Grove Vale Primary School

Mental and Written Calculation Progression

Calculation Policy

Date: November 2022

Review: November 2024

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Grove Vale Primary School

Calculation Progression

Addition

Addition

EYFS

National Curriculum:

Pupils should be taught to:

- Know one more than a number
- Using quantities and objects, they add two single-digit numbers and count onto find the answer.

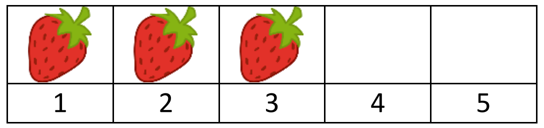
Adding One More Than a Number

Use of concrete objects, cubes, fingers and counters to find one more than any given number to 5, 10 and then 20 starting with the biggest number.

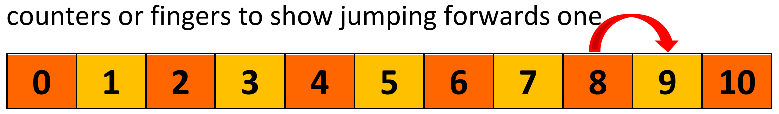
9 blocks + 1 more block equals 10 blocks

Use of pictorial representations to count one more than a number (using 5 frame).

Ask children to add one more, using a 5 frame and a number track underneath, pointing to the new number (part whole):



Use of a number track to count on or jump one more than a number. Use counters or fingers to show jumping forwards one

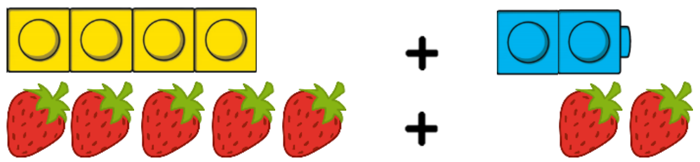


Begin to count on one from a number between 1 and 9 mentally.

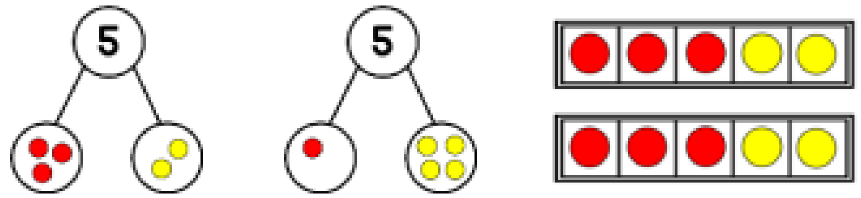
8….9 through simple songs and nursery rhymes.

Add Two Single-Digit Numbers

Use a range of concrete objects and pictorial representations (including drawing own pictures) counting on from one, progressing to counting on from the biggest number using number track. Re-enforced through real life role play, games, songs, playful practical indoor and outdoor math activities focusing on use of mathematical language like 'altogether'. Encourage discussion while solving practical problems to discuss ideas and processes



Number bonds to 5 progressing to number bonds to 10 pictorially (part whole) progressing to show this as an abstract calculation.



Year 1

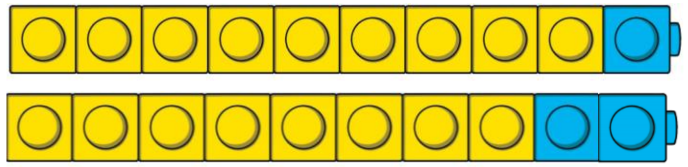
National Curriculum:

Pupils should be taught to:

- Read, write and interpret mathematical statements involving addition
- Represent and use all number bonds within 20
- Add one-digit and two-digit numbers to 20, including 0
- Solve one-step problems that involve addition using concrete objects and pictorial representations, and missing number problems

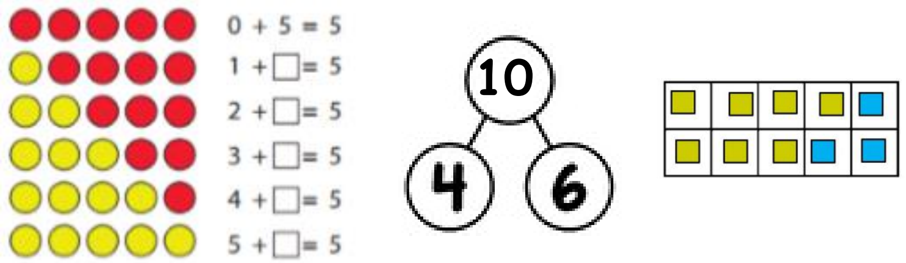
Number Bonds

Use of concrete objects (hands, washing line, cubes, counters etc) and pictorial representations to establish number bonds for 5, 10 progressing to 20, using part-part-whole, developing as a mental method.



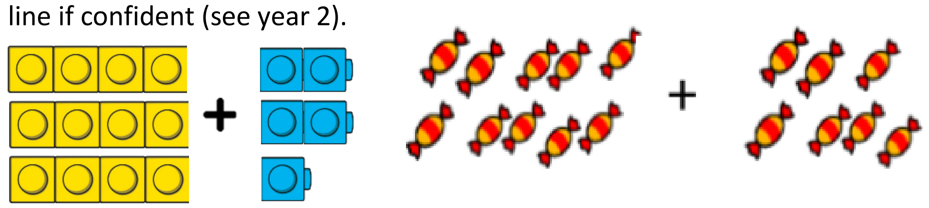
9 and 1 equal 10

8 and 2 equal 10



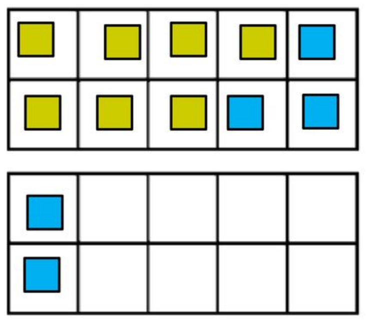
Adding One and Two-Digit Numbers (Up to 20)

Use a range of concrete objects (Using cubes to show part=part-whole, counters, real life objects) to add one and two-digits up to 20.. Then, progress this as a pictorial representation, starting with the biggest number and adding on the smaller number. Finally progress to showing this abstractly as a calculation or using number bond circles to show part-part-whole while beginning to count on mentally from the larger number. Progress to number line if confident (see year 2).



Children should then apply this skill to **missing number problems** by counting on from the first number. First using concrete objects (washing lines) then to pictorial (circle number on number line, count on) to abstract use of number bonds to partition when counting on to find the missing number.

Regrouping to Make 10



Use a 10 frame concretely then pictorially to regroup by partitioning the smaller number to make 10 and then counting on the left-over amount.

Start with the biggest number (7) then partition that number to make 10 (5 = 3 + 2). Then place the remaining amount.

Year 2

National Curriculum:

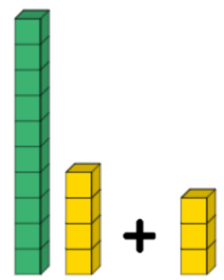
Pupils should be taught to:

- Solve problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures and applying their increasing knowledge of mental and written methods
- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and 1s; a two-digit number and 10s; 2 two-digit numbers and adding 3 one-digit numbers
- Show that addition of 2 numbers can be done in any order (commutative) and subtraction can't.
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

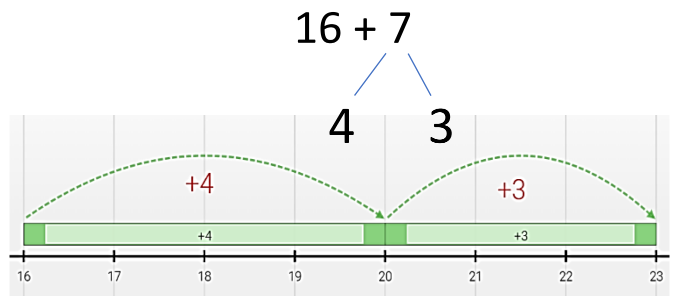
Adding a Two Digit and Ones

Use of concrete objects such as numicon, beads, or dienes etc to add on ones. Progress this to using part-part-whole concept to enable children to use knowledge of number binds to partition to support adding. Mentally secure number facts to 20/100

Concrete (14 + 3)



Abstract (16 + 7)

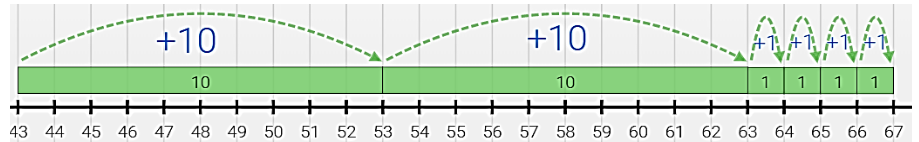


Adding Ten to a Number

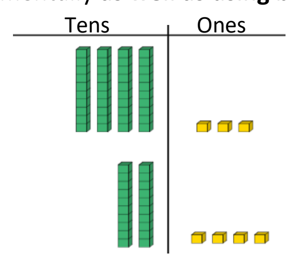
Start showing this concretely using dienes and a hundred square to jump a multiple of ten. Progress this to showing pictorially using dienes how adding a ten changes the place value. Encourage children to notice the pattern that the tens column increases by one each time.

Adding Two Two-Digit Numbers

Use of **number line** initially to add tens individually and then ones: (43 + 24)



When children are confident, move to groups of jumps i.e. +20 rather than +10 +10. Progress to **partitioning** numbers into tens and ones and adding them individually using concrete and pictorial representations moving onto doing this abstractly and mentally **as well as using bar model to show the inverse to check answers.**



43 + 24

40 3 20 4

Add tens and ones in column format

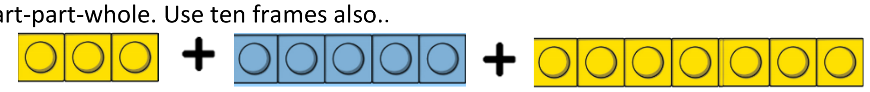
40 + 3

20 + 4

60 + 3 = 63

Adding Three Single Digit Numbers

Make sure two of the numbers add to ten. (3 + 5 + 7) Then encourage children to find the number bonds to ten first. (3 + 7 = 10) Then, make a new calculation adding the remaining number (10 + 5 = 15). Use blocks concretely to show this in relation to part-part-whole. Use ten frames also..



Addition

Year 3

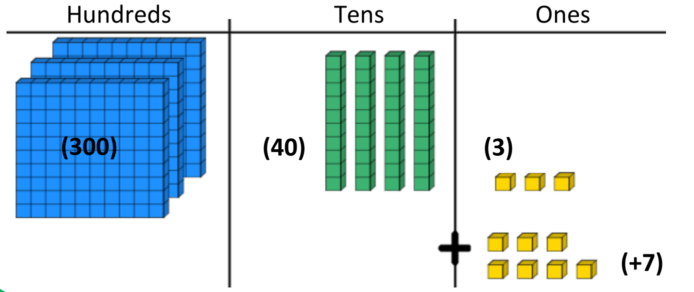
National Curriculum:

Pupils should be taught to:

- Add numbers mentally, including: a three-digit number and 1s, a three-digit number and 10s, a three-digit number and 100s
- Add numbers with up to 3 digits, using formal written methods of columnar addition

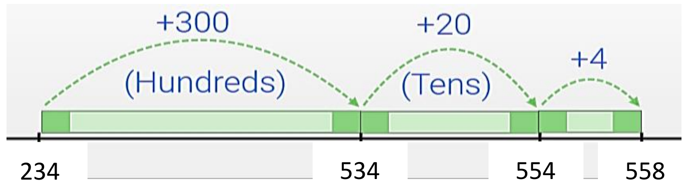
Adding Mentally

Use place value knowledge when adding ones, tens or hundreds to a three-digit number, initially using dienes concretely and pictorially to show that you only need to increase the column that is being added to. (Example: 343+7)



Adding Three-Digit Numbers

Then, use the **number line** to add three-digit numbers, in place value steps) using concrete and pictorial representations to assist understanding, using place value vocabulary to reinforce.



Further develop **partitioning** strategy with calculations beyond a 100, partitioning and adding the groups of hundreds, tens and ones using columns where possible.

234 + 324

200 + 30 + 4 + 300 + 20 + 4

200 + 300 = 500 *(Add hundreds)*
30 + 20 = 50 *(Add tens)*
4 + 4 = 8 *(Add ones)*

Follow this strategy with the **expanded column method**, adding ones, tens and hundreds individually, then adding those totals up to get the final answer. Develop this to bridge a barrier in preparation for the **formal written method** using relevant vocabulary to reinforce.

```
   234
+  422
     8   Add Ones (4+3)
+   50   Add Tens (30+50)
   600   Add Hundreds (200+300)
   658   Then, add all the groups for the final answer
```

ATTEMPT ADDING DECIMALS MAKING IT EXPLICIT THAT IT IS TENTHS AND THE DECIMAL PLACE STAYS ROOTED.

If children are confident and ready to move on, they can begin to introduce the formal written method (see year 4).

Year 4

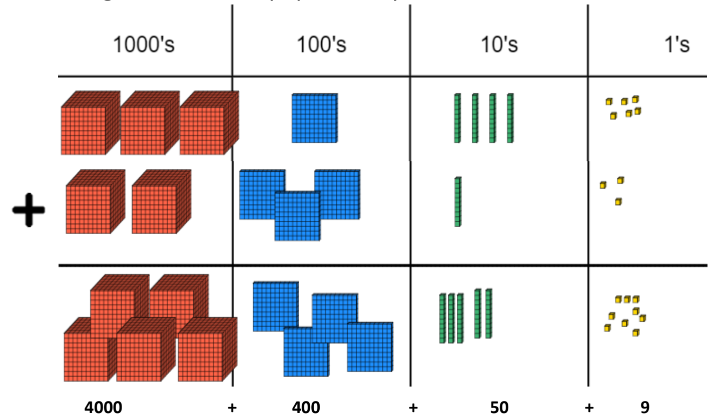
National Curriculum:

Pupils should be taught to:

- Add numbers with up to 4 digits using the formal written methods of columnar addition

Adding Four-Digit Numbers (no re-grouping)

Following recapping of **expanded method**, teach this method for all **without re-grouping** to ensure understanding, initially teaching this concretely/ pictorially with the use of dienes.



Then, show abstractly, adding each column and placing the answer directly underneath in that place value column. Ensure use of mathematical vocabulary i.e altogether there are 4 hundreds.

```
  7436
+ 2162
  9588
```

Adding Four-Digit Numbers (with re-grouping)

Importantly, model the process of re-grouping with a large place value display and concrete or pictorial dienes to show how the number bridge from one column to the other. I.e. 10 ones make a ten, 10 tens make a hundred so we have to re-group them into the right column. Start with re-grouping one column and as children become more confident, progress to more complex re-grouping, in particular if there are zero's involved and how re-grouping works then.

```
 7349     7349     7349     7349
+6785    +6785    +6785    +6785
    4       34      134    14134
   1       11      111      111
```

Column Addition with Decimals

Include real life math elements here, using money and measurements where possible as the context. When adding decimals make it explicit that it is tenths and the decimal place stays rooted. When re-grouping, make it clear 10 tenths make a one using dienes to show this concretely/ pictorially.

```
 73.49    73.49    73.49    73.49
+67.85   +67.85   +67.85   +67.85
    .4      .34    13.4    141.34
   1       11      111      111
```

Year 5

National Curriculum:

Pupils should be taught to:

- Add whole numbers with more than 4 digits, including using formal written methods (columnar addition)
- Continue to revisit mental strategies of years 3 and 4
- Continue to solve missing number problems

Adding More Than Four Digit Numbers

```
 85683    85683    85683    85683    85683
+45978   +45978   +45978   +45978   +45978
     1       61      661     1661   131661
    1       11      111     1111     1111
```

Children should use the column method with re-grouping to add two numbers in the tens and hundreds of thousands (five and six-digit numbers). Continue to use concrete/ pictorial methods to reinforce re-grouping.

Children should also use the column method to more than two values.

```
  48216
+ 37452
  11367
  97035
  1 1 1 1
```

Column Addition with Decimals

Childrens place value knowledge is key here when adding decimals, recognising they are adding tenths and hundredths and understanding this is a part of a whole.

So, 21.01 + 210.6 = becomes:

```
  210.60
+  21.01
  231.61
```

```
  210.60
+  21.01
  231.61
```

It is also important to add then, that 210.6 is actually 210.60 when putting it in to the column method and zero is added as a place value. (Again, it is important children understand that six tenths is equal to sixty hundredths.

Children should be familiar in using the column method to add more than two decimal values while also applying in real life maths like time, money and measurements.

44.1cm + 22.25cm + 2.05cm

```
  44.10cm
  22.25cm
+  2.05cm
  68.40cm
```

34.2ml + 12.65ml + 0.2ml

```
  34.20ml
  12.65ml
+  0.20ml
  47.05
   1
```

£19.01 + £3.65 + 70p

```
  £19.01
  £ 3.65
+   0.70p
  £23.36
   1 1
```

Year 6

National Curriculum:

In year six children continue to practise column method for addition for bigger numbers and decimal numbers up to three decimal places

Addition of Numbers to One Million

Children should be confident in adding several numbers together with an increasing level of complexity. The numbers should be a combination of thousands, tens of thousands and hundreds of thousands. **(Children should have a secure understanding of the place value involved)**

```
   81059       423721
    3668        47890
   15301        31133
+  20551     + 413214
  120579       915958
   11 11        11 11
```

Children should be efficient in their use of vocabulary to explain their process when calculating.

Column Addition with Decimals

Children should be taught to add decimals of a range of values up to 3 decimal places. Embedded in a range of real-life contexts (money, time, measurements etc see year 5), using their place value knowledge ensure the digits and decimal point are correctly aligned. Children use the column method to add several numbers with different numbers of decimal places

```
   5.400
   3.060
+ 12.421
   9.900
  30.781
```

Zero **(0)** should be used as a place holder to ensure that the numbers are to the same decimal place. Zero is added to show there is no value to add. **(Children should have a sound understanding of the values and how many tenths, hundredths and thousandths make up a whole one)**

Useful Pictorial Resources:

Dienes Generator
(used to make the illustrations in this policy)
https://mathsbot.com/manipulatives/blocks

Number Line Generator
(used to make the illustrations in this policy)
https://apps.mathlearningcenter.org/number-line/

Maths Frame Online Interactive Addition Games
https://mathsframe.co.uk/en/resources/category/9/addition-and-subtraction

White Rose Reasoning and Problem Solving
https://whiterosemaths.com/resources/schemes-of-learning/primary-sols/

Grove Vale Primary School

Calculation Progression

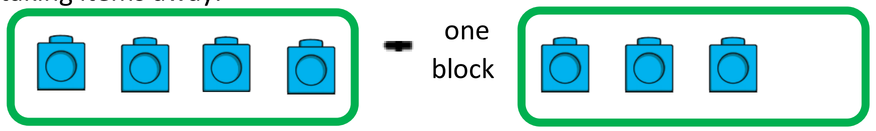
Subtraction

Subtraction

EYFS

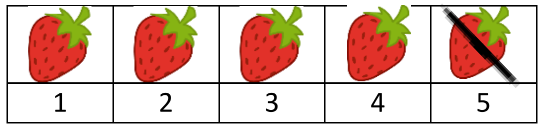
National Curriculum:
Pupils should be able to:
- Know one less than a number
- Using quantities and objects, they subtract two single-digit numbers and count back to find the answer

Finding One Less Than a Number
Use of concrete resources (everyday objects, cubes, beads, counters, toys etc) to show that the quantity of a group (5, 10 and then 20) can be changed by taking items away.

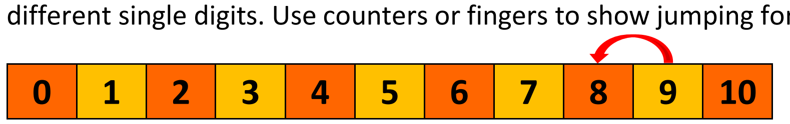


4 blocks take away 1 block leaves 3 blocks. Discuss it is one less.
Also, use pictorial representations to cross objects off to show one less.

Ask children to cross one out, using a 5 frame and a number track underneath, pointing to the new number (part whole):



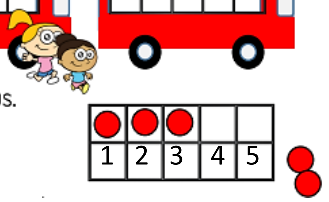
Use of a number track to count or jump backwards one number from different single digits. Use counters or fingers to show jumping forwards one.

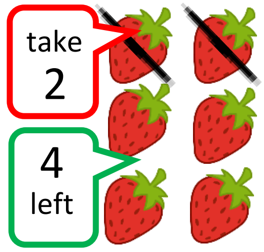


Begin to count on one backwards from a number between 1 and 9 mentally. 9....8 through simple songs, actions and nursery rhymes.

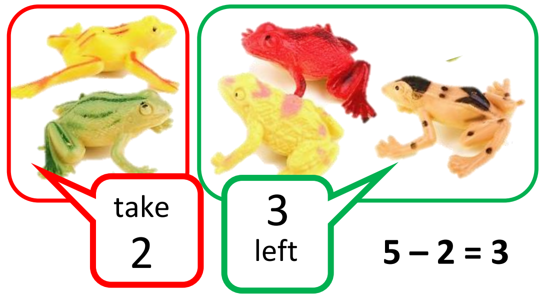
Subtracting Two Single Digit Number
Use a range of concrete objects and pictorial representations (including drawing own pictures) counting back from 5, progressing to counting on from the bigger numbers using number track. Re-enforced through real life role play, games, songs, stories, role play and playful practical indoor and outdoor math activities focusing on use of mathematical language like take away.

First there were 5 people on the bus.
Then 2 people got off the bus.
Now there are 3 people on the bus.

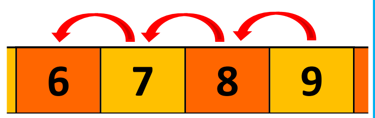




6 – 2 = 4



5 – 2 = 3

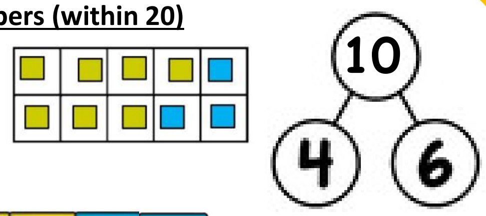


9 – 3 = 6

Year 1

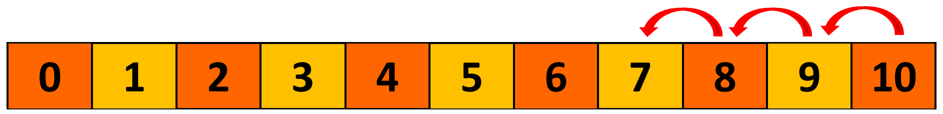
National Curriculum:
Pupils should be able to:
- Read, write and interpret mathematical statements involving subtraction
- Represent and use all number bonds within 20
- Subtract one-digit and two-digit within 20, including 0
- Solve one-step problems that involve subtraction using concrete objects and pictorial representations, and missing number problems

Subtracting One and Two-Digit Numbers (within 20)
Use of concrete objects, in particular, cubes to represent bar model and part-part whole model – making links with number bonds.



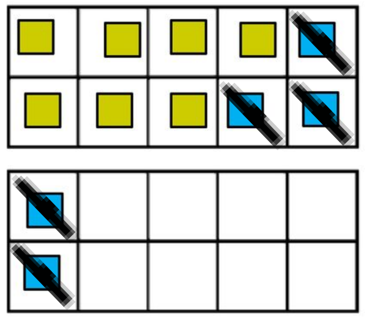
10 – 2 = 8

Use a number track to show this by counting back from the largest number from 10 progressing to showing this on a number from 20. (See year 2)



Once subtracting two-digits within 20, move to solving missing number problems using a number line practically to count backwards.

Regrouping to Make 10



Use a 10 frame concretely then pictorially to regroup by subtracting to make ten first and then subtracting the left over amount.

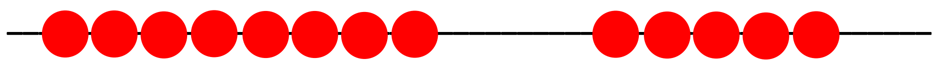
12 – 5 =

12 – 2 = 10

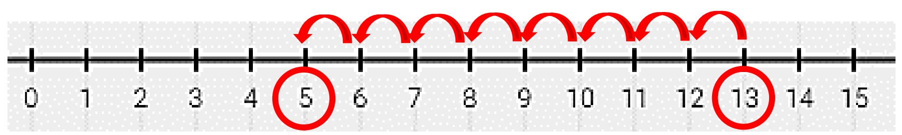
10 – 3 = 7

Missing Number Problems
Following on from children using concrete objects to count back, do this with missing number problems by starting with the biggest number and then counting back.

13 – 8 = 5 **8 missing beads** **Count back until 5**



Children can then use the support of a number line to support counting back to find missing numbers, as well as trying this mentally counting back if secure.



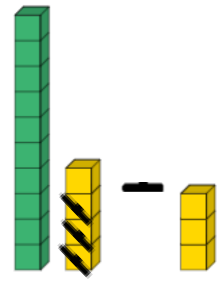
Count back to 5 (8 jumps)

Year 2

National Curriculum:
Pupils should be able to:
- Solve problems with subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures Applying their increasing knowledge of mental and written methods
- Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and 1s, a two-digit number and 10s, 2 two-digit numbers
- Show that subtraction is not commutative as addition is
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems

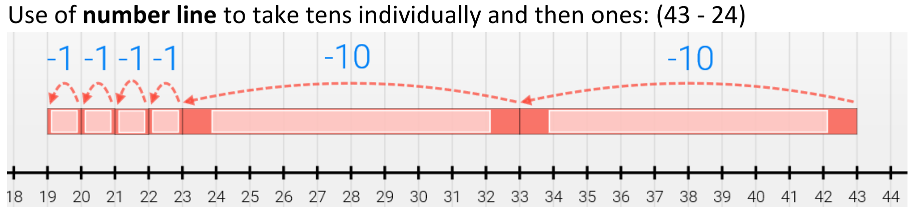
Subtracting a Two Digit Number and Ones
Use of concrete objects such as numicon, beads, or dienes etc to take away ones. Progress this to using part-part-whole concept to enable children to use knowledge of number bonds to partition to support subtracting on a number line in chunks. **Also, become secure in number facts of 20 and 100 to aid subtraction.**

Concrete (14 - 3) Abstract (14 - 7)

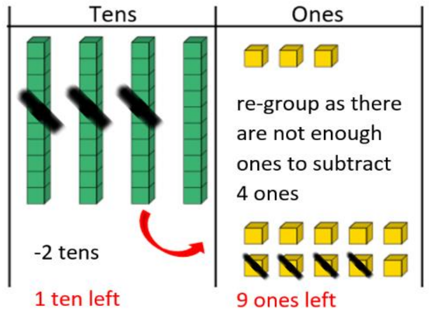


Subtracting 10 from a Number
Start showing this concretely using dienes and a hundred square to jump backwards a multiple of ten. Progress this to showing pictorially using dienes how subtracting a ten changes the place value. Encourage children to notice the pattern that the tens column decreases by one each time a multiple of ten is taken.

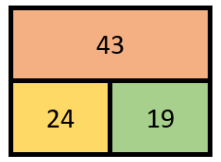
Subtracting Two Two-Digit Numbers
Use of **number line** to take tens individually and then ones: (43 - 24)



When children are confident, move to groups of jumps i.e. -20 rather than -10 -10.
Progress to **partitioning** numbers into tens and ones and subtract tens then ones, re-grouping if not enough ones using concrete and pictorial representations (see below) moving onto doing this abstractly and mentally **as well as using bar model to show the inverse to check answers combined with addition.**



Inverse Operation Use bar model/ part-part-whole to demonstrate the inverse relationship between addition and subtraction to then further help with problems i.e. missing number.



43 – 24 = 19
19 + 24 = 43
43 – 19 = 24
24 + 19 = 43

Subtraction

Year 3

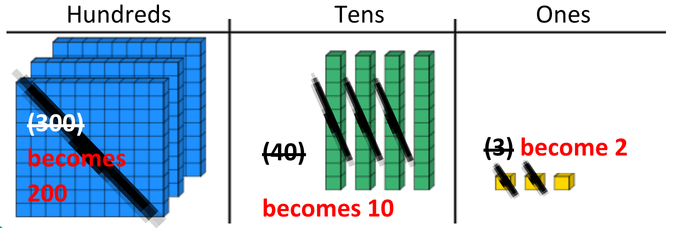
National Curriculum:

Pupils should be taught to:

- Subtract numbers mentally, including: a three-digit number and 1s, a three-digit number and 10s, a three-digit number and 100s
- Subtract numbers with up to 3 digits, using formal written methods of columnar subtraction
- Estimate, Inverse and solve problems involving subtract

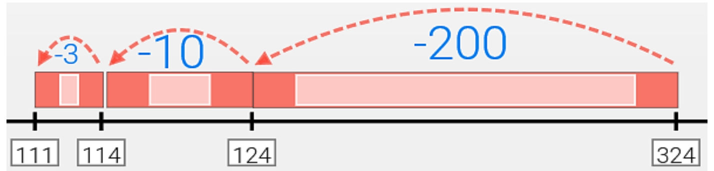
Subtracting Mentally

Use place value knowledge when adding ones, tens or hundreds to a three-digit number, initially using dienes concretely and pictorially to show that you only need to increase the column that is being added to. **(E.g. 343 - 131)**



Subtract 3 Digit Numbers

Then, use the **number line** to subtract three-digit numbers, in place value steps) using concrete (like above) and pictorial representations to assist understanding, using place value vocabulary to reinforce. **E.g. 324 - 213**



Apply this when solving problems and use bar model/ part-part whole to re-enforce inverse relationship. (See year 2)

324 - 213

300 + 20 + 4 - 200 + 10 + 3

300 – 200 = 100 *(Subtract hundreds)*
20 – 10 = 10 *(Subtract tens)*
4 – 3 = 1 *(Subtract ones)*

Further develop **partitioning** strategy with calculations, partitioning and subtracting the groups of hundreds, tens and ones using columns where possible.

Follow this strategy with the **expanded column method**, subtracting ones, tens and hundreds individually, then subtracting those totals up to get the final answer. Develop this to bridge a barrier in preparation for the **formal written method** using relevant vocabulary to reinforce. Use dienes or other concrete resources to reinforce. Move on to doing this with re-grouping (see year 4)

300 – 200 = 100 ***(Subtract hundreds)***
20 – 10 = 10 ***(Subtract tens)***
4 – 3 = 1 ***(Subtract ones)***
Then add what is left 100 + 10 + 1 = 111

ATTEMPT SUBTRACTING DECIMALS MAKING IT EXPLICIT THAT IT IS TENTHS AND THE DECIMAL PLACE STAYS ROOTED.

If children are confident and ready to move on, they can begin to introduce the formal written method (see year 4).

Year 4

National Curriculum:

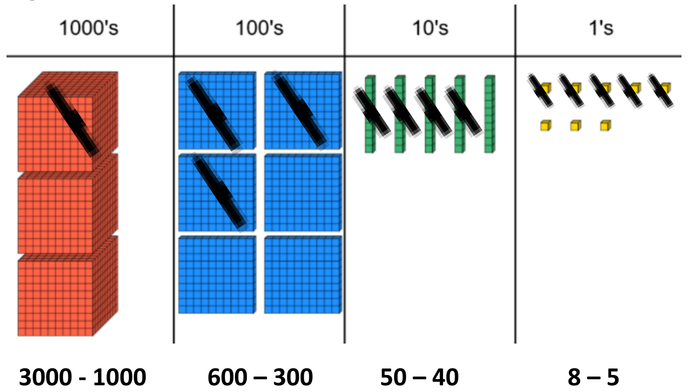
Pupils should be taught to:

- Subtract numbers with up to 4 digits, using formal written methods of columnar subtraction

Subtracting Four Digit Numbers (without re-grouping)

Following recapping of **expanded method**, teach this method for all **without re-grouping** to ensure understanding, initially teaching this concretely/ pictorially with the use of dienes.

E.g. 3658 – 1345



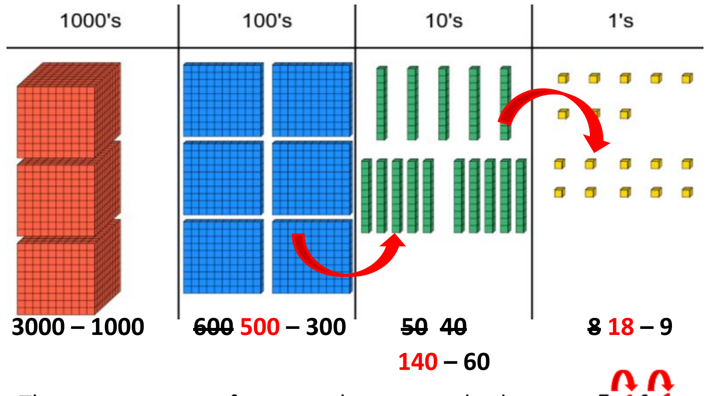
Then, show abstractly, subtracting each column and placing the answer directly underneath in that place value column. Ensure use of mathematical vocabulary i.e take away/ difference.

```
  3658
- 1345
  2313
```

Subtract Four-Digit Numbers (with re-grouping)

Importantly, model the process of re-grouping with a large place value display and heavy use of concrete or pictorial dienes to show if there isn't enough in that column to subtract then they re-group so they can do the calculation.

E.g. 3658 – 1369



Then, once secure, focus on abstract method: Expose children to a range of problems to solve, reason and apply calculation knowledge for both – and +

```
   5 14 1
  3 6 5 8
- 1 3 6 9
  2 2 8 9
```

Subtracting Decimals Include real life math (money and measurements) as the context. When adding decimals make it explicit that it is tenths and the decimal place stays rooted. When re-grouping, make it clear 10 tenths make a one using dienes to show this concretely/ pictorially. **E.g. 365.8 – 136.9**

```
   5 14 1
  3 6 5.8
- 1 3 6.9
  2 2 8.9
```

Year 5

National Curriculum:

Pupils should be taught to:

- Subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition)
- Continue to revisit mental strategies of years 3 and 4
- Continue to solve missing number problems

Subtracting More Than Four Digit Numbers

Children should use the column method with re-grouping to add two numbers in the tens and hundreds of thousands (five and six-digit numbers). Continue to use concrete/ pictorial methods to reinforce re-grouping as it becomes more challenging (especially if there are 0's or multiple re-groupings needed). Children should also use the column method to subtract more than two values.

```
      5 14 1
  3 3 6 5 8
-   4 1 3 6 9
    9 2 2 8 9
```

Column Addition with Decimals

Childrens place value knowledge is key here when adding decimals, recognising they are adding tenths and hundredths and understanding this is a part of a whole.

So, 658.8 – 369.02 = becomes:

```
  5 14 1
  6 5 8 . 8
- 3 6 9 . 0 2
  2 8 9 . 8 2
```

It is also important to add then, that 658.8 is actually 658.80 when putting it in to the column method and zero is added as a place value. (Again, it is important children understand that eight tenths is equal to eighty hundredths.

```
  5 14 1
  6 5 8 . 8 0
- 3 6 9 . 0 2
  2 8 9 . 8 2
```

Children should be familiar in using the column method to add more than two decimal values while also applying in real life maths like time, money and measurements.

Year 6

National Curriculum:

In year six children continue to practise column method for subtraction for bigger numbers and decimal numbers up to three decimal places

Subtraction of Larger Numbers

Children should be confident in subtracting several numbers with an increasing level of complexity. The numbers should be a combination of thousands, tens of thousands and hundreds of thousands. **(Children should have a secure understanding of the place value involved)**

```
  3 1     5 14 1
  4 3 3 6 5 8
- 2 4 1 3 6 9
  1 9 2 2 8 9
```

Children should be efficient in their use of vocabulary to explain their process when calculating.

Column Subtraction with Decimals

Children should be taught to subtract decimals of a range of values up to 3 decimal places. Embedded in a range of real-life contexts (money, time, measurements etc), using their place value knowledge ensure the digits and decimal point are correctly aligned. Children use the column method to subtract several numbers with different numbers of decimal places

```
    5 14 1
  3 6 5 8 . 8 0
- 1 3 6 9 . 0 2
  2 2 8 9 . 8 2
```

Zero (0) should be used as a place holder to ensure that the numbers are to the same decimal place. Zero is added to show there is no value to subtract. **(Children should have a sound understanding of the values and how many tenths, hundredths and thousandths make up a whole one)**

Useful Pictorial Resources:

Dienes Generator
(used to make the illustrations in this policy)
https://mathsbot.com/manipulatives/blocks

Number Line Generator
(used to make the illustrations in this policy)
https://apps.mathlearningcenter.org/number-line/

Maths Frame Online Interactive Subtraction Games
https://mathsframe.co.uk/en/resources/category/9/addition-and-subtraction

White Rose Reasoning and Problem Solving
https://whiterosemaths.com/resources/schemes-of-learning/primary-sols/

Grove Vale Primary School

Calculation Progression

Multiplication

Multiplication

EYFS

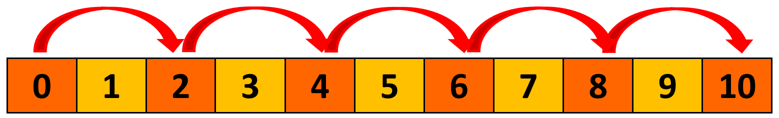
National Curriculum:

Pupils should be able to:

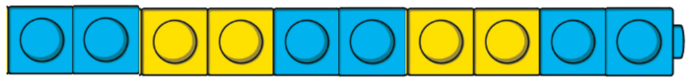
- Can solve problems involving doubling

Counting On

Provide opportunities for children to count beginning with emphasising on multiples of two progressing to counting in twos and tens using a range of concrete resources embedded in a real life context (indoors and outdoors).

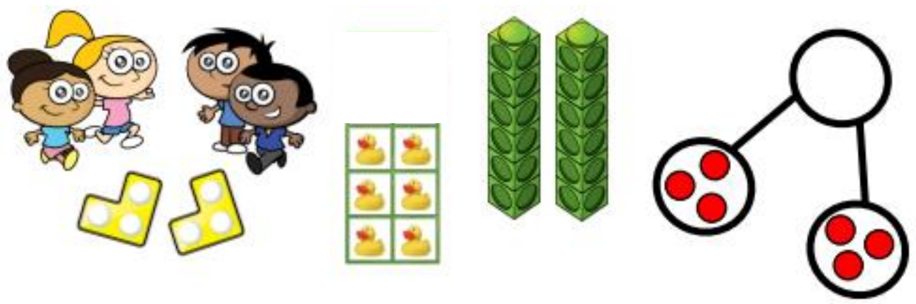


Progress to counting on sets of two objects.



Doubling

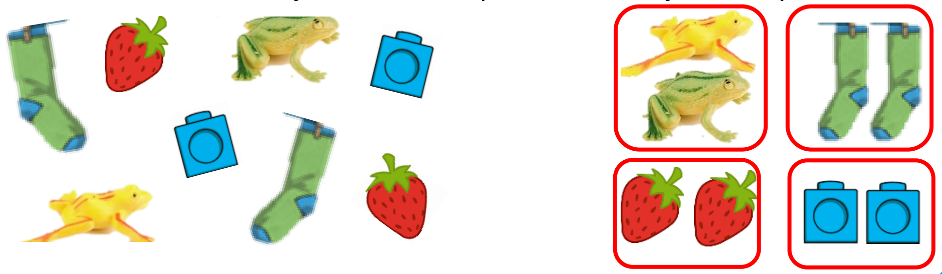
Children should learn that double means twice as many, having access to an array of real-life objects and mathematical equipment to build doubles progressing to numbers up to ten. Children should have access to mirrors to double the items they build understanding each side is equal or the same.



Making Equal Groups

Children will have access to a range of concrete and pictorial versions of real life objects to begin to make sets of objects by counting the objects in a set and making more sets. For example, this could be the pairing of socks. How many groups of two have you made?

Match the same two objects. How many sets of two objects can you make?



Year 1

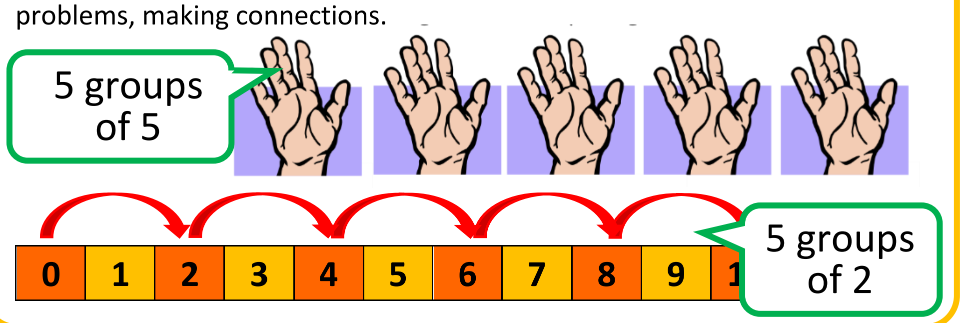
National Curriculum:

Pupils should be able to:

- Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
- Recall doubles and count on/ back in multiples of 1, 2, 5 and 10

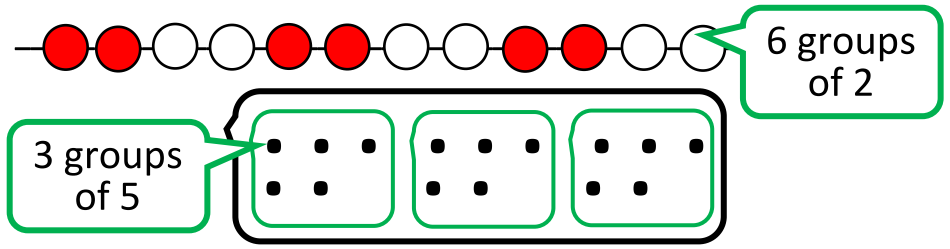
Counting in Multiples

Children should use a range of concrete and pictorial representations of every day/ real life object, used to count how many groups of 2, 5 or 10 there are. This should progress to children being able to count on independently in 1, 2, 5 and 10 while understanding multiplication link and applying to simple problems, making connections.



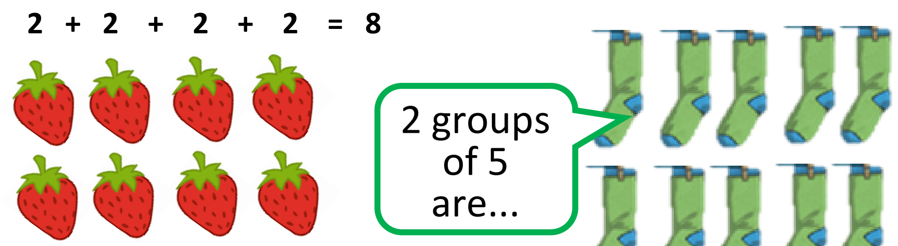
Grouping/ Repeated Addition

Use concrete resources/ everyday objects as well as pictorial representation on whiteboards to show the visuals of making groups/multiples of 2, 5 or 10 while solving simple 1 step problems.



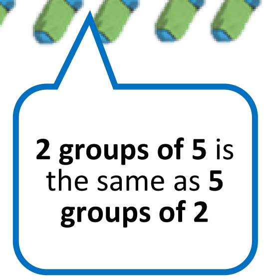
Arrays

Use of array in maths, arranging objects, numbers or pictures in columns or rows. The purpose of an array is to help children understand and identify groups and reinforce multiplication is grouping and repeated addition.



Commutative Relationship

Following on from arrays, this should be used to install a commutative understanding of multiplication, showing that one array can create two calculations. I.e. 4 groups of 3 or 3 groups of 4.



Year 2

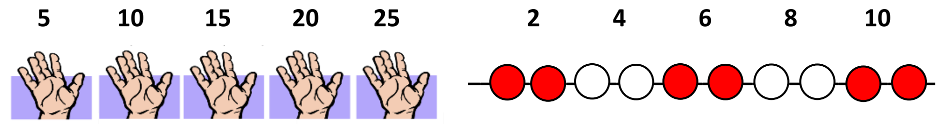
National Curriculum:

Pupils should be able to:

- Recall and use multiplication facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs
- Show that multiplication of two numbers can be done in any order (commutative)
- Solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in contexts

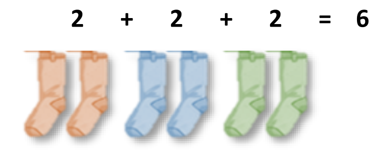
Counting in Multiples (Timetables)

Use practical concrete real-life objects, as well as pictorial representation to count in 2, 5, 10 and beginning 3. Practical methods should reinforce the connection between multiplication and timetables. Children should be progress to doing this mentally.



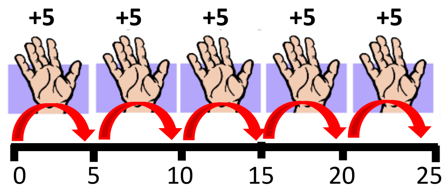
Repeated Addition

Children should use the above concrete and pictorial methods to understand multiplication as repeated addition.



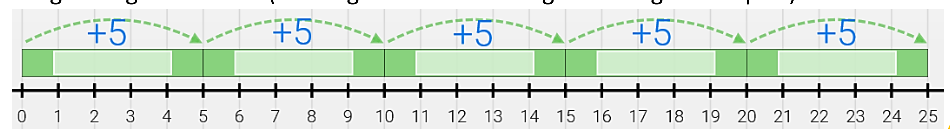
2 x 6 = 8 (2 groups of 6)

Number Line



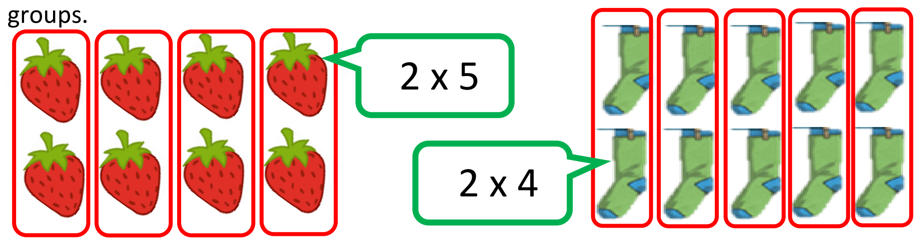
Once secure this can then be shown above a number line, progress to combination of pictorial and abstract use of number line to multiply by repeatedly adding the groups.

Progressing to abstract (starting at 0 and counting on in single multiples):



Arrays

Begin with use of objects and pictures within arrays in columns and rows by recognising equal groups and counting the multiple/ groups. Then, progress to use of pictorial representation of arrays enabling count the total amount of multiples/ groups.



Commutative Relationship and Inverse

Following on from arrays, this should be used to install a commutative understanding of multiplication, showing that one array can create two calculations. I.e. 2 groups of 5 is the same as 5 groups of 2. Further this understanding by teaching the inverse operation and the 2 multiplication and 2 division calculations that can be made from one array. **USE THE BAR MODEL TO DEMONSTRATE THIS.**

Multiplication

Year 3

National Curriculum:

Pupils should be taught to:

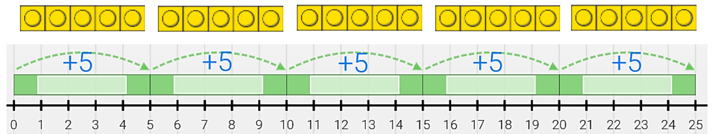
- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- Solve problems, including missing number problems

Counting in Multiples

See year 2 for methods to explicitly teach to reinforce timetable knowledge of 2, 5 and 10. Build on this to secure 3, 4 and 8. Use concrete resources to aid fluent mental recall.

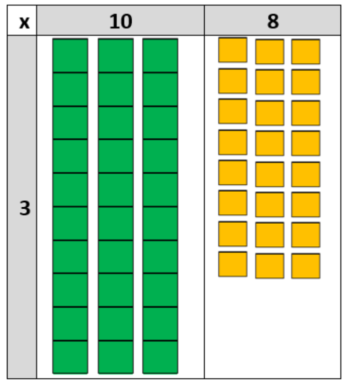
Number Line

Use a number line to multiply by through repeated addition, reinforce this by teaching with use of concrete objects.



Partitioning and Grid Method

Ensure place value understanding is secure so children are competent in portioning a 2-digit number into its T and O.
Chn will first practise this method concretely and pictorially using dienes.
Then, count what is there.



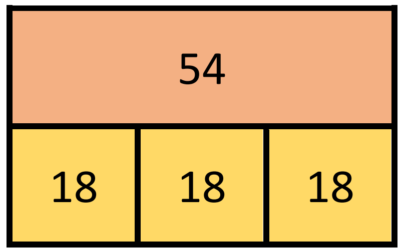
x	10	8
3	30	24

Then add the values 30 + 24 = 54

This will progress to using the grid method abstractly, and then adding the two values to get the answer. Children should be confident drawing these independently.

Problem Solving (Bar Model/ Inverse)

Use the bar model to reinforce understanding, reinforce inverse operation knowledge as well as to assist with problem solving when multiplying.



Inverse:
54 ÷ 18 = 3
54 ÷ 3 = 18
3 x 18 = 54
18 x 3 = 54

Year 4

National Curriculum:

Pupils should be taught to:

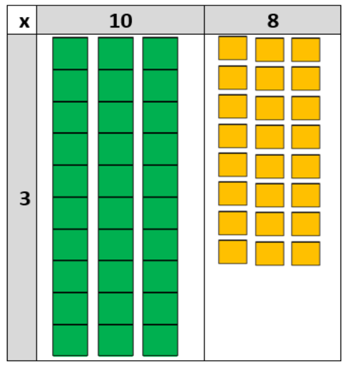
- Recall and use multiplication and division facts for multiplication tables up to 12
- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout

Timetables

Timetables should be embedded within mental math sessions, with regular explicitly timetable teaching – refer to counting in multiple methods if needed.

Grid Method

Ensure place value understanding is secure so children are competent in partitioning a 3-digit number into its T and O.
Chn will first practise this method concretely and pictorially using dienes.
Then, count what is there.



x	10	8
3	30	24

Then add the values 30 + 24 = 54

This will progress to using the grid method abstractly, and then adding the two values to get the answer. Children should be confident drawing these independently.

Expanded Column Method

Column method of addition should be taught initially using the expanded method with the use of concrete and pictorial dienes to reinforce relational understanding.

	H	T	O	
		3	6	
X			4	
		2	4	(4 X 6) Digits are multiplied, starting
	1	2	0	(4 X 30) with the lowest value digit.
	1	4	4	(24 + 120) The new columns are added, starting with the digit of least value.

Short Column Method

The expanded method facilitates a good practical understanding of the process of short column multiplication. When initially teaching short method, show without regrouping. Then, once understanding is secure, progress to teaching it with regrouping, ensuring explicit teaching using dienes to enable a concrete understanding of regrouping.

	T	O
	3	6
X		4
1	4	4
	2	

BAR MODEL SHOULD STILL BE USED TO REENFORCE INVERSE UNDERSTANDING AND ASSIST PROBLEM SOLVING (SEE YEAR 3)

Year 5

National Curriculum:

Pupils should be taught to:

- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- Continue to revisit mental strategies, timetables and bar modelling during problem solving

Short Column Method

The expanded method facilitates a good practical understanding of the process of short column multiplication. When initially teaching short method, show without regrouping. Then, once understanding is secure, progress to teaching it with regrouping, ensuring explicit teaching using dienes to enable a concrete understanding of regrouping.

	T	O
	3	6
X		4
1	4	4
	2	

Expanded Long Multiplication/ Grid Method

	T	H	T	O	
			3	6	
X			7	4	
			2	4	(4 X 6)
		1	2	0	(4 X 30)
		4	2	0	(70 x 6)
	2	1	0	0	(70 x 30)
	2	6	6	4	

Initially model the Grid Method for long multiplication to demonstrate practically how the expanded method works. Use the same process as in Y4, just add another row.

Then, move on to using the expanded long multiplication method to teach multiplying up to 4 digits by a 2-digit number. Initially, use dienes concretely or pictorially to enable a more relational understanding of the method.

Long Multiplication (Column Method)

Once secure with the expanded method, ensure children are secure by the end of the year in using the compact long method with regrouping including the use of decimals. This should be applied within problems that focus on real life maths particular measurement decimals. Begin with 2 digit by 2 digit progressing to 3 or 4 digit by 2 digit.

	H	T	O
		2	4
X		1	6
	1	4	4
		2	
	2	4	0
	3	8	4

Multiplying Decimals

As well as multiplying up to 4 digit whole numbers by a 2 digit number, children should be exposed to similar calculations involving decimals, ensuring place value knowledge is secure and decimal stays rooted.

	T	O	. t	
		3	. 6	
X		4		
		2	. 4	(4 X 0.6)
	1	2	. 0	(4 X 3)
	1	4	. 4	(2.4 + 12)

Year 6

National Curriculum:

Pupils should be taught to:

- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

Short Column Method

Children should be efficient in the use of the short method for multiplying up to 4 digits by 1 digit including decimals from a range of real-life contexts.
Ensure that children are using pictorial methods as shown in previous year groups to support their continued and more complex understanding. If secure, use larger numbers.

342 × 7 becomes

	3	4	2
×			7
2	3	9	4
	2	1	

Answer: 2394

2741 × 6 becomes

	2	7	4	1
×				6
1	6	4	4	6
	4	2		

Answer: 16 446

Long Column Method

Children should consolidate long multiplication by being exposed to more complex calculation up to 4 digits by 2 digits. Including decimals from a range of real-life contexts.

	324
x	46
	1944
+	12960
	14904

Problem Solving

Access to regular problem solving and reasoning where children can apply these methods is essential. Use of bar model is still required as it should be a familiar aid being embedded across the school.

Useful Pictorial Resources:

Dienes Generator
(used to make the illustrations in this policy)
https://mathsbot.com/manipulatives/blocks

Number Line Generator
(used to make the illustrations in this policy)
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Maths Frame Online Interactive Subtraction Games
https://mathsframe.co.uk/en/resources/category/7/multiplication-and-division

White Rose Reasoning and Problem Solving
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Grove Vale Primary School

Calculation Progression

Division

Division

EYFS

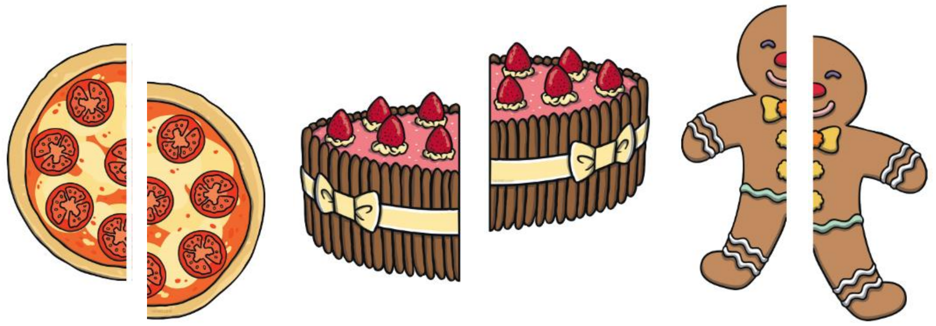
National Curriculum:

Pupils should be able to:

- Understanding the concept of a fair share

Halving

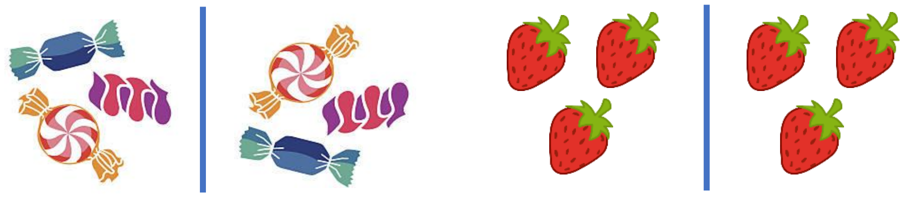
Children should be encouraged to use a range of everyday objects in the classroom and outdoor environments to share fairly. Children should start to understand the concept that halving is sharing equally and fairly between two people, both having the same amount. Use language half for you, half for me.



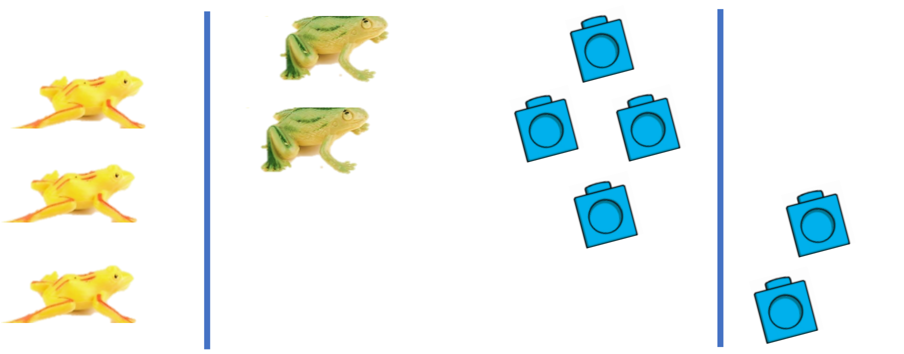
Also, demonstrate when something is not shared equally and discuss why:

Fair Sharing

Children will have access to a range of concrete and pictorial versions of real-life objects and also engage in a variety of songs and rhymes to share quantities of objects equally between two people. If confident, begin to explore sharing between three and four. Use language half, sharing equally and fairly.



Also, demonstrate when something is not shared equally and discuss why:



Year 1

National Curriculum:

Pupils should be able to:

- Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
- Recall doubles and count on/ back in multiples of 1, 2, 5 and 10

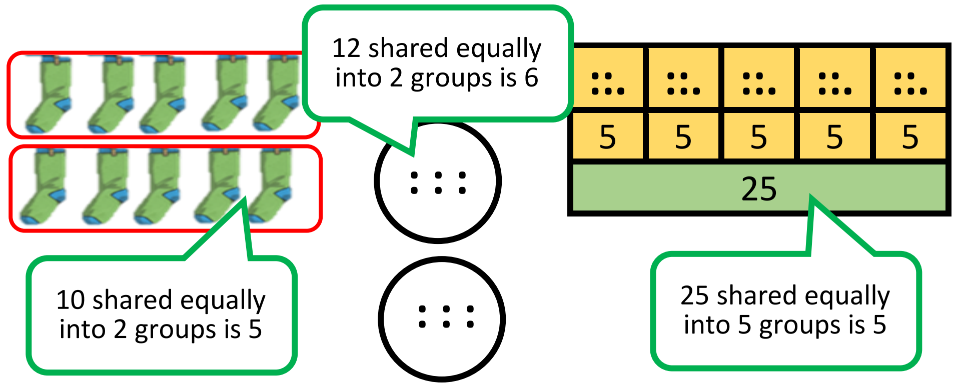
Fair Sharing

Children will start with practical sharing using a range of concrete resources and everyday objects. They will share objects in to equal groups in a variety of real-life situations. Encourage use of vocabulary (share, equally, fair) associated with division in practical, real-life contexts.

'Share 20 crayons between 2 pots.'

'How many crayons are in each pot?'

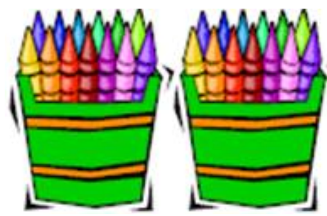
Children can then move on to representing pictorial in books either them drawing themselves or sharing circles provided on a worksheet.



If children are ready, they could be pushed on to solve more abstractly through use of a bar model. Or provided with a bar and show their representations. (See above)

Grouping

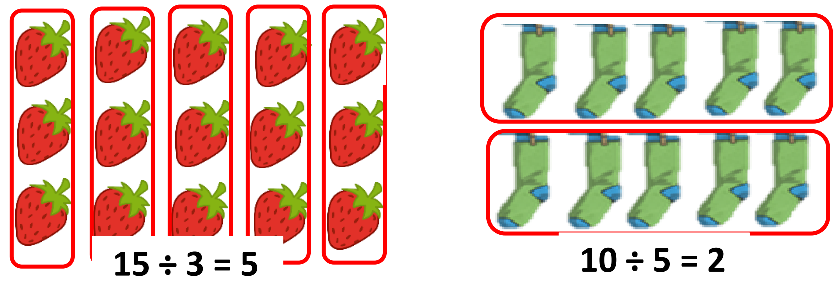
Children will move from sharing to grouping in a practical, real-life context.



'Put 20 crayons into groups of 10.'

'How many pots do we need?'

Children should experience grouping objects into groups of the multiple. E.g. placing objects into groups of 5 and seeing how many groups there are through use of arrays. Rather than children drawing arrays in their books, they may be provided pictorially with an array, circling to group. If children are ready they can draw their own.



Year 2

National Curriculum:

Pupils should be able to:

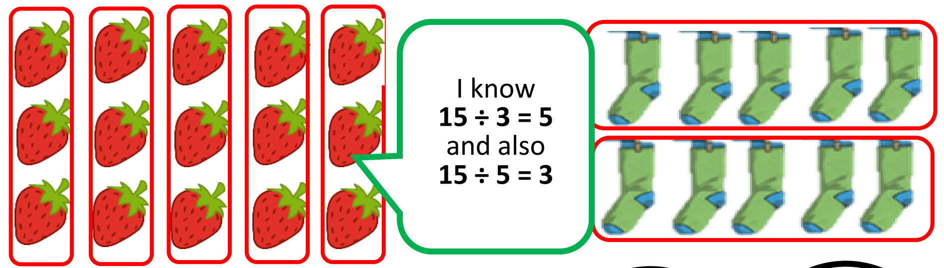
- Recall and use division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- Calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs
- Show that division of one number by another cannot
- Solve problems involving division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Division Facts/ Inverse Operation

Children should be encouraged to count in multiples as seen in the multiplication policy; using their timetable knowledge to make division fact links (link to inverse). As seen in multiplication, the inverse operation link between ÷ / x should constantly be referred to throughout.

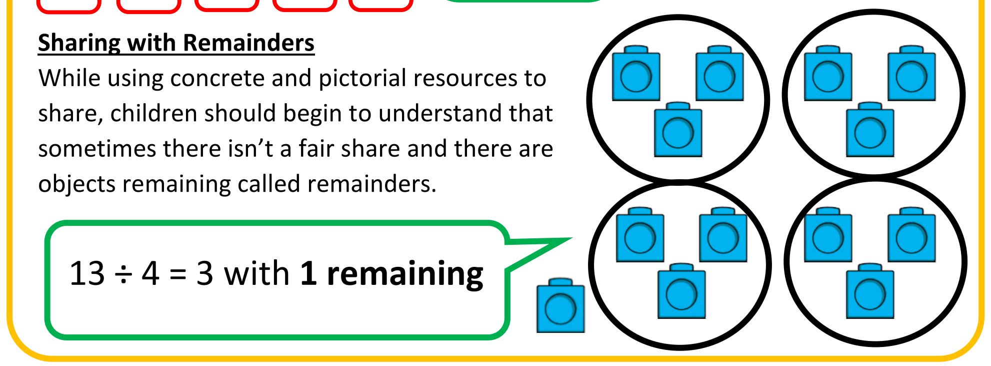
Fair Sharing and Grouping

Children will continue with embedding understanding of sharing and grouping with concrete objects and pictorial arrays (see year 1). This should progress to enable children to see the two division relationships. This should be done to share between 2, 5 and 10 equally.



Sharing with Remainders

While using concrete and pictorial resources to share, children should begin to understand that sometimes there isn't a fair share and there are objects remaining called remainders.

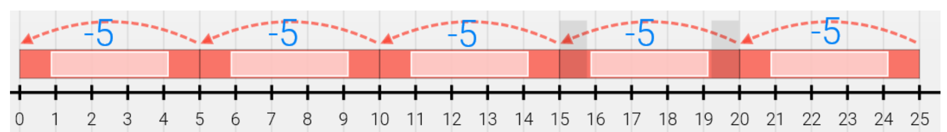


Number Line (Repeated Addition)

Children should start mental and formal division methods by using the number line method learnt in multiplication, counting on in groups to reach the total then logging the number of groups. E.g. 25 ÷ 5 = How many groups of 5 do we count to make 25? See multiplication for pictorial representation.

Number Line (Repeated Subtraction)

Following on from counting in multiples of 2, 5 and 10, use this timetable knowledge to make the same steps backwards. You could use a meter stick or beads to reinforce this concretely. This would be further on in the year and greater depth children could even use this to incorporate remainders. (2,5 and 10 multiples)



E.g. 25 ÷ 5 = 5 Start at 25, jump backwards in groups of 5 until you reach 0. If confident, children could begin to chunk the groups to make less jumps (se year 3).

BAR MODEL SHOULD CONTINUE TO BE USED FOR PROBLEM SOLVING (see year 1)

Division

Year 3

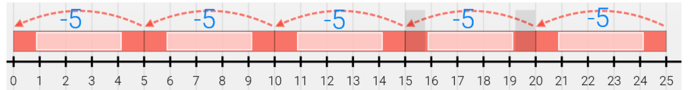
National Curriculum:

Pupils should be taught to:

- Recall and use division facts for the 3, 4 and 8 multiplication tables
- Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

Repeated Subtraction

Children use previous methods learned in year 2, but focus on aspect of repeated subtraction to prepare for subtracting when chunking.



Formal Layout

Introduce the formal division layout using multiplication/ division facts that the children know to prepare them for formal division methods.

24 ÷ 3 = 8

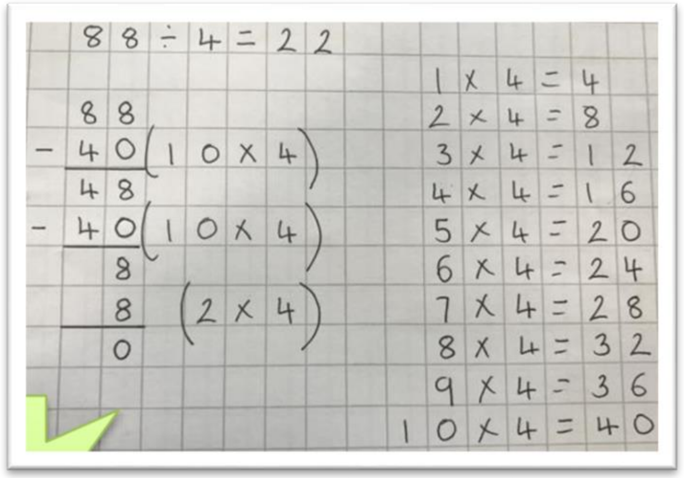
This can also be recorded as…

```
      8
3 ) 24
```

Chunking

Begin by showing chunking on a number line, rather than using repeated subtraction, jumping back in chunks of groups. Children should be encouraged to write down the related time tables facts to support them with the formal method of chunking. **Use concrete resources to reinforce.**

Then, begin to use chunking vertically to demonstrate conceptual understanding of short division (subtracting chunks of the divisor rather than individual jumps). Children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.



If confident move to dividing with remainders using this method, progressing to short method without remainders.

Year 4

National Curriculum:

Pupils should be taught to:

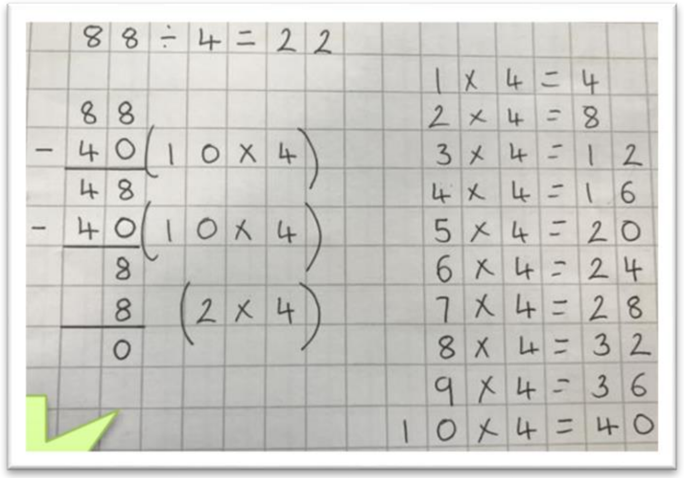
- Recall multiplication and division facts for multiplication tables up to 12 × 12
- Use place value, known and derived facts to divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers
- Recognise and use factor pairs and commutativity in mental calculations Multiply two-digit and three-digit numbers by a one-digit number using formal written layout

Division Facts

Consolidation of timetables and recall of related division facts should be incorporated into mental maths regularly to support division. Concrete resources can be used to reinforce.

Chunking (with and without remainders)

Like in year 3, children will begin chunking vertically to demonstrate conceptual understanding of short division (subtracting chunks of the divisor rather than individual



jumps). Children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.
If confident move to dividing with remainders using this method, once consolidated, children should progress to short method without remainders. Reinforce with concrete resources like dienes.

Short Column Division (no remainders)

Continue to use the formal division layout using multiplication/ division facts that the children know to prepare them for formal division methods. This could progress to remainders within timetable division facts.

25 ÷ 3 = 8 r1

```
     8 r 1
3 ) 25
```

Year 5

National Curriculum:

Pupils should be taught to:

- Divide numbers mentally, drawing upon known facts
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Divide whole numbers and those involving decimals by 10, 100 and 1,000
- Solve problems involving division, as well as a combination of all 4 calculations and understanding the meaning of the equals sign

Short Column Method

When children are secure with division, they are taught the compact column method, starting from the left and regrouping each remainder. Progress to more complex numbers with more complex regrouping.

$$\begin{array}{r|l} & 0\ \ 8\ \ 8\ \ \text{R }1 \\ \hline 4 & 3\ \ {}^{3}5\ \ {}^{3}3 \end{array}$$

Children should revisit chunking as a class reinforce short method as well as aiding children to make an informed estimation or to check their answers.

Dividing Decimals

As well as dividing up to 4 digit whole numbers by a 1 digit number, children should be exposed to similar calculations involving decimals, ensuring place value knowledge is secure and decimal stays rooted.

$$\begin{array}{r|l} & 0\ \ 8\ \ 8\ \ \text{R }1 \\ \hline 4 & 3\ \ {}^{3}5\ \ {}^{3}\!.3 \end{array}$$

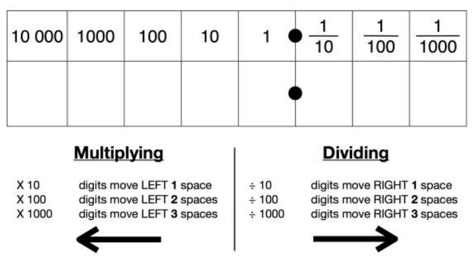
Remainders

$$\begin{array}{r|l} & 0\ \ 8\ \ 8\ \ \text{R }1 \\ \hline 4 & 3\ \ {}^{3}5\ \ {}^{3}3 \end{array}$$

Pupils will begin to be taught how to calculate the remainder as a fraction or decimal, relating to how far they are in that area of maths. **The remainder becomes ¼ then children will use their conversion knowledge that the answer will be 88 ¼ or 88.25.**

Dividing/ Multiplying by 10, 100 and 1000

Place value knowledge should be secure so children are aware the decimal stays rooted and the place value moves to the right or left depending on the operation.



Year 6

National Curriculum:

Pupils should be taught to:

- Divide numbers up to 4-digits by a two-digit whole number using the formal written method of short division
- Where appropriate for the context divide numbers up to 4 digits by a two digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- Solve problems involving division
- Use written division methods in cases where the answer has up to two decimal places

Short Division

$$\begin{array}{r|l} & 0\ \ 8\ \ 8\ \ \text{R }1 \\ \hline 4 & 3\ \ {}^{3}5\ \ {}^{3}3 \end{array}$$

Same applies as year 5, children should have access to problems in a wide range of real-life contexts, and should be dividing more complex 4 to 5-digit numbers (inc decimals).

Remainders

$$\begin{array}{r|l} & 0\ \ 8\ \ 8\ \ \text{R }1 \\ \hline 4 & 3\ \ {}^{3}5\ \ {}^{3}3 \end{array}$$

Pupils will be taught how to calculate the remainder as a fraction or decimal, understanding the remainder as a fraction of a group left over.. **The remainder becomes ¼ then children will use their conversion knowledge that the answer will be 88 ¼ or 88.25.**

Long Division

Children are taught initially to divide up to 4 digit numbers by a 2 digit number subtracting large chunks, this will consolidate understanding ready to progress to formal long division method.

```
        2 8  r12
15 )  4 3 2
      3 0 0   (20 x 15)
      1 3 2
      1 2 0   (8 x 15)
        1 2   (remainder)
```

Multiples of the divisor (15) have been subtracted from the dividend (432)

'20 (chunks of 15) + 8 (chunks of 15) = 28

12/15 is the remainder

Then, teach the formal long method ONLY when children are completely secure with the prior method and have a relational understanding of the method. Dienes could be used to reinforce.

432 ÷ 15 = 28·8

```
          2 8 · 8
15 )  4 3 2 · 0
      3 0
      1 3 2
      1 2 0
        1 2 0
        1 2 0
            0
```

Grove Vale Primary School

Mental Calculation Progression

Mental Calculation

Year Group	Mental calculation National curriculum objectives Key Stage 1	Strategies/ Teaching points
1	○ represent and use number bonds and related subtraction facts within 20 ○ add and subtract one-digit and two-digit numbers to 20, including 0 ○ recognise, find and name a half as 1 of 2 equal parts of an object, shape or quantity ○ recognise, find and name a quarter as 1 of 4 equal parts of an object, shape or quantity	○ count on or back in ones; ○ reorder numbers in a calculation; ○ begin to bridge through 10, and later 20, when adding a single-digit number; ○ use known number facts and place value to add or subtract pairs of single-digit numbers; ○ add 9 to single-digit numbers by adding 10 then subtracting 1; ○ identify near doubles, using doubles already known; ○ use patterns of similar calculations.

2	○ count in steps of 2, 3, and 5 from 0, and in 10s from any number, forward and backward
	    ○ applying their increasing knowledge of mental and written methods
	○ recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
	○ add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
	    ○ a two-digit number and 1s
	    ○ a two-digit number and 10s
	    ○ 2 two-digit numbers
	    ○ adding 3 one-digit numbers
	○ show that addition of 2 numbers can be done in any order (commutative) and subtraction of 1 number from another cannot
	○ recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
	○ calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
	○ show that multiplication of 2 numbers can be done in any order (commutative) and division of 1 number by another cannot
	○ recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity
		– count on or back in tens or ones;
		○ find a small difference by counting up from the smaller to the larger number;
		○ reorder numbers in a calculation;
		○ add three small numbers by putting the largest number first and/or find a pair totalling 10;
		○ partition additions into tens and units then recombine;
		○ bridge through 10 or 20;
		○ use known number facts and place value to add or subtract pairs of numbers;
		○ partition into '5 and a bit' when adding 6, 7, 8 or 9, then recombine;
		○ add or subtract 9, 19, 11 or 21 by rounding and compensating; identify near doubles;
		○ use patterns of similar calculations;
		○ use the relationship between addition and subtraction;
		○ use knowledge of number facts and place value to multiply or divide by 2, 5 or 10;
		○ use doubles and halves and halving as the inverse of doubling

Non-Negotiables – End of Key Stage 1

- add or subtract a single-digit to or from a single-digit, without crossing 10, eg 4 + 5, 8 – 3;

– add or subtract a single-digit to or from 10;

– add or subtract a single-digit to or from a ‘teens’ number, without crossing 20 or 10, eg 13 + 5, 17 – 3;

– doubles of all numbers to 10, eg 8 + 8, double 6.

– add or subtract any single-digit to or from any two-digit number, without crossing the tens boundary, eg 62 + 4, 38 – 7;

– add or subtract any single-digit to or from a multiple of 10, eg 60 + 5, 80 – 7;

– add or subtract any ‘teens’ number to any two-digit number, without crossing the tens boundary, eg 23 + 14, 48 – 13;

– find what must be added to any two-digit multiple of 10 to make 100, eg 70 + ? = 100;

– add or subtract a multiple of 10 to or from any two-digit number, without crossing 100, eg, 47 + 30, 82 – 50;

– subtract any two-digit number from any two-digit number when the difference is less than 10, eg 78 – 71, or 52 – 48;

– doubles of all numbers to at least 15, eg double 14;

– double any multiple of 5 up to 50, eg double 35;

– halve any multiple of 10 up to 100, eg halve 50

3	○ count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number ○ add and subtract numbers mentally, including: ○ a three-digit number and 1s ○ a three-digit number and 10s ○ a three-digit number and 100s ○ recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables ○ write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods ○ count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10 ○ recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators ○	○ count on or back in tens or ones; ○ find a small difference by counting up from the smaller to the larger number; ○ reorder numbers in a calculation; ○ add three or four small numbers by putting the largest number first and/or by finding pairs totalling 9, 10 or 11; ○ partition into tens and units then recombine; ○ bridge through a multiple of 10, then adjust; ○ use knowledge of number facts and place value to add or subtract pairs of numbers; ○ partition into '5 and a bit' when adding 6, 7, 8 or 9; ○ add or subtract mentally a 'near multiple of 10' to or from a two-digit number; ○ identify near doubles; ○ use patterns of similar calculations; ○ say or write a subtraction statement corresponding to a given addition statement; ○ to multiply a number by 10/100, shift its digits one/two places to the left; ○ use knowledge of number facts and place value to multiply or divide by 2, 5, 10, 100; ○ use doubling or halving; ○ say or write a division statement corresponding to a given multiplication statement.

4	○ count in multiples of 6, 7, 9, 25 and 1,000 ○ find 1,000 more or less than a given number ○ count backwards through 0 to include negative numbers ○ recall multiplication and division facts for multiplication tables up to 12 × 12 ○ use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together 3 numbers ○ recognise and use factor pairs and commutativity in mental calculations ○ count up and down in hundredths; recognise that hundredths arise when dividing an object by 100 and dividing tenths by 10 ○ find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths ○ round decimals with 1 decimal place to the nearest whole number	○ count on or back in repeated steps of 1, 10 and 100; ○ count up through the next multiple of 10, 100 or 1000; ○ reorder numbers in a calculation; ○ add 3 or 4 small numbers, finding pairs totalling 10; ○ add three two-digit multiples of 10; ○ partition into tens and units, adding the tens first; ○ bridge through 100; ○ use knowledge of number facts and place value to add or subtract any pair of two-digit numbers; ○ add or subtract 9, 19, 29, 11, 21 or 31 by rounding and compensating; ○ add or subtract the nearest multiple of 10 then adjust; identify near doubles; ○ continue to use the relationship between addition and subtraction; ○ double any two-digit number by doubling tens first; ○ use known number facts and place value to multiply or divide, including multiplying and dividing by 10 and then 100; ○ partition to carry out multiplication; ○ use doubling or halving; ○ use closely related facts to carry out multiplication and division; use the relationship between multiplication and division.

Non-Negotiables – End of Lower Key Stage 2

- find what must be added to any multiple of 100 to make 1000, eg 300 + ? = 1000;
- add or subtract any pair of two-digit numbers, without crossing a tens boundary or 100, eg 33 + 45, 87 – 2;
- add or subtract any single-digit to any two-digit number, including crossing the tens boundary, eg 67 + 5, 82 – 7;
- find what must be added to/subtracted from any two-digit number to make the next higher/lower multiple of 10, eg 64 + ? = 70, 56 – ? = 50;
- subtract any three-digit number from any three-digit number when the difference is less than 10, eg 458 – 451, or 603 – 597;
- find what must be added to/subtracted from any three-digit number to make the next higher/lower multiple of 10, eg 647 + ? = 650, 246 – ? = 240;
- doubles:
- double any number to at least 20, eg double 18, and corresponding halves, eg halve 36;
- double 60, halve 120;
- double 35, halve 70;
- double 450, halve 900;
- multiply single-digit numbers by 10 or 100, eg 6 x 100;
- divide any multiple of 10 by 10, eg 60 ÷ 10, and any multiple of 100 by 100, eg 700 ÷ 100.
- find what must be added to any two-digit number to make 100, eg 37 + ? = 100;
- add or subtract any pair of two-digit numbers, eg 38 + 85, 92 – 47;
- find out what must be added to/subtracted from any two- or three-digit number to make the next higher/lower multiple of 100, eg 374 + ? = 400, 826 – ? = 800;
- subtract any four-digit number from any four-digit number when the difference is small, eg 3641 - 3628, 6002 – 5991;
- doubles and halves:
- double any whole number from 1 to 50, eg double 36, and find all the corresponding halves, eg 96 ÷ 2;
- double any multiple of 10 to 500, eg 380 x 2, and find all the corresponding halves, eg 760 ÷ 2, 130 ÷ 2;
- double any multiple of 5 to 100, eg 65 x 2;
- multiply any two-digit number by 10, eg 26 x 10;
- divide a multiple of 100 by 10, eg 600 ÷ 10;
- multiply any two-digit multiple of 10 by 2, 3, 4 or 5, eg 60 x 4, 80 x 3.

5	○ count forwards or backwards in steps of powers of 10 for any given number up to 1,000,000 ○ interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through 0 ○ round any number up to 1,000,000 to the nearest 10, 100, 1,000, 10,000 and 100,000 ○ add and subtract numbers mentally with increasingly large numbers ○ identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers ○ establish whether a number up to 100 is prime and recall prime numbers up to 19 ○ multiply and divide numbers mentally, drawing upon known facts ○ multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000 ○ recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3}) ○ round decimals with 2 decimal places to the nearest whole number and to 1 decimal place	○ count up through the next multiple of 10, 100 or 1000; ○ reorder numbers in a calculation; ○ partition into hundreds, tens and units, adding the most significant digit first; ○ use known number facts and place value to add or subtract pairs of three-digit multiples of 10 and two-digit numbers with one decimal place; ○ add or subtract the nearest multiple of 10 or 100 then adjust; ○ identify near doubles; ○ add several numbers; ○ develop further the relationship between addition and subtraction; ○ use factors; ○ partition to carry out multiplication; ○ use doubling and halving; ○ use closely related facts to carry out multiplication and division; ○ use the relationship between multiplication and division; ○ use knowledge of number facts and place value to multiply or divide.
6	○ round any whole number to a required degree of accuracy ○ use negative numbers in context, and calculate intervals across 0 ○ perform mental calculations, including with mixed operations and large numbers ○ identify common factors, common multiples and prime numbers ○ use common factors to simplify fractions; use common multiples to express fractions in the same denomination ○ identify the value of each digit in numbers given to 3 decimal places and multiply and divide numbers by 10, 100 and 1,000 giving answers up to 3 decimal places	○ consolidate all strategies from previous years; ○ use knowledge of number facts and place value to add or subtract pairs of three-digit multiples of 10 and two-digit numbers with one decimal place; ○ add or subtract the nearest multiple of 10, 100 or 1000, then adjust; ○ continue to use the relationship between addition and subtraction; ○ use factors; ○ partition to carry out multiplication; ○ use doubling and halving; ○ use closely related facts to carry out multiplication and division; ○ use the relationship between multiplication and division; ○ use knowledge of number facts and place value to multiply or divide.

Non-Negotiables – End of Key Stage 2

- add or subtract any pair of three-digit multiples of 10, eg 570 + 250, 620 – 380;
- find what must be added to a decimal fraction with units and tenths to make the next higher whole number, eg. 4.3 + ? = 5;
- add or subtract any pair of decimal fractions each with units and tenths, or each with tenths and hundredths, eg 5.7 + 2.5, 0.63 – 0.48;
- subtract a four-digit number just less than a multiple of 1000 from a four-digit number just more than a multiple of 1000, eg 5001 – 1997;
- multiply any two- or three-digit number by 10 or 100, eg 79 x 100, 363 x 100;
- divide a multiple of 100 by 10 or 100, eg 4000 ÷ 10, 3600 ÷ 100;
- multiply any two-digit multiple of 10 by a single-digit, eg 60 x 7, 90 x 6;
- double any whole number from 1 to 100, multiples of 10 to 1000, and find corresponding halves;
- find 50%, 25%, 10% of a small whole numbers or quantities, eg 25% of £8.
- multiply any two-digit number by a single-digit, eg 34 x 6;
- multiply any two-digit number by 50 or 25, eg 23 x 50, 47 x 25;
- multiply or divide any whole number by 10 or 100, giving any remainder as a decimal, eg 47 ÷ 10 = 4.7, 1763 ÷ 100 = 17.63;
- find squares of multiples of 10 to 100;
- find any multiple of 10% of a whole number or quantity, eg 70% of £20, 50% of 5kg, 20% of 2 metres.

Mental Calculation Strategies Progression

Counting Forwards and Backwards	
Example Calculation	Possible counting strategy
4 + 5	count on in ones from 4 (or in ones from 5)
8 – 3	count back in ones from 8
10 + 7	count on in ones from 10 (or use place value)
13 + 5	count on in ones from 13
17 – 3	count back in ones from 17
18 – 6	count back in twos
23 + 5	count on in ones from 23
57 – 3	count back in ones from 57
60 + 5	count on in ones or use place value
80 – 7	count back in ones from 80 (or use knowledge of number fact to 10 and place value)
27 + 60	count on in tens from 27
72 – 50	count back in tens from 72
50 + 38	count on in tens then in ones from 50
90 – 27	count back in tens then in ones from 90
34 + 65	count on in tens then ones from 34
87 – 23	count back in tens then ones from 87
35 + 15	count on in steps of 5 from 35
73 – 68	count up from 68, counting 2 to 70 then 3 to 73
47 + 58	count on 50 from 47, then 3 to 100 then 5 to 105
124 – 47	count back 40 from 124, then 4 to 80, then 3 to 77
570 + 300	count on in hundreds from 570
960 – 500	count back in hundreds from 960
3.2 + 0.6	count on in tenths
1.7 + 0.55	Count on in tenths and hundredths

Re-ordering	
Example Calculation	Possible reordering strategy
2 + 7	7 + 2
5 + 13	13 + 5
10 + 2 + 10	10 + 10 + 2
5 + 34	34 + 5
5 + 7 + 5	5 + 5 + 7
23 + 54	54 + 23
12 – 7 – 2	12 – 2 - 7
13 + 21 + 13	13 + 13 + 21 (using double 13)
6 + 13 + 4 + 3	6 + 4 + 13 + 3
17 + 9 – 7	17 – 7 + 9
28 + 75	75 + 28 (thinking of 28 as 25 + 3)
12 + 17 + 8 + 3	12 + 8 + 17 + 3
25 + 36 + 75	25 + 75 + 36
58 + 47 - 38	58 – 38 + 47
200 + 567	567 + 200
3 + 8 + 7 + 6 + 2	3 + 7 + 8 + 2 + 6
34 + 27 + 46	34 + 46 + 27
180 + 650	650 + 180 (thinking of 180 as 150 + 30)
1.7 + 2.8 + 0.3	1.7 + 0.3 + 2.8
4.7 + 5.6 – 0.7	4.7 – 0.7 + 5.6 = 4 + 5.6

Partitioning: counting on or back:	
Example Calculation	Possible counting on or back strategy
30 + 47	30 + 40 + 7
78 – 40	70 + 8 – 40 = 70 – 40 + 8
17 + 14	10 + 7 + 10 + 4 = 10 + 10 + 7 + 4
23 + 45	40 + 5 + 20 + 3 = 40 + 20 + 5 + 3
68 – 32	60 + 8 – 30 – 2 = 60 – 30 + 8 - 2
55 + 37	50 + 5 + 30 + 7 = 85 + 7
365 – 40	300 + 60 + 5 – 40 = 300 + 60 – 40 + 5
43 + 28 + 51	40 + 3 + 20 + 8 + 50 + 1 = 40 + 20 + 50 + 3 + 8 + 1
5.6 + 3.7	5.6 + 3 + 0.7 = 8.6 + 0.7
4.7 – 3.5	4.7 – 3 – 0.5
540 + 280	540 + 200 + 80
276 – 153	276 – 100 – 50 – 3

Partitioning: bridging through multiples of 10:	
Example Calculation	Possible bridging through ten strategy
5 + 8 or 12 – 7	5 + 5 + 3 or 12 – 2 – 5
65 + 7 or 43 – 6	65 + 5 + 2 or 43 – 3 – 3
24 – 19	19 + 1 + 4
49 + 32	49 + 1 + 31
90 – 27	27 + 3 + 60
57 + 34 or 92 – 25	57 + 3 + 31 or 92 – 2 – 20 – 3
84 – 35	35 + 5 + 40 + 4
607 – 288	288 + 12 + 300 + 7
6070 – 4987	4987 + 13 + 1000 + 70
1.4 + 1.7 or 5.6 – 3.7	1.4 + 0.6 + 1.1 or 5.6 – 0.6 – 3 – 0.1
0.8 + 0.35	0.8 + 0.2 + 0.15
8.3 – 2.8	2.8 + 0.2 + 5.3 or 8.3 – 2.3 – 0.5

Partitioning: compensation	
Example Calculation	Possible compensating strategy
34 + 9 34 + 19 34 + 29 and so on	34 + 10 – 1 34 + 20 – 1 34 + 30 – 1 and so on
34 + 11 34 + 21 34 + 31 and so on	34 + 10 + 1 34 + 20 + 1 34 + 30 +1 and so on
70 – 9	70 – 10 + 1
53 + 12	53 + 10 + 2
53 – 12	53 – 10 – 2
53 + 18	53 + 20 – 2
84 – 18	84 – 20 + 2
38 + 68	38 + 70 – 2
95 – 78	95 – 80 + 2
58 + 32	58 + 30 + 2
64 – 32	64 – 30 – 2
138 + 69	138 + 70 – 1
405 – 399	405 – 400 + 1
2½ + 1¾	2½ + 2 - ¼
5.7 + 3.9	5.7 + 4.0 – 0.1
6.8 – 4.9	6.8 – 5.0 + 0.1

Partitioning: using near doubles:	
Example Calculation	Possible compensating strategy
6 + 7	is double 6 and add 1 or double 7 and subtract 1
13 + 14	is double 13 and add 1 or double 14 and subtract 1
39 + 40	is double 40 and subtract 1
18 + 16	is double 18 and subtract 2 or double 16 and add 2
60 + 70	is double 60 and add 10 or double 70 and subtract 10
76 + 75	is double 76 and subtract 1 or double 75 and add 1
160 +170	is double 150 then add 10, then add 20 or double 160 and add 10 or double 170 and subtract 10
2.5 + 2.6	is double 2.5 and add 0.1 or double 2.6 and subtract 0.1

Partitioning: bridging through 60 to calculate time interval:
Example Calculation
It is 10.30am. How many minutes to 10.45am?
It is 3.45pm. How many minutes to 4.15pm?
I get up 40 minutes after 6.30am. What time is that?
What is the time 50 minutes before 1.10pm?
It is 4.25pm. How many minutes to 5.05pm?
What time will it be 26 minutes after 3.30am?
What was the time 33 minutes before 2.15pm?
It is 4.18pm. How many minutes to 5.00pm? 5.26pm?
It is 08.35. How many minutes is it to 09.15?
It is 11.45. How many hours and minutes is it to 15.20?
A train leaves Dumfries for Dundee at 22.33. The journey takes 2 hours 47 minutes. What time does the train arrive?

Multiplication and division facts to 10 x 10:
Expectations of learners with examples
Count on, from and back to zero in ones, twos, fives and tens.
Recognise odd and even numbers to 20
Recall the doubles of all numbers to 10
Derive and recall doubles of all numbers to 20, and doubles of multiples of 10 to 50, and corresponding halves.
Derive and recall multiplication facts for 2, 4; 5 and 10 times tables and corresponding division facts.
Recognise odd and even numbers to 100
Recognise multiples of 2, 4; 5 and 10
Derive and recall doubles multiples of 10 to 100 and corresponding halves.
Derive and recall multiplication facts for the 2, 4; 5, 10; 3 and 6 times tables and corresponding division facts.
Recognise multiples of 2, 4; 5, 10; 3 and 6 up to the tenth multiple.
Identify doubles of two digit numbers and corresponding halves.
Derive doubles of multiples of 10 and 100 and corresponding halves.
Derive and recall multiplication facts up to 10 x 10 and corresponding division facts.
Recognise multiples of 2, 4, 8; 5, 10; 3, 6, 9; and 7 up to the tenth multiple.
Recall squares of numbers to 10 x 10
Use multiplication facts to derive products of pairs of multiples of 10 and 100 and corresponding division facts.
Recall squares of numbers to 12 x 12 and derive corresponding squares of multiples of 10.
Use place value and multiplication facts to derive related multiplication and division fact involving decimal fractions (e.g. 0.8 x 7, 4.8 ÷ 6)
Identify factor pairs of two digit numbers.
Identify prime numbers less than 100

Doubling and halving:
Expectations of learners with examples
Double all numbers to 10 e.g. double 9
Double all numbers to 20 and find the corresponding halves e.g. double 7, half of 14
Double multiples of 10 to 50 e.g. double 40 and find corresponding halves.
Double multiples of 5 to 50 and find the corresponding halves e.g. double 35, half of 70.
Double multiples of 10 to 100 e.g. double 90 and corresponding halves
Double multiples of 5 to 100 and find the corresponding halves e.g. double 85, halve 170
Double any two digit number and find the corresponding halves e.g. double 47 , half of 94
Double multiples of 10 and 100 and find the corresponding halves e.g. double 800, double 340, half of 1600, half of 680
Form equivalent calculations and use doubling and halving such as: • Multiply by 4 by doubling twice e.g. 16 x 4: 16 x 2 = 32, 32 x 2 = 64 • Multiply by 8 by doubling three times e.g. 12 x 8: 12 x 2 = 24, 24 x 2 = 48, 48 x 2 = 96 • Divide by 4 by halving twice e.g. 104 ÷ 4: 104 ÷ 2 = 52, 52 ÷ 2 = 26 • Divide by 8 by halving three times e.g. 104 ÷ 8: 104 ÷ 2 = 52, 52 ÷ 2 = 26, 26 ÷ 2 = 13 • Multiply by 20 by doubling then multiplying by 10 e.g. 53 x 20: 53 x 2 = 106, 106 x 10 = 1060
Multiply by 50 by multiplying by 100 and halving
Multiply by 25 by multiplying by 100 and halving twice
Double decimal fractions with units and tenths e.g. double 7.6 and find the corresponding halves e.g. half of 15.2
Form equivalent calculations and use doubling and halving such as: • Divide by 25 by dividing by 100 then multiplying by 4 e.g. 460 ÷ 25 = 4.6 x 4 = 18.4 • Divide by 50 by dividing by 100 then doubling e.g. 270 ÷ 50 = 2.7 x 2 = 5.4

Multiplying and dividing by multiples of 10:
Expectations of learners with examples
Recall multiplication and division fact for the 10 times table e.g. 7 x 10, 60 ÷ 10
Multiply one-digit and two-digit numbers by 10 or 100 e.g. 7 x 100, 46 x 10, 54 x 100
Change pounds to pence e.g. £1.50 to 150 pence, £6 to 600 pence
Multiply numbers to 1000 by 10 and 100 e.g. 325 x 10, 42 x 100
Divide numbers to 1000 by 10 and 100 (whole number answers) e.g. 120 ÷ 10, 600 ÷ 100, 850 ÷ 10
Multiply a multiple of 10 to 100 by a single-digit number e.g. 60 x 3, 50 x 7
Change hours to minutes; convert between units involving multiples of 10 and 100 e.g. centimetres and millimetres, centilitres and millimetres and convert between pounds and pence, metres and centimetres.
Multiply and divide whole numbers and decimal fractions by 10, 100 or 1000 e.g. 4.3 x 10, 0.75 x 100, 25 ÷ 10, 673 ÷ 100
Divide a multiple of 10 by a single-digit number (whole number answers) e.g. 80 ÷ 4, 270 ÷ 3
Multiply pairs of multiples of 10 and a multiple of a 100 by a single-digit number e.g. 60 x 30, 900 x 8
Convert larger to smaller units of measurement using decimal fractions to one place e.g. change 2.6kg to 2600g, 3.5cm to 35mm, 1.2m to 120cm
Multiply by 25 or 50 using equivalent calculations e.g. 48 x 100 ÷ 4, 32 x 100 ÷ 2
Multiply pairs of multiples of 10 and 100 e.g. 50 x 30, 600 x 20
Divide multiples of 100 by a multiple of 10 and 100 e.g. 300 ÷ 50, 600 ÷ 20
Divide by 25 or 50
Convert between units of measurement using decimal fractions to two places e.g. change 2.75l to 2750 ml or vice versa

Multiplying and dividing by single-digit numbers and multiplying by two-digit numbers:
Expectations of learners with examples
Find one quarter by halving one half
Multiply numbers to 20 by a single-digit number e.g. 17 x 3
Multiply and divide two-digit numbers by 4 or 8 e.g. 26 x 4, 96 ÷ 8
Multiply two-digit numbers by 5 or 20 e.g. 32 x 5, 14 x 20 .
Multiply by 25 or 50 e.g. 48 x 25, 32 x 50
Multiply a two-digit and a single digit number e.g. 28 x 7
Divide a two-digit number by a single-digit number e.g. 68 ÷ 4
Divide by 25 or 50 e.g. 480 ÷ 25, 3200 ÷ 50
Find new facts from given facts e.g. given that three pears cost 24p, find the cost of 4 pears

Fractions, decimal fractions and percentages:
Expectations of learners with examples
Find half of any even number to 40 or multiple of 10 to 100 e.g. halve 80
Find half of any multiple of 10 up to 200 e.g. halve 170
Find 1/2 , 1/3 , 1/4 , 1/5 and 1/10 of numbers in the 2, 3, 4, 5 and 10 times tables
Find half of any even number to 200
Find unit fractions and simple non-unit fractions of whole numbers or quantities e.g. 3/8 of 24
Recall fractions and decimal fraction equivalents from one-half, quarters, tenths and hundredths e.g. recall the equivalence of 0.3 and 3/10 and 0.03 and 3/100
Recall percentage equivalents of one-half, one-quarter, three-quarters, tenths and hundredths
Find fractions of whole numbers or quantities e.g. 2/3 of 27, 4/5 of 70kg
Find 50%, 25% or 10% of whole numbers or quantities e.g. 25% of 20kg, 10% of £80
Recall equivalent fractions, decimal fractions and percentages for hundredths e.g. 35% is equivalent to 0.35 or 35/100
Find half of decimal fractions with units and tenths e.g. half of 3.2
Find 10% or multiples of 10% of whole numbers and quantities e.g. 30% of 50ml, 40% of £30, 70% of 200g
Recall commonly used equivalent fractions for 33 1/3% and 66 2/3 %