

Ratio and Proportion

Guidance for Teachers

Highfield Schools

The following guidance aims to provide a consistent approach to the Ratio and Proportion, and includes guidance on fractions, decimals and percentages

The main aims of this guidance is to:

- Create love and enthusiasm for Maths
- Develop a deeper understanding of ratio and proportion
- Improve pupils' ability to work with fractions, decimals and percentages
- Ensure a consistent approach within schools
- Support Medium Term planning at Highfield Schools

Fractions

Definition: a fraction is a numerical quantity that is not a whole number, consists of numerator (top) and denominator (bottom)

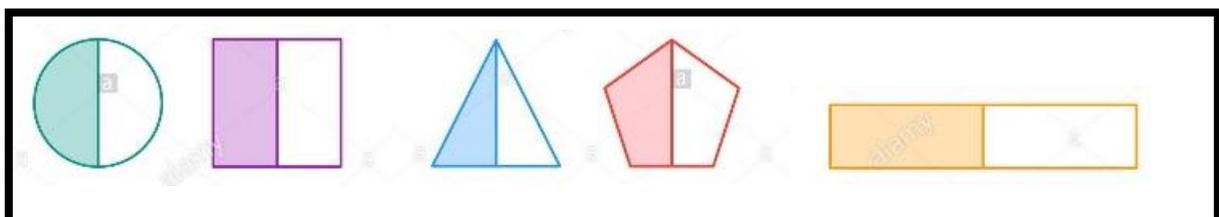
Core skills

To progress and use fractions effectively, pupils will need:

- Basic numeracy skills
- Understanding of equal portions/sizes
- Understanding of symmetry
- Knowledge of multiples and factors

Progression

Half as 1 of 2 equal parts of shapes



Half of quantities

Pictorial

Ladybird Halving to 10

Half of 2 is _____

Half of 4 is _____

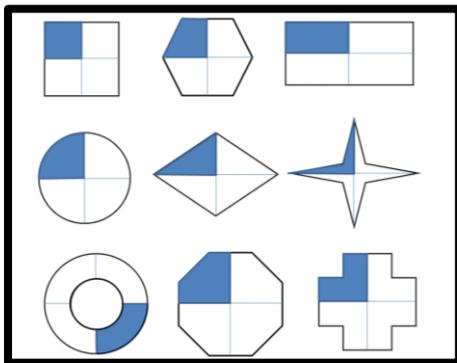
Half of 6 is _____

Half of 8 is _____

Concrete

Find half of six

Quarter as 1 of 4 equal parts of shapes



Quarter of quantities

Pictorial

kites  $\frac{1}{4}$ of =

Quarter of is

cats  $\frac{1}{4}$ of =

Quarter of is

flowers  $\frac{1}{4}$ of =

Quarter of is

trees  $\frac{1}{4}$ of =

Quarter of is

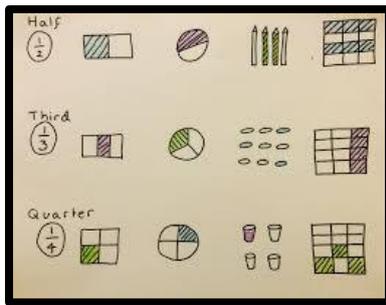
stars  $\frac{1}{4}$ of =

Quarter of is

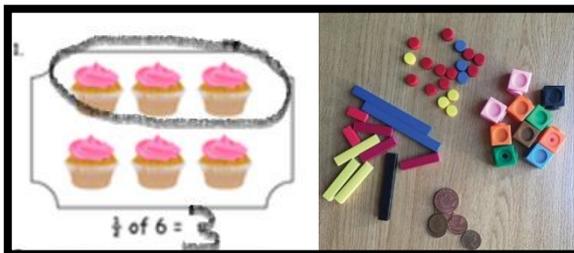
Concrete

Find one quarter of 12

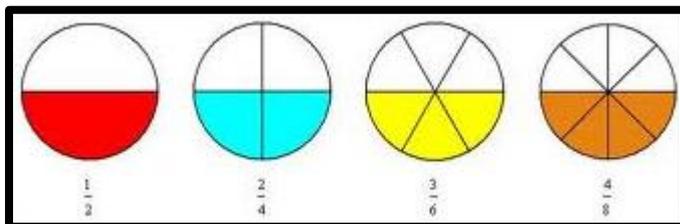
$\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of length, shape, quantities



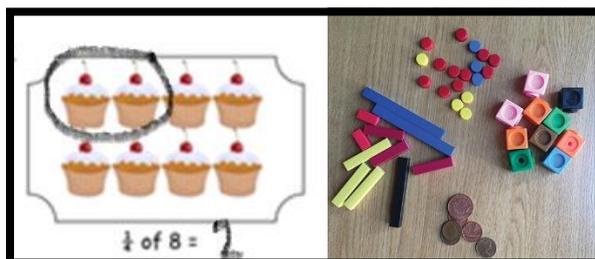
Write simple fractions $\frac{1}{2}$ of 6 = 3



Recognise equivalence of $\frac{2}{4}$ and $\frac{1}{2}$



Find unit fractions of numbers e.g. what is $\frac{1}{4}$ of 8 and show pictorially



Add fractions with same denominator

Example $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$

Ask 'how many sevenths altogether?' Add the numerators

Compare and order fractions with the same denominators

Example Put these fractions in size order $\frac{5}{7}, \frac{1}{7}, \frac{3}{7}$

Order the numerators $\frac{1}{7}, \frac{3}{7}, \frac{5}{7}$

Subtract fractions with the same denominator

Example $\frac{5}{7} - \frac{1}{7} = \frac{4}{7}$

Ask 'how many sevenths left?' Subtract the numerators

Find equivalent fractions using common multiples

Example

$\frac{1}{2} = \frac{2}{4} = \frac{6}{12} = \frac{30}{60}$

$\frac{3}{4} = \frac{12}{16} = \frac{120}{160}$

multiply top and bottom by same number

Compare and order fractions where denominators are factors of the same number

Example Put these fractions in size order $\frac{5}{6}, \frac{2}{3}, \frac{3}{4}$

Use knowledge of equivalent fractions to give common denominators

$$\frac{5}{6} = \frac{10}{12}$$

$$\frac{2}{3} = \frac{8}{12}$$

$$\frac{3}{4} = \frac{9}{12}$$

Order the equivalent fractions $\frac{8}{12}, \frac{9}{12}, \frac{10}{12}$

Use common factors to simplify fractions

Example

$$\frac{21}{24} = \frac{7}{8}$$

$\div 3$ (on numerator and denominator)

$$\frac{36}{40} = \frac{18}{20} = \frac{9}{10}$$

$\div 2$ (on numerator and denominator), $\div 2$ (on numerator and denominator)

Add and subtract fractions with different denominators

Example

$\frac{1}{4} + \frac{2}{5}$ use equivalent fractions so that fractions have a common denominator

$$\frac{1}{4} = \frac{5}{20}$$
$$\frac{2}{5} = \frac{8}{20}$$
$$\frac{5}{20} + \frac{8}{20} = \frac{13}{20}$$

Multiplications with fractions

Example - fraction \times fraction

$$\frac{2}{5} \times \frac{3}{7} = \frac{6}{35}$$

- multiply the numerators
- multiply the denominators

Example - fraction \times integer

$$\frac{1}{6} \times 3$$

All integers have 'invisible' denominator of 1

$$\frac{1}{6} \times \frac{3}{1} = \frac{3}{6} = \frac{1}{2}$$

- multiply numerators
- multiply denominators

(simplify answer if possible)

Dividing fractions

Example - fraction \div fraction

$$\frac{5}{12} \div \frac{1}{2}$$

'Flip' second fraction, swap \div for \times

$$\frac{5}{12} \times \frac{2}{1} = \frac{10}{12} = \frac{5}{6}$$

- multiply numerators
- multiply denominators

(simplify answer if possible)

Example - fraction \div integer

$$\frac{1}{6} \div 2$$

'Flip' integer (all integers have 'invisible' denominator of 1). Swap \div for \times

$$\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

- multiply numerators
- multiply denominators

(simplify answer if possible)

Decimals

Definition: a number that uses a **decimal point** followed by digits that show a value smaller than one

Core skills

To progress and use decimals effectively, pupils will need:

- Basic numeracy skills
- Understanding of decimal number system
- Understanding of place value
- Understanding of fractions

Progression

Count up and down in 10ths, 100ths

Counting in 10ths and 100ths
Fill in the correct 10th or 100th that completes each sequence.

1) 0.4 0.5 0.6 0.7 ___
2) 0.9 0.8 0.7 0.6 ___
3) 0.9 1 1.1 1.2 ___
4) 0.7 ___ 0.9 ___ 1.1
5) 2.4 2.3 ___ ___ 2

6) 0.73 0.74 0.75 0.76 ___
7) 0.32 0.31 0.3 0.29 ___
8) 5.67 ___ 5.69 ___ 5.71

10ths and 100ths Answer Sheet
A that completes each sequence.

0.5 0.6 0.7 **0.8**
0.8 0.7 0.6 **0.5**
1 1.1 1.2 **1.3**
0.8 0.9 **1** 1.1
2.3 **2.2** **2.1** 2

0.74 0.75 0.76 **0.77**
0.31 0.3 0.29 **0.28**
5.68 5.69 **5.7** 5.71

Recognise and write decimal equivalents to $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$

Fractions, Decimals and Percentages

	= 1	= 1	= 100%
	= $\frac{1}{2}$	= 0.5	= 50%
	= $\frac{1}{3}$	= 0.33	= 33.3%
	= $\frac{1}{4}$	= 0.25	= 25%

Fractions, Decimals and Percentages

	= 1	= 1	= 100%
	= $\frac{1}{2}$	= 0.5	= 50%
	= $\frac{1}{4}$	= 0.25	= 25%
	= $\frac{1}{5}$	= 0.2	= 20%
	= $\frac{1}{8}$	= 0.125	= 12.5%
	= $\frac{1}{10}$	= 0.1	= 10%
	= $\frac{1}{100}$	= 0.01	= 1%

Demonstrate the effect of dividing a one or 2 digit number by 10 or 100, recognising the value of the digits

method 1 - move digits to the right

$$\begin{array}{r} \text{T} \\ 2 \end{array} \quad \begin{array}{r} \text{1's} \\ 7 \end{array} \cdot \frac{1}{10} \quad \div 10$$

$$2 \cdot 7$$

$$\begin{array}{r} \text{T} \\ 3 \end{array} \quad \begin{array}{r} \text{1's} \\ 6 \end{array} \cdot \frac{1}{10} \quad \frac{1}{100} \quad \div 100$$

$$. 3 \quad 6$$

method 2 - move decimal place to the left

$$2 \cdot 7 \quad \div 10 \quad (\text{1 jump})$$

$$. 3 \quad 6 \quad \div 100 \quad (\text{2 jumps})$$

Round decimals to 1 or 2 decimal places

Rounding to 1 decimal place

17.8387891254 cm

17.8 cm

Rounding to 2 decimal places

17.8387891254 cm

17.84 cm

Solve simple measure and money problems involving fractions and decimals to 2 decimal places

Example

Suni buys a reading book that costs £5 and two notebooks costing £1.50 each. She pays with a £10 note. How much change does she get?

Read and write decimal numbers as fractions

Decimals to fractions
Examples

1) 0.71 has two decimal places

$$\frac{71}{100} \leftarrow \begin{array}{l} \text{digits from decimal} \\ \text{two decimal places} = 2 \text{ zeros} \end{array}$$

2) 0.397 has 3 decimal places

$$\frac{397}{1000} \leftarrow \begin{array}{l} \text{digits from decimal} \\ \text{3 decimal places} = 3 \text{ zeros} \end{array}$$

SIMPLIFY IF POSSIBLE

Read, write, order and compare numbers with up to 3 decimal places

Ordering decimals
Example method

order 0.41, 0.401, 0.041, 0.4

1) Add zeros at the end to give the numbers the same amount of decimal places

$$0.410 \quad 0.401 \quad 0.041 \quad 0.400$$

2) Ignore the leading zero and decimal place

$$410 \quad 401 \quad 041 \quad 400$$

3) Order these numbers

$$041 \quad 400 \quad 401 \quad 410$$

4) Re-write in original decimal format

$$0.041 \quad 0.4 \quad 0.401 \quad 0.41$$

Multiply one digit numbers with 2 decimal places by whole numbers

2.76×3 method 1

1) Ignore decimals and calculate

$$276 \times 3$$

$$\begin{array}{r} 276 \times \\ 3 \\ \hline 828 \\ \hline \end{array}$$

2) Give your answer the same number of decimal places as in the question

$$\underbrace{2.76}_{2} \times 3 = 8.\underbrace{28}_{2}$$

2.76×3 method 2

Use lattice method of multiplication

Percentages

Definition: parts per one hundred, denoted by the symbol %

Core skills

To progress and use percentages effectively, pupils will need:

- Basic numeracy skills, including multiplying and dividing by 10, 100
- Understanding of simple fractions
- Knowledge of fraction and decimal equivalents

Progression

Recognise the % symbol and recognise this is the number of parts per 100

Example

2% means 2 out of every one hundred

35% is 35 in every one hundred

Write percentages as a fraction with the denominator 100

Example

$$2\% = \frac{2}{100}$$

$$35\% = \frac{35}{100}$$

Write percentages as simplified fractions

Example

$$2\% = \frac{2}{100} = \frac{1}{50}$$

$$35\% = \frac{35}{100} = \frac{7}{20}$$

Write percentages as a decimal

Example

$$2\% = 2 \div 100 = 0.02$$

$$35\% = 35 \div 100 = 0.35$$

Find simple percentages of amounts

Example - find 20% of 120

Method 1 - unitary method

Find 1% by dividing 120 by 100 $120 \div 100 = 1.2$

Multiply by percentage required $1.2 \times 20 = 24$

Method 2 - fraction equivalents

$$20\% = \frac{1}{5} \quad \frac{1}{5} \text{ of } 120 = 24$$

Method 3 - calculator method

Enter $120 \times 20\%$

Solve problems involving percentage increase or decrease

Example - a pair of jeans, costing £30, are reduced by 20% in a sale. How much do they cost now?

Method 1

Find 20% of £30 using one of the methods above

Deduct the answer from the original £30 $30 - 6 = \text{£}24$

Method 2 - calculator

Use a decimal multiplier to find the answer

Enter 30×0.8

Ratio and Proportion

Definition:

Ratio - the relationship between two groups or amounts that expresses how much bigger one is than the other

Proportion - a part, share, or number considered in comparative relation to a whole

Ratio compares part-to-part, proportion compares part-to-whole

Core skills

To progress and use ratio and proportion effectively, pupils will need:

- Basic numeracy skills
- An understanding of simple fractions
- An understanding of factors and multiples

Progression

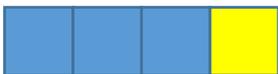
Recognise a ratio says how much of one thing there is to another

Example - 3 blue squares for every 1 yellow square is written as 3:1

Keep the ratio parts in the same order as the words

Write a ratio as a fraction

Example - The ratio of blue squares to yellow squares is 3:1. What fraction of the squares are blue?

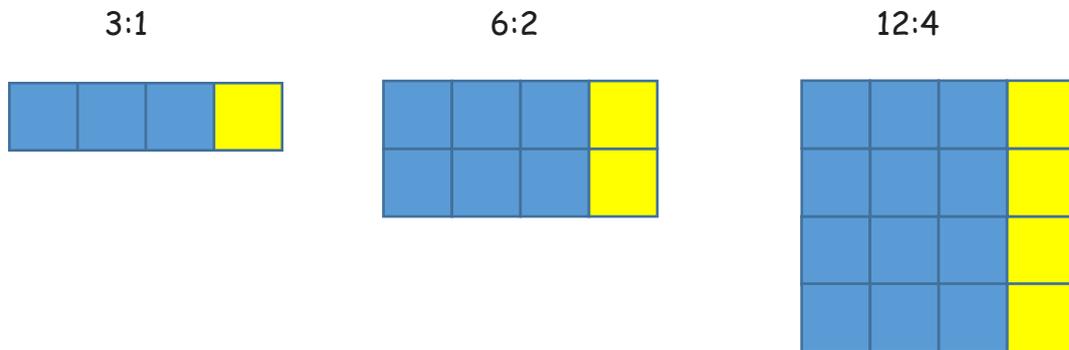


There are 4 squares altogether, 3 of which are blue.

The fraction of blue squares is $\frac{3}{4}$

Recognise ratios can be scaled up by doubling and scaled down by halving

Example - a ratio of 3:1 is equivalent to 6:2, 12:4 etc.

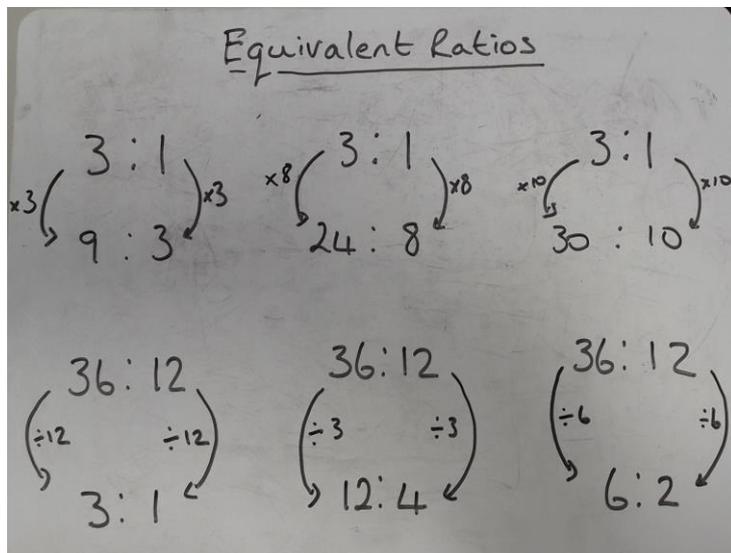


Find equivalent ratios by multiplying or dividing

Multiply or divide both parts of the ratio to find equivalents

Example - a ratio of 3:1 is equivalent to 9:3, 24:8, 30:10 etc

36:12 is equivalent to 3:1, 12:4, 6:2 etc.



Divide amounts by a given ratio

Example - 27 sweets are shared between Anne and Bill in the ratio 4:5.
How many sweets does each child get?

Method 1 - numerical

Add the parts of the ratio

$$4 + 5 = 9$$

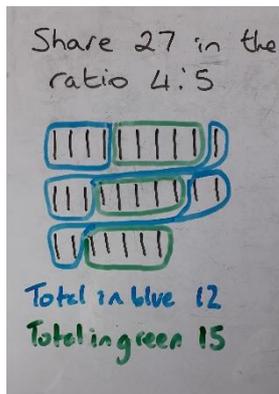
Divide the amount to be shared by this total $27 \div 9 = 3$

Multiply both parts of the ratio by this answer $4 \times 3 = 12$

$$5 \times 3 = 15$$

Answer Anne gets 12 sweets, Bill gets 15 sweets

Method 2 - pictorial



Method 3 - concrete



Useful Websites

Activities for all year groups: www.ixl.com

Go Gordons Interactive Maths: <http://www.wldps.com/gordons/>

Top Marks Games: <http://www.topmarks.co.uk/maths-games/5-7-years/counting>

Algebra tiles: <http://technology.cpm.org/general/tiles/>

Interactive Cuisenaire rods: <https://nrich.maths.org/4348>

Interactive bar modelling:

http://www.mathplayground.com/ThinkingBlocks/thinking_blocks_modeling%20tool.html

Problem solving activities/Maths games: <http://www.transum.org/Software/>

Starters, Practice questions, Videos: <https://corbettmaths.com/>

www.mathsisfun.com

<http://nrich.maths.org/frontpage>

<http://www.mathematicshed.com/>

<https://whiterosemaths.com/>