LONGBENTON HIGH SCHOOL

MATHS

GCSE To A Level

Bridging Booklet ANSWERS

EXERCISE ANSWERS

1. Indices and Surds

1 a 7 b 3 c $\frac{1}{5}$ d $\frac{1}{4}$

e 27 f 8 g $\frac{1}{25}$ h $\frac{1}{8}$

i 81 j $\frac{2}{3}$ k $\frac{4}{3}$ l $\frac{4}{9}$

2 a $2\sqrt{2}$ b $5\sqrt{3}$ c $4\sqrt{6}$

d $12\sqrt{3}$ e $3\sqrt{5}$ f $\sqrt{3}$

g $14\sqrt{2}$ h $5\sqrt{2}+15\sqrt{5}$ i $5\sqrt{17}$

j $14\sqrt{2}$ k $12\sqrt{2}-2\sqrt{3}$ l $6\sqrt{5}+5\sqrt{2}$

3 a $\frac{\sqrt{7}}{7}$ b $\frac{\sqrt{2}}{2}$ c $4\sqrt{3}$

d $\frac{\sqrt{6}}{3}$ e $\frac{1}{2}(\sqrt{3}-1)$ f $2(\sqrt{2}-1)$

g $-2(1+\sqrt{5})$ h $\frac{1}{2}(\sqrt{5}+1)$ i $2\sqrt{2}-\sqrt{6}$

j $3\sqrt{2}+2\sqrt{3}$ k $-3-2\sqrt{2}$ l $-\frac{3}{2}\sqrt{5}-\frac{7}{2}$

4 a $5+4\sqrt{2}$ b $1+2\sqrt{2}$

c $1-2\sqrt{2}$ d $5-4\sqrt{2}$

e $6\sqrt{3}+11$ f $2\sqrt{3}+5$

g $2\sqrt{3}-5$ h $6\sqrt{3}-11$

i $2\sqrt{3}+3\sqrt{6}+\sqrt{2}+3$ j $2\sqrt{3}-3\sqrt{6}+\sqrt{2}-3$

k $2\sqrt{3}+3\sqrt{6}-\sqrt{2}-3$ l $2\sqrt{3}-3\sqrt{6}-\sqrt{2}+3$

5 a x^{10} b $21x^{11}$ c $40x^{11}$

d x^{6} e $4x^{-2}$ f $\frac{1}{4}x$

g x^{35} h x^{-10} i $81x^{8}$

j $36x^{10}$ k $x^{\frac{3}{2}}$ l $x^{\frac{5}{4}}$

m $5x^{-\frac{1}{2}}$ n $2x^{\frac{3}{2}}$ o $\frac{1}{3}x^{\frac{3}{2}}$

p $x^{8}-x^{3}$ q $x^{\frac{7}{2}}+2x^{3}$ r $x^{-2}+2x^{-3}$

s $x^{-\frac{1}{2}}+3x^{-1}$ t $3x^{-\frac{1}{2}}-x^{\frac{5}{2}}$ u $x+6\sqrt{x}+9$

v $3x^{-2}+x^{-\frac{3}{2}}$ w $\frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{2}x^{\frac{1}{2}}$ x $\frac{1}{3}x^{-\frac{5}{2}}+\frac{2}{3}x^{-3}$

2. Linear equations and rearranging formulae

1 a $x=-\frac{10}{3}$ b $x=-\frac{5}{3}$ c $x=31$

d $x=\frac{13}{3}$ e $x=2.5$ f $x=2.1$

g $x=7$ h $x=\frac{5}{13}$

2 a $x\geq -4$ b $x>-3$ c $x>\frac{17}{4}$

d $x>5$ e $x\leq -\frac{2}{3}$ f $x\geq -1.8$

g $x<\frac{16}{7}$ h $x\leq -\frac{5}{3}$

3 a $x=\frac{3A-6}{2}$ b $x=\frac{3-u}{1-v}$ c $x=\frac{1}{3-2k}$

d $x=\frac{15m-4}{5-2n}$ e $x=\frac{1\pm\sqrt{t}}{3}$ f $x=\frac{pq}{p+q}$

g $x=\pm\sqrt{\frac{1}{10}-k}$ h $x=4B^{2}-A$

4 a $y=2, x=-6$ b $x=-3, y=7$ c $y=2, x=7$

d $=\frac{3}{5}, y=-\frac{1}{10}$ e $y=12, x=7$ f $y=-\frac{1}{2}, x=\frac{1}{4}$

5 a $(-3, 17)$ b $\left(\frac{1}{2}, -\frac{1}{2}\right)$ c $\left(\frac{2}{3}, \frac{13}{3}\right)$

d $(1.5, -0.5)$ e $(0.8, 3.4)$ f $(0.2, 6.6)$

EXERCISE ANSWERS

3. Factorising quadratics and simple cubics

1 **a** $x(3x+5)$ **b** $4x(2x-1)$
c $17x(x+2)$ **d** $6x(3x-4)$

2 **a** $(x+2)(x+3)$ **b** $(x-5)(x-2)$
c $(x-6)(x+1)$ **d** $(x+7)(x-4)$
e $(x-9)(x+8)$ **f** $(x+8)(x-6)$
g $(x-11)(x-1)$ **h** $(x-8)(x+3)$

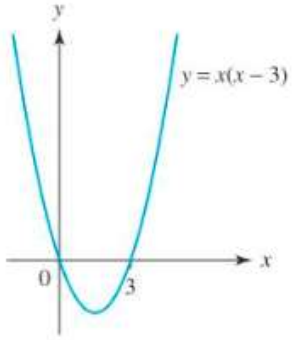
3 **a** $(x+10)(x-10)$ **b** $(x+9)(x-9)$
c $(2x+3)(2x-3)$ **d** $(8+3x)(8-3x)$

4 **a** $(3x+1)(x+2)$ **b** $(3x+4)(2x+3)$
c $(4x-1)(x-3)$ **d** $(2x+3)(x-5)$
e $(2x+5)(x-1)$ **f** $(7x-3)(x+4)$
g $(4x-5)(2x-3)$ **h** $(4x-1)(3x+5)$

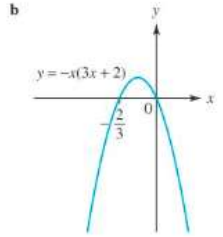
5 **a** $(4x+5)(4x-5)$ **b** $4x(x-4)$
c $(x+12)(x+1)$ **d** $(3x-5)(x+7)$
e $(x+4)(x-3)$ **f** $(10+3x)(10-3x)$
g $2x(x-7)$ **h** $(5x-2)(4x+1)$

6 **a** $x=0$ or $x=\frac{1}{3}$ **b** $x=-6$ or $x=6$
c $x=0$ or $x=-2$ **d** $x=-\frac{1}{2}$ or $x=-\frac{5}{3}$
e $x=-\frac{7}{2}$ or $x=\frac{7}{2}$ **f** $x=9$ or $x=-2$
g $x=6$ or $x=1$ **h** $x=\frac{2}{7}$ or $x=-\frac{1}{3}$
i $x=\frac{2}{5}$ or $x=3$ **j** $x=-\frac{3}{4}$
k $x=\frac{2}{3}$ **l** $x=\frac{3}{8}$ or $x=-\frac{2}{5}$

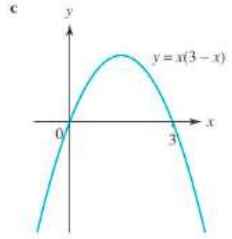
7 **a**



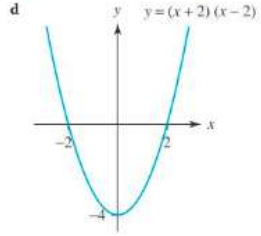
7 **b**



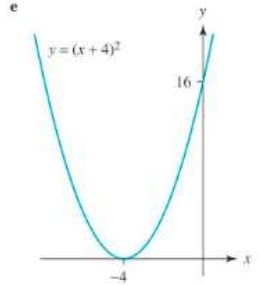
c



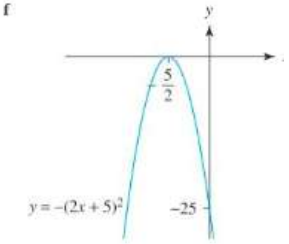
d



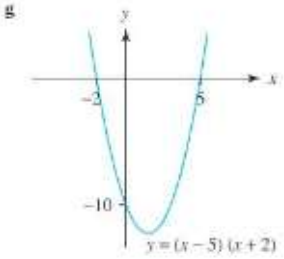
e



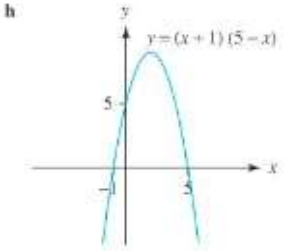
f



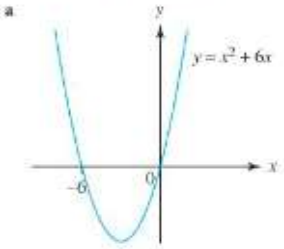
7 **g**



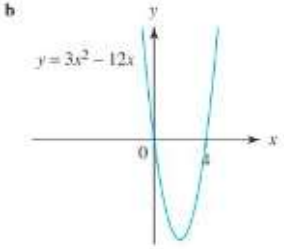
h



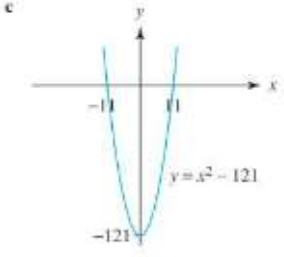
8 **a**



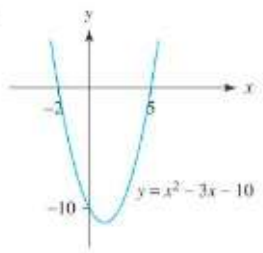
b



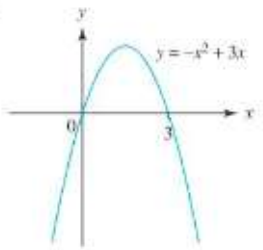
c



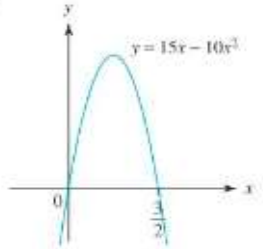
d



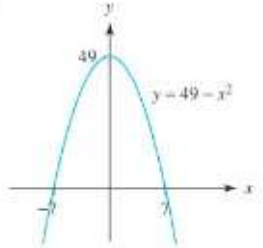
e



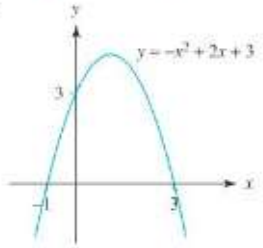
f



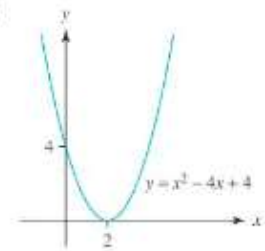
g



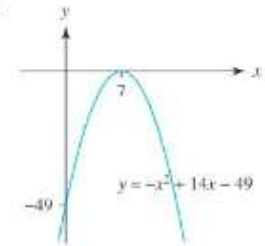
h



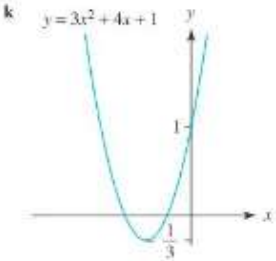
i



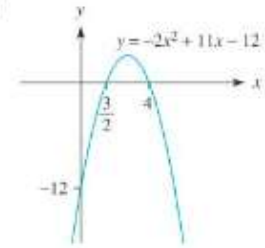
j



k



l



EXERCISE ANSWERS

4. Completing the square

1 a $(x+4)^2-16$ b $(x-9)^2-81$
c $(x+3)^2-6$ d $(x+6)^2-41$
e $\left(x-\frac{7}{2}\right)^2-\frac{9}{4}$ f $\left(x+\frac{5}{2}\right)^2+\frac{11}{4}$
g $2(x+2)^2-4$ h $3(x+2)^2-33$
i $2\left(x-\frac{5}{2}\right)^2-\frac{19}{2}$ j $-(x-6)^2+35$
k $-\left(x-\frac{9}{2}\right)^2+\frac{69}{4}$ l $-2\left(x-\frac{5}{4}\right)^2+\frac{17}{8}$

2 a $(-7, -49)$ is a minimum point
b $(9, -78)$ is a minimum point
c $\left(\frac{9}{2}, -\frac{81}{4}\right)$ is a minimum point
d $(2, 4)$ is a maximum point
e $\left(-\frac{11}{2}, -\frac{1}{4}\right)$ is a minimum point
f $(3, 2)$ is a maximum point
g $(-4, -37)$ is a minimum point
h $\left(\frac{5}{2}, \frac{67}{4}\right)$ is a maximum point.

5. The Quadratic Formula

1 a $x = 0.88$ or $x = -1.30$
b $x = 3.41$ or $x = 0.59$
c $x = 11.66$ or $x = 0.34$

2 a No real solutions.
b Two (distinct) real solutions.
c One real solution (coincidental solutions).

3 a $y = 7x^2 - 5x + 4$ b $y = -4x^2 + 12x - 9$
c $y = 6x^2 - x - 15$ d $y = -x^2 + 2x - 4$

4 a $-\frac{1}{3}$ b $\frac{1}{16}$ c $\frac{65}{8}$

5 a $k \geq -\frac{3}{4}$ b $k \leq \frac{49}{16}$ c $k \leq 7$

6 a $k > \frac{1}{40}$ b $k < -\frac{4}{5}$ c $k < \frac{47}{48}$

6. Line Graphs

1 a -1 b $\frac{4}{9}$ c -6
d $\frac{4}{3}$ e $\sqrt{3}$ f -2

2 a $5\sqrt{2}$ b $\sqrt{481}$ c 14.2
d $\frac{\sqrt{13}}{5}$ e $\sqrt{41}$ f $k\sqrt{10}$

3 a $(2, 8)$ b $(-0.5, -6.5)$
c $(4.2, -0.1)$ d $(-0.5, -1)$
e $\left(\frac{5}{2}\sqrt{5}, \frac{3}{2}\sqrt{5}\right)$ f $(2m, 0)$

4 a gradient is 7, y-intercept is -4
b gradient is -2, y-intercept is 3
c gradient is 1, y-intercept is -4
d gradient is $-\frac{3}{2}$, y-intercept is $\frac{7}{2}$
e gradient is $\frac{5}{2}$, y-intercept is $-\frac{9}{2}$
f gradient is $\frac{3}{5}$, y-intercept is 0
g gradient is $-\frac{1}{6}$, y-intercept is $-\frac{1}{2}$
h gradient is $\frac{4}{3}$, y-intercept is $\frac{2}{3}$

5 a $y = -\frac{1}{2}x + 6$ b $y = -2x - 1$
c $11x + 3y - 56 = 0$ d $y = \frac{1}{4}x - 4$
e $y = 2x - 1$ f $y = \frac{5}{2}x - \frac{7}{2}\sqrt{2}$

6 a perpendicular
b neither parallel nor perpendicular
c parallel

7 a perpendicular
b neither parallel nor perpendicular
c parallel

8 a parallel
b neither parallel nor perpendicular
c perpendicular

9 a $5x - y - 18 = 0$ b $x + 5y - 1 = 0$
10 a $x - 2y + 11 = 0$ b $2x + y - 14 = 0$
11 a $3x + y - 22 = 0$ b $x - 3y - 2 = 0$
12 a $6x + 5y - 24 = 0$ b $5x - 6y - 42 = 0$
13 a $6x - 2y - 1 = 0$ b $2x + 6y + 5 = 0$
14 a $2x - 3y - 5 = 0$
b $5x + 7y + 14 = 0$
c $y = -x + 6$
d $x - 3y - 7 = 0$
e $8x - 2y - 25 = 0$

15 a $(1, 1)$ b $(-2, 16)$
c $\left(\frac{4}{3}, \frac{13}{3}\right)$ d $\left(2, \frac{15}{2}\right)$

16 a $(2, -1)$ b $(4, 4)$
c $\left(\frac{5}{6}, \frac{1}{6}\right)$ d $\left(-\frac{1}{3}, -\frac{1}{3}\right)$

EXERCISE ANSWERS

7. Circles

1 **a** $(x-2)^2+(y-5)^2=49$
b $(x+1)^2+(y+3)^2=16$
c $(x+3)^2+y^2=2$
d $(x-4)^2+(y+2)^2=5$

2 **a** centre (5, 3), radius 4
b centre (−3, 4), radius 6
c centre (9, −2), radius 10
d centre (−3, −1), radius $4\sqrt{5}$
e centre $(\sqrt{2}, -2\sqrt{2})$, radius $4\sqrt{2}$
f centre $\left(-\frac{1}{4}, -\frac{1}{3}\right)$, radius $\frac{5}{2}$

3 **a** centre (−1, 0), radius 5
b centre (0, −6), radius 7
c centre (2, 0), radius 1
d centre (−3, −4), radius is $\sqrt{23}$
e centre (4, 5), radius $2\sqrt{11}$
f centre (−7, 1), radius $\sqrt{55}$
g centre $\left(-\frac{5}{2}, 2\right)$, radius $\frac{1}{2}\sqrt{29}$
h centre $\left(\frac{3}{2}, \frac{9}{2}\right)$, radius $\frac{7}{2}\sqrt{2}$
i centre $\left(\frac{1}{2}, -\frac{7}{2}\right)$, radius $\frac{\sqrt{2}}{2}$

4 **a** $(x-2)^2+(y-6)^2=2$
b $(x-3)^2+(y+3)^2=5$
c $(x+4)^2+(y+4.5)^2=27.25$
d $(x-2.5)^2+(y+11.5)^2=50.5$
e $x^2+(y-5)^2=3$
f $(x-\sqrt{3})^2+(y+3\sqrt{3})^2=39$

5 **a** does not lie on the circle
b does lie on the circle
c does not lie on the circle
d does lie on the circle

6 **a** lies on this circle
b doesn't lie on this circle
c lies on this circle

7 $x-3y-14=0$

8 $3x+5y+10=0$

9 $y=\frac{1}{4}x-\frac{19}{4}$

10 $y=9x-5$

11 **a** (−2, 7) and (7, −2)
b (−3, −1) and (5, −1)
c (−1.6, 3.8) and (−4, −1)
d (−2.2, −3.4) and (−5, −9)

12 **a** (−4, −2) and (−10, −4)
b $2\sqrt{10}$

13 **a** (−7, 6) and (−0.6, 2.8)
b $\frac{16}{5}\sqrt{5}$

14 $(x-3)^2+(x-3+2)^2=2 \Rightarrow (x-3)^2+(x-1)^2=2$
$\Rightarrow x^2-6x+9+x^2-2x+1=2$
$\Rightarrow 2x^2-8x+8=0$
$b^2-4ac=(-8)^2-4\times2\times8=0$ so only one solution
hence a tangent

15 $y=34-4x \Rightarrow (x+1)^2+(34-4x-4)^2=68$
$\Rightarrow (x+1)^2+(30-4x)^2=68$
$\Rightarrow x^2+2x+1+900-240x+16x^2=68$
$\Rightarrow 17x^2-238x+833=0$
$b^2-4ac=(-238)^2-4\times17\times833=0$ so only one solution
hence a tangent

16 $x=25-3y \Rightarrow (25-3y)^2+(y-5)^2=10$
$\Rightarrow 625-150y+9y^2+y^2-10y+25=10$
$\Rightarrow 10y^2-160y+640=0$
$b^2-4ac=(-160)^2-4\times10\times640=0$ so only one solution
hence a tangent

17 $(x-1)^2+(2x+3+4)^2=1$
$\Rightarrow (x-1)^2+(2x+7)^2=1$
$\Rightarrow x^2-2x+1+4x^2+28x+49=1$
$\Rightarrow 5x^2+26x+49=0$
$b^2-4ac=26^2-4\times5\times49=-304$ negative so no solutions
hence they do not intersect

18 $3x=-2-4y \Rightarrow x=-\frac{2}{3}-\frac{4}{3}y$
$\Rightarrow \left(-\frac{2}{3}-\frac{4}{3}y+3\right)^2+(y-6)^2=9$
$\Rightarrow \left(\frac{7}{3}-\frac{4}{3}y\right)^2+(y-6)^2=9$
$\Rightarrow \frac{49}{9}-\frac{56}{9}y+\frac{16}{9}y^2+y^2-12y+36=9$
$\Rightarrow \frac{25}{9}y^2-\frac{164}{9}y+\frac{292}{9}=0$
$b^2-4ac=\left(-\frac{164}{9}\right)^2-4\times\frac{25}{9}\times\frac{292}{9}=-\frac{256}{9}$ negative so no
solutions hence they do not intersect

TRY IT ANSWERS

1. Indices and Surds

1 a $10x^{10}$ b $6x^{7}$ c $16x^{24}$ d $\frac{x^6}{9}$

2 a 6 b 9 c $\frac{1}{8}$ d $\frac{1}{16}$

3 a $x^{\frac{2}{5}}$ b $3x^{-\frac{1}{2}}$ c $3x^{\frac{3}{2}}$ d $\frac{1}{3}x^{-\frac{1}{2}}$

4 a $5\sqrt{7}$ b $\frac{4\sqrt{3}}{3}$ c $-3+3\sqrt{2}$ d $5+2\sqrt{5}$

2. Linear equations and rearranging formulae

1 $x=7$

2 $x>2$

3 $x=\frac{1-3A}{3-B}$

4 $y=3, x=-7$

5 (2, 10)

3. Factorising Quadratics and simple cubics

1 a $7x(2x-1)$ b $(x-4)(x-1)$ c $(x+5)(x-5)$

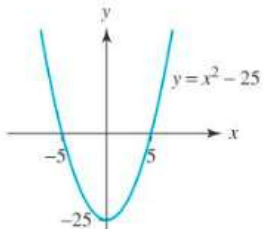
2 a $(5x+1)(x+4)$

b $(3x-1)(2x+3)$

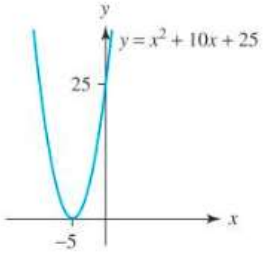
c $(2x-5)(4x-1)$

3 a $x=0$ or $x=2$ b $x=\frac{3}{4}$ or $x=5$

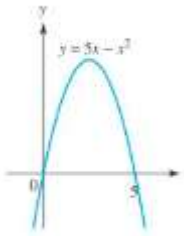
4 a



b



c



4. Completing The Square

1 a $(x+11)^2-121$ b $2(x-2)^2-14$

c $-(x-5)^2+25$

2 a $\left(\frac{3}{2},-\frac{5}{4}\right)$ is a minimum

b $\left(-\frac{7}{2},\frac{1}{4}\right)$ is a maximum

c (−1, −3) is a minimum

5. The Quadratic Formula

1 $x=1.25$ or $x=-0.68$

2 $\frac{1}{20}$

3 $k\geq-\frac{9}{4}$

4 $k>\frac{49}{4}$

6. Line Graphs

1 a $\frac{1}{3}$ b −2 c $-\frac{7}{2}$

2 a $2\sqrt{2}$ b $3\sqrt{10}$ c $3\sqrt{11}$

3 a (1.5, 7) b (−3.5, −2) c (1.9, −6.5)

4 a gradient is −2, y-intercept is 8

b gradient is $-\frac{1}{2}$, y-intercept is $\frac{3}{2}$

c gradient is $\frac{2}{3}$, y-intercept is $-\frac{4}{9}$

5 a $y=-2x+13$ b $y=3x-16$ c $5y=3x-11$

6 $3x-2y-13=0$

7 a neither parallel nor perpendicular

b perpendicular

c parallel

8 $3x-2y+13=0$

9 $7x-4y+39=0$

7. Circles

1 a centre (−2, 8), radius is 5

b $(x-7)^2+(y+9)^2=64$

2 a centre (0, 5), radius 3

b centre (−3, 6), radius $3\sqrt{5}$

3 $(x-3)^2+(y-1)^2=26$

4 a $(6-1)^2+(1+4)^2=5^2+5^2=50$ so (6, 1) lies on the circle

b $y=-x+7$

5 a (1.6, 0.2), (−1, 8)

b $\frac{13}{5}\sqrt{10}$

6 $y=2x+11\Rightarrow(x-5)^2+(2x+11-1)^2=80$

$\Rightarrow(x-5)^2+(2x+10)^2=80$

$\Rightarrow x^2-10x+25+4x^2+40x+100=80$

$\Rightarrow 5x^2+30x+45=0$

exactly one solution

Therefore the line and the circle touch once, hence the line is a tangent to the circle.