Goring-by-Sea C of E Primary School

Progression through calculation policy

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Goring C of E Primary School
Calculations Policy: Introduction

The revised National Curriculum (2014) provides a structured and systematic approach to the teaching of calculation. There is considerable emphasis on formal written methods, alongside which we will practice teaching mental calculation strategies and reasoning activities.

Up to the age of 7 (Year 2), informal written recording should take place regularly and is an important part of learning and understanding. From KS2 onwards there will be a greater emphasis on formal written methods. **However, it should be noted that more formal written methods should follow only when the child is able to use a wide range of mental calculation strategies.** This will help communicate methods and solutions.

Goring Primary School has developed a consistent approach to the teaching of written calculations in order to establish continuity and progression throughout the school. To develop this policy we have used guidance and images from the 'White Rose Mathematics Hub' calculation policy, alongside our own progression and images.

Although the focus of this policy is on written recording, it is important to recognise that the ability to calculate mentally lies at the heart of calculation. At Goring Primary School, we use a mastery approach to the teaching of mathematics; this approach embeds foundations in calculations and place value using a wide range of strategies to develop children's mathematical fluency. We therefore do not introduce formal written methods until a child has shown they have a secure understanding of place value and are secure in number fluency.

Fluency is not simply about recalling facts and numbers quickly and accurately - it is far more than that. Fluency is the ability to recall and apply mathematical knowledge rapidly and accurately; it requires flexible thinking and an awareness of relationships between numbers. To be truly fluent, children need to consider effective and efficient methods to find a way to solve a problem – e.g. when presented with 18 x 5, children will adapt their thinking to find a method which works for them.

20 x 5 = 100 2 x 5 = 10 100 - 10 = 90	10 x 5 = 50 8 x 5 = 40 50 + 40 = 90	18 x 5 = 9 x 10 9 x 10 = 90	18 x 2 = 36 2 x 36 = 72 18 + 72 = 90	9 x 5 = 45 45 x 2 = 90

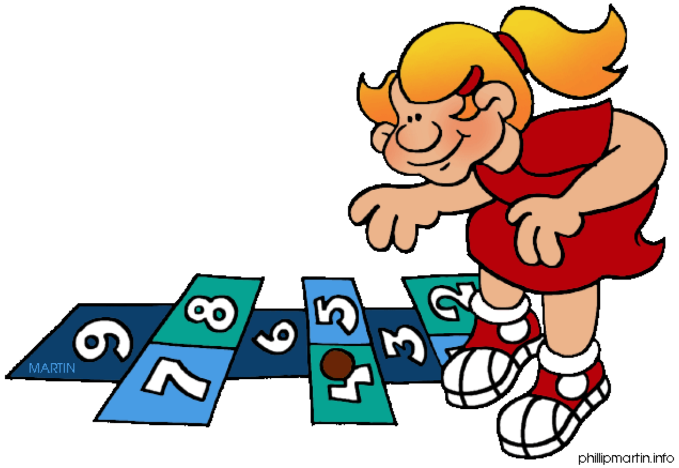
This policy is divided into stages, which shows the development through children's learning within calculation and the progression through the methods they will use. It is important to note that children progress at *individual* rates within mathematics, therefore children will move naturally through the stages when they are able in line with their development.

Why do we need this policy?

- Consistency in methods taught throughout the school
- Progression from informal/practical methods of recording to written methods within each of the four operations
- An aid to parents' understanding in their child's stages of learning.

Recording in mathematics

Children need to be introduced to the processes of calculation through practical, oral and mental activities. As children begin to understand the underlying ideas behind calculations, they develop ways of recording to support their thinking and methods. Children learn how to use models and images, such as number lines, to support their mental and informal written methods of calculation. As children's mental methods are strengthened and refined, so too are their informal written methods. These methods become more efficient and succinct and lead to written methods that can be used more generally.



Early knowledge of number

It is important to note that children cannot progress through calculations without a clear understanding of early number and pattern. At the time children enter school, and throughout their schooling, they will need to be able to continuously access all the skills outlined below. These form the foundations of mathematical fluency.

- **Understand the value of a number.**

We cannot stress enough the importance of children understanding the value of a number. Children need to be able to understand that 4 objects are the same amount, no matter the objects (e.g. eggs or pencils) and whichever formation they fall into. Without this knowledge children will not understand the skills required within calculations.

- **Count reliably up to 10 objects.**

Children need to be able to count objects accurately, touching them as they say the number. They need to understand that if you put the same amount of objects into a different array that the number will remain the same.

- **Say the number names in order.**

Children need to be able to understand the number names and relate these to a number, although they do not need to be introduced to numerals at this stage. They should recognise number order and count from 1 to 20. Regular number rhymes and games can help to support the development of early number.

- **Recognise differences in quantities**.

Children need to be able to see that one quantity is larger than another by sight, e.g. to see an array of 4 cubes and an array of 9 and say that 9 is bigger. If they cannot do this then they do not have a secure understanding of the value of a number.

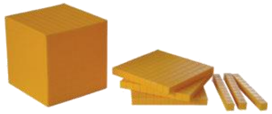
Vocabulary

A secure understanding of mathematical vocabulary underpins the foundation of calculation and children's understanding of number. If children understand the vocabulary, they can apply their knowledge when solving problems. At Goring Primary School, we focus on vocabulary in every maths lesson. Below are some of the definitions we may use within the school, adapting as necessary for each year group.

	Definitions
operation	A mathematical process. The most common are add, subtract, multiply and divide (+, −, ×, ÷).
calculate	To use an operation/s to work out a missing value/s
addition	Combining two or more sets of objects to find a total. *Other words meaning the same thing include:* add, plus, sum, increase, more, total
subtraction	Taking one number away from another. *Other words meaning the same thing include:* minus, take away, subtract, less, difference, decrease, deduct
multiplication	The same as repeated addition. *Other words meaning the same thing include:* multiply, times, groups of, lots of
division	Division is sharing, grouping or repeatedly subtracting numbers or objects equally. *Other words meaning the same thing include:* divide, share, group, split
exchange	To trade one thing for another – e.g. 10 ones for 1 ten.
decompose	To break something into parts, that together are the same value as the original. Example: We can decompose 349 like this: 349 → 300 + 40 + 9

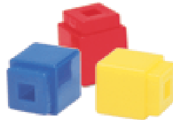
Concrete

The term 'concrete' in this policy refers to the use of practical equipment and hands on manipulatives to represent mathematical ideas and calculations. This could be either real-life objects, such as oranges and marbles, or mathematical based resources, like Numicon or dienes.



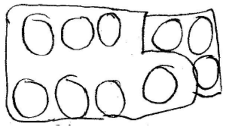
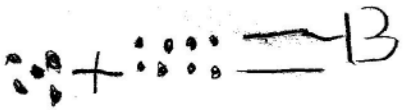


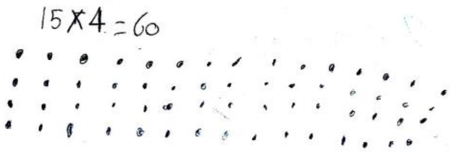


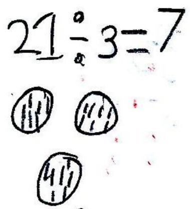



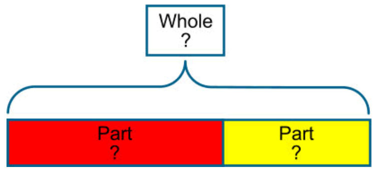

Pictorial

The term 'pictorial' in this policy refers to the use of images and pictures to represent mathematical ideas and calculations. Children may be presented with images of equipment or they may draw their own representations. Throughout Goring CE Primary School, children will also be taught how to use bar models as a way represent their calculations; these are particularly useful when solving problems.







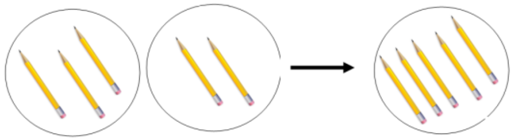
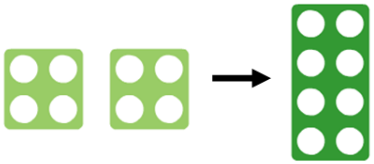
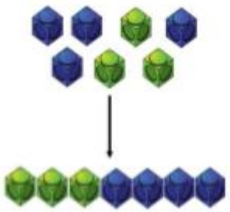
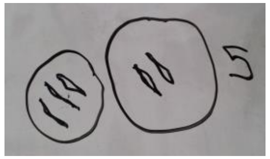
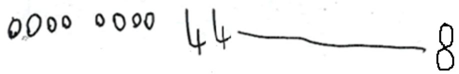
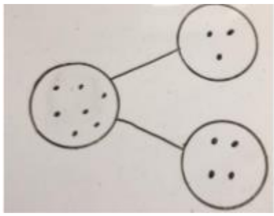
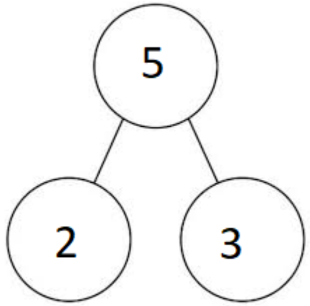
Abstract

The term 'abstract' in this policy refers to the use of numbers/digits and symbols to represent mathematical ideas and calculations. It is important that children understand the idea of number, and the values behind a number, before they progress into writing calculations in abstract form.

Addition:

Stage 1:

At every stage, practical equipment and images are imperative to providing a secure foundation for calculation. At stage 1, children are not expected to use numerals or operation signs unless they are able to associate them with a value or operation.

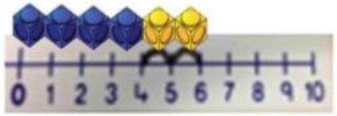
Concrete	Pictorial	Abstract
Combing **two parts** to make a **whole** (using many practical resources)   	Children will choose how they represent their own mathematics and are encouraged to explain what it shows.    Children may be taught to use a **part-whole** model to represent each part.	A teacher or adult may model a number sentence but the children are not expected to record using numerals or operation signs unless they are able to associate them with a value or operation. 3 + 2 = 5 "3 is a part, 2 is a part and the whole is 5". 

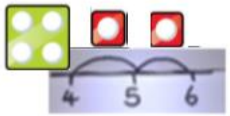
Stage 2:

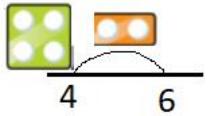
Practical equipment and images are still incredibly important at this stage and children are now beginning to use signs and numerals to record their number sentences. As their knowledge of number value grows, children will also begin to use number lines and number squares to support their understanding of calculation and numerical value.

Concrete

Counting on using **number lines** (including with practical equipment such as cubes or Numicon.)

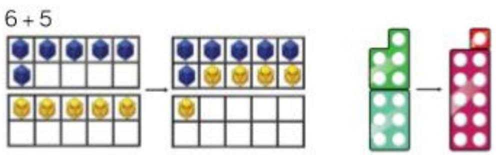






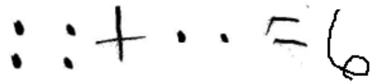
4 + 2 = 6

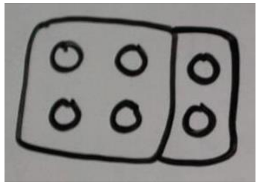
Regrouping to make 10; using ten frames and counters/cubes or Numicon

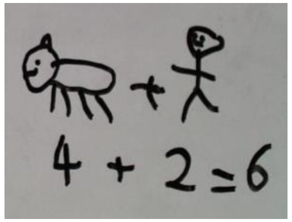


Pictorial

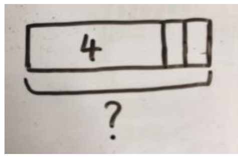
Children will begin to draw to support their understanding of calculation and represent this in their own way. They will begin to use signs and numerals to show number sentences and they may begin to group numbers when drawing to make it easier to record.



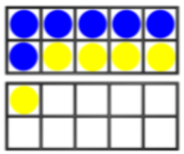




Children may use a **bar model** which encourages them to count on, rather than count all.

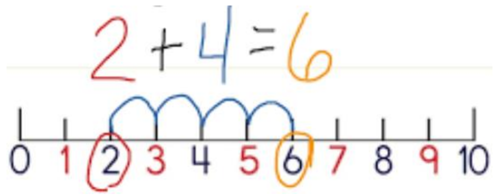


Children may draw **tens frames**:



Abstract

Representing on a **number line**:



The abstract number line:

What is 2 more than 4?
What is the sum of 2 and 4?
What is the total of 4 and 2?
4 + 2

Children will develop an understanding of **equality**, e.g.

6 + __ = 11
6 + 5 = 5 + __
6 + 5 = __ + 4

<u>Stage 3:</u>

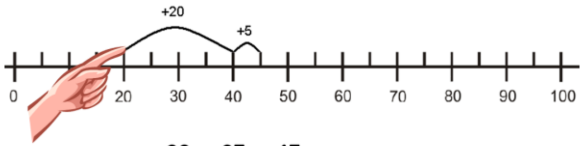
At stage 3, children will be continuously accessing practical equipment and using visual drawings to support their calculations. They will begin to know number facts off by heart and be developing a repertoire of strategies to support their mental calculations.

By stage 3, children should be becoming more confident with their mental calculations. Children therefore need to be able to access the following before moving towards other stages of calculations. These again form the <u>foundations of fluency.</u>

- Recognition and recall of number pairs to 10 and 20.
- Knowledge of place value (tens and ones).
- Understanding of number values from 0-100.
- Visualise a number or mathematical image in their heads.

Concrete

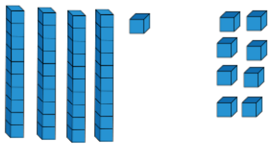
Children will continue to use **number squares** and **number lines** to support their addition. They will make *practical* jumps using their knowledge of tens and ones and learn strategies to use these successfully.



20 + 25 = 45

Tens and ones + ones using base 10, Numicon and place value counters. Continue to develop understanding of **partitioning** and **place value**.

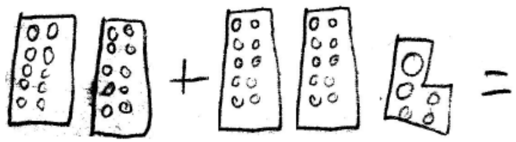
E.g. 41 + 8 =

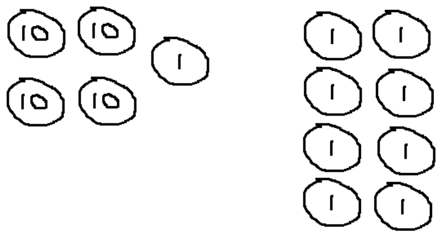


Pictorial

Children will continue to draw their own representations for number calculation sentences but now these will begin to understand the importance of **grouping tens** to ensure accuracy.

20 + 25 = 45





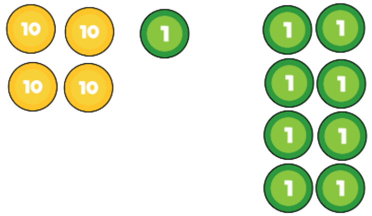
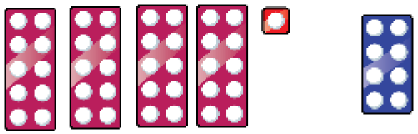
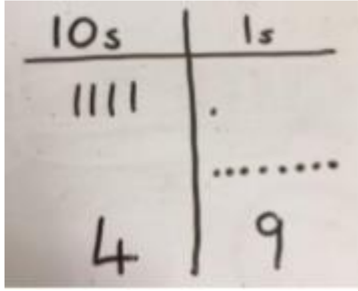
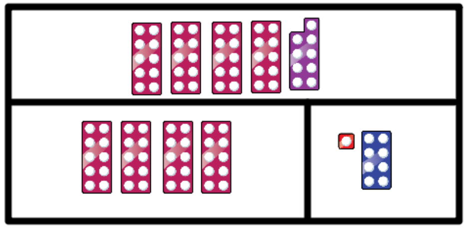
Abstract

Children may use **bar models** to support/visualise their calculations.

45	
25	20

They will begin to **partition** numbers to support mental calculation and fluency, e.g:

1 + 8 = 9
40 + 9 = 49

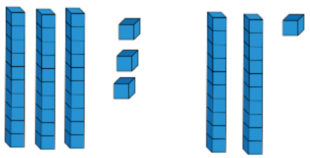
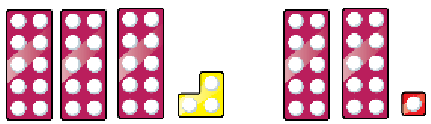
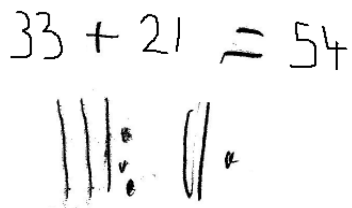
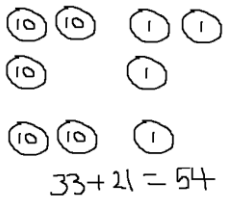
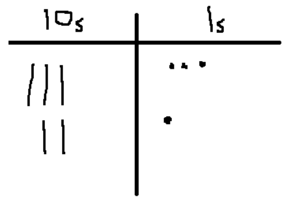
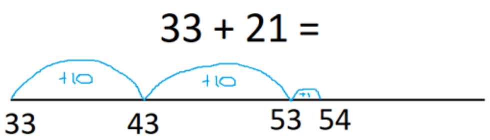
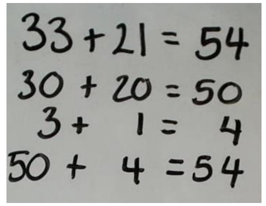
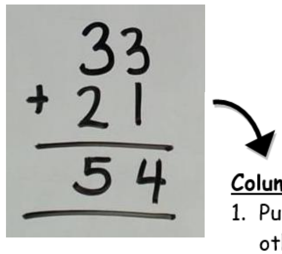
 	 They may also use bar models to show **partitioning** into **tens** and **ones**. 	

<u>Stage 4:</u>

By stage 4, children should be confident with mental calculation of tens and ones + ones where exchanging does not take place. Children should have a greater fluency and number fact knowledge, and a greater understanding of relationships between numbers and calculations. Children will begin to use their knowledge flexibly and apply in a number of different situations.

At stage 4, children may record maths in methods they deem appropriate, but they will be introduced to more formal methods of calculation; first <u>without</u> exchanging.

Children will continue to access practical equipment - it is important children have this support at *all times* throughout their development of mathematics and working practically should be encouraged when needed.

Concrete	Pictorial	Abstract
Tens and ones + tens and ones (without exchanging) using base 10, Numicon and place value counters. Continue to develop understanding of **partitioning** and **place value**. E.g. 33 + 21   	Children to represent the base 10, Numicon and place value counters to support addition of tens and ones + tens and ones without exchanging. They may also count on using a number line.     	**Partitioning** to **add tens and ones + tens and ones** For this method children will be encouraged to use place value equipment where necessary to support their understanding of partitioning. They will partition each number into tens and ones before adding up the total amount.   <u>**Column method addition**</u> 1. Put the numbers one above the other showing clear place value columns. 2. Draw the equal lines. 3. Add the ones, then the tens.

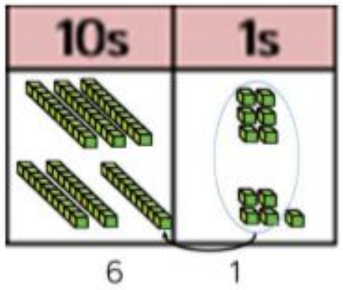
Stage 5:

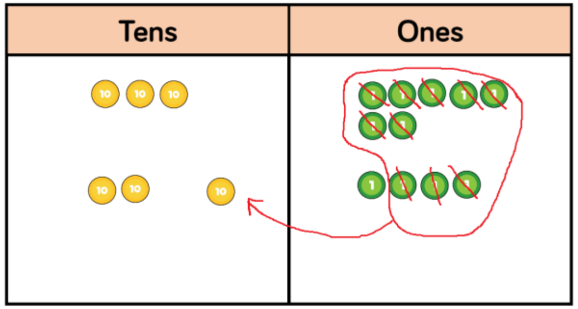
To build on from stage 4, the children will continue to access their knowledge of partitioning to help support their calculations. They should be beginning to calculate tens and ones + tens and ones mentally (without exchanging).

At stage 5, as their confidence with place value and calculating grows, children will be taught to exchange; first with concrete methods, before moving onto a formal written calculation. <u>Children should not progress onto formal written methods until their number fact knowledge is secure.</u>

Concrete

Tens and ones + tens and ones (with exchanging) using base 10, Numicon and place value counters. Continue to develop understanding of **partitioning** and **place value**.

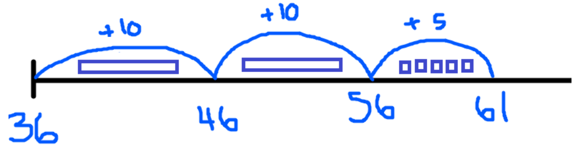




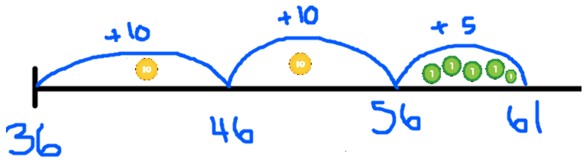
Pictorial

Where needed, children will use an open number line to calculate before moving onto formal written methods. They may use base 10 equipment to support their understanding.

36 + 25 =

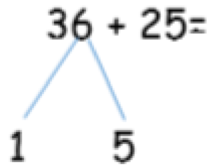


36 + 25 =



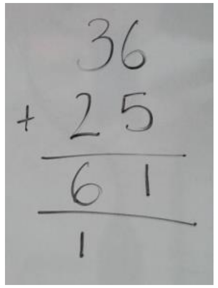
Abstract

Looking for ways to **make 10**.



30 + 20 = 50
5 + 5 = 10
50 + 10 + 1 = 61

Formal method:



<u>**Column method addition**</u>

1. Put the numbers one above the other showing clear place value columns.
2. Draw the equal lines.
3. Add the ones, then the tens and hundreds.
4. If the number goes over the next boundary, children will learn to exchange ten ones for one ten underneath the tens column.

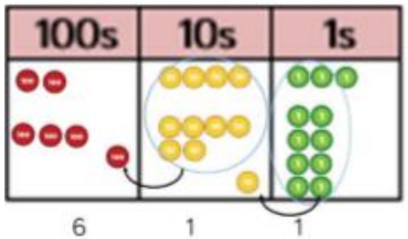
Stage 6:

Stage 6 builds on from stage 5 and children should now be secure with the concept of exchanging within 2-digits. Children will now be taught to apply their knowledge to calculations including hundreds and beyond. Below are the expectations set out in the National Curriculum:

Year 3 – addition with 3 digits
Year 4 – addition with 4 digits
Year 5 and 6 – addition with more than 4 digits; application of this to decimals

Concrete

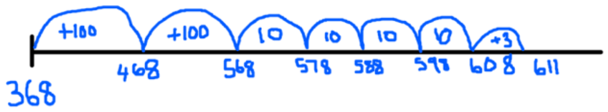
Use of place value counters and base 10 to **add HTO + TO, HTO + HTO, ThHTO + HTO etc and ones + tens and ones (with exchanging).**



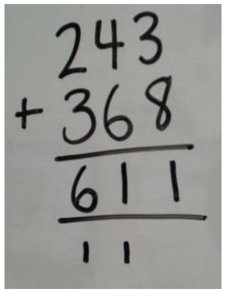
Pictorial

Where needed, children will use an open number line to calculate before moving onto formal written methods.

243 + 368 =



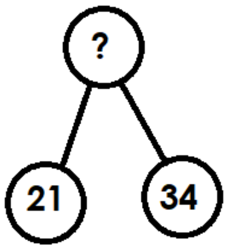
Abstract



Column method addition

1. Put the numbers one above the other showing clear place value columns.
2. Draw the equal lines.
3. Add the ones, then the tens and hundreds.
4. If the number goes over the next boundary, children will learn to exchange ten ones for one ten underneath the tens column.

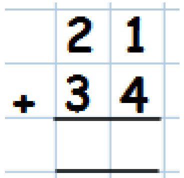
Conceptual variation; different ways to ask children to solve 21 + 34



?	
21	34

In Year 3, there are 21 children and in Year 4, there are 34 children. How many children in total?

21 + 34 = 55. Prove it.

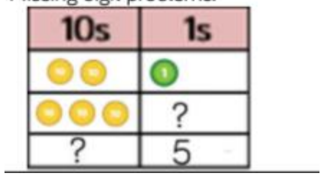


21 + 34 =

☐ = 21 + 34

Calculate the sum of twenty-one and thirty-four.

Missing digit problems:

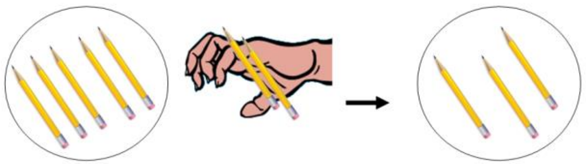
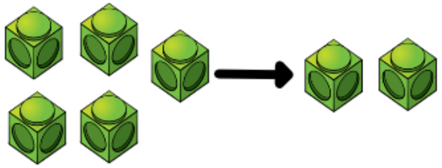
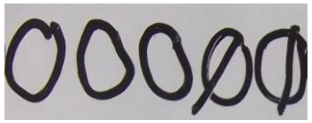
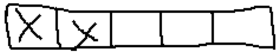
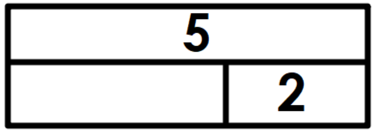
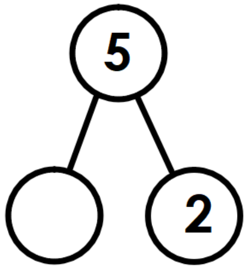


Subtraction:

Key language: take away, less than, the difference, subtract, minus, fewer, decrease

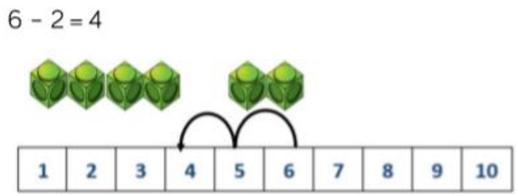
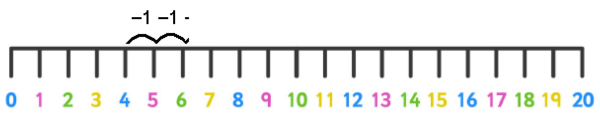
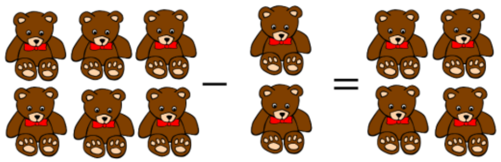
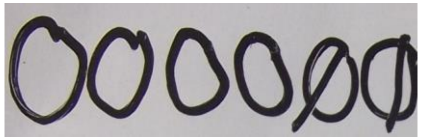
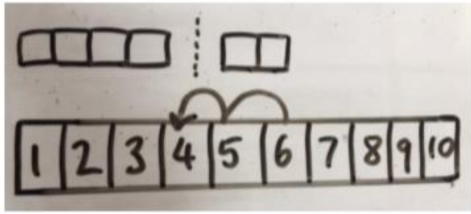
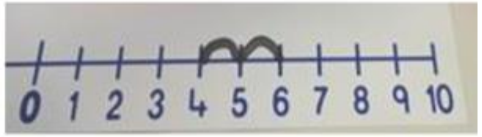
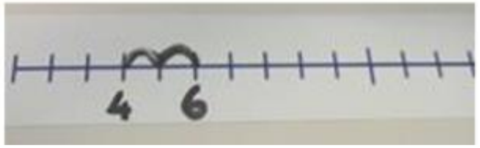
Stage 1:

At every stage, practical equipment and images are imperative to providing a secure foundation for calculation. At stage 1, children are not expected to use numerals or operation signs unless they are able to associate them with a value or operation.

Concrete	Pictorial	Abstract
Physically taking away and **removing** objects from a whole (Tens Frames, Numicon, cubes etc.) 5 – 2 =   	Children will choose how they **represent** their own subtraction and are encouraged to **explain** what it shows.  Children could draw the **concrete** resources they are using and cross out the correct amount. The **bar model** can also be used. 5 - 2 =  	5 – 2 = ____ = 5 – 2  

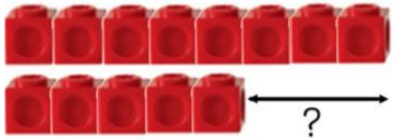
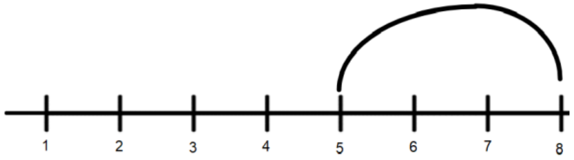
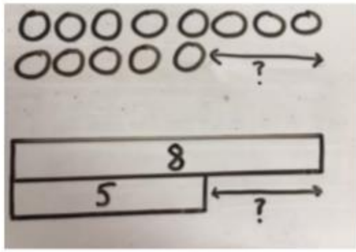
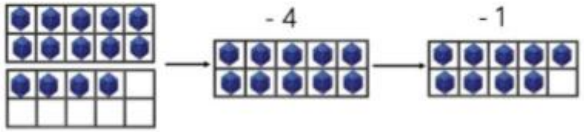
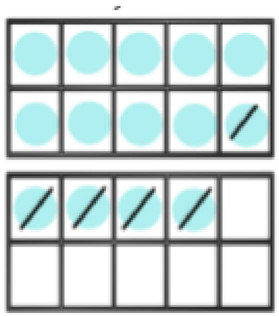
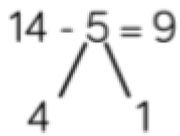
Stage 2:

Practical equipment and images are still incredibly important at this stage and children are now beginning to use signs and numerals to record their number sentences. As their knowledge of number value grows, children will also begin to use number lines and number squares to support their understanding of calculation and numerical value.

Concrete	Pictorial	Abstract
Counting back (using number lines or number tracks) children start with 6 and count back 2.   Beginning to use signs: 	Children will begin to draw to support their understanding of subtraction and represent this in their own way. They will begin to use signs and numerals to show number sentences and they may begin to group numbers when drawing to make it easier to record.  Children to represent what they see **pictorially** e.g.  	Children to **represent** the calculation on a **number line** or **number track** and show their jumps. Encourage children to use an empty number line.  

Stage 3:

At stage 3, children are continuing to develop their understanding of mental methods to support their calculations. This extends to include the concept of 'finding the difference' and 'making ten'.

Concrete	Pictorial	Abstract
Children will begin to **find the difference** between two numbers using different apparatus, e.g. cubes, Numicon, number lines etc. Calculate the difference between 8 and 5.  "The difference between 8 and 5 is ___."	Children to draw the cubes/other concrete objects which they have used or use the **bar model** to illustrate what they need to calculate.   	Find the **difference** between 8 and 5. 8 – 5, the difference is _____. "The difference between 8 and 5 is ___." Children to explore why **9 - 6 = 8 – 5 = 7 – 4** have the same difference.
Children will understand and **demonstrate place value** of **tens** and **ones** to **subtract**. Making 10 using ten frames. 14 – 5 =  	Children to present the **ten frame pictorially** and discuss what they did to make 10. 	Children to show how they can make 10 by **partitioning** the subtrahend.  14 – 4 = 10 10 – 1 = 9

Stage 4:

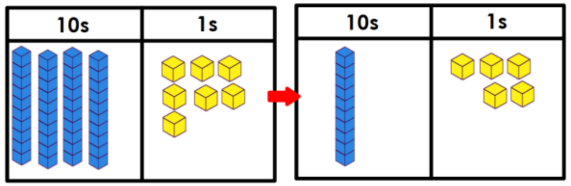
By Stage 4, children will be continuously accessing practical equipment and using visual drawings to support their subtraction calculations. They should now be developing their fluency of number facts and be developing a repertoire of strategies to support their mental calculations, including, counting back, finding the difference by counting on and 'making 10'.

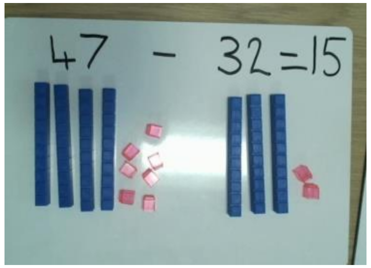
Children will now use their knowledge of place value and partitioning into tens and ones to help them subtract, which will help to build a secure foundation for formal written methods later on.

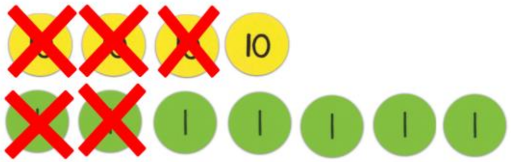
Concrete

Partitioning using base 10. Children can use base 10 resources to subtract, using their knowledge of tens and ones.

47 – 32 =

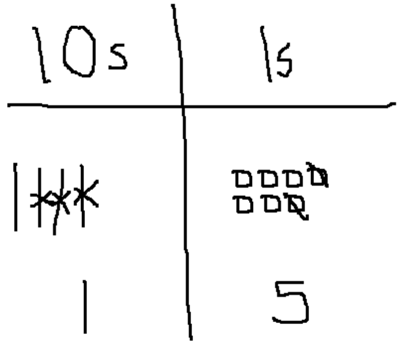




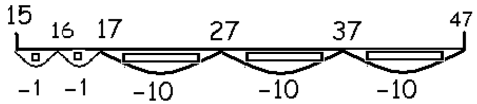


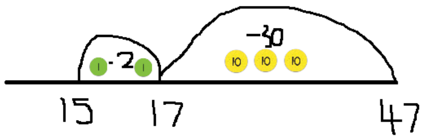
Pictorial

Children to represent the base 10 **pictorially**.



As with addition, for this method children are encouraged to use base 10 to support their understanding using an open number-line.





Abstract

Children will use their knowledge of **partitioning** to subtract.

47 – 32 = 15
40 – 30 = 10
7 – 2 = 5
10 + 5 = 15

Stage 5:

Before moving towards Stage 5, children need to be able to understand and apply the following concepts of subtraction:

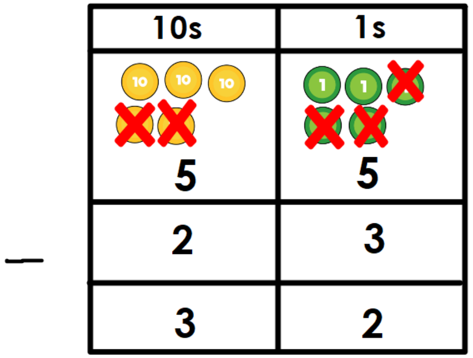
- Understand that subtraction makes a number smaller.
- Understand that the biggest number needs to go first.
- Reliably count backwards from any given number below 100.
- Know that "finding the difference" is subtraction.
- Understand that subtraction is the inverse of addition.
- Knowledge of place value (tens and ones).
- Visualise a number or mathematical image in their heads.

At stage 5, as their confidence with place value and calculating grows, children will be taught to decompose; first with concrete methods, before moving onto a formal written calculation. <u>Children should not progress onto formal written methods until their number fact knowledge is secure.</u>

Concrete

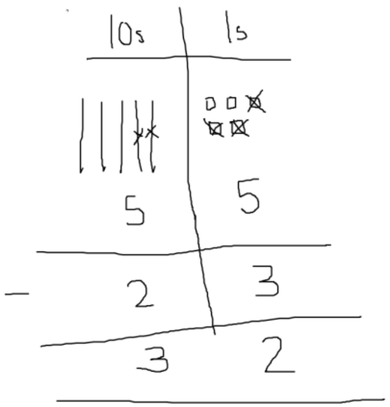
Column method using base 10 without decomposition.

55 – 23 =



Pictorial

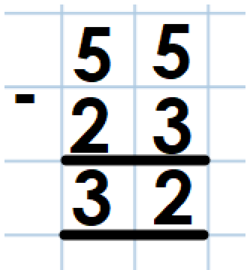
Represent **column method** using base 10 pictorially.



Abstract

Children can now use their knowledge from concrete and pictorial work to use the formal written method to subtract. It is important that the children still understand the significance of the place value tens and ones in the columns. Children should only use the formal written method once they are confident with place value and what the tens and ones represent.

Column method without decomposition



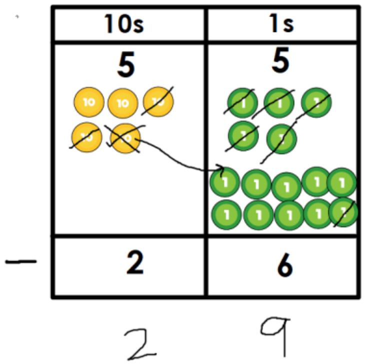
Stage 6:

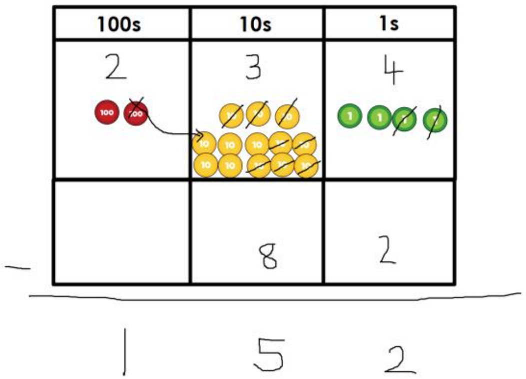
Stage 6 builds on from stage 5 and children should now be secure with the concept of column subtraction. Children will now be taught to decompose in columns and then to apply their knowledge to calculations including hundreds and beyond. Below are the expectations set out in the national curriculum:

Year 3 – subtraction mentally and then with up to 3 digits
Year 4 – subtraction with up to 4 digits
Year 5 and 6 – subtraction with more than 4 digits; application of this to decimals

Concrete

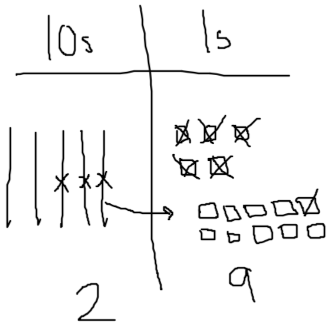
Column method using base 10 with decomposition.

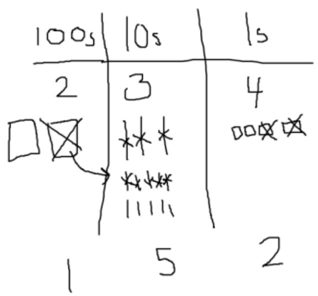




Pictorial

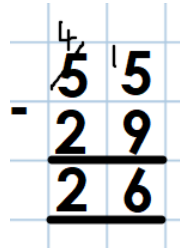
Represent the place value counters pictorially; remembering to show what has been decomposed.

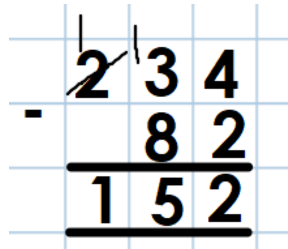


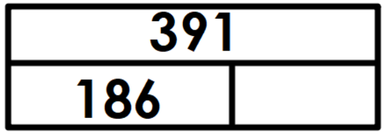
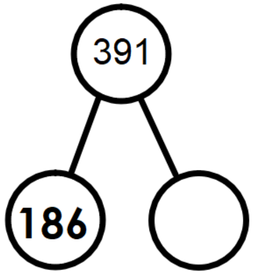
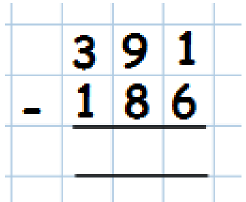
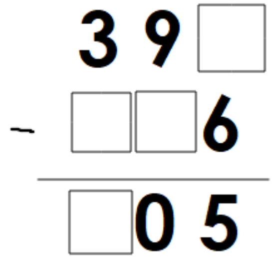


Abstract

Formal column method. Children should be able to explain what has happened where they have crossed out the digits to decompose.





Conceptual variation; different ways to ask children to solve 391 - 186			
 	Raj spent £391, Timmy spend £186. How much more did Raj spend? Calculate the difference between 391 and 186.	☐ = 391 – 186  What is 186 less than 391?	Missing digit calculations 

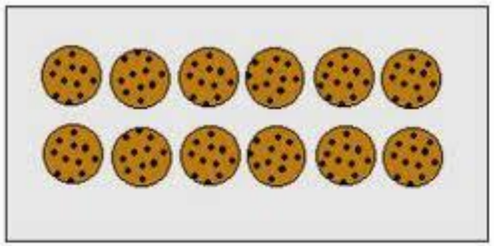
Multiplication:

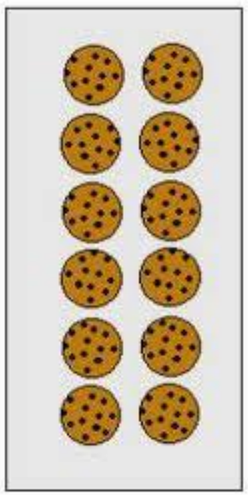
Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups

The use of the multiplication sign can cause difficulties. Mathematically 3 x 4 means 3 multiplied by 4 or three, 4 times (not 3 lots). When teachers are modelling the process of multiplication to children it is important that the vocabulary used matches the image being shown. Ultimately children should come to recognise that multiplication is commutative (3 x 4 = 4 x 3) so it can be done in any order.

6 cookies, 2 times

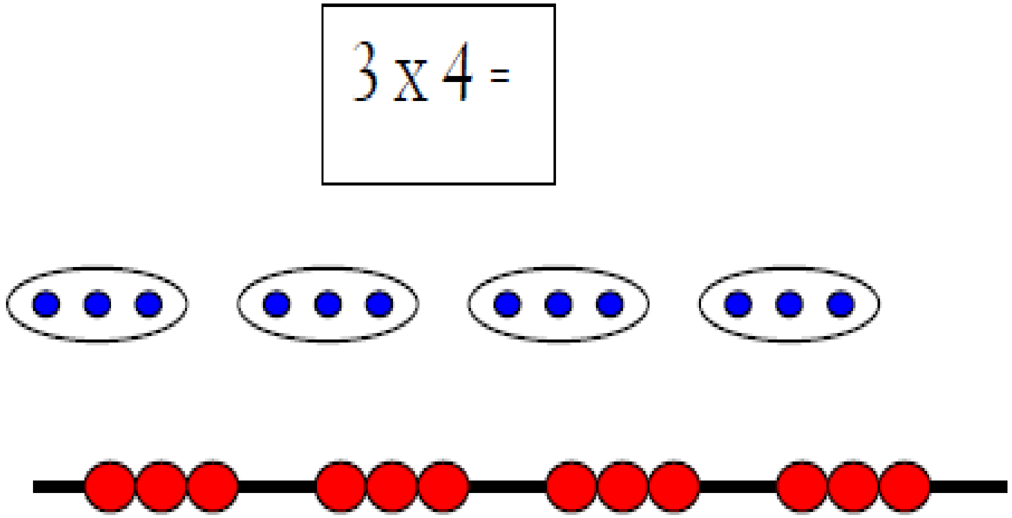
6 x 2 = 12





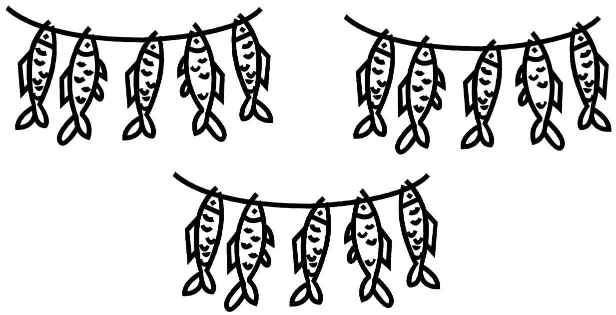
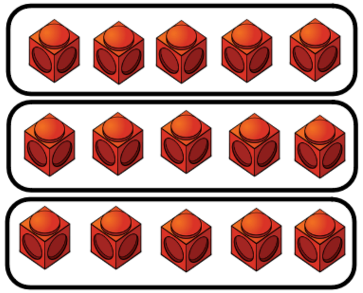
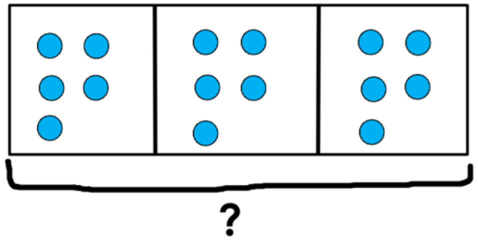
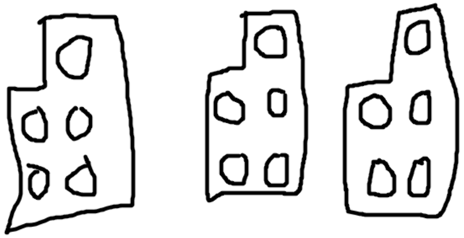
2 cookies, 6 times

2 x 6 = 12



Stage 1:

At stage 1, children are introduced to multiplication as groups of numbers. As with other calculations, practical equipment and images are imperative to providing a secure foundation for multiplication and children are not expected to use numerals or operation signs unless they are able to associate them with a value or operation.

Concrete	Pictorial	Abstract
Children will understand **counting in groups** practically **e.g. 5 x 3** **5 fish, 3 times**   There are 5 cubes, 3 times 5 x 3 = 15	Children to represent the practical resources in a **picture** and use a **bar model**.  	Children will use their understanding of equal groups to see the link between multiplication and **repeated addition**. 5 x 3 = 15 5 + 5 + 5 = 15

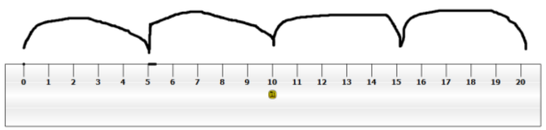
Stage 2:

Practical equipment and images are still incredibly important at this stage and children are now beginning to use signs and numerals to record their number sentences. Children will continue to explore the idea that multiplication is the same repeated addition and will use number lines to show repeated groups.

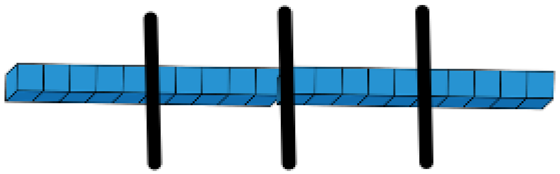
Concrete

Use number lines to show **repeated groups**.

This represents groups of 5 being multiplied 4 times.

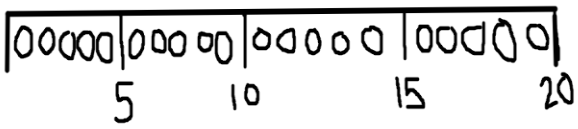


Rods can also be used to represent repeated addition, in this example showing groups of 5, multiplied 4 times.



Pictorial

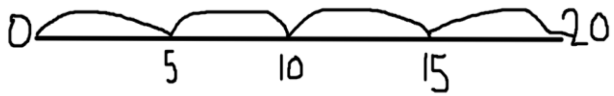
Children can represent this pictorially alongside a **number line** e.g.



Children can also **draw pictures** to represent repeated addition and multiplication.

Abstract

Children can use an abstract number line to show 5 being multiplied 4 times.

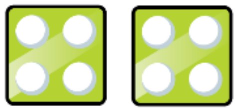
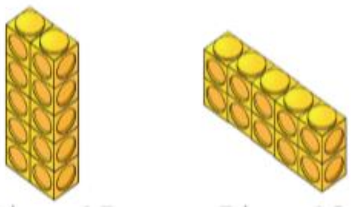
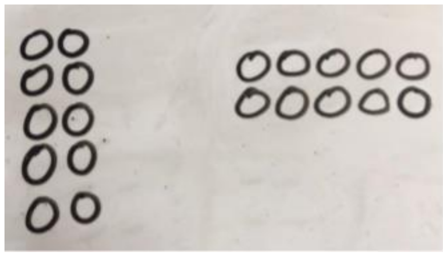


Stage 3:

By now children are beginning to know number facts off by heart and be developing a repertoire of strategies to support their mental calculations. By the end of Year 4, children are expected to know all of their times tables off by heart.

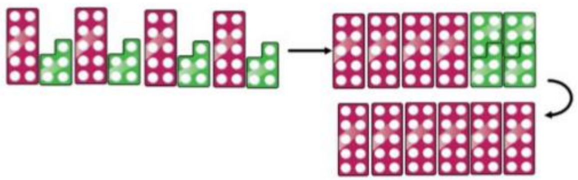
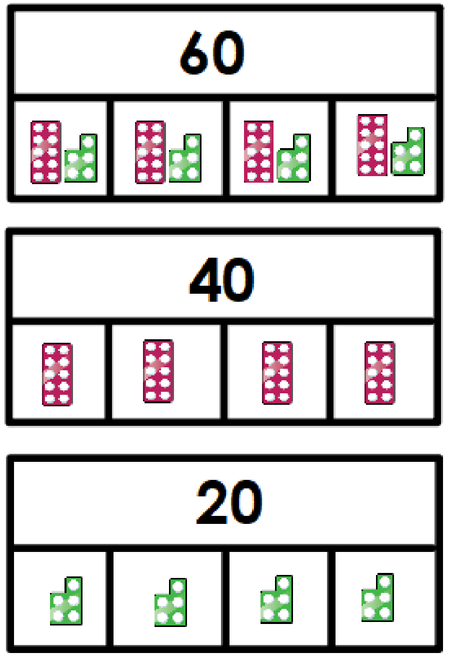
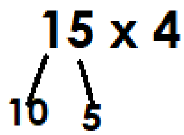
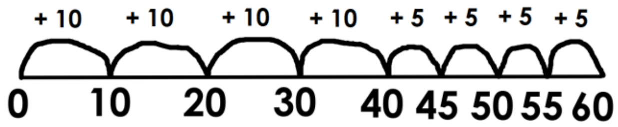
Children need to be able to understand and access the following before moving towards other stages of calculations.

- Know that doubling is the same as adding the same number twice
- Know doubles of numbers up to and including 10.
- Recognise repeated addition as multiplication
- Understand how to count in 2s, 3s, 5s and 10s and begin to relate this to multiplication

Concrete	Pictorial	Abstract
Children to demonstrate an understanding of **doubling** by using equipment. This shows understanding that doubling is the same as adding the number twice.  **Double 4 is the same as 4 + 4**	Children to use drawings to demonstrate doubling. 	Children are able to write these calculations to support their understanding of doubling. $4 + 4 = 8$ $4 \times 2 = 8$
Children to use **arrays** to illustrate commutivity, e.g. 2 x 5 = 5 x 2  2 multiplied by 5 **and** 5 multiplied by 2	Children to represent **arrays** pictorially, understanding that multiplication can be done both ways, it is commutative. 	Children to be able to use an **array** to write a range of calculations and see the **relationships** between those e.g. $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$

Stage 4:

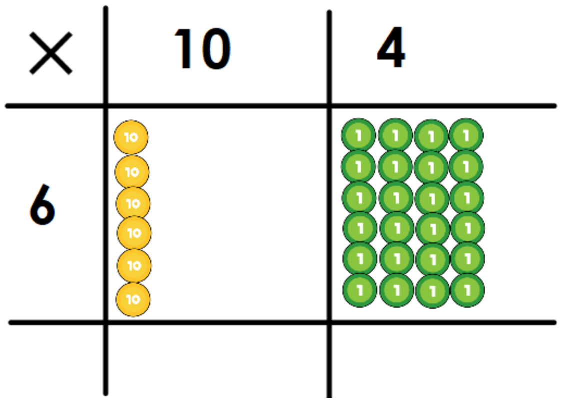
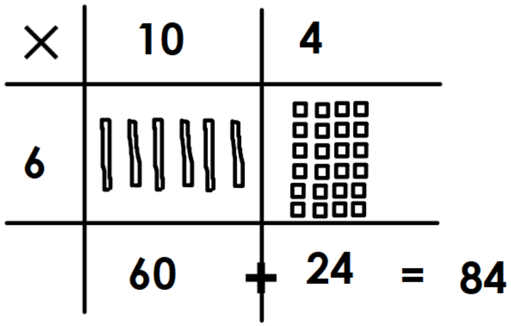
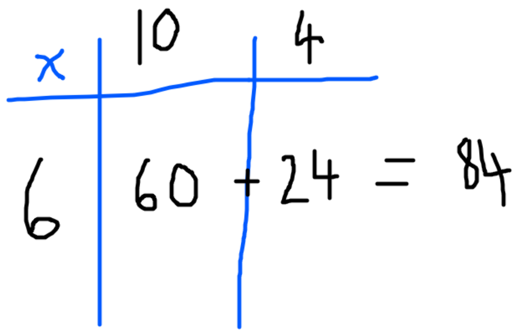
Practical equipment and images are still incredibly important at this stage and children are now beginning to use signs and numerals to record their number sentences. Children will continue to use arrays to support their understanding of multiplication and will begin to use their knowledge of place value and partitioning to support this further.

Concrete	Pictorial	Abstract
Partition to multiply using Numicon or base 10. 15 x 4 = 	Children to use **bar models** to represent **partitioning** into tens and ones to solve multiplication problem. 	Children to be encouraged to show the steps they have taken.  **10 x 4 = 40** **5 x 4 = 20** **20 + 40 = 60** A number line can also be used. 

Stage 5:

The National Curriculum (2014) sets out guidelines that children in KS2 onwards must use formal written methods for calculation. At this stage, it is still important that children use both practical and informal methods and recording in maths to support their understanding of formal written calculations. We will therefore still encourage children to use methods in line with their development.

In order to gain confidence working with larger multiplication calculations, children will be taught to use the grid method. The grid method uses knowledge of number facts and the idea of partitioning a number into its values to help children understand the process of multiplication.

Concrete	Pictorial	Abstract
Children to use place value counters to create the **grid method**. 	Children to represent place value counters or base 10 pictorially. 	Use grid method to multiply by **partitioning**. 

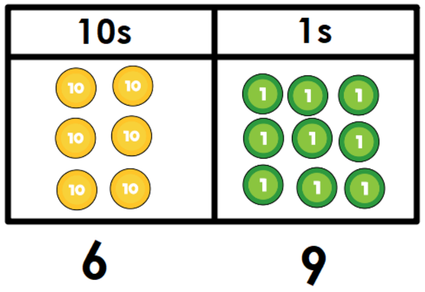
Stage 6:

To build on from stage 5 the children will continue to access their knowledge of grid method to help support their calculations. However, once this is clear, children will be taught 'shorter' methods to find answers. Children will learn the short column method but still be encouraged to choose the method which works best for them.

Concrete

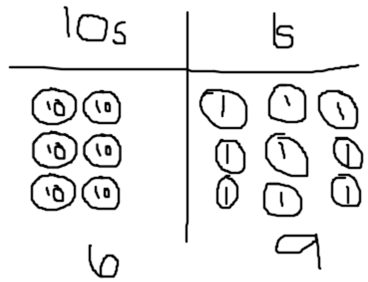
Formal column method with place value counters or base 10, first without exchange

23 x 3



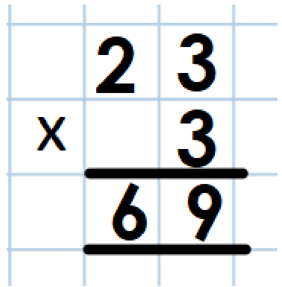
Pictorial

Children to represent pictorially, first without exchange



Abstract

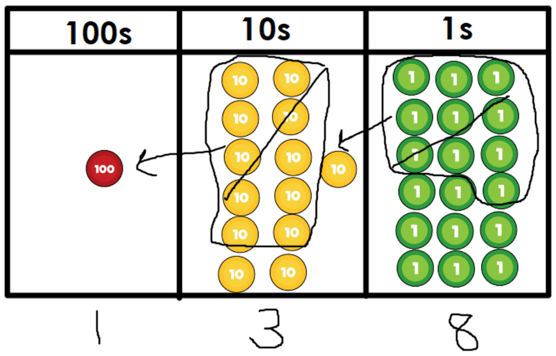
Formal written method, first without exchange



Concrete

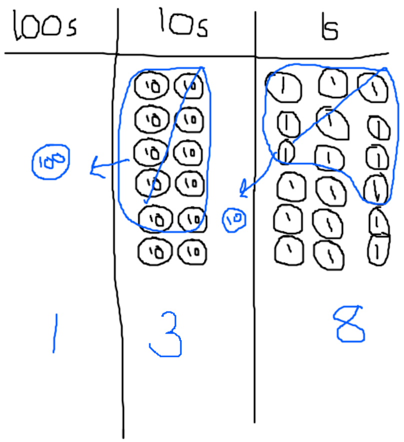
Formal column method with place value counters or base 10, next with exchange

23 x 6



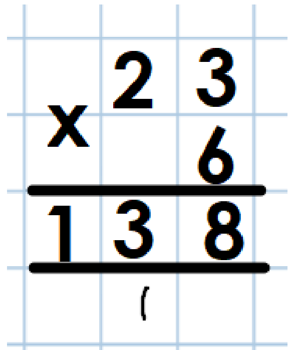
Pictorial

Children to represent pictorially, next with exchange. It is useful to do this alongside bar modelling to support understanding.



Abstract

Formal written method, next with exchange



342 x 7

Th	H	T	U
	3	4	2
x			7
2	3	9	4
	2	1	

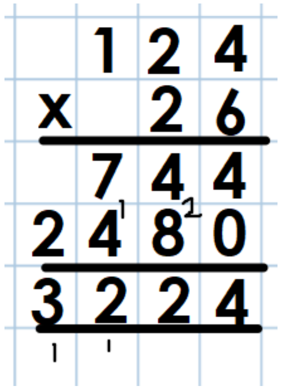
Stage 7:

In Years 5 and 6, children will be introduce to long multiplication.

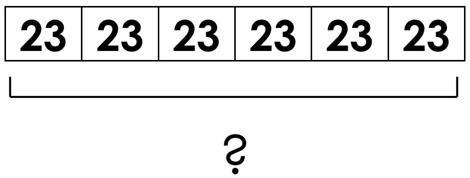
Long multiplication formal written method.

When children start to multiply 3d x 3d and 4d x 2d etc, they should be confident with the abstract:

To get 744 children have solve 6 x 124
To get 2480 they have solved 20 x 124



Conceptual variation; different ways to ask children to solve 23 x 6



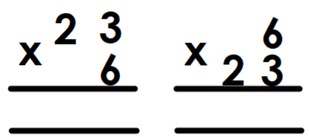
Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

With the counters, prove that 23 x 6 = 138.

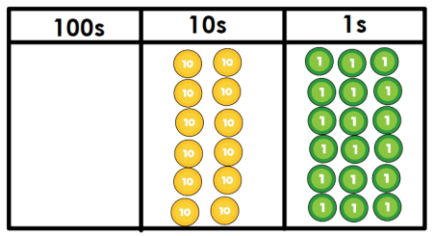
Find the product of 6 and 23

23 x 6 =

□ = 23 x 6



What is the calculation?
What is the product?

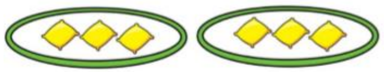
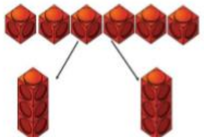
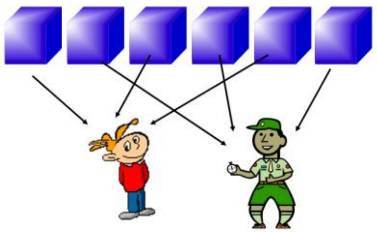
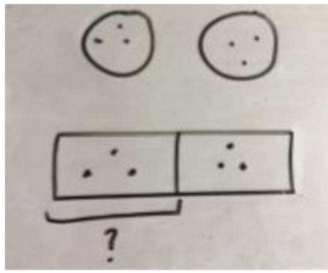
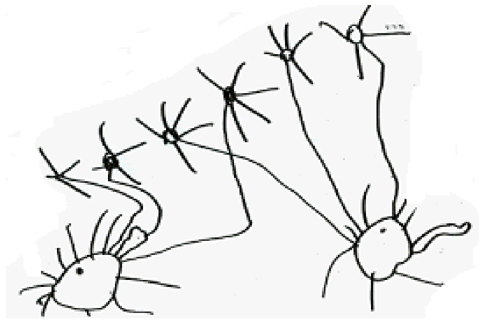


Division:

Key language: share, group, divide, divided by, half

Stage 1:

At stage 1, children are introduced to division as sharing. As with other calculations, practical equipment and images are imperative to providing a secure foundation for division and children are not expected to use numerals or operation signs unless they are able to associate them with a value or operation.

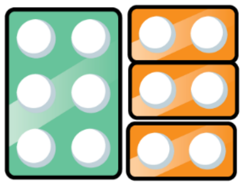
Concrete	Pictorial	Abstract
Children will physically **share** out a range of objects, one at a time, until all the objects have been shared **equally**.   	Children will represent sharing pictorially  	6 ÷ 2 = 3 | 3 | 3 | From Year 2, children should also be encouraged to use their 2 times table facts as and when they learn these.

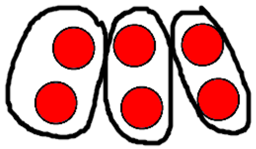
Stage 2:

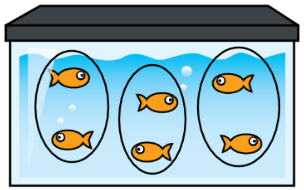
Practical equipment and images are still incredibly important at this stage and children may begin to use signs and numerals to record their number sentences. Children will continue as in stage 1 sharing between an amount. They may begin to understand that sharing between 2 people is the same as half.

Concrete

Children will physically **group** a range of objects.
6 ÷ 2 = 3





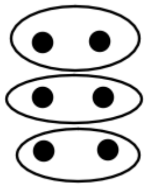


"There are 3 groups of 2 in 6."

Pictorial

Children will represent grouping pictorially.

6 ÷ 2 = 3



Abstract

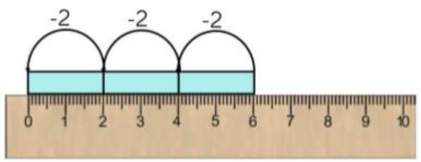
6 ÷ 2 = 3

2	2	2

Children should also be encouraged to use their **multiplication (times table) facts** as and when they learn these.

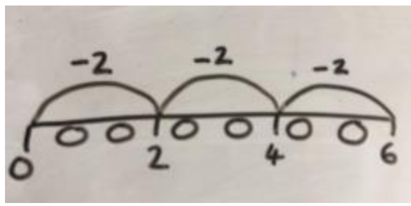
Concrete

Children will **repeatedly subtract**



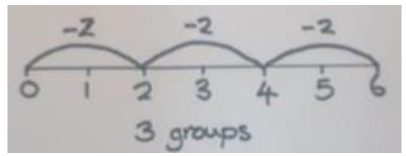
Pictorial

Represent **repeated subtraction** pictorially



Abstract

Abstract number line to represent the equal groups that have been **repeatedly subtracted**.

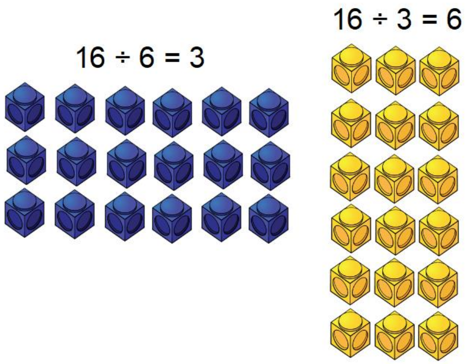
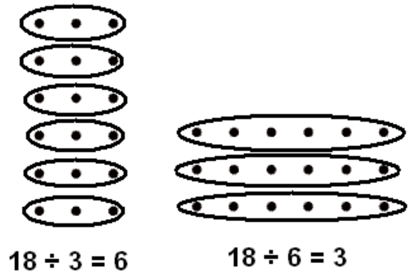


<u>Stage 3:</u>

At stage 3, children will be continuously accessing practical equipment and using visual drawings to support their calculations. They will be independently using accurate mathematical symbols to represent division calculations. Children will begin to associate division as the inverse of multiplication and use this knowledge to find answers.

By stage 3, children should be becoming more confident with their mental calculations. Children therefore need to be able to access the following before moving towards other stages of calculations. These again form the <u>foundations of fluency.</u>

- Understand division as sharing equally
- Know that halving is the same as sharing between 2.
- Understand that division is the inverse of multiplication
- Understand that division by 1 leaves a number unchanged.

Concrete	Pictorial	Abstract
To reinforce division and multiplication as **inverse** operations, children will be introduced to using **arrays** to support their division. 	Children will draw **arrays** to support their division. 	Children will be encouraged to check their results using their knowledge of **multiplication facts**. 6 x 3 = 18 3 x 6 = 18 18 ÷ 3 = 6 18 ÷ 6 = 3

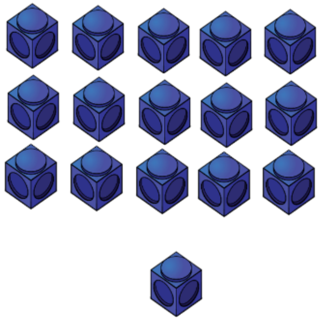
Stage 4:

The National Curriculum (2014) sets out guidelines that children in KS2 onwards must use formal written methods for calculation. At Goring CE Primary, the focus for division calculations in Year 3 will be using multiplication facts to support division before moving to more written methods in Year 4. We will still encourage children to use concrete and pictorial methods in line with their development. Children will now be introduced to the idea of 'remainders', although they may have come across this concept previously in their exploration.

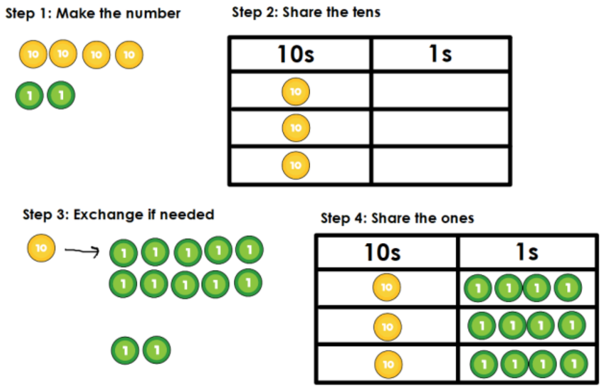
Concrete

2d ÷1d with **remainders**.

16 ÷ 5 = 3 with one left over



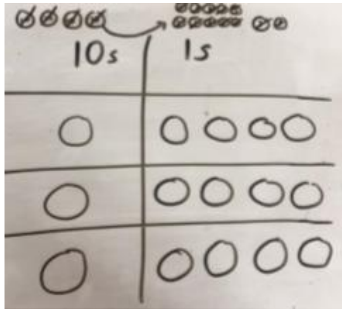
Sharing or grouping using place value counters.
E.g. 42 ÷ 3 = 14



Pictorial

16 ÷ 5 = 3 with 1 left over

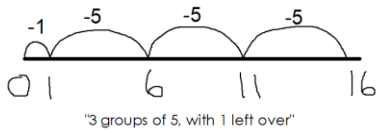
Children to represent place value counters pictorially.



Abstract

Children should be encouraged to use their times tables facts; they could also represent repeated subtraction on a numberline.

"I know that 3 x 5 = 15 and 5 x 3 = 15. 16 is one more than 15 so…. 16 divided by 5 would be 3 with one left over"



Children will be able to make sense of the place value counters and explain the process. E.g.

"42 ÷ 3
42 partitioned is 30 and 12
I know 30 divided by 3 is 10
I know 12 divided by 3 is 4
So 42 divided by 3 is 14."

"Chunking" using known times tables may also benefit some children:

96 ÷ 6 =

96	
- 60	(6 x 10)
36	
- 36	(6 x 6)
0	

Answer: 16

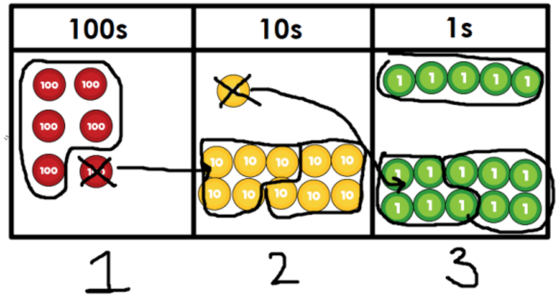
Stage 5:

From Year 4 and beyond, children will be introduced to more formal methods such as 'short division'.

Concrete

Short division using place value counters to group.
615 ÷ 5 =

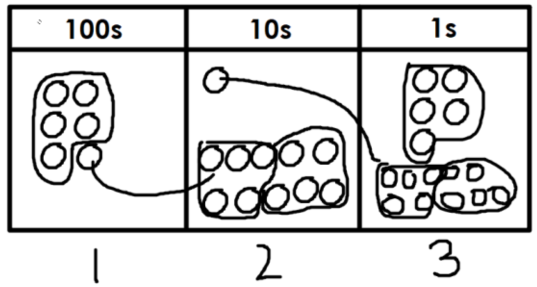
(Children will initially be introduced to short division without remainders)



1. Make 615 with place value counters
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

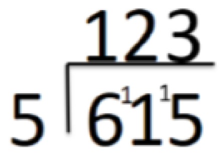
Pictorial

Represent the place value counters pictorially.



Abstract

Children to complete the calculation using the short division scaffold.



1. How many 500s are there in 600? (One with 1 hundred left over).
2. How many 50s in 110? (2, with one ten left over.)
3. How many 5s in 15? (3)

In Year 5 children will then **interpret any remainders for the context**.

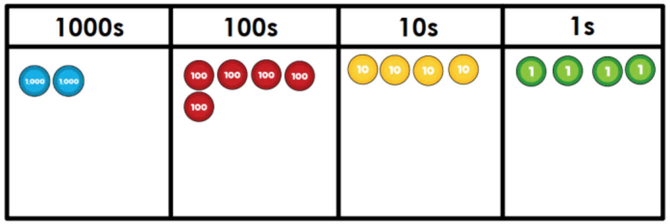
Year 6 will show a remainder as a **decimal or fraction.**

Stage 6:

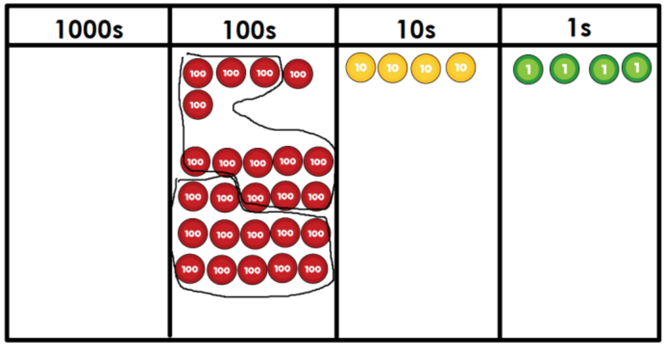
In Year 6, children will be introduced to the formal method of 'long division'.

Long division using place value counters

2544 ÷ 12 =

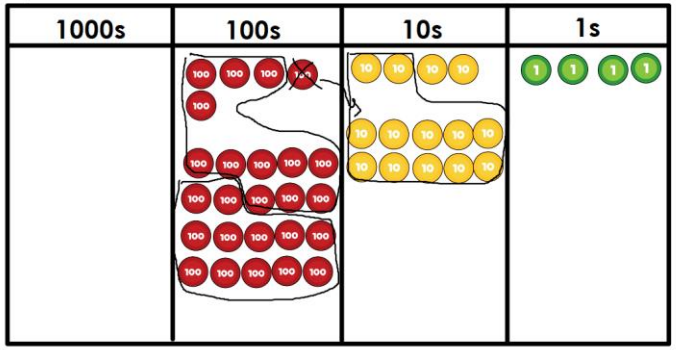


We can't group 2 thousands into groups of 12 so we will have to exchange them.



We can group 24 hundreds into groups of 12 which leaves us with 1 hundred.

```
      0 2
12 | 2²5 4 4
     2 4
       1
```



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

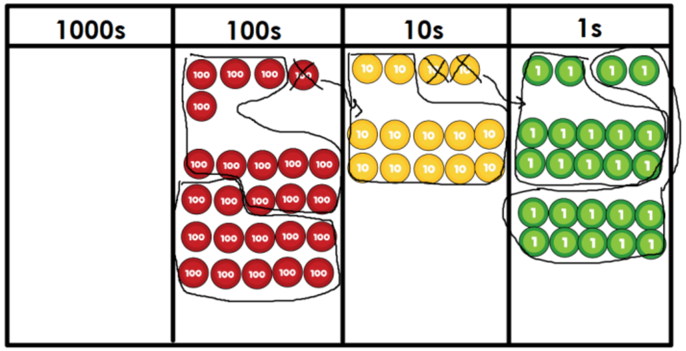
```
      0 2 1
12 | 2²5¹4 4
     2 4
       1 4
       1 2
         2
```

Abstract

Long division

432 ÷ 15 =

```
      H  T  U
         2  8 r12
15 |  4  3  2
      3  0  0
      1  3  2
      1  2  0
         1  2
```



After exchanging 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

```
     0 2 1 2
12 | 2 ²5 ¹4 ²4
     2 4
       1 4
       1 2
         2 4
         2 4
           0
```

Children will be encouraged to show a multiplication fact box to supports their calculations and ensure accuracy, e.g.

12
24
36
48
60
72
84 etc…

Long division **with decimals.**

$432 \div 15 = 28.5$

```
      H  T  U
         2  8 . 8
15 |  4  3  2
      3  0
      1  3  2
      1  2  0
         1  2 . 0
         1  2 . 0
               0
```

Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division?

I have £615 and share it equally between 5 bank accounts. How much will be in each bank account?

615 pupils need to be put into 5 groups. How many will be in each group?

$5\overline{)615}$

615 ÷ 5 =

☐ = 615 ÷ 5

What is the calculation?
What is the answer?