Owner	**Maths Subject Lead**
Date Updated	**January 2020**
Audience	**All teaching staff**

Calculations Policy

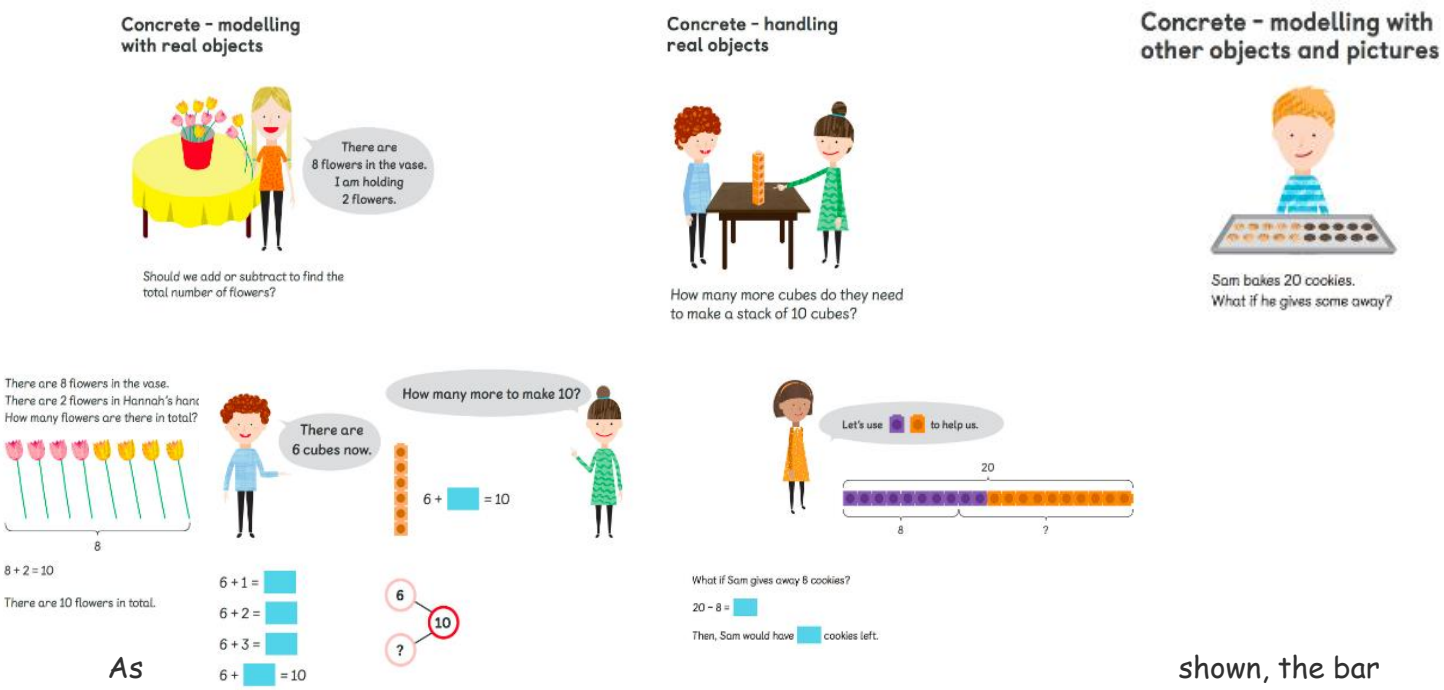
Maths at Harrow Gate – our intent

Bar Modelling and Concrete, Pictorial, Abstract (CPA) approach

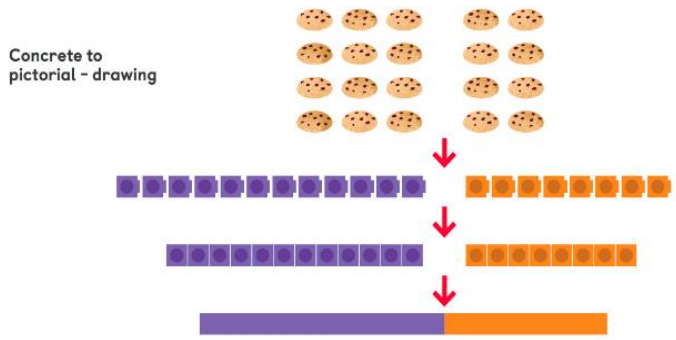
Throughout this policy, and our mathematics teaching, is the idea of representation, and using concrete objects / manipulatives to represent numbers & concepts in different ways.

The bar model method draws on the Concrete, Pictorial, Abstract (CPA) approach — an essential maths mastery concept. The process begins with pupils exploring problems via concrete objects. Pupils then progress to drawing pictorial diagrams, and then to abstract algorithms and notations (such as the +, -, x and / symbols).

The example below explains how bar modelling moves from concrete maths models to pictorial representations.



As shown, the bar method is primarily pictorial. Pupils will naturally develop from handling **concrete** objects, to drawing **pictorial** representations, to creating **abstract** rectangles to illustrate a problem. With time and practice, pupils will no longer need to draw individual boxes/units. Instead, they will label one long rectangle/bar with a number. At this stage, the bars will be somewhat proportional. So, in the example above, the purple bar representing 12 cookies is longer than the orange bar representing 8 cookies.

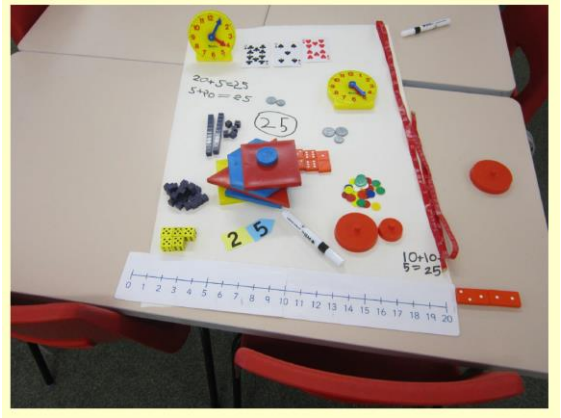


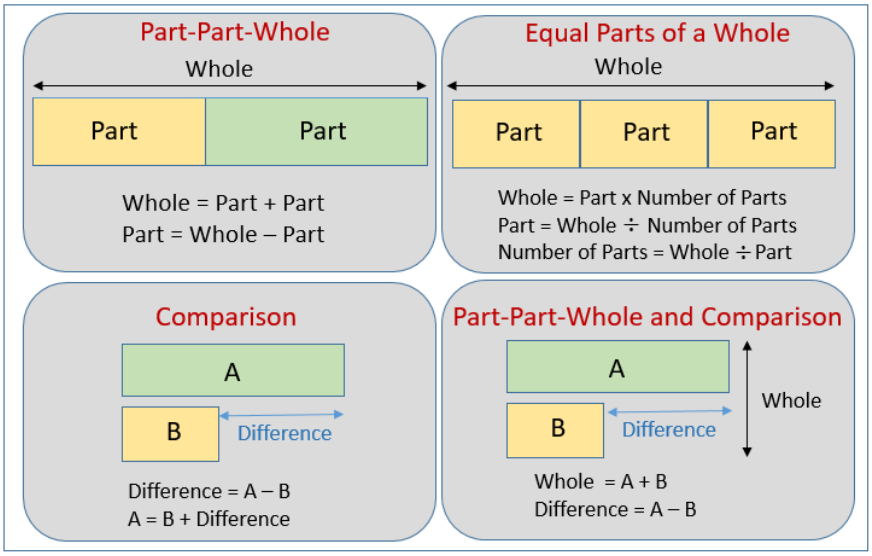
The lasting advantages of bar modelling

On one hand, the Singapore maths model method — bar modelling — provides pupils with a powerful tool for solving word problems. However, the lasting power of bar modelling is that once pupils master the approach, they can easily use bar models year after year across many maths topics. For example, bar modelling is an excellent technique (but not the only one!) for tackling ratio problems, volume problems, fractions, and more.

Importantly, bar modelling leads students down the path towards mathematical fluency and number sense. Maths models using concrete or pictorial rectangles allow pupils to understand complex formulas (for example, algebra) on an intuitive, conceptual level. Instead of simply following the steps of any given formula, students will possess a strong understanding of what is actually happening when applying or working with formulas.

The result? A stable, transferable, and solid mathematical framework for approaching abstract concepts. Combined with other essential maths mastery strategies and concepts, bar modelling sets students up for long-term maths success.





Addition & Subtraction

KS1

Y1 Objectives:

Pupils should be taught to:

- *read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs*
- *represent and use number bonds and related subtraction facts within 20*
- *add and subtract one-digit and two-digit numbers to 20, including zero*
- *solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ☐ – 9.*

Y2 Objectives:

Pupils should be taught to:

- *solve problems with addition and subtraction:*
    - *using concrete objects and pictorial representations, including those involving numbers, quantities and measures*
    - *applying their increasing knowledge of mental and written methods*
- *recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100*
- *add and subtract numbers using concrete objects, pictorial representations, and mentally, including:*
    - *a two-digit number and ones*
    - *a two-digit number and tens*
    - *two two-digit numbers*
    - *adding three one-digit numbers*
- *show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot*
- *recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.*

Structures - addition as aggregation and augmentation:

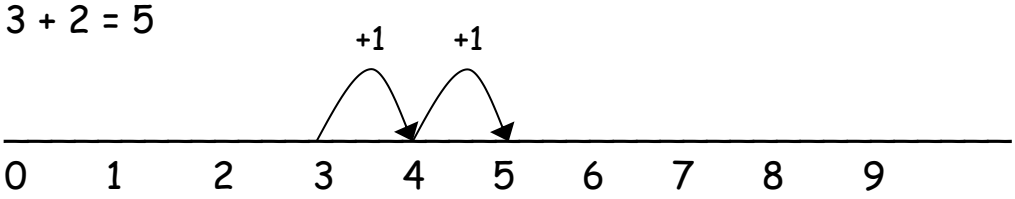
Aggregation - Joining two groups and then recounting all objects using one-to-one correspondence

3+4 = 7

Augmentation – adding more to a number. The number line can be used.
12+3=15

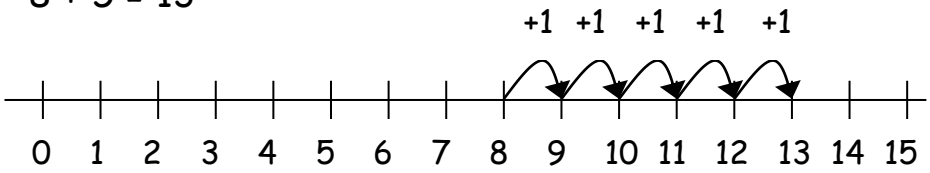
Children can use a numbered line to count on in ones. They use number lines and practical resources to support calculation and teachers *demonstrate* the use of the number line.

3 + 2 = 5

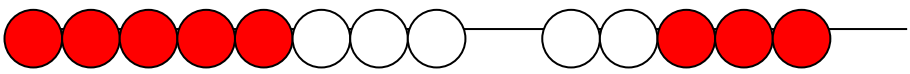


Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.

8 + 5 = 13



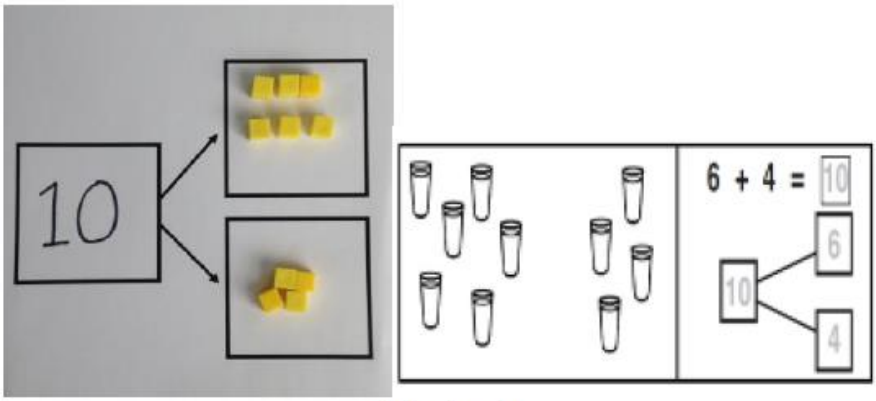
Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.



Children can use Numicon pieces to put together and find the total.

Part-part whole model:
Teach both addition and subtraction alongside each other, as pupils will use this model to identify the inverse link between them.

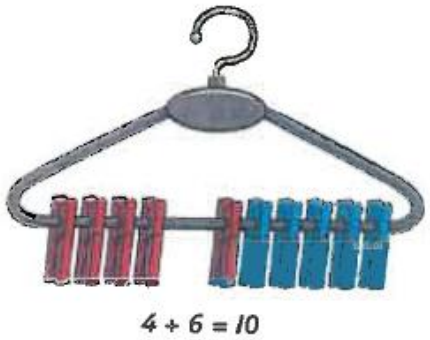
Pupils could place ten on top of the whole as well as writing it down. The parts could also be written in alongside the concrete representation.
This model begins to develop the understanding of the commutativity of addition, as pupils become aware that the parts will make the whole in any order.



10 = 6 + 4
10 – 6 = 4
10 – 4 = 6
10 = 4 + 6

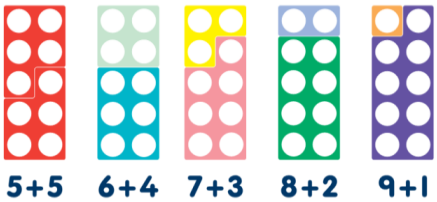
<u>Using number bonds & facts:</u>

Children use a variety of representations to learn number bonds within 20.



This includes the 'story' of a number
e.g. 7 = 7+0, 6+=1, 5+2,

Numicon can be used to create the story of a number eg this is the story of 10



Subtraction – as take away

Children will use Numicon pieces and pegs to take away by physically removing the pegs.



7-4=3

Subtraction – as difference

Children will use numicon pieces to solve problems e.g.

7 = ☐ +4

By placing the 3 piece on top of the 7 piece you can see that the missing number is 4.

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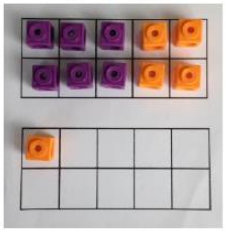
'Make ten' strategy

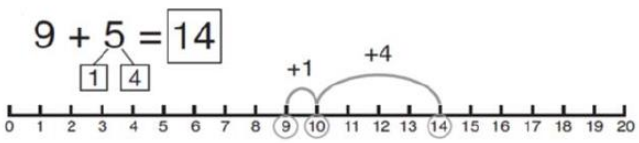
Pupils should be encouraged to start at the bigger number and use the smaller number to make ten. The colours of the beads on the bead string make it clear how many more need to be added to make ten.

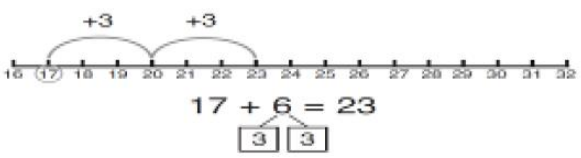
Also, the empty spaces on the ten frame make it clear how many more are needed to make ten.

6 + 5 = 11

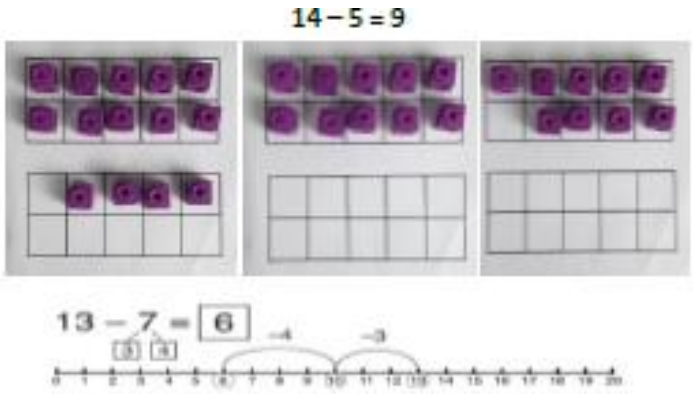


4 + 9 = 13





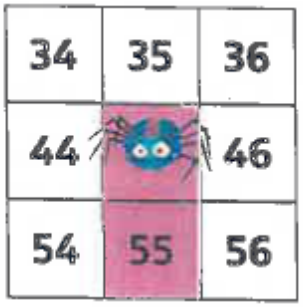
Single digit number from a 2-digit number - Pupils identify how many need to be taken away to make ten first. Then they take away the rest to reach the answer.



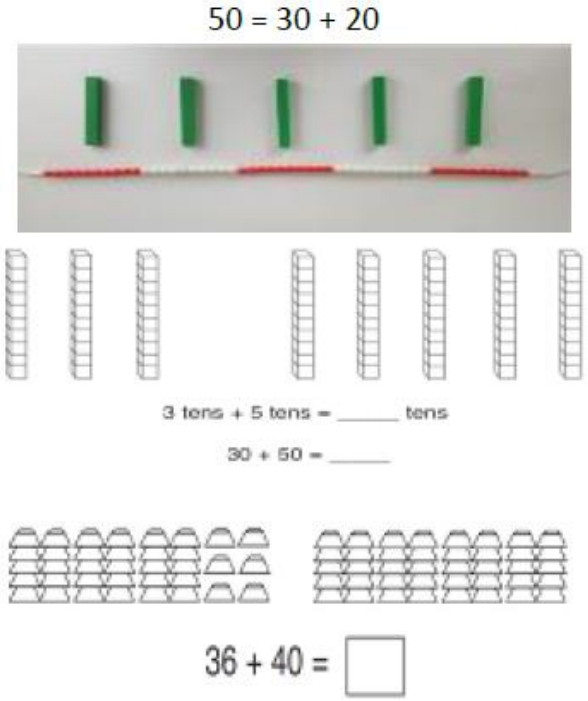
Multiples of ten:

The 100 square can be used to demonstrate how to count on in 10s from any starting number.

45+10=55
55-10=45



Using the vocabulary of 1 ten, 2 tens, 3 tens etc. alongside 10, 20, 30 is important as pupils need to understand that it is a ***ten*** *not a one that is being taken away / added.*
It also emphasises the link to known number facts. E.g. '2 + 3 is equal to 5. So 2 tens + 3 tens is equal to 5 tens.



+ = signs and missing numbers:

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

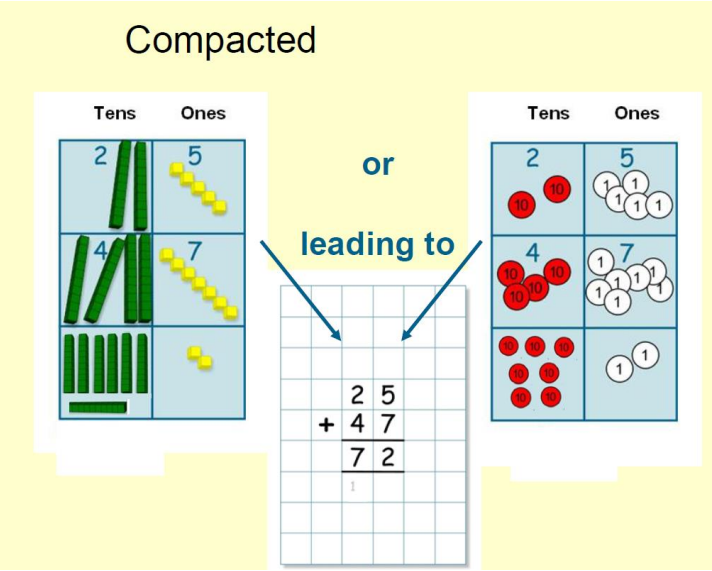
2 = 1+ 1
2 + 3 = 4 + 1
3 = 3
2 + 2 + 2 = 4 + 2

Missing numbers need to be placed in all possible places.

3 + 4 = □	□ = 3 + 4
3 + □ = 7	7 = □ + 4
□ + 4 = 7	7 = 3 + □
□ + ∇ = 7	7 = □ + ∇
7 - 3 = □	□ = 7 - 3
7 - □ = 4	4 = □ - 3
□ - 3 = 4	4 = 7 - □
□ - ∇ = 4	4 = □ - ∇

Continue using a range of equations as in Year 1 but with appropriate, larger numbers.
Extend to 14 + 5 = 10 + □
and 32 + □ + □ = 100 35 = 1 + □ + 5

Teaching TO+TO and TO-TO:



Use concerte materials (Numicon, PV Counters) to model TO+TO leading to the compacted formal method. Ensure to reinforce that ten ones equals one ten.

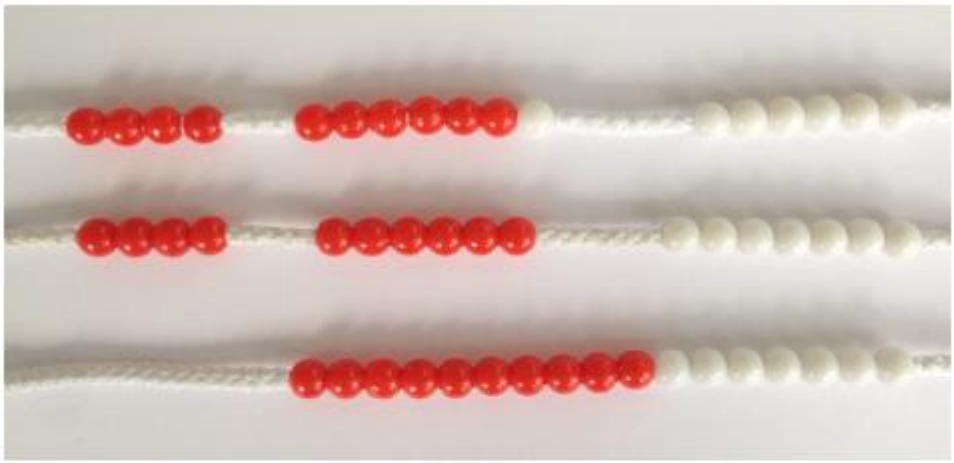
Adding three one-digit numbers:

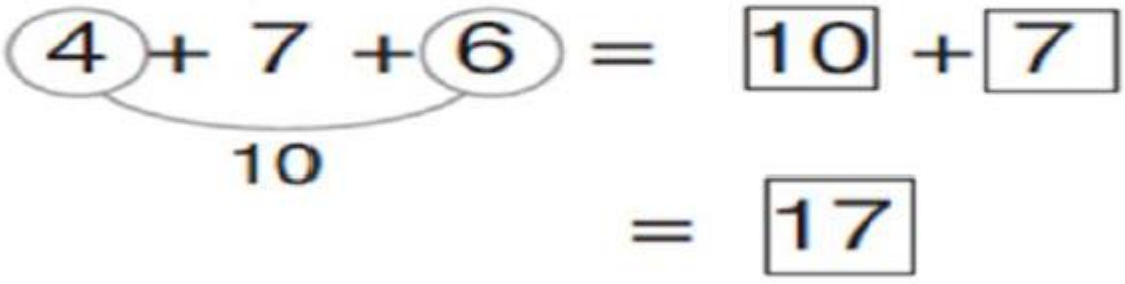
Pupils may need to try different combinations before they find the two numbers that make 10.

The first bead string shows 4, 7 and 6. The colours of the bead string show that it makes more than ten.

The second bead string shows 4, 6 and then 7.

The final bead string shows how they have now been put together to find the total.





Lower Key Stage 2

Year 3 Objectives:

Pupils should be taught to:

- *add and subtract numbers mentally, including:*
    - *a three-digit number and ones*
    - *a three-digit number and tens*
    - *a three-digit number and hundreds*
- *add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction*
- *estimate the answer to a calculation and use inverse operations to check answers*
- *solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.*
-

Year 4 Objectives:

Pupils should be taught to:

- *add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate*
- *estimate and use inverse operations to check answers to a calculation*
- *solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.*

See Year 2 – using the same strategies and, where required, manipulatives with numbers appropriate to the unit pupils are working within (3-digit numbers) .

Pupils learn that they are able to use known number-facts and other mental strategies to complete mental calculations with accuracy.
To begin with, some pupils will prefer to use this strategy only when there is no need to regroup, using number facts within 10 and derivations. More confident pupils might choose from a range of mental strategies that avoid regrouping, including (but not exhaustively): known number facts within 20, derived number facts, make ten, compensation (see Year 2 guidance for exemplification of these – the use of concrete manipulatives other than Dienes blocks is important in reinforcing the use of these other strategies).
It is important that pupils are given plenty of (scaffolded) practice at choosing their own strategies to complete calculations efficiently and accurately. Explicit links need to be made between familiar number facts and the calculations that they can be useful for and pupils need to be encouraged to aim for efficiency.

+ = signs and missing numbers

Continue using a range of equations as in Year 1 but with appropriate, larger numbers.

Formal written method (columnar addition and subtraction):

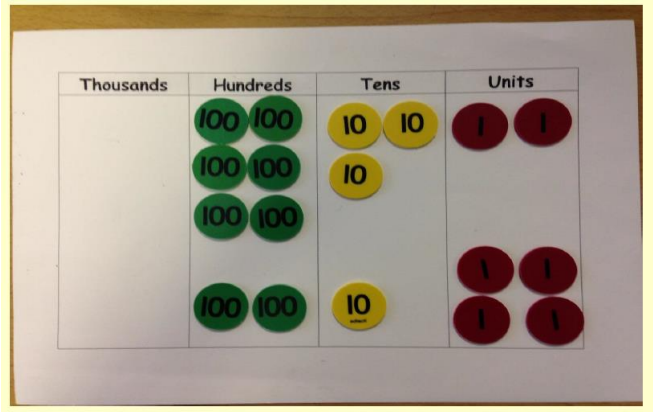
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction. Still use concrete and manipulatives, particularly place value counters, to model this (see Year 2).

From this, children will begin to carry below the line.

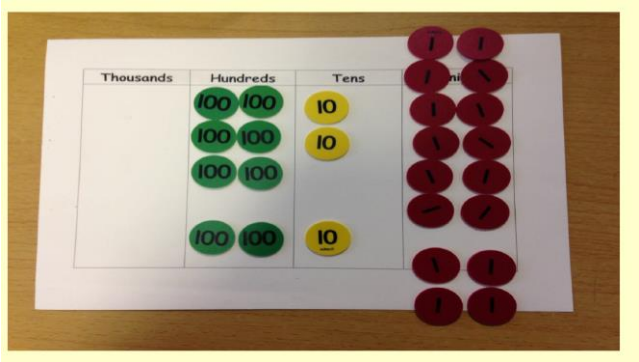
625	783	367
+ 48	+ 42	+ 85
673	825	452
1	1	1 1

Children subtract numbers of up to three-digits using the formal written method. Their work on partitioning in different ways will aid their understanding of this.

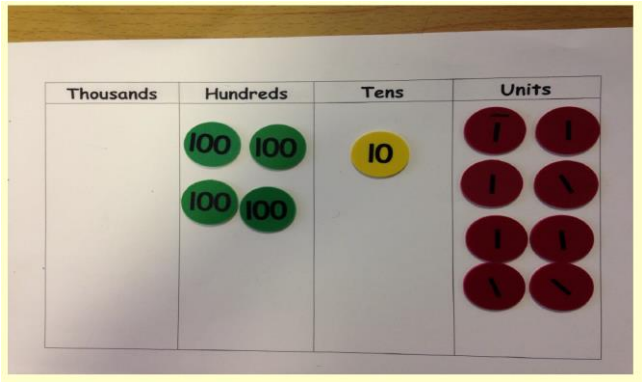
E.g. 632-214



We can't do 2-4 so we exchange one ten for ten ones:



Now we can do the calculation as we have 12 ones take 4 ones, 2 tens take 1 ten and 6 hundreds take 2 hundreds leaving us with:



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This method would be refined by the children over time like so:

```
  614 1
   7 5 4
 -   8 6
 ---------
   6 6 8
```

As they progress into Year 4, children develop their understanding of the column method, extending to four-digit numbers.

```
    7  8  9
+   6  4  2
-----------
1   4  3  1
    1  1
```

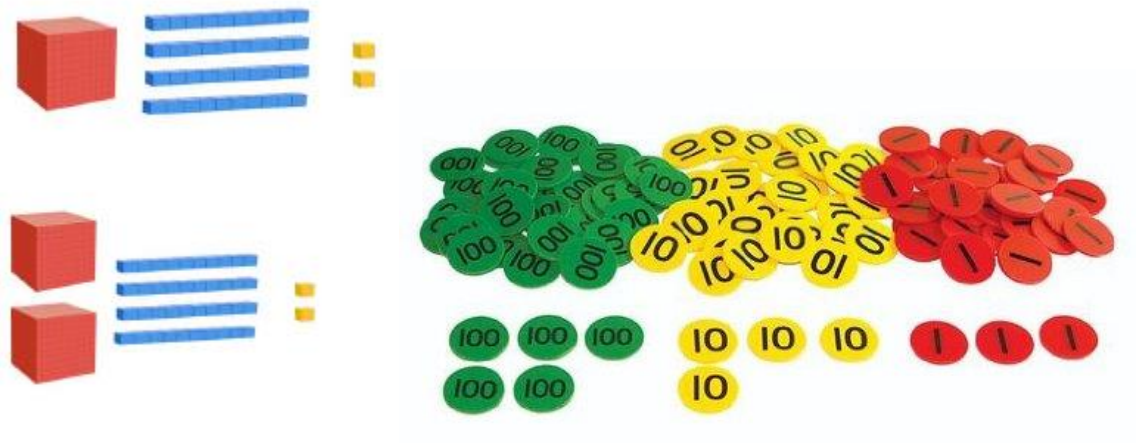
Answer:
1431

In Year 4, using the method, children will:

- ✓ extend to four-digit numbers
- ✓ add and subtract several numbers with different numbers of digits;
- ✓ begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;
- ✓ know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.

Find 10, 100 and 1000 more or less than a given number

As pupils become familiar with numbers up to 1000, place value should be emphasised and comparisons drawn between adding tens, hundreds (revision - see Year 2 guidance) and thousands, including use of concrete manipulatives and appropriate images.
After initial teaching, this should be incorporated into transition activities and practised regularly.



Upper Key Stage Two

Year 5 Objectives:

Pupils should be taught to:

- *add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)*
- *add and subtract numbers mentally with increasingly large numbers*
- *use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy*
- *solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.*

Formal written method (columnar addition):

Children should extend the carrying method to numbers with at least four digits.

```
   587          3587
 + 475        +  675
  1062         4262
   1 1          1 1 1
```

Using the method, children will:

- ✓ add several numbers with different numbers of digits;
- ✓ begin to add two or more decimal fractions with up to three digits and the same and/or different number of decimal places;
- ✓ know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m – 280 cm.

There are no formal addition objectives in Year 6. Children should practise learned methods with larger numbers and in different contexts.

Children should extend the carrying method to number with any number of digits and including decimals.

```
   7648          6584
 + 1486        + 5848
   9134         12432
   1 1 1        1 1 1
```

Using the method, children will:

- ✓ add several numbers with different numbers of digits;
- ✓ begin to add two or more decimal fractions with up to four digits and either one or two decimal places;
- ✓ know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 401.2 + 26.85 + 0.71.

Multiplication & Division

Key Stage One:

Year 1 Objectives:

Pupils should be taught to:

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

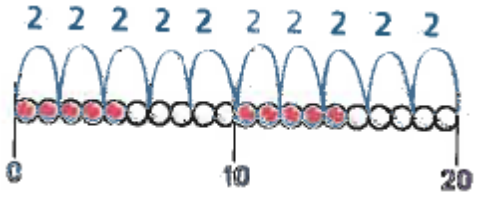
Year 2 Objectives

Pupils should be taught to:

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

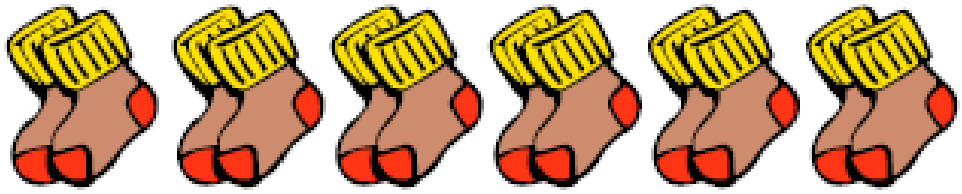
Counting in 2's, 5's and 10's in different ways:

1	2	3	4	5	6	7	8	9	
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



Children will experience equal groups of objects and will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups. Then using that counting with grouping
E.g. there are six pairs of socks. How many all together?

2, 4, 6, 8, 10, 12



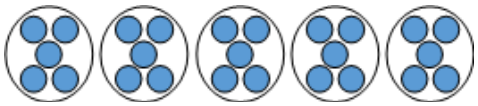
Counting in 2's, 5's and 10's. Count in different ways, leading to children learning the x-table facts in Year 2.

Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.

Develop ability to group objects practically and mentally.

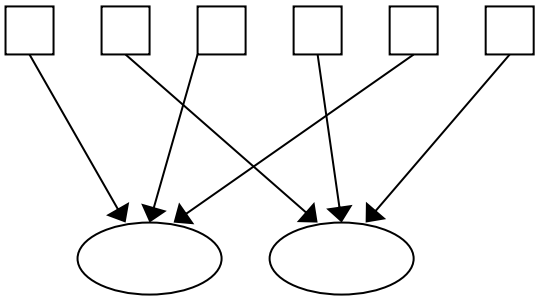
Share twenty crayons between two pots. How many in each pot?

e.g. 25 ÷ 5 =



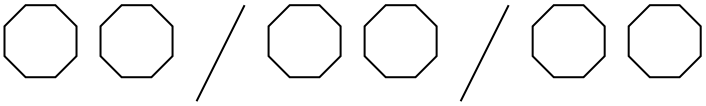
Sharing equally

6 sweets shared between 2 people, how many do they each get?



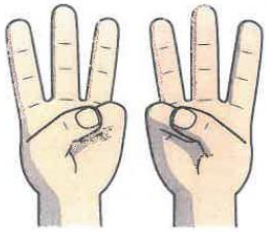
Grouping or repeated subtraction

There are 6 sweets, how many people can have 2 sweets each?

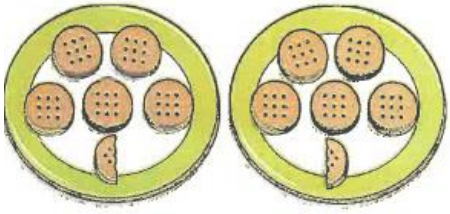


Doubles & halves:

Using fingers to find doubles – e.g. Double 3 is 6. Halve of 6 is 3.



Use concrete representations to find halves of add numbers. E.g. half of 11 = 5.5



Progress to learning basic doubles and halves by heart (Rapid Recall).

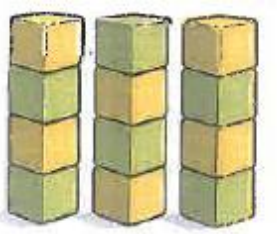
Fractions

Relate finding unit fractions of numbers to division (e.g. divide by 4 to find a quarter).

Arrays:

Setting out objects or marks in arrays - this can be used to teach the family of multiplication facts:

Three lots of four is 12



3 x 2 = 6 and 2 x 3 = 6

6 ÷ 2 = 3 and 6 ÷ 3 = 2

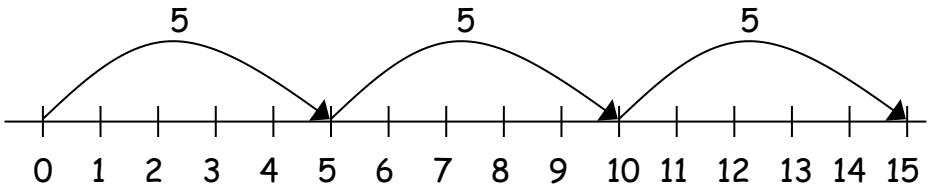
The commutative law and concept of inverse are explored through arrays.

Repeated addition

3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x 3

Repeated addition can be shown easily on a number line:

5 x 3 = 5 + 5 + 5



Mental Methods – larger numbers

Use partitioning into tens and units e.g.

17 x 3 = 10 x 3 = 30

7 x 3 = 21

30 + 21 = 51

x ÷ = signs and missing numbers

7 x 2 = □	□ = 2 x 7
7 x □ = 14	14 = □ x 7
□ x 2 = 14	14 = 2 x □
□ x ∇ = 14	14 = □ x ∇
6 ÷ 2 = □	□ = 6 ÷ 2
6 ÷ □ = 3	3 = 6 ÷ □
□ ÷ 2 = 3	3 = □ ÷ 2
□ ÷ ∇ = 3	3 = □ ÷ ∇

Lower Key Stage Two

Objectives:

Pupils should be taught to:

- *recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables*
- *write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods*
- *solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.*

Objectives:

Pupils should be taught to:

- *recall multiplication and division facts for multiplication tables up to 12 × 12*
- *use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers*
- *recognise and use factor pairs and commutativity in mental calculations*
- *multiply two-digit and three-digit numbers by a one-digit number using formal written layout*
- *solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.*

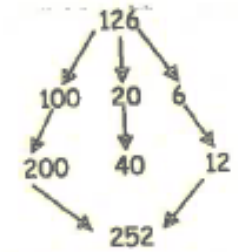
Counting:

In a variety of different ways, specifically in 3's, 4's and 8's in Year 3. By year 4, they should be able to count in any multiples up to 12, and 50, 100 and 1000.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Doubling & halving:

Use partitioning e.g. double 126

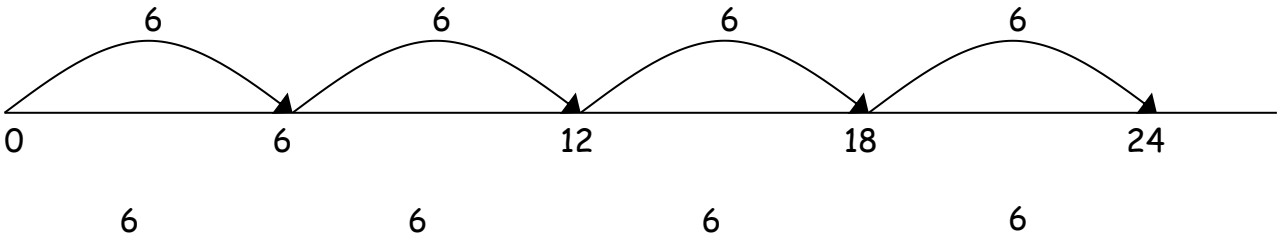


e.g. half of 48

Repeated addition

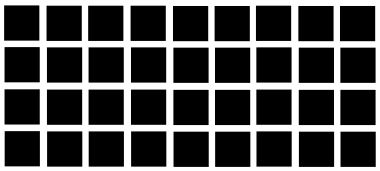
4 times 6 is 6 + 6 + 6 + 6 = 24 or 4 lots of 6 or 6 x 4

Children should use number lines or bead bars to support their understanding.



Arrays

Children should be able to model a multiplication calculation using an array. This knowledge will support learning that multiplication and division are the inverse.



4 x 9 = 36

9 x 4 = 36

Children will also develop an understanding of

Scaling

e.g. Find a ribbon that is 4 times as long as the blue ribbon

5 cm

20 cm

x = signs and missing numbers

□ x 5 = 20 3 x △ = 18 □ x O = 32

Mental methods for larger numbers:

17x 5 = (10x 5) + (7x5)

= 50+35
=85

Formal written method (columnar multiplication - short)

Vertical format, compact version for multiplication by units:
eg 24 x 6 becomes

$$\begin{array}{r} 2\ 4 \\ \times\quad 6 \\ \hline 1\ 4\ 4 \\ \hline \quad 2 \end{array}$$

Answer:
144

In Year 4, Extend up to a three-digit number a by a one-digit number:

Formal written methods – short division

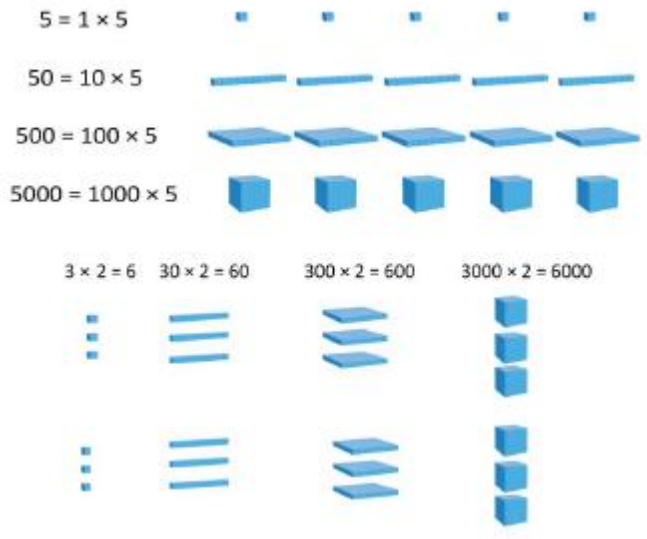
School have decided that the formal written method for short division will be taught in Year 4
Begin short division once understanding of remainders is secure
98 ÷ 7 becomes

$$\begin{array}{r} 1\ 4 \\ 7\overline{)\ 9\ 8} \\ \quad\ 2 \end{array}$$

Answer: 14

This should be modelled with concrete apparatus first e.g. Cuisenaire rods or place value counters.
Extend short division to three digits including with remainders.

Using known facts for multiplying by multiples of 10, 100 and 1000
Pupils' growing understanding of place value, allows them to make use of known facts to derive multiplications using powers of 10.
It is important to use tables with which they are already familiar (i.e. not 7 or 9 tables in Year 3)



Upper Key Stage 2:

Year 5 Objectives:

Pupils should be taught to:

- *identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers*
- *know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers*
- *establish whether a number up to 100 is prime and recall prime numbers up to 19*
- *multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers*
- *multiply and divide numbers mentally drawing upon known facts*
- *divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context*
- *multiply and divide whole numbers and those involving decimals by 10, 100 and 1000*
- *recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})*
- *solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes*
- *solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign*
- *solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.*

Year 6 Objectives:

Pupils should be taught to:

- *multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication*
- *divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context*
- *divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context*
- *perform mental calculations, including with mixed operations and large numbers*
- *identify common factors, common multiples and prime numbers*
- *use their knowledge of the order of operations to carry out calculations involving the four operations*
- *solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why*

Formal written method (columnar multiplication – short & long)

Consolidate short multiplication (as Year 4 but with appropriate numbers) and extent to long multiplication, first with two-digit number by two-digit numbers then three-digit numbers by two-digit numbers.

56 x 27 = 1512

$$\begin{array}{r} 56 \\ \times\ 27 \\ \hline 392 \\ +\ 1120 \\ \hline 1512 \\ \hline 1 \end{array} \quad \begin{array}{l} \\ \\ (7\times56) \\ (20\times56) \\ \\ \\ \end{array}$$

124 x 26 = 3224

$$\begin{array}{r} 124 \\ \times\ 26 \\ \hline 744 \\ +\ 2480 \\ \hline 3224 \\ \hline 11 \end{array} \quad \begin{array}{l} \\ \\ (6\times124) \\ (20\times124) \\ \\ \\ \end{array}$$

In year 6, consolidate from Year 5 and extend to decimal numbers.

$$\begin{array}{r} 53{\cdot}2 \\ \times\ 24{\cdot}0 \\ \hline 212{\cdot}8 \\ 1064{\cdot}0 \\ \hline 1276{\cdot}8 \end{array} \quad \begin{array}{l} \\ \\ (53{\cdot}2\times4) \\ (53{\cdot}2\times20) \\ \\ \end{array}$$

Formal written methods – short division

Extend short division to four digits including with remainders. Express the remainders in different ways e.g. fractions or decimals.

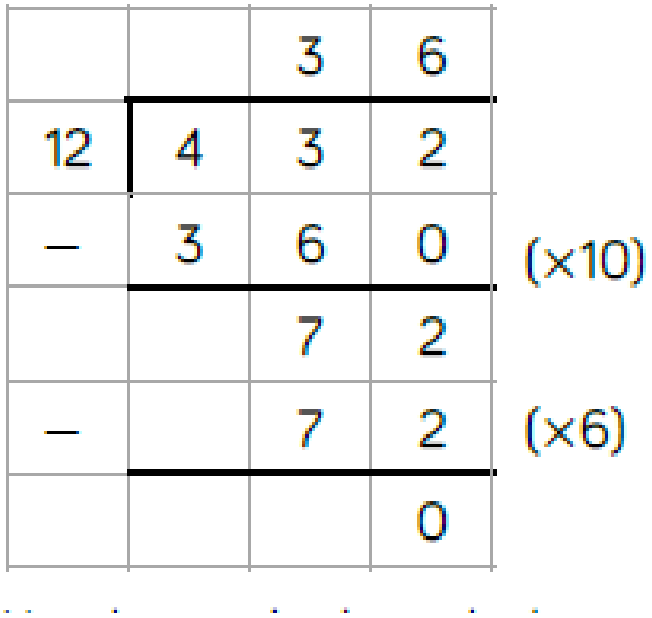
496 ÷ 11 becomes

$$\begin{array}{r} 45\ r1 \\ 11\overline{)\,496} \\ 5 \end{array}$$

Answer: $45\frac{1}{11}$

Formal written methods – long division

In year 6, extend to formal long division. Express the remainders in different ways e.g. fractions or decimals.



		3	6	
12	4	3	2	
–	3	6	0	(×10)
		7	2	
–		7	2	(×6)
			0	

Multiples to help

$12 \times 1 = 12$

$12 \times 2 = 24$

$12 \times 5 = 60$

$12 \times 10 = 120$

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Children should not be made to go onto the next stage if:

1) **they are not ready.**
2) **they are not confident.**

Children should be encouraged to approximate their answers before calculating.
Children should be encouraged to check their answers after calculation using an appropriate strategy.
Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.