

Teaching Calculations

Guidance for Teachers

Highfield Schools

The following guidance aims to provide a consistent approach to the written and mental calculation methods. Guidance was taken from 'The Calculation Guidance: NCETM October 2015'.

The main aims of the calculations guidance are to:

- Create love and enthusiasm for Maths
- Develop a deeper understanding of calculations
- Improve pupils' ability to calculate accurately
- Ensure a consistent approach within schools
- Support Medium Term planning at Highfield Schools

Addition

Definition: Addition is finding or calculating the total of two or more sets, numbers or amounts. It is the inverse of subtraction.

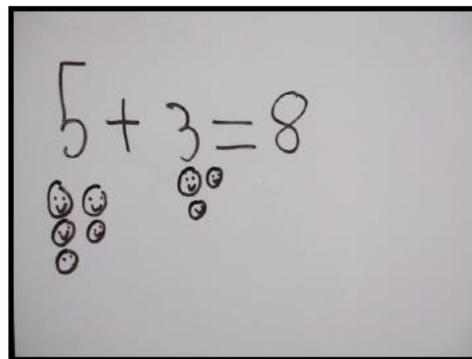
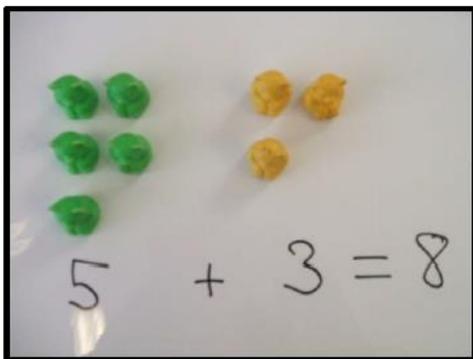
Core skills

To progress and use efficient methods of addition, pupils will need to be able to:

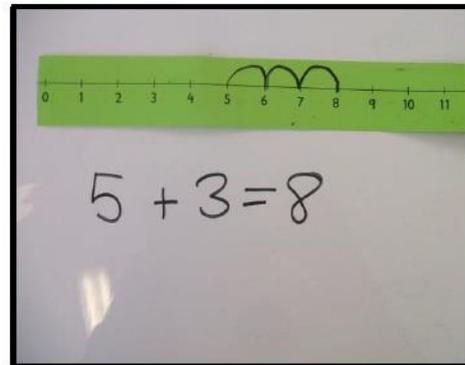
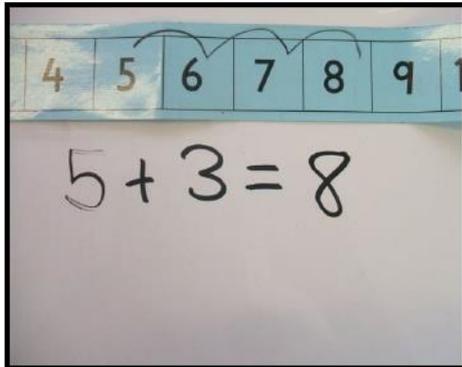
- Use one to one correspondence
- Count forwards and backwards
- Understand that addition is commutative
- Partition
- Have rapid recall of number bonds
- Understand addition as the inverse of subtraction.

Early learning

Pupils use concrete and pictorial methods for calculating.

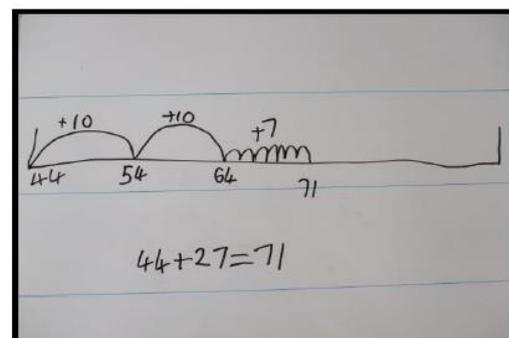
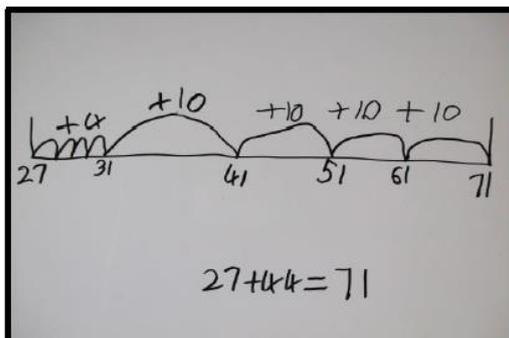
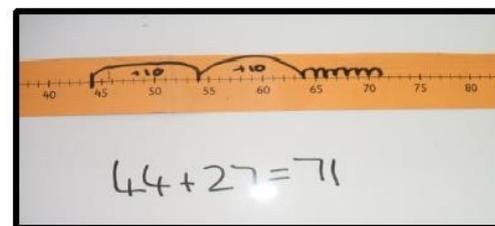
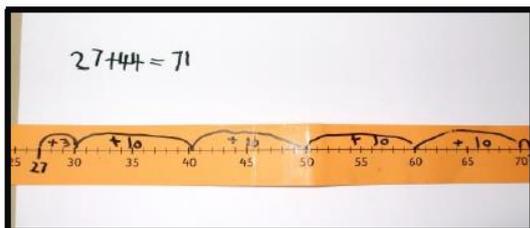


Using the core skill of counting forwards, pupils use number tracks and marked number lines to find the total. This can be shown with a bar model as well as using concrete resources.



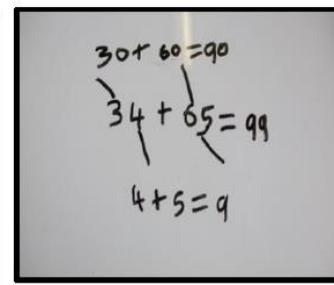
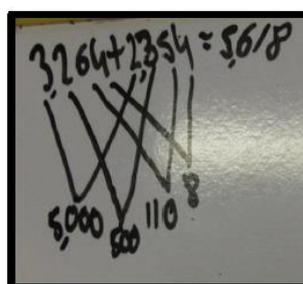
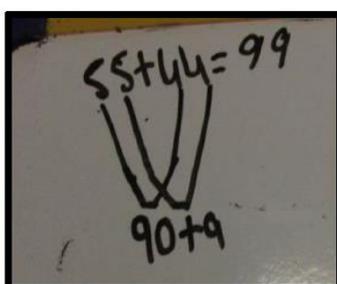
Written methods

Counting on an unmarked number line using various efficient methods



Progress to Partitioning

Partition and add horizontally:



Partition horizontally and add vertically:

$$34 + 65 = 99$$

$$30 + 60 = 90$$

$$5 + 4 = 9$$

$$90 + 9 = 99$$

$$25.6 + 32.2 = 57.8$$

$$20 + 30 = 50$$

$$30 + 2 + 0.2 = 57.8$$

$$1000 + 9000 = 10,000$$

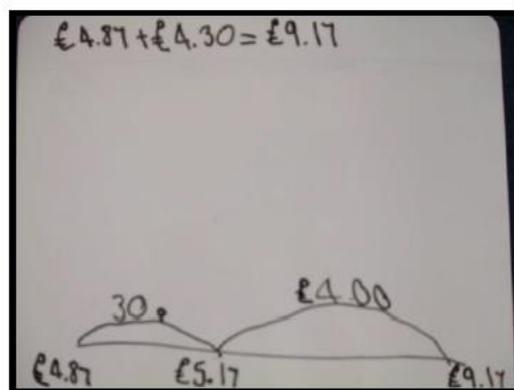
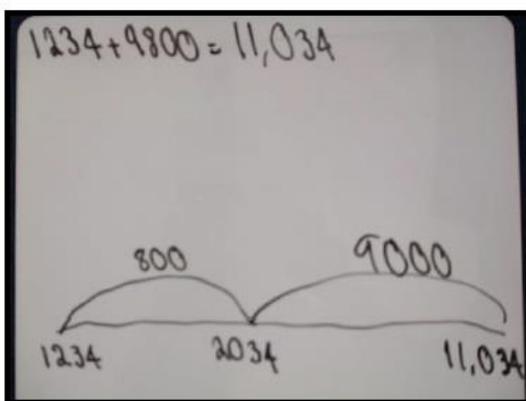
$$200 + 600 = 800$$

$$30 + 30 = 60$$

$$4 + 1 = 5$$

$$1234 + 9631 = 10,865$$

Add on an empty number line:



Pupils continue to partition and move towards vertical methods to increase efficiency and fluency.

$$\begin{array}{r} 53 \\ + 46 \\ \hline 9 \quad (3+6) \\ 90 \quad (50+40) \\ \hline 99 \end{array}$$

$$\begin{array}{r} 26.4 \\ + 32.6 \\ \hline 1.0 \quad (0.4+0.6) \\ 8.0 \quad (6+2) \\ 50.0 \quad (20+30) \\ \hline 59.0 \end{array}$$

Students then progress on to the formal written method. Students are then given the choice to choose the most appropriate method for the calculation they are completing for which they are most confident.

Subtraction

Definition: Subtraction is the taking away of one number or amount from another, or finding the difference between two numbers or amounts.

Core Skills:

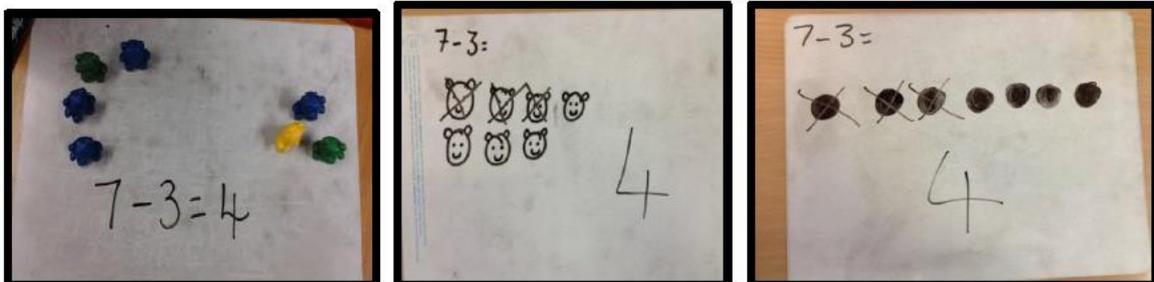
To progress and use efficient methods of subtraction, pupils will need to be able to:

- Count forwards and backwards
- Understand that subtraction is not commutative
- Partition numbers
- Rapidly recall number bonds
- Understand that subtraction is the inverse of addition
- Recognise the number to be subtracted.

Early Learning

Children are encouraged to use a range of concrete resources. In subtraction, resources are physically taken away. They then develop ways of recording calculations using pictures, crossing out these images to represent subtraction. These pictures may begin as representations of the problem but will develop to more informal representations.

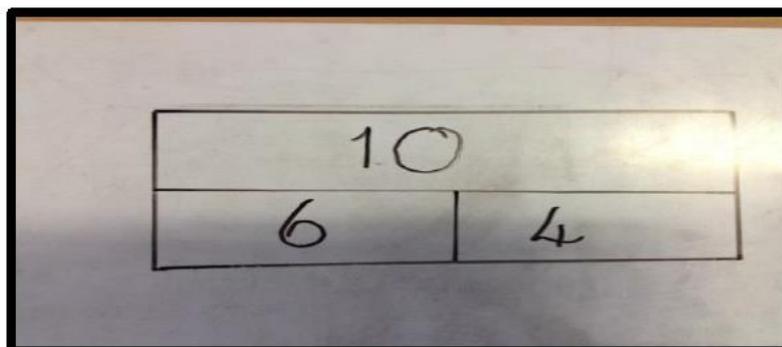
e.g. 7 bears are having a party. 3 bears leave. How many bears are left at the party?



Bar Model

A further method for more abstract pictorial representation is the bar model, which supports children to visualise the part-part-whole model.

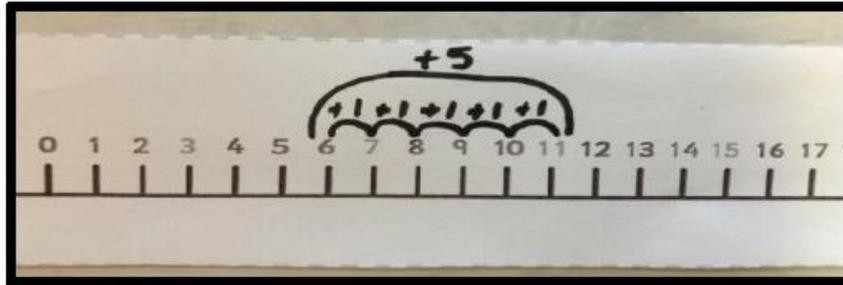
e.g. $6 + 4 = 10$



Number Tracks and Number Lines

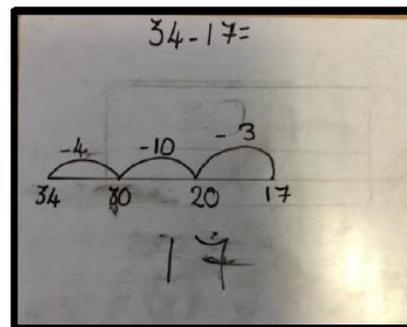
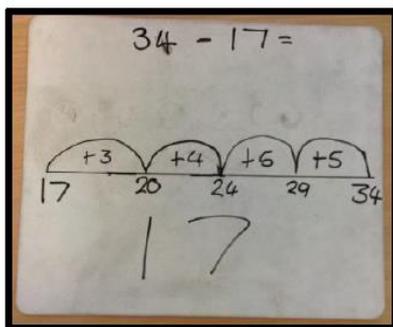
Use marked number lines alongside concrete objects and pictorial representations. Include both counting up to find the difference and counting back to subtract.

e.g: counting up on a marked number line to show $11 - 5 = 6$



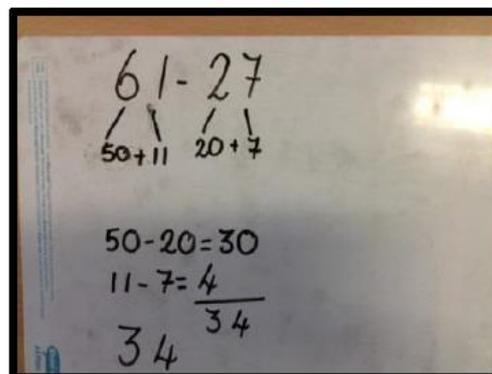
Written Methods – 2, 3 and 4 Digits

Use number lines to count on or back in ones to begin with, and develop more efficient jumps. As understanding increases, progress to empty number lines.



Progress to partitioning:

Initially, the children will be taught using examples where exchange is not required. They begin with partitioning the numbers and subtracting the tens and ones separately before recording underneath:



Begin with examples where 'exchanging' is necessary and encourage partitioning of the number in different ways. This method should be taught in a range of contexts to increase understanding, including larger numbers.

$$61 - 27 = 34$$

60	11
20	7
30	4

$$341 - 186$$

300	130	11
100	80	6
100	50	5

155

$$5793 - 967$$

5000	1700	90	13
	900	60	7
4000	800	20	6

4826

$$£6.50 - £2.81$$

5.00	14	0.4	0.10
6.00	0.5	0.00	
2.00	0.8	0.01	
3.00	0.6	0.09	

£3.69

Develop an understanding of subtracting negative numbers:

$$341 - 273 =$$

$$300 - 200 = 100$$

$$40 - 70 = -30$$

$$1 - 3 = -2$$

68

Decimals

All written methods can be applied to different contexts, such as decimal subtraction.

$$£10.00 - £4.28$$

10.00	5.00	4.50	4.30	4.28
	-5.00	-0.50	-0.20	-0.02

£5.72

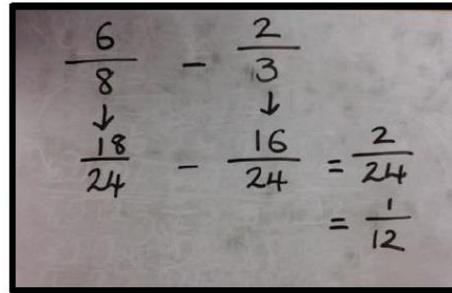
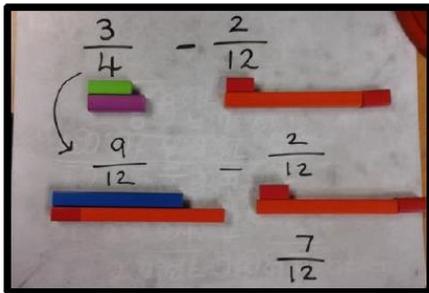
$$£12.50 - £4.67$$

12.50
- 4.67
7.83

£7.83

Fractions

Use concrete and pictorial methods along with abstract to subtract fractions.



Multiplication

Definition: Multiplication is the product of two numbers or repeatedly adding the same set of number as many times as the other number. Therefore 3 multiplied by 4, is 4 lots of 3, or 3 added repeatedly 4 times. It is an inverse of division.

Core Skills:

To progress and use efficient methods of multiplication, pupils will need to be able to:

- Count forwards and backwards in multiples e.g. 2's, 4's 10's.
- Use repeated addition
- Rapidly recall multiplication facts
- Partition
- Understand place value
- Multiply and divide by 10, 100 and 1000
- Double and halve
- Use division as the inverse of multiplication.

Early Learning

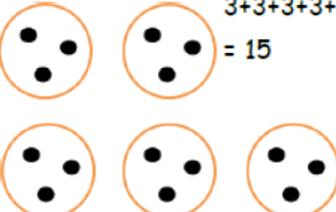
Pupils manipulate and experience a range of resources in real life contexts and play. They are encouraged to solve real life problems.

How many legs will 3 teddies have?



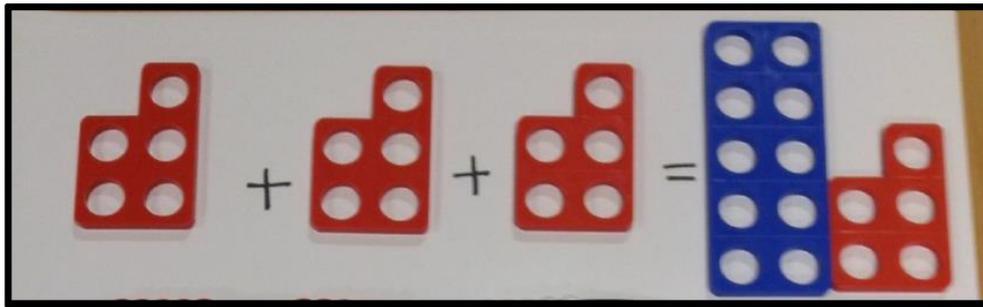
$2 + 2 + 2 = 6$

There are 3 sweets in one bag.
How many sweets are in 5 bags altogether?



$3+3+3+3+3 = 15$

They draw pictures and show their mathematical thinking through various representations, e.g. pennies, cubes, bead strings, Numicon.

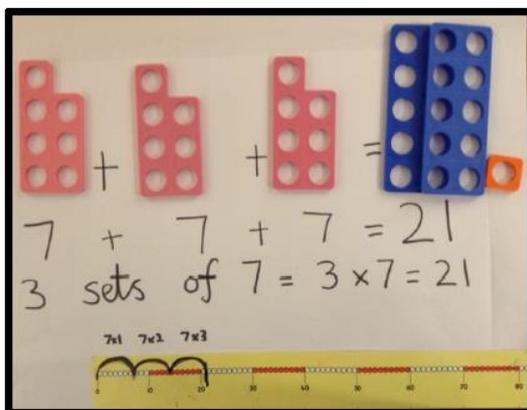


Children recall multiplication facts and develop strategies to use known facts for further calculations.

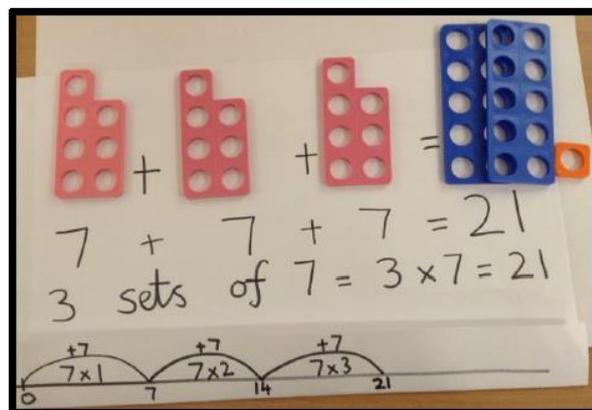
Number lines

Verbalise multiplication as 'lots of' or 'groups of' e.g. $5 \times 3 =$ "3 lots of 5" to ensure that the children make the connection with repeated addition. Use number lines alongside concrete and pictorial, drawing links with measure and money.

On a marked number line:



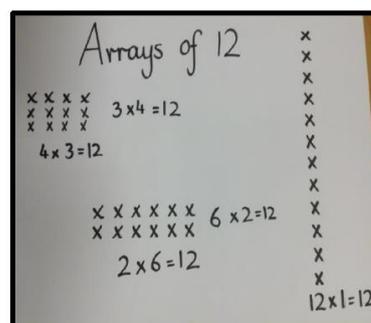
On an empty number line:



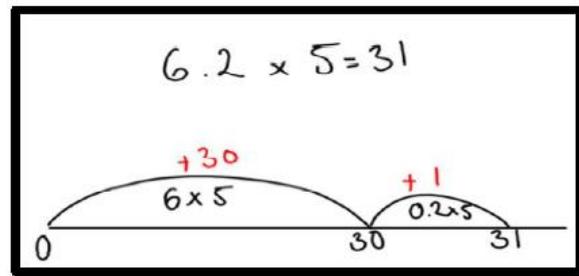
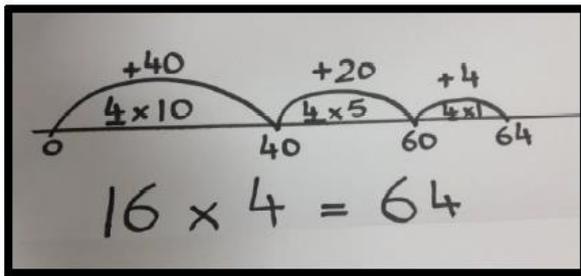
Arrays

Arrays are taught alongside number lines to help prepare for grid method. Count groups of objects in each row or column to find the product. Give a number of objects to arrange in an array of various dimensions and also give an unknown number of objects to count by arranging in arrays.

3 lots of 5 = 5 groups of 3
 $5 \times 3 = 3 \times 5$
 $5 + 5 + 5 = 3 + 3 + 3 + 3 + 3 = 15$



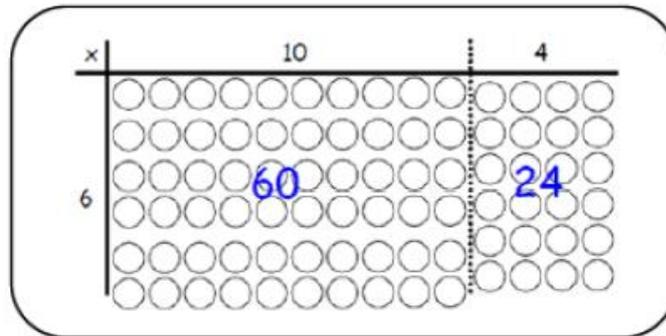
Partitioning using Number Lines and Grids



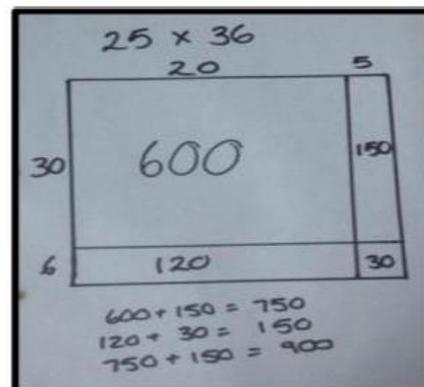
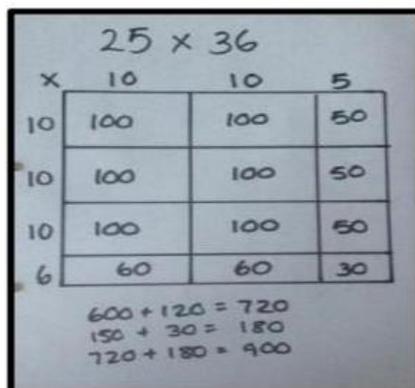
This should be taught at the same time as arrays and can be done using decimals as well.

Grid method

Progress to grid method, using increasingly larger numbers making links with arrays.



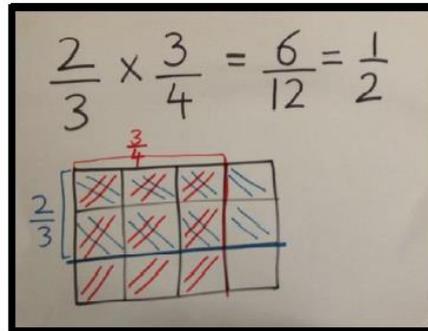
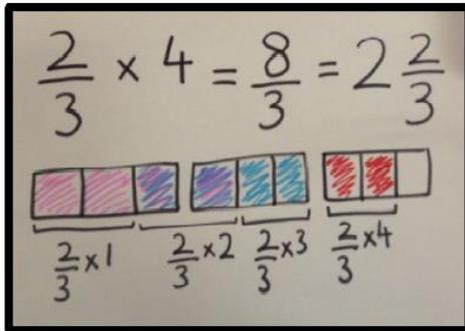
Multiply two digit numbers by partitioning and keeping it proportional.



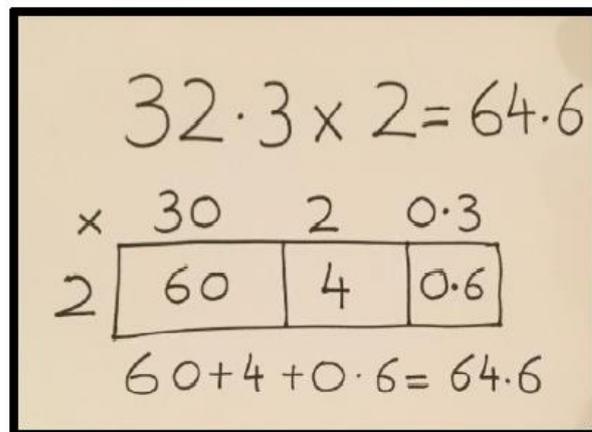
NOTE: when teaching grid method. NEVER say take off 0's and add them back on. Develop fluency skills using known facts e.g. $20 \times 30 = 2 \times 10 \times 3 \times 10$.

Multiplication of fractions and decimals:

Use of bar model is encouraged to develop understanding:



Multiply with decimals using grid method and partitioning.



Division

Definition: Division is repeated subtraction or splitting into equal parts or groups. It is the result of "fair sharing".

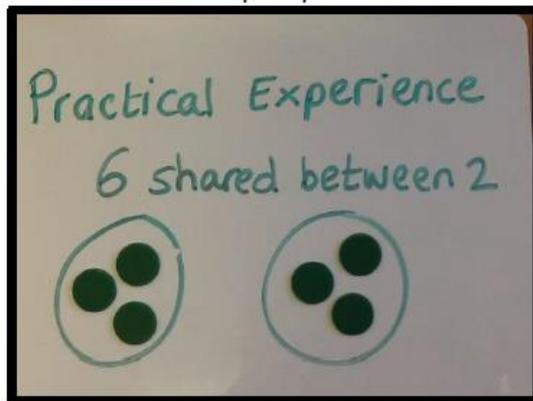
Core Skills:

To progress and use efficient methods of division, pupils will need to be able to:

- Recall multiplication facts
- Understand division as the inverse of multiplication
- Count accurately
- Understand place value
- Multiply and divide by 10, 100 and 1000
- Double and halve

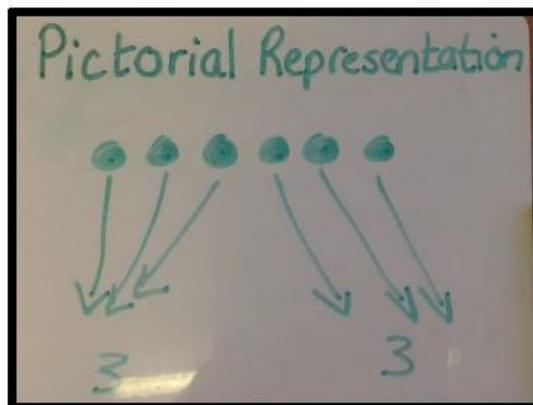
Early Learning

Children use concrete resources to share equally in real life contexts.



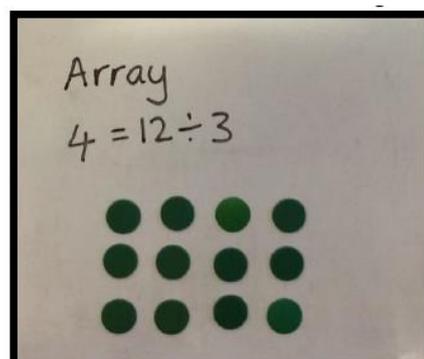
Pictorial Representation

Children share equally using pictures to support their understanding.



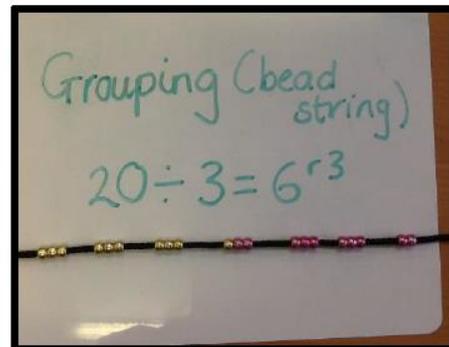
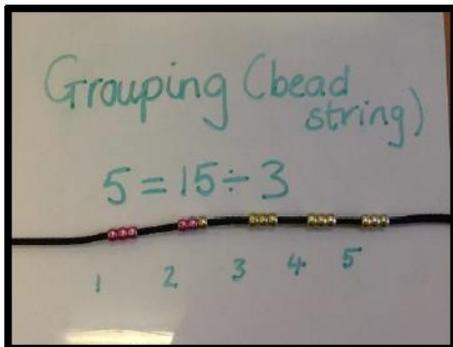
Arrays

Children use concrete and pictorial representations for division calculations (including recognising multiplication as the inverse of division through arrays).



Grouping (bead string)

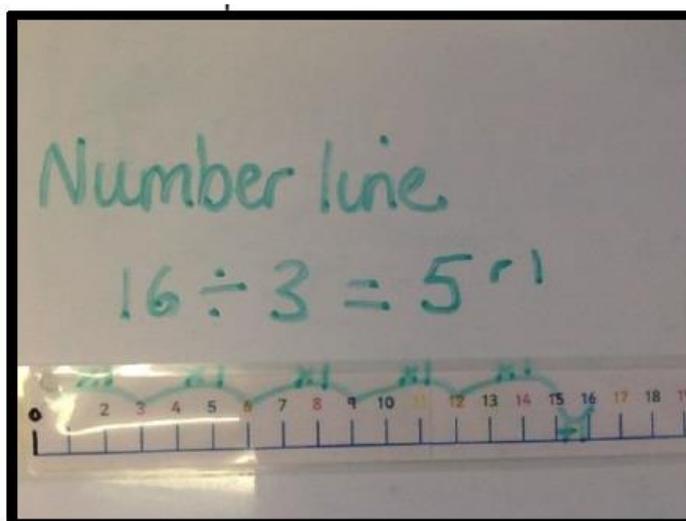
Children learn that dividing can be done as groups including equal sharing and those with remainders.



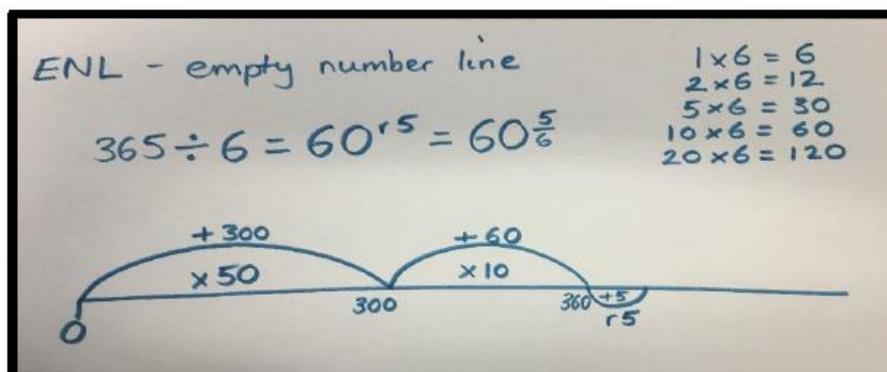
Number lines:

(Counting on or back – addition or subtraction)

Children will count in 'lots of' or 'multiples' of the divisor on marked number lines.



Children will progress on to an empty number line, using the strategies above of counting in 'lots of' or multiples (chunks). To support this method, children list known facts to help them (can be known as coin multiples – 1x/2x/5x/10x/20x/50x/100x).



Chunking

Subtract multiples of the divisor from the starting number. Again, the children list the 'coin multiples' to support their chunking.

Chunking can be done by subtracting each chunk or by beginning at 0 and adding on each multiple until the total is reached (possibly with a remainder).

Chunking
Top down

$$2365 \div 67 = 35^{r20} \text{ or } 35\frac{20}{67}$$

	2365	
20 x 67 =	1340	
	<u>1025</u>	
10 x 67 =	670	
	<u>355</u>	
2 x 67 =	134	
	<u>221</u>	
2 x 67 =	134	
	<u>87</u>	
1 x 67 =	67	
	<u>20</u>	

Chunking
counting up

$$536 = 695 \div 13$$

+ 650	(13 x 50)	1 x 13 = 13
<u>650</u>		2 x 13 = 26
+ 13	(13 x 1)	5 x 13 = 65
<u>663</u>		10 x 13 = 130
+ 13	(13 x 1)	20 x 13 = 260
<u>676</u>		50 x 13 = 650
+ 13	(13 x 1)	
<u>689</u>		
		695
		<u>689</u>
		remainder 6

50 + 1 + 1 + 1 = 53

Formal Written Method:

$$\begin{array}{r} 121.5 \\ 6 \overline{) 729.0} \end{array}$$

$$\begin{array}{r} 143 \\ 6 \overline{) 858} \end{array}$$

$$888 \div 37 = 24$$

10	x 37 =	888
		<u>370</u>
10	x 37 =	4518
		<u>370</u>
2	x 37 =	0x148
		<u>74</u>
2	x 37 =	<u>74</u>
		<u>74</u>
		0

Algebra

What is algebra?

Algebra is a way of thinking and a set of concepts and skills that enable pupils to generalise, model and analyse mathematical situations.

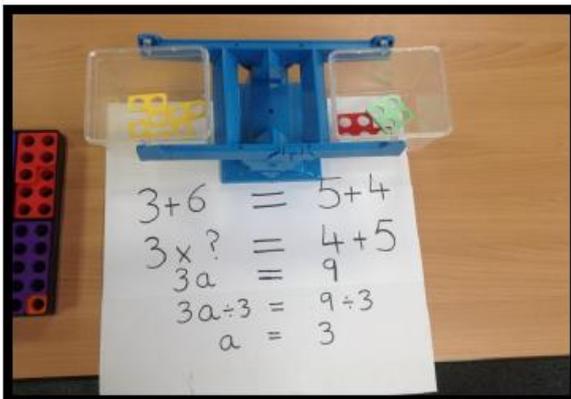
Why is it important?

Algebra provides a systematic way to investigate relationships, helping to describe, organise, and understand the world.

Although learning to use algebra makes students powerful problem solvers, these important concepts and skills take time to develop. Understanding of Algebra begins early and should be a focus of mathematics instruction throughout students education.

Equals sign =

In Maths equality (=) means balance between two sets and inequality (\neq) means an imbalance.



Algebra requires children to solve simple equations that involve addition, subtraction, multiplication and division with a deeper understanding of the equality symbol. Using concrete resources to start with, they should be able to explore the equality and inequality of values of numbers.

A helpful pedagogy to use is, '**What's the same and what's different** on both sides of the equation?' There must be an opportunity to experience some examples of inequality to appreciate equality in a greater sense.

Generalisation

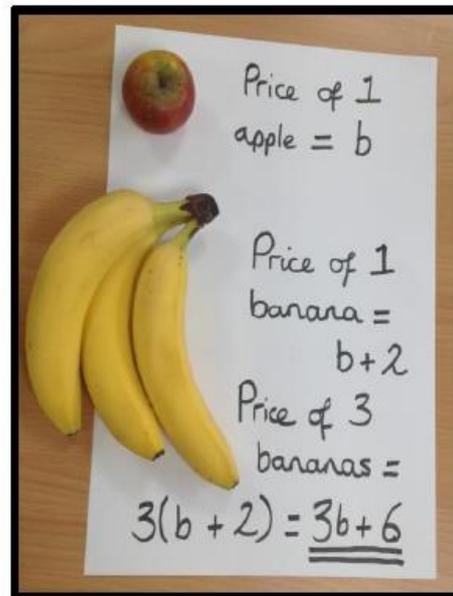
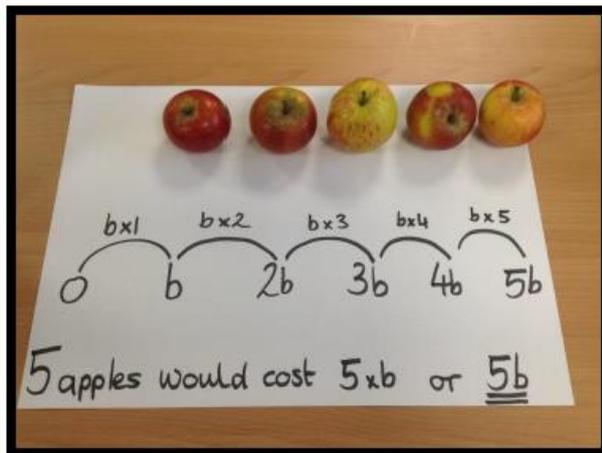
Children should be encouraged to complete the sequence to the nth term by generalising.

e.g. 5, 8, 11, 14, 17, ___ so the nth term = $3n + 2$

Children find the unknown or the missing numbers in all areas of calculations.

e.g. If each banana costs 2p more than an apple, what is the cost of 3 bananas?

What is the cost of 5 apples?



Pupils should be encouraged to make links e.g.

$$2 \times 10 + 3 \times 10 = 5 \times 10$$

Find the missing numbers to solve problems e.g.

$10 \div 2 = 20 \div a$
 $10/2 = 20/a$
 $2 \times 10/2 = 20/a \times 2$
 $10a = 40/a \times a$
 $a = 4$
(Multiply both sides by 2)
(Multiply both sides by a)
(Divide both sides by 10)

Useful IT Resources

Curriculum, Pedagogy and Planning

The National Curriculum: <https://www.gov.uk/government/publications/national-curriculum-in-england-mathematics-programmes-of-study#history>

My Maths Planning and Resources: www.mymaths.co.uk

NCETM: <https://www.ncetm.org.uk/>

STEM: <https://www.stem.org.uk/>

Problem Solving

NRich site for problem solving: <http://nrich.maths.org/frontpage>

The Mathematics Shed: <http://www.mathematicshed.com/>

Games and Activities

Activities for all year groups: www.ixl.com

Go Gordons Interactive Maths: <http://www.wldps.com/gordons/>

Top Marks Games: <http://www.topmarks.co.uk/maths-games/5-7-years/counting>

Algebra tiles: <http://technology.cpm.org/general/tiles/>

Interactive Cuisenaire rods: <https://nrich.maths.org/4348>

Interactive bar modelling:

[http://www.mathplayground.com/ThinkingBlocks/thinking_blocks_modeling%20 tool.html](http://www.mathplayground.com/ThinkingBlocks/thinking_blocks_modeling%20tool.html)

Problem solving activities/Maths games: <http://www.transum.org/Software/>

Starters, Practice questions, Videos: <https://corbettmaths.com/>