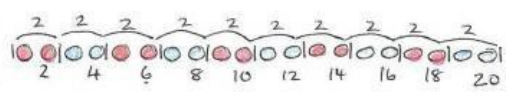


# Y4-6 SUBTRACTION -

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Subtracting tens and ones</p> <p>Year 4 subtract with up to 4 digits.</p> <p><i>Introduce decimal subtraction through context of money</i></p>	<p>234 - 179</p>  <p>Model process of exchange using Numicon, base ten and then move to PV counters.</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	 <p>Use the phrase 'take and make' for exchange</p>
<p>Year 5- Subtract with at least 4 digits, including money and measures.</p> <p><i>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal</i></p>	<p>As Year 4</p>	<p>Children to draw PV counters and show their exchange—see Y3</p>	 <p>Use zeros for place-holders.</p>
<p>Year 6—Subtract with increasingly large and more complex numbers and decimal values.</p>			




# MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Doubling</p>	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling</p>  <p>double 4 is 8 <math>4 \times 2 = 8</math></p>	<p>Draw pictures to show how to double numbers</p> <p>Double 4 is 8</p> 	<p>Partition a number and then double each part before recombining it back together.</p>  <p><math>16</math> <math>\begin{matrix} 10 &amp; 6 \\ \times 2 &amp; \times 2 \\ \hline 20 &amp; +12 \\ \hline 32 \end{matrix}</math></p>
<p>Counting in multiple</p>	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting.</p> 	 <p>Children make representations to show counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>

Making equal groups and counting the total



Use manipulatives to create equal groups.

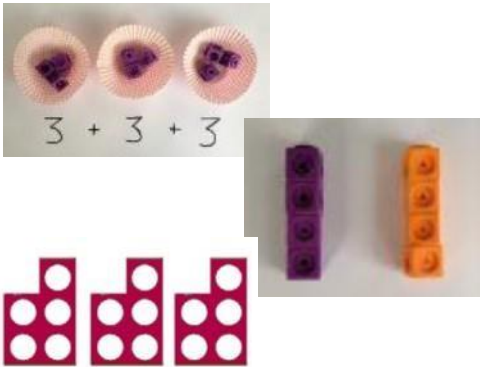
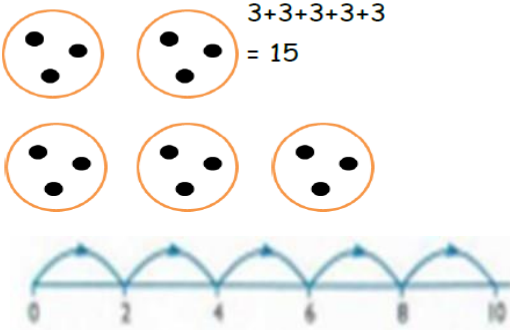

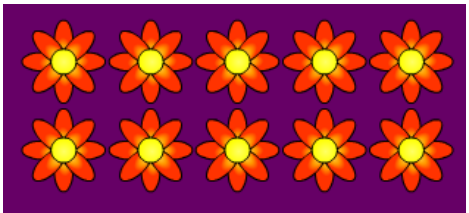
Draw  to show  $2 \times 3 = 6$

Draw and make representations

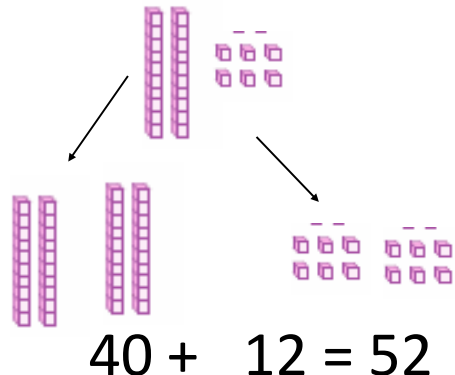
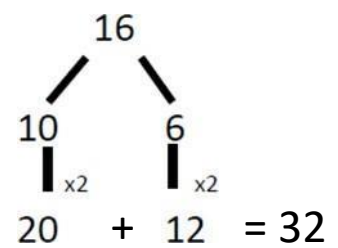
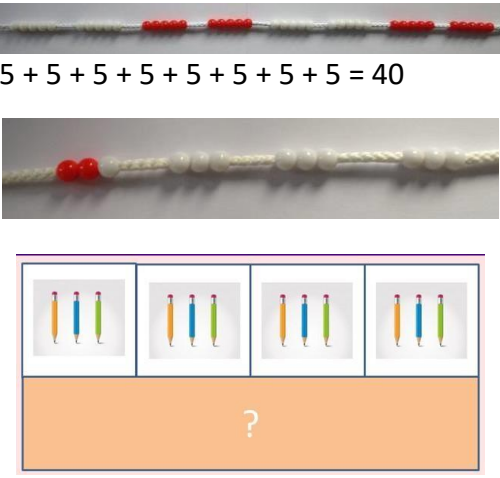
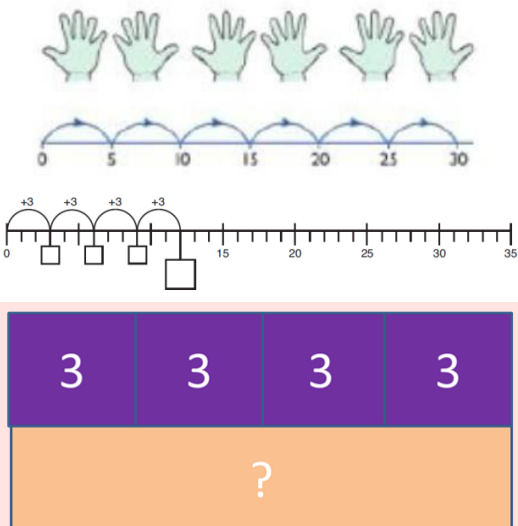
$$2 \times 4 = 8$$



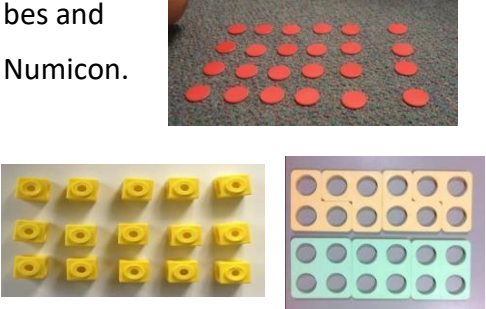
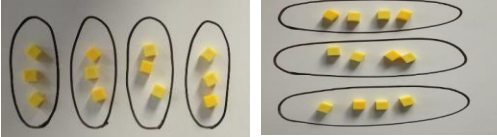
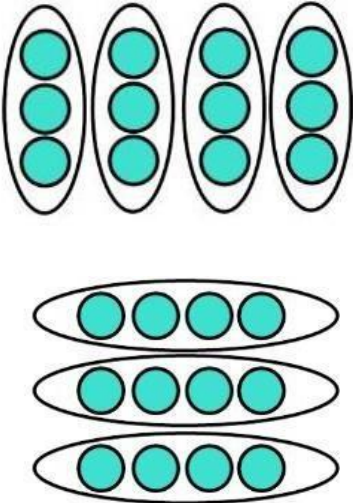


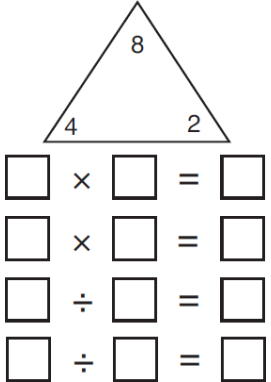
# MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Repeated addition</p>	 <p>Use different objects to add equal groups</p>	<p>Use pictorial including number lines to solve problems There are 3 sweets in one bag. How many sweets are in 5 bags altogether?</p> 	<p>Write addition sentences to describe objects and pictures.</p> 
<p>Understanding arrays</p>	<p>Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.</p> 	<p>Draw representations of arrays to show understanding</p>	$3 \times 2 = 6$ $2 \times 5 = 10$

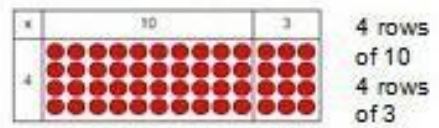
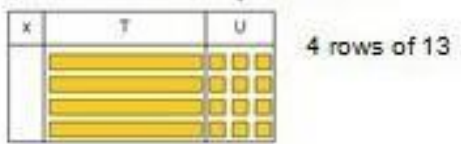
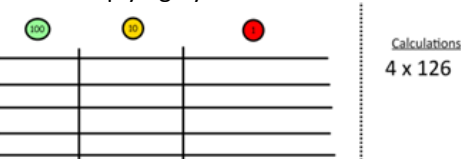
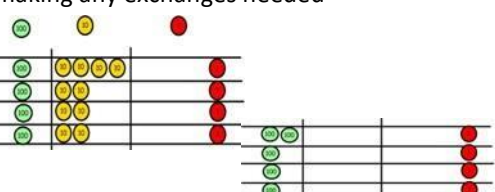
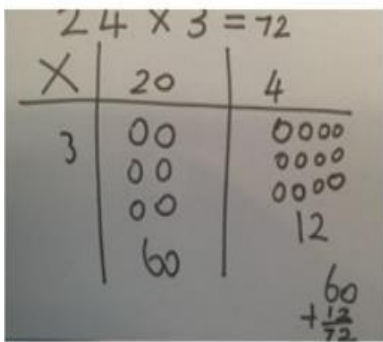
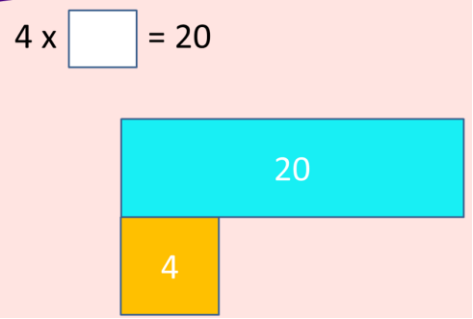
# MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Doubling</p>	<p>Model doubling using dienes and PV counters.</p>  <p><math>40 + 12 = 52</math></p>	<p>Draw pictures and representations to show how to double numbers</p>	<p>Partition a number and then double each part before recombining it back together.</p>  <p><math>20 + 12 = 32</math></p>
<p>Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)</p>	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.</p>  <p><math>5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40</math></p>	<p>Number lines, counting sticks and bar models should be used to show representation of counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30</p> <p><math>4 \times 3 = \square</math></p>

# MULTIPLICATION X

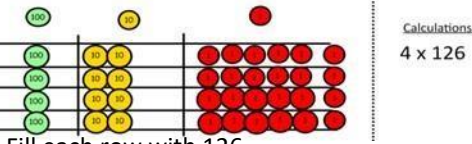
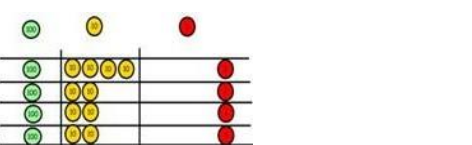
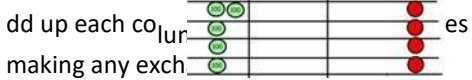
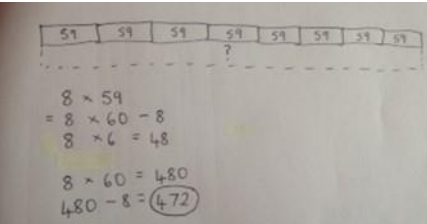
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Multiplication is commutative</p>	<p>Create arrays using counters and cubes and Numicon.</p>  <p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</p> 	<p>Use representations of arrays to show different calculations and explore commutativity.</p> 	<p><math>12 = 3 \times 4</math>  <math>12 = 4 \times 3</math></p> <p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p><math>5 + 5 + 5 = 15</math>  <math>3 + 3 + 3 + 3 + 3 = 15</math>  <math>5 \times 3 = 15</math>  <math>3 \times 5 = 15</math></p>
<p>Using the Inverse</p> <p><i>This should be taught alongside division, so pupils learn how they work alongside each other.</i></p>			<p><math>2 \times 4 = 8</math>  <math>4 \times 2 = 8</math>  <math>8 \div 2 = 4</math>  <math>8 \div 4 = 2</math>  <math>8 = 2 \times 4</math>  <math>8 = 4 \times 2</math>  <math>2 = 8 \div 4</math>  <math>4 = 8 \div 2</math></p> <p>Show all 8 related fact family sentences.</p>

# MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract															
<p>Grid method</p>	<p>Show the links with arrays to first introduce the grid method.</p>  <p>Move onto base ten to move towards a more compact method.</p>  <p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>  <p>Calculations 4 x 126</p> <p>Fill each row with 126</p> <p>Add up each column, starting with the ones making any exchanges needed</p>  <p>Then you have your answer.</p>	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p>  <p>Bar model are used to explore missing numbers</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1556 359 1892 462"> <tr> <td>x</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p><math>210 + 35 = 245</math></p> <p>Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</p> <table border="1" data-bbox="1568 766 1881 973"> <tr> <td></td> <td>10</td> <td>8</td> </tr> <tr> <td>10</td> <td>100</td> <td>80</td> </tr> <tr> <td>3</td> <td>30</td> <td>24</td> </tr> </table>	x	30	5	7	210	35		10	8	10	100	80	3	30	24
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# MULTIPLICATION



Objective & Strategy	Concrete	Pictorial	Abstract																																													
<p>Grid method recap from year 3 for 2 digits x 1 digit</p> <p>Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)</p>	<p>Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>  <p>Calculations 4 x 126</p> <p>Fill each row with 126</p>  <p>Add up each column making any exchanges</p> 	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p>	<p>When multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1545 391 1881 494"> <tr> <td>x</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p>210 + 35 = 245</p>	x	30	5	7	210	35																																							
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<p>Column multiplication</p>	<p>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642</p> <table border="1" data-bbox="392 1045 728 1452"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </tbody> </table>	Hundreds	Tens	Ones													<table border="1" data-bbox="1008 925 1355 1013"> <tr> <td>x</td> <td>300</td> <td>20</td> <td>7</td> </tr> <tr> <td>4</td> <td>1200</td> <td>80</td> <td>28</td> </tr> </table> <p>The grid method may be used to show how this relates to a formal written method.</p>  <p>Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</p>	x	300	20	7	4	1200	80	28	<table data-bbox="1624 941 1803 1332"> <tr> <td>327</td> </tr> <tr> <td>4</td> </tr> <tr> <td>8</td> </tr> <tr> <td>80</td> </tr> <tr> <td>1200</td> </tr> <tr> <td>1308</td> </tr> </table> <p>This may lead to a compact method.</p> <table border="1" data-bbox="1534 1380 1691 1500"> <tr> <td></td> <td>3</td> <td>2</td> <td>7</td> </tr> <tr> <td>x</td> <td></td> <td></td> <td>4</td> </tr> <tr> <td></td> <td>1</td> <td>3</td> <td>0</td> </tr> <tr> <td></td> <td></td> <td>2</td> <td>8</td> </tr> </table>	327	4	8	80	1200	1308		3	2	7	x			4		1	3	0			2	8
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# MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract																																																	
<p>Column Multiplication for 3 and 4 digits x 1 digit.</p>	<div style="display: flex; align-items: center;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td style="background-color: #ff0000; color: white;">Hundreds</td> <td style="background-color: #00ff00; color: white;">Tens</td> <td style="background-color: #0000ff; color: white;">Ones</td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </table> <div style="margin-left: 10px;"> <p>It is important at this stage that they always multiply the ones first.</p> <p>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. <math>321 \times 2 = 642</math> The corresponding expanded method is modelled alongside</p> </div> </div>	Hundreds	Tens	Ones													<div style="display: flex; align-items: center;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td>x</td> <td>300</td> <td>20</td> <td>7</td> </tr> <tr> <td>4</td> <td>1200</td> <td>80</td> <td>28</td> </tr> </table> <div style="margin-left: 20px; color: red; font-size: 2em;">➔</div> </div>	x	300	20	7	4	1200	80	28	<div style="text-align: right;"> <math display="block">\begin{array}{r} 327 \\ \times 4 \\ \hline 28 \\ 80 \\ 1200 \\ \hline 1308 \end{array}</math> </div> <div style="text-align: center; color: red; font-size: 2em; margin: 10px 0;">↻</div> <div style="display: flex; align-items: center;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td></td><td>3</td><td>2</td><td>7</td></tr> <tr><td>x</td><td></td><td></td><td>4</td></tr> <tr><td></td><td>1</td><td>3</td><td>0</td></tr> <tr><td></td><td></td><td>1</td><td>2</td></tr> </table> <div style="margin-left: 10px;"> <p>This will lead to a compact method.</p> </div> </div>		3	2	7	x			4		1	3	0			1	2										
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<p>Column multiplication</p>	<p>Manipulatives may still be used with the corresponding long multiplication modelled alongside.</p>	<div style="display: flex; align-items: center;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td></td> <td>10</td> <td>8</td> </tr> <tr> <td>10</td> <td style="background-color: #ff4500;">100</td> <td style="background-color: #ff4500;">80</td> </tr> <tr> <td>3</td> <td style="background-color: #ff4500;">30</td> <td style="background-color: #ff4500;">24</td> </tr> </table> <div style="margin-left: 20px; color: red; font-size: 2em;">➔</div> </div> <p style="margin-top: 20px;">Continue to use bar modelling to support problem solving</p>		10	8	10	100	80	3	30	24	<div style="display: flex; align-items: center;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td></td><td>1</td><td>8</td></tr> <tr><td>x</td><td>1</td><td>3</td></tr> <tr><td></td><td>5</td><td>4</td></tr> <tr><td></td><td>1</td><td>8</td></tr> <tr><td></td><td>2</td><td>3</td></tr> </table> <div style="margin-left: 10px;"> <p>18 x 3 on the first row (8 x 3 = 24, carrying the 2 for 20, then 1 x 3) 18 x 10 on the 2nd row. Show multiplying by 10 by putting zero in units first</p> </div> </div> <div style="margin-top: 20px;"> <table border="1" style="border-collapse: collapse; text-align: center;"> <tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr> <tr><td>x</td><td></td><td></td><td>1</td><td>6</td></tr> <tr><td></td><td>7</td><td>4</td><td>0</td><td>4</td></tr> <tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr> <tr><td></td><td>1</td><td>9</td><td>7</td><td>4</td></tr> </table> <p>(1234 x 6) (1234 x 10)</p> </div>		1	8	x	1	3		5	4		1	8		2	3		1	2	3	4	x			1	6		7	4	0	4		1	2	3	4		1	9	7	4
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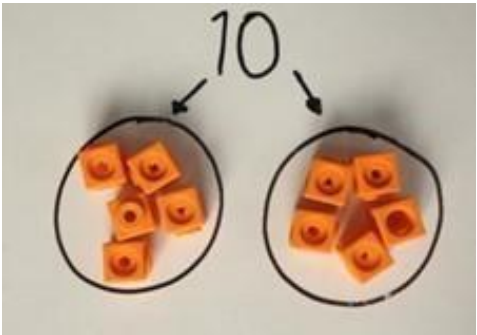
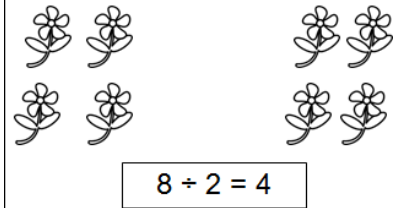
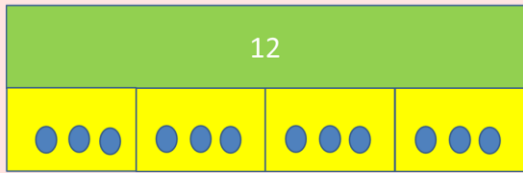
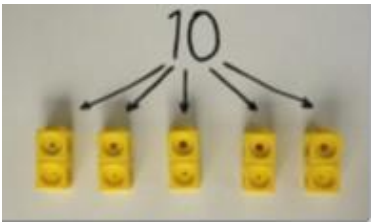
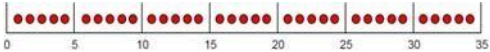
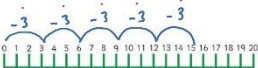
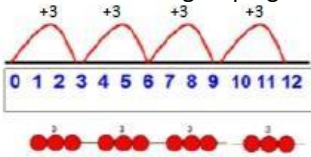
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Multiplying decimals up to 2 decimal places by a single digit.</p>			<p>Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer.</p> $  \begin{array}{r}  3.19 \\  \times 8 \\  \hline  25.52  \end{array}  $







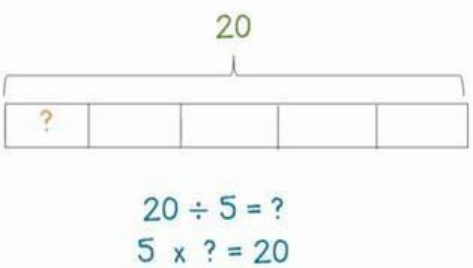
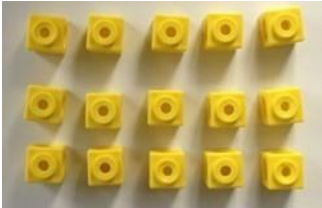
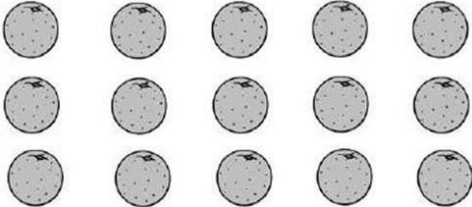
# DIVISION



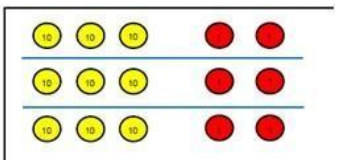

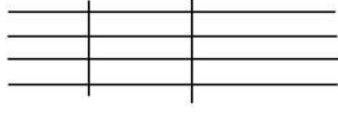

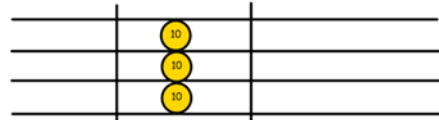
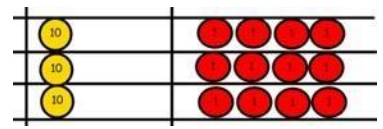
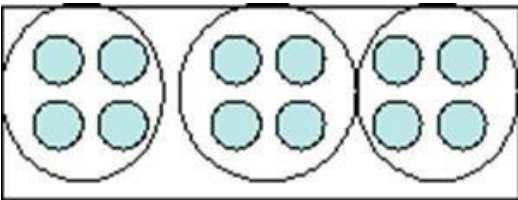
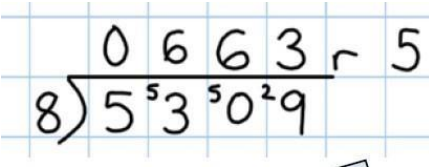
Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division as sharing</p>	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <p>Children use bar modelling to show and support understanding.</p>  <p><math>12 \div 4 = 3</math></p>	<p><math>12 \div 3 = 4</math></p>
<p>Division as grouping</p>	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p>  	<p>Use number lines for repeated subtraction</p> <p><math>15 \div 3 = 5</math></p>  <p>Use number lines for grouping</p>  <p><math>12 \div 3 = 4</math></p> <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>	<p><math>28 \div 7 = 4</math></p> <p>Divide 28 into 7 groups. How many are in each group?</p>

# DIVISION



Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division as grouping</p>	<p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> $96 \div 3 = 32$ 	<p>Continue to use bar modelling to aid solving division problems.</p>  $20 \div 5 = ?$ $5 \times ? = 20$	<p>How many groups of 6 in 24?</p> $24 \div 6 = 4$
<p>Division with arrays</p>	 <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg <math>15 \div 3 = 5</math>    <math>5 \times 3 = 15</math></p> <p>     <math>15 \div 5 = 3</math>    <math>3 \times 5 = 15</math></p>	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences</p> 	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences.</p> $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$



Objective & Strategy	Concrete	Pictorial	Abstract
<p>Divide at least 3 digit numbers by 1 digit.</p> <p>Short Division</p>	<p><math>96 \div 3</math></p> <p>Tens      Units</p> <p>3            2</p>  <p>Use place value counters to divide using the bus stop method alongside</p>  <p>Calculations <math>42 \div 3 =</math></p>  <p><math>42 \div 3 =</math></p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p>   <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p>  <p>We look how much in 1 group so the answer is 14.</p>	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p>  <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Begin with divisions that divide equally with no remainder.</p> $\begin{array}{r} 218 \\ 3 \overline{) 654} \end{array}$ <p>Move onto divisions with a remainder.</p> $\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \end{array}$ <p>Finally move into decimal places to divide the total accurately.</p> $\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$ 

Y4-6

9-4-1

# DIVISION



## Long Division

Step 1—a remainder in the ones

$$\begin{array}{r} \text{h t o} \\ 041\text{R}1 \\ \hline 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400\text{R}7 \\ \hline 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ( $3,200 \div 8 = 400$ )

8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.

Y6

DIVISION



## Long Division

Step 1 continued...

$$\begin{array}{r} \text{h t o} \\ 061 \\ 4 \overline{) 247} \\ \underline{-4} \\ 3 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply  $1 \times 4 = 4$ , write that four under the 7, and subtract. This finds us the remainder of 3.

Check:  $4 \times 61 + 3 = 247$

$$\begin{array}{r} \text{th h t o} \\ 0402 \\ 4 \overline{) 1609} \\ \underline{-8} \\ 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply  $2 \times 4 = 8$ , write that eight under the 9, and subtract. This finds us the remainder of 1.

Check:  $4 \times 402 + 1 = 1,609$

Y6

DIVISION





# Long Division

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ \underline{4} \phantom{0} \\ 18 \end{array}$ <p>Two goes into 5 two times, or 5 tens <math>\div 2 = 2</math> whole tens -- but there is a remainder!</p>	$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ \underline{-4} \phantom{0} \\ 18 \end{array}$ <p>To find it, multiply <math>2 \times 2 = 4</math>, write that 4 under the five, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ \underline{-4} \downarrow \\ 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.</p>

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ \underline{-4} \phantom{0} \\ 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ \underline{-4} \phantom{0} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>Multiply <math>9 \times 2 = 18</math>, write that 18 under the 18, and subtract.</p>	$\begin{array}{r} \text{t o} \\ 29 \\ 2 \overline{) 58} \\ \underline{-4} \phantom{0} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>The division is over since there are no more digits in the dividend. The quotient is 29.</p>

Y6

20

DIVISION



# Long Division

Step 2—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{)278} \end{array}$ <p>Two goes into 2 one time, or 2 hundreds <math>\div 2 = 1</math> hundred.</p>	$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{)278} \\ -2 \\ \hline 0 \end{array}$ <p>Multiply <math>1 \times 2 = 2</math>, write that 2 under the two, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 18 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \end{array}$ <p>Next, drop down the 7 of the tens next to the zero.</p>
Divide.	Multiply & subtract.	Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \end{array}$ <p>Divide 2 into 7. Place 3 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 1 \end{array}$ <p>Multiply <math>3 \times 2 = 6</math>, write that 6 under the 7, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the 1 leftover ten.</p>
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>Multiply <math>9 \times 2 = 18</math>, write that 18 under the 18, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{)278} \\ -2 \\ \hline 07 \\ -6 \\ \hline 18 \\ -18 \\ \hline 0 \end{array}$ <p>There are no more digits to drop down. The quotient is 139.</p>