YEAR 9 – REASONING WITH ALGEBRA…

Straight Line Graphs

What do I need to be able to do?

By the end of this unit you should be able to:

- Compare gradients
- Compare intercepts
- Understand and use y= mx + c
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs

Keywords

Gradient: the steepness of a line

Intercept: where two lines cross. The y-intercept: where the line meets the y-axis.

Parallel: two lines that never meet with the same gradient.

Co-ordinate: a set of values that show an exact position on a graph.

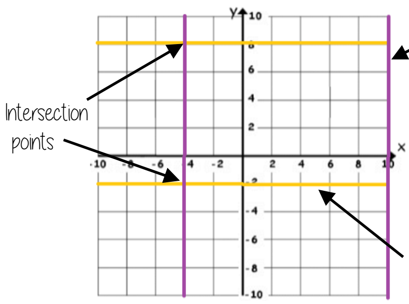
Linear: linear graphs (straight line) – linear common difference by addition/ subtraction

Asymptote: a straight line that a graph will never meet.

Reciprocal: a pair of numbers that multiply together to give 1.

Perpendicular: two lines that meet at a right angle.

Lines parallel to the axes



All the points on this line have a x coordinate of 10

'a' can be ANY positive or negative value including 0

Lines parallel to the **y axis** take the form **x = a** and are **vertical**

Lines parallel to the **x axis** take the form **y = a** and are **horizontal**

All the points on this line have a y coordinate of -2

eg (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

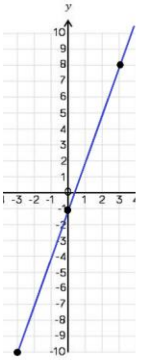
Plotting y = mx + c graphs

$y = 3x - 1$ → 3 x the x coordinate then – 1

x	−3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

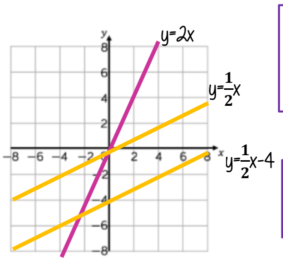
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Compare Gradients

$y = mx + c$

The **coefficient** of x (the number in front of x) tells us the gradient of the line



The greater the gradient – the **steeper** the line

Parallel lines have the **same** gradient

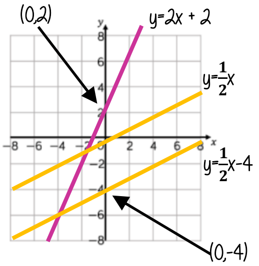
Positive gradients

Negative gradients

Compare Intercepts

$y = mx + c$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**



The coordinate of a y intercept will always be (0,c)

Lines with the **same** y-intercept cross in the **same** place

y = mx + c

The **coefficient** of x (the number in front of x) tells us the gradient of the line

$y = mx + c$

The value of **c** is the point at which the line crosses the y-axis. **Y intercept**

y and x are **coordinates**.

The equation of a line can be rearranged. Eg:

y = c + mx

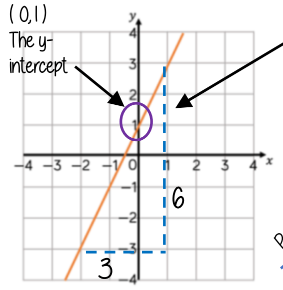
c = y – mx

Identify which coefficient you are identifying or comparing

Find the equation from a graph

(0,1)

The y-intercept



The Gradient

$\frac{6}{3} = 2$

$y = 2x + 1$

The direction of the line indicates a positive gradient

Positive gradients

Negative gradients

Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

The y-intercept shows the minimum charge.
The gradient represents the price per mile

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

Direct Proportion graphs To represent direct proportion the graph must start at the origin.

When you have 0 pens this has 0 cost.
The gradient shows the price per pen.

A box of pens costs £2.30
Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

YEAR 9 — REASONING WITH ALGEBRA...

Forming & Solving Equations

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve inequalities with negative numbers
- Solve equations with unknowns on both sides
- Solve inequalities with unknowns on both sides
- Substitute into formulae and equations
- Rearrange formulae

Keywords

Inequality: an inequality compares who values showing if one is greater than, less than or equal to another

Variable: a quantity that may change within the context of the problem

Rearrange: Change the order

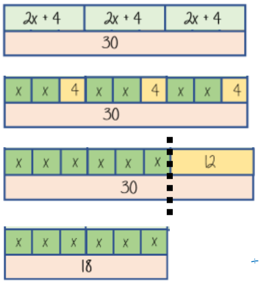
Inverse operation: the operation that reverses the action

Substitute: replace a variable with a numerical value

Solve: find a numerical value that satisfies an equation

Solve equations with brackets

R



$3(2x + 4) = 30$

Expand the brackets

$6x + 12 = 30$

$-12 \qquad -12$

$6x = 18$

$\div 6 \qquad \div 6$

$\underline{x = 3}$

Form and solve inequalities

R

Two more than treble my number is greater than 11

Find the possible range of values

$3x + 2 > 11$

Solve

$x \leftarrow -3 \leftarrow -2 \leftarrow 11$

$x > 3$

Inequalities with negatives

Method 1 Make x positive first

$2 - 3x > 17$

$+3x \qquad +3x$

$2 > 17 + 3x$

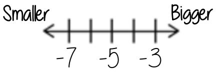
$-17 \qquad -17$

$-15 > 3x$

$\div 3 \qquad \div 3$

$\underline{-5 > x}$

x is true for any value smaller than -5



✓ CHECK IT!

$2 - 3(-6) = 20$

TRUE/ CORRECT

Method 2 Keep the negative x

$2 - 3x > 17$

$-2 \qquad -2$

$-3x > 15$

$\div -3 \qquad \div -3$

$\underline{x > -5}$

x is true for any value bigger than -5

This cannot be true...

$\underline{x < -5}$

When you multiply or divide x by a negative you need to reverse the inequality

Equations with unknown on both sides

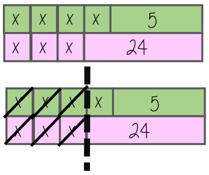
$4x + 5 = 3x + 24$

$-3x \qquad -3x$

$x + 5 = 24$

$-5 \qquad -5$

$\underline{x = 19}$



Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$5(x + 4) < 3(x + 2)$

$5x + 20 < 3x + 6$

$2x + 20 < 6$

$2x < -14$

$\underline{x < -7}$

Check it!

$5(-8 + 4) < 3(-8 + 2)$

$5(-4) < 3(-6)$

$-20 < -18$

✓ -20 IS smaller than -18

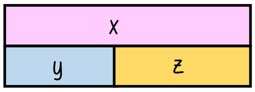
Formulae and Equations

Formulae – all expressed in symbols

Substitute in values

Equations – include numbers and can be solved

Rearranging Formulae (one step)



$x = y + z$

Rearrange to make y the subject.

$\underline{y = x - z}$

$y \rightarrow +z \rightarrow x$

$y \leftarrow -z \leftarrow x$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution.

Language of rearranging...

Change the subject

Make XXX the subject

Rearrange

Rearranging Formulae (two step)

In an equation (find x)

$4x - 3 = 9$

$+3 \qquad +3$

$4x = 12$

$\div 4 \qquad \div 4$

$\underline{x = 3}$

In a formula (make x the subject)

$xy - s = a$

$+s \qquad +s$

$xy = a + s$

$\div y \qquad \div y$

$x = \dfrac{a + s}{y}$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g. Find the gradient of the line $2y - 4x = 9$

Make y the subject first $y = \dfrac{4x + 9}{2}$ Gradient $= \dfrac{4}{2} = 2$

YEAR 9 – CONSTRUCTING IN 2D/3D…

3D Shapes

What do I need to be able to do?

By the end of this unit you should be able to:

- Name 2D & 3D shapes
- Recognise Prisms
- Sketch and recognise nets
- Draw plans and elevations
- Find areas of 2D shapes
- Find Surface area for cubes, cuboids, triangular prisms and cylinders
- Find the volume of 3D shapes

Keywords

2D: two dimensions to the shape e.g. length and width

3D: three dimensions to the shape e.g. length, width and height

Vertex: a point where two or more line segments meet

Edge a line on the boundary joining two vertex

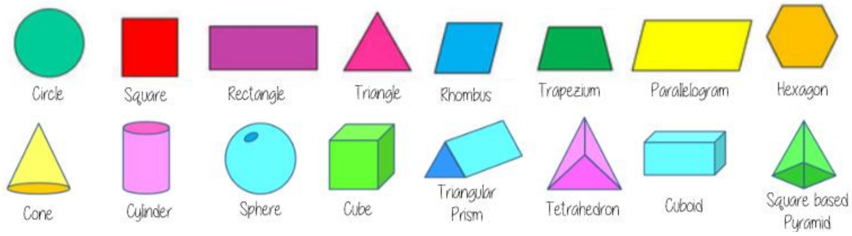
Face: a flat surface on a solid object

Cross-section: a view inside a solid shape made by cutting through it

Plan: a drawing of something when drawn from above (sometimes birds eye view)

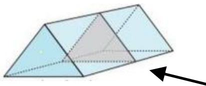
Perspective: a way to give illustration of a 3D shape when drawn on a flat surface.

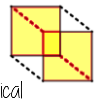
Name 2D & 3D shapes



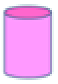
Recognise prisms

A solid object with two identical ends and flat sides



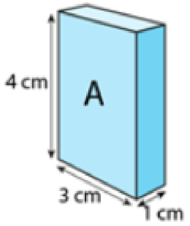


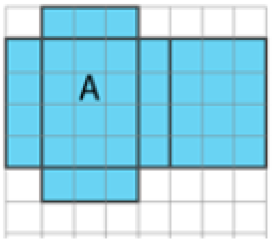
The cross section will also be identical to the end faces.



A cylinder although with very similar properties does not have flat faces so is not categorised as a prism

Nets of cuboids

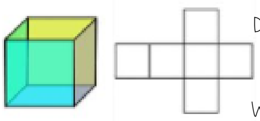




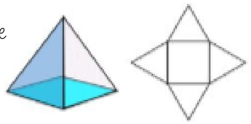
1cm grids help to draw accurately

Visualise the folding of the net. Will it make the cuboid with all sides touching

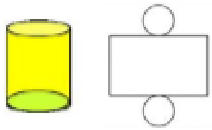
Sketch and recognise nets



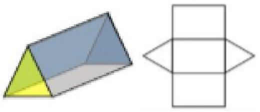
Do they have the same number of faces?



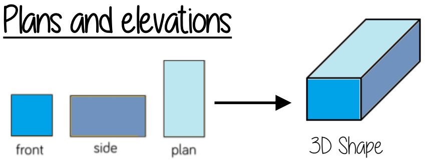
Where do the edges join?



Are the shapes of the faces correct?



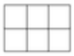
Plans and elevations



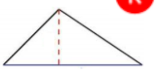
The direction you are considering the shape from determines the front and side views

Area of 2D shapes

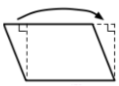
Rectangle
Base x Height



Triangle
½ x Base x Perpendicular height

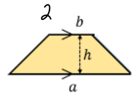


Parallelogram/ Rhombus
Base x Perpendicular height

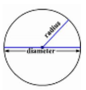


Area of a trapezium

$$\frac{(a+b) \times h}{2}$$

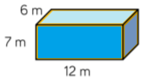


Area of a circle
π x radius²

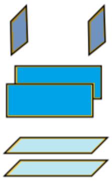


Surface area

Sketching nets first helps you visualise all the sides that will form the overall surface area

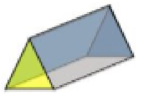


For cubes and cuboids you can also find one of each face and double it



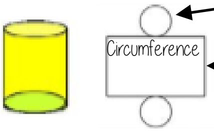
Sides	6 x 7 6 x 7
Front and back	12 x 7 12 x 7
Top and Bottom	12 x 6 12 x 6

Sum of all sides is surface area



For other shapes – not all the sides are the same, so calculate the individually

Surface area - cylinders



The area of the circle
π x radius²

The width of this face is the same as the circumference
π x diameter x height

2 x π x radius² + π x diameter x height

Volumes

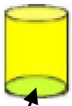
Volume is the 3D space it takes up – also known as capacity if using liquids to fill the space

Counting cubes

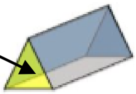
Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape.

Cubes/ Cuboids = base x width x height

Remember multiplication is commutative



Cross section



Cross section

Prisms and cylinders = area cross section x height

Height can also be described as depth

Areas – square units
Volumes – cube units

Areas and volumes can be left in terms of pi π

YEAR 9 – CONSTRUCTING IN 2D/3D...
Constructions & Congruency

What do I need to be able to do?

By the end of this unit you should be able to:

- Draw and measure angles
- Construct scale drawings
- Find locus of distance from points, lines, two lines
- Construct perpendiculars from points, lines, angles
- Identify congruence
- Identify congruent triangles

Keywords

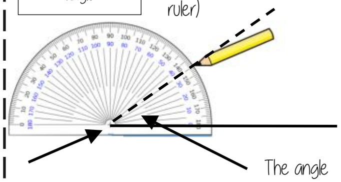
Protractor: piece of equipment used to measure and draw angles
Locus: set of points with a common property
Equidistant: the same distance
Discorectangle: (a stadium) – a rectangle with semi circles at either end
Perpendicular: lines that meet at 90°
Arc: part of a curve
Bisector: a line that divides something into two equal parts
Congruent: the same shape and size

Draw and measure angles

R

Draw a 35° angle

Make a mark at 35° with a pencil And join to the angle point (use a ruler)



The angle

Make sure the cross is at the end of the line (where you want the angle)

Scale drawings

R

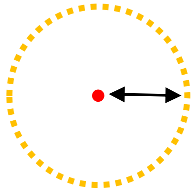
A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

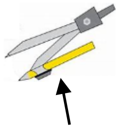
The car image is 10cm

Image : Real life
× 10 1cm : 30cm × 10
10cm : 300cm

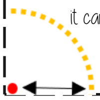
Locus of a distance from a point



All points are equidistant (the same distance) from the fixed point in the middle.

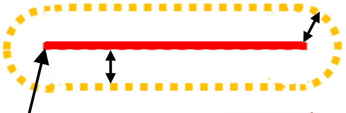


Equipment needed
The radius is the distance from the fixed point



If the point is in the corner it can only make a quarter circle

Locus of a distance from a straight line

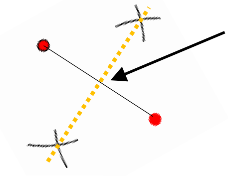


All points are equidistant (the same distance) from line

The ends of the line are fixed points

Equipment needed
The line is straight so a ruler is used for the straight lines parallel to your original line

Locus equidistant from two points



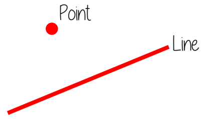
Also a perpendicular bisector

Because if the points are joined, this new line intersects it at a 90°

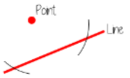
Keep the compass the same size and draw two arcs from each point

Join the intersections with a ruler.
All points on this line are equidistant from both points

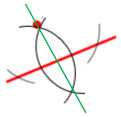
Construct a perpendicular from a point



Use a compass and draw an arc that cuts the line. Use the point to place the compass



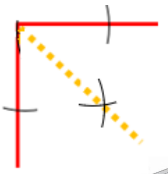
Keep the compass the same distance and now use your new points to make new interconnecting arcs



Connecting the arcs makes the bisector

If P is a point on the line the steps are the same

Locus of a distance from two lines



Also an angle bisector
This cuts the angle in half

From the angle vertex draw two arcs that cut the lines forming the angle

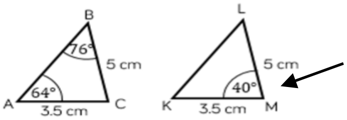
Keep the compass the same size and use the new arcs as centres to draw intersecting arcs in the middle

Join the vertex to the intersection

Congruent figures

Congruent figures are identical in size and shape – they can be reflections or rotations of each other

Congruent shapes are identical – all corresponding sides and angles are the same size



$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**

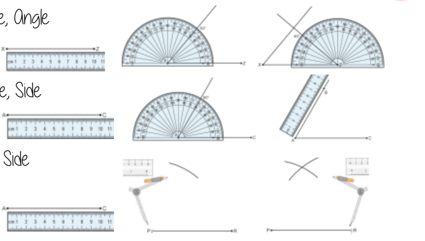
Constructing Triangles

Link to steps → R

Side, Angle, Angle

Side, Angle, Side

Side, Side, Side



Congruent triangles

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

YEAR 9 – REASONING WITH NUMBER…

Using Percentages

What do I need to be able to do?

By the end of this unit you should be able to:

- Use FDP equivalence
- Calculate percentage increase and decrease
- Express percentage change
- Solve reverse percentage problems
- Solve percentage problems (calculator and non calculator problems)

Keywords

Percent: parts per 100 – written using the % symbol.

Decimal: a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.

Fraction: a fraction represents how many parts of a whole value you have.

Equivalent: of equal value.

Reduce: to make smaller in value.

Growth: to increase/ to grow.

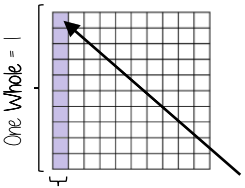
Integer: whole number, can be positive, negative or zero.

Invest: use money with the goal of it increasing in value over time (usually in a bank).

Multiplier: the number you are multiplying by.

Profit: the income take away any expenses/ costs.

FDP Equivalence R



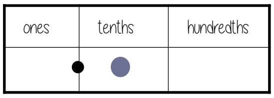
Percentage

100% = a whole = 100 hundredths

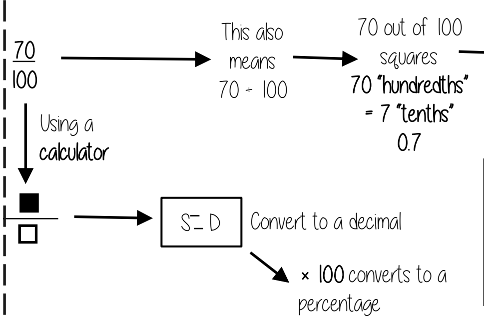
10 hundredths
10 out of 100
10%

$\frac{10}{100} = \frac{1}{10} = 0.10$

One **hundredth**
(one whole split into 100 equal parts)



Converting FDP R



$\frac{70}{100}$ → This also means 70 ÷ 100 → 70 out of 100 squares **70 "hundredths" = 7 "tenths"** 0.7 → → 70 hundredths = 70%

Using a **calculator**

S⇌D Convert to a decimal

× 100 converts to a percentage

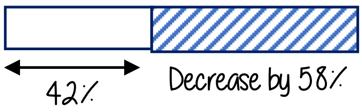
Be careful of recurring decimals

e.g $\frac{1}{3}$ = 0.3333333
= $0.\dot{3}$

The dot above the 3

Percentage Increase/ Decrease R

Decrease



Decrease by 58%

Multiplier
Less than 1

1.00 – 0.58 = 0.42

Increase

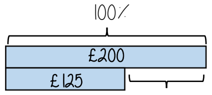
Increase by 12%

100% + 12% = 112%
1.00 + 0.12 = 1.12

Multiplier
More than 1

Percentage change R

I bought a phone for £200.
A year later sold it for £125



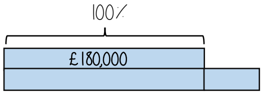
All values of change compare to the ORIGINAL value

Percentage loss

$\frac{75}{200} \times 100 = 37.5\%$

$$\frac{\textit{Difference in values}}{\textit{Original value}} \times 100$$

I bought a house for £180,000, I later sold it for £216,000.

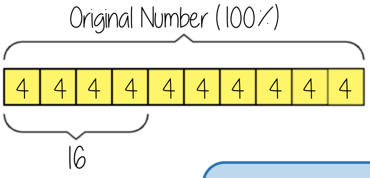


Percentage profit

Money made (profit value) → $\frac{36000}{180000} \times 100 = 20\%$

Reverse Percentages

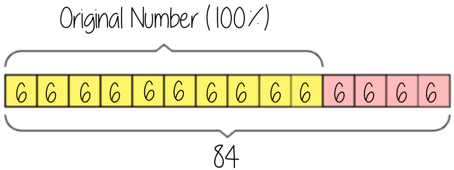
40% of my number is 16.
What am I thinking of?



40% = 16
10% - 4
100% = 40

Try to scale down to 10% or 1% and then scale back up to 100%

140% of my number is 84. What is the original number?



140% = 84
10% - 6
100% = 60

YEAR 9 — REASONING WITH GEOMETRY…

Rotation & Translation

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify the order of rotational symmetry
- Rotate a shape about a point on the shape
- Rotate a shape about a point not on a shape
- Translate by a given vector
- Compare rotations and reflections

Keywords

Rotate: a rotation is a circular movement.

Symmetry: when two or more parts are identical after a transformation.

Regular: a regular shape has angles and sides of equal lengths.

Invariant: a point that does not move after a transformation.

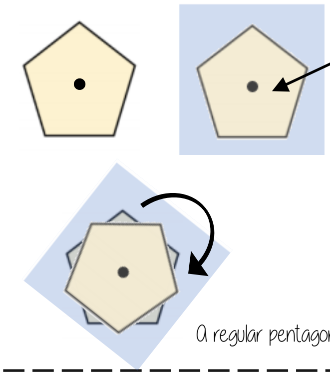
Vertex: a point two edges meet.

Horizontal: from side to side

Vertical: from up to down

Rotational Symmetry

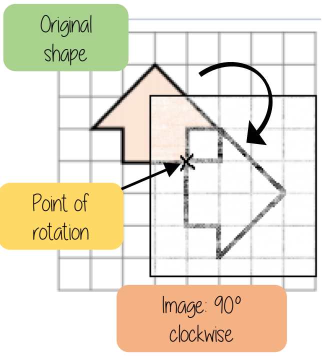
Tracing paper helps check rotational symmetry.



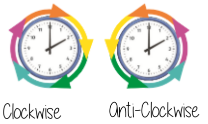
1. Trace your shape (mark the centre point)
2. Rotate your tracing paper on top of the original through 360°
3. Count the times it fits back into itself

A regular pentagon has rotational symmetry of order 5

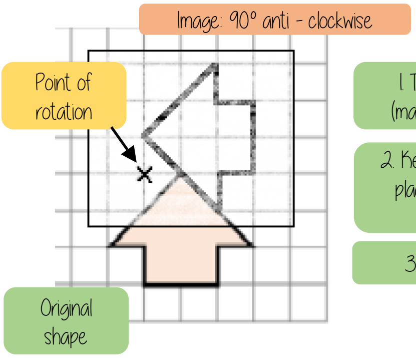
Rotate from a point (in a shape)



1. Trace the original shape (mark the point of rotation)
2. Keep the point in the same place and turn the tracing paper
3. Draw the new shape



Rotate from a point (outside a shape)



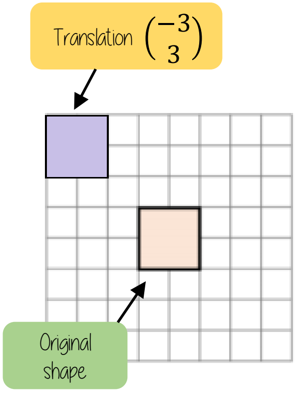
1. Trace the original shape (mark the point of rotation)
2. Keep the point in the same place and turn the tracing paper
3. Draw the new shape

Translation and vector notation

Vector Notation → $\begin{pmatrix}1\\-2\end{pmatrix}$

How far left or right to move
Negative value (left)
Positive value (right)

How far up or down to move
Negative value (down)
Positive value (up)

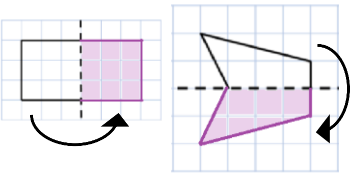


Every vertex has been translated by the same amount

$$\begin{pmatrix}-3\\3\end{pmatrix}$$

The image has been moved 3 squares to the left and 3 squares up

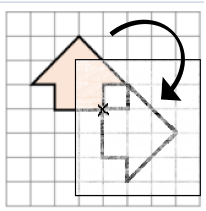
Compare rotations and reflections



Reflections are a mirror image of the original shape.

Information needed to perform a reflection:

- Line of reflection (Mirror line)



Rotations are the movement of a shape in a circular motion

Information needed to perform a rotation:

- Point of rotation
- Direction of rotation
- Degrees of rotation

YEAR 9 – REASONING WITH GEOMETRY...

Reflection & Symmetry

What do I need to be able to do?

By the end of this unit you should be able to:

- Recognise line symmetry
- Reflect in a horizontal line
- Reflect in a vertical line
- Reflect in a diagonal line

Keywords

Mirror line: a line that passes through the center of a shape with a mirror image on either side of the line

Line of symmetry: same definition as the mirror line

Reflect: mapping of one object from one position to another of equal distance from a given line.

Vertex: a point where two or more-line segments meet.

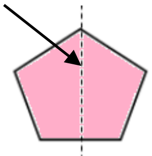
Perpendicular: lines that cross at 90°

Horizontal: a straight line from left to right (parallel to the x axis)

Vertical: a straight line from top to bottom (parallel to the y axis)

Lines of symmetry

Mirror line (line of reflection)

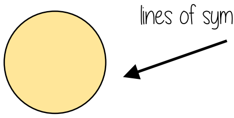


Rhombus
two lines of symmetry

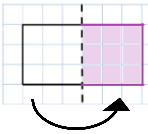
Parallelogram
No lines of symmetry

Shapes can have more than one line of symmetry.... This regular polygon (a regular pentagon has 5 lines of symmetry)

a circle has an infinite amount of lines of symmetry

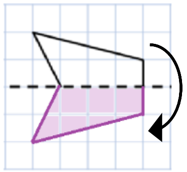


Reflect horizontally/ vertically (1)



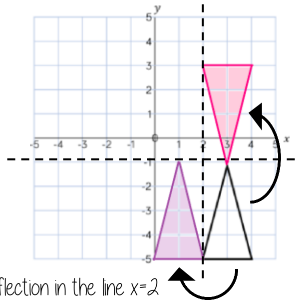
Note: a reflection doubles the area of the original shape

Reflection in a vertical line



Reflection in a horizontal line

Reflection on an axis grid

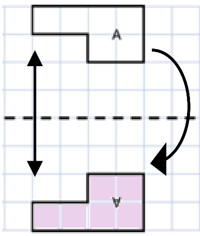


Reflection in the line y=-2

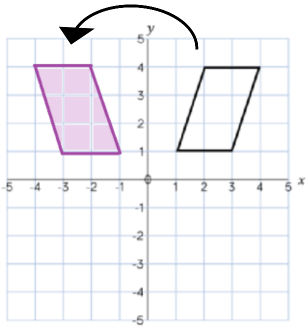
Reflection in the line x=2

Reflect horizontally/ vertically (2)

All points need to be the same distance away from the line of reflection



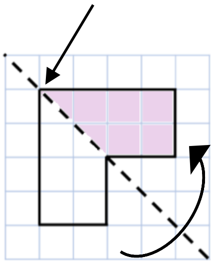
Reflection in the line y axis – this is also a reflection in the line x=0



Lines parallel to the x and y axis
REMEMBER
Lines parallel to the x-axis are y = ____
Lines parallel to the y-axis are x = ____

Reflect Diagonally (1)

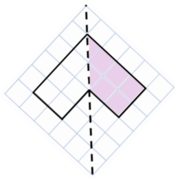
Points on the mirror line don't change position

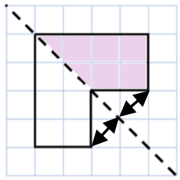


Fold along the line of symmetry to check the direction of the reflection

Turn your image

If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)



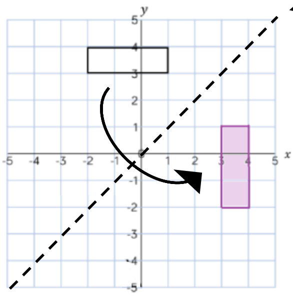


Drawing perpendicular lines

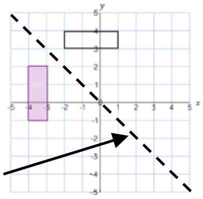
Perpendicular lines to and from the mirror line can help you to plot diagonal reflections

Reflect Diagonally (2)

This is the line **y = x** (every y coordinate is the same as the x coordinate along this line)

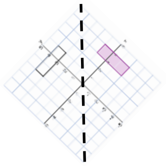


This is the line **y = - x**
The x and y coordinate have the same value but opposite sign



Turn your image

If you turn your image it becomes a vertical/ horizontal reflection (also good to check your answer this way)



YEAR 9 — REASONING WITH GEOMETRY
Enlargement & Similarity

What do I need to be able to do?

By the end of this unit you should be able to:
- Enlarge by a positive scale factor
- Enlarge by a fractional scale factor
- Identify similar shapes
- Work out missing sides and angles in similar shapes
- Use parallel lines to find missing angles
- Understand similarity and congruence

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)
Scale Factor: the multiplier of enlargement
Centre of enlargement: the point the shape is enlarged from
Similar: when one shape can become another with a reflection, rotation, enlargement or translation.
Congruent: the same size and shape
Corresponding: items that appear in the same place in two similar situations
Parallel: straight lines that never meet (equal gradients)

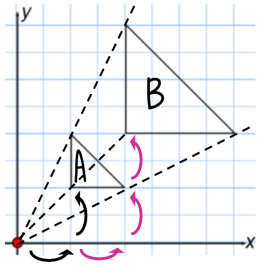
Positive scale factors R

Enlargement from a point

Enlarge shape A by SF 2 from (0,0)

The shape is enlarged by 2

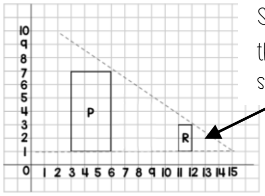
The distance from the point enlarges by 2



Fractional scale factors R

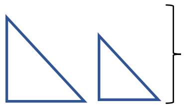
Fractions less than 1 make a shape SMALLER

R is an enlargement of P by a scale factor $\frac{1}{3}$ from centre of enlargement (15,1)



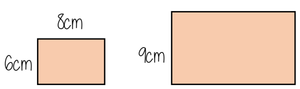
SF: $\frac{1}{3}$ - R is three times smaller than P

Identify similar shapes



Angles in similar shapes do not change.
e.g if a triangle gets bigger the angles can not go above 180°

Similar shapes

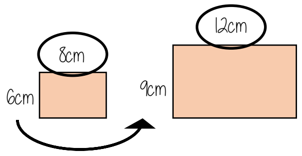


Scale Factor: Both sides on the bigger shape are 1.5 times bigger

Compare sides:	6 : 9	8 : 12
	2 : 3	2 : 3

Both sets of sides are in the same ratio

Information in similar shapes



x 1.5

Compare the equivalent side on both shapes

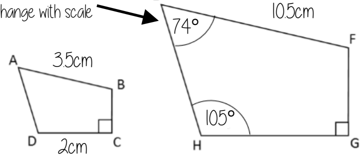
Scale Factor is the multiplicative relationship between the two lengths

Shape ABCD and EFGH are similar

Notation helps us find the corresponding sides

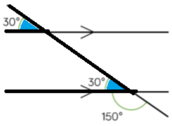
AB and EF are corresponding

Remember angles do not increase or change with scale



Angles in parallel lines R

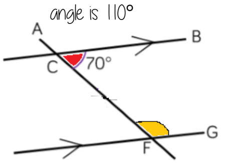
Alternate angles



Because alternate angles are equal the highlighted angles are the same size

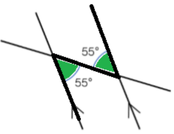
Co-interior angles

Because co-interior angles have a sum of 180° the highlighted angle is 110°



Corresponding angles

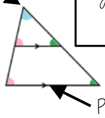
Because corresponding angles are equal the highlighted angles are the same size



As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

Similar triangles

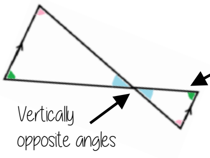
Shares a vertex



Because corresponding angles are equal the highlighted angles are the same size

Parallel lines — all angles will be the same in both triangle

As all angles are the same this is similar — it only one pair of sides are needed to show equality

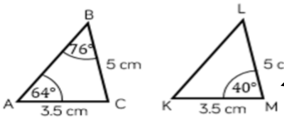


Vertically opposite angles

All the angles in both triangles are the same and so similar

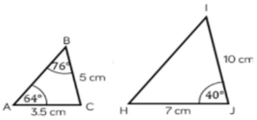
Congruence and Similarity

Congruent shapes are identical — all corresponding sides and angles are the same size



$A\hat{C}B = K\hat{M}L$

Because all the angles are the same and AC=KM BC=LM triangles ABC and KLM are **congruent**



Because all angles are the same, but all sides are enlarged by 2 ABC and HIJ are **similar**

Conditions for congruent triangles

Triangles are congruent if they satisfy any of the following conditions

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

YEAR 9 – REASONING WITH GEOMETRY…

Pythagoras' Theorem

What do I need to be able to do?

By the end of this unit you should be able to:

- Use square and cube roots
- Identify the hypotenuse
- Calculate the hypotenuse
- Find a missing side in a Right angled triangle
- Use Pythagoras' theorem on axes
- Explore proofs of Pythagoras' theorem

Keywords

Square number: the output of a number multiplied by itself

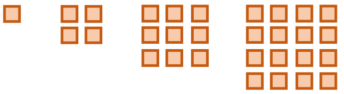
Square root: a value that can be multiplied by itself to give a square number

Hypotenuse: the largest side on a right angled triangle. Always opposite the right angle.

Opposite: the side opposite the angle of interest

Adjacent: the side next to the angle of interest

Squares and square roots R



This can also be written as 6^2

$\sqrt{\ }$ is the square root symbol

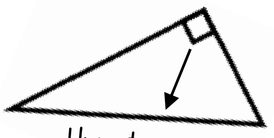
eg $\sqrt{64} = 8$

Because 8 × 8 = 64

1 × 1	2 × 2	3 × 3	4 × 4	5 × 5	6 × 6	7 × 7	8 × 8	9 × 9	10 × 10
1	4	9	16	25	36	49	64	81	100

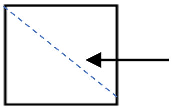
Square numbers

Identify the hypotenuse



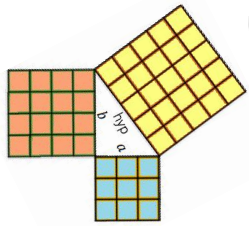
Hypotenuse

The hypotenuse is always the longest side on a triangle because it is opposite the biggest angle.



Polygons can still have a hypotenuse if it is split up into triangles and opposite a right angle

Determine if a triangle is right-angled



a = 3 b = 4 c = 5

If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse.

$$a^2 + b^2 = \text{hypotenuse}^2$$

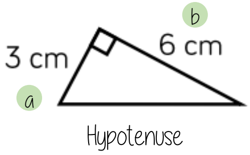
eg $a^2+b^2 = hypotenuse^2$

$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

Substituting the numbers into the theorem shows that this is a right-angled triangle

Calculate the hypotenuse



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

1. Substitute in the values for a and b

$3^2+6^2 = \text{hypotenuse}^2$

$9 + 36 = \text{hypotenuse}^2$

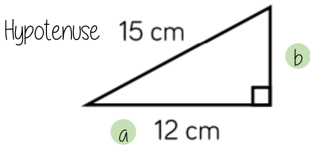
$45 = \text{hypotenuse}^2$

2. To find the hypotenuse square root the sum of the squares of the shorter sides.

$\sqrt{45} = \text{hypotenuse}$

$\mathbf{6.71}\boldsymbol{cm} = \text{hypotenuse}$

Calculate missing sides



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

$12^2+b^2 = 15^2$

1. Substitute in the values you are given

$\underset{-144}{144} + b^2 = \underset{-144}{225}$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

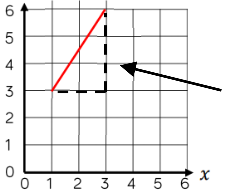
Square root to find the length of the side

$b^2 = 111$

$b = \sqrt{111} = 10.54\ cm$

Pythagoras' theorem on a coordinate axis

Find the **length** of the line segment



The segment can be made into a right-angled triangle by adding the sides on the diagram.

The line segment is the **hypotenuse**

$$\underbrace{a^2 + b^2} = \text{hypotenuse}^2$$

The lengths of a and b are the sides of the triangle.

Be careful to check the scale on the axes

YEAR 9 – REPRESENTATIONS...

Algebraic Representation

What do I need to be able to do?

By the end of this unit you should be able to:

- Draw quadratic graphs
- Interpret quadratic graphs
- Interpret other graphs including reciprocals
- Represent inequalities

Keywords

Quadratic: a curved graph with the highest power being 2. Square power.

Inequality: makes a non equal comparison between two numbers

Reciprocal: a reciprocal is 1 divided by the number

Cubic: a curved graph with the highest power being 3. Cubic power.

Origin: the coordinate (0, 0)

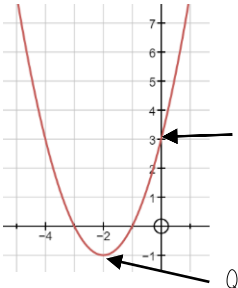
Parabola: a 'u' shaped curve that has mirror symmetry

Quadratic Graphs

$y = x^2 + 4x + 3$

If x^2 is the highest power in your equation then you have a quadratic graph.

It will have a parabola shape



Intersection with the y axis

Quadratic graphs are always symmetrical with the turning point in the middle

Substitute the x values into the equation of your line to find the y coordinates

x	-4	-3	-2	-1	0	1
y	3	0	-1	0	3	8

Coordinate pairs for plotting $(-3, 0)$

Plot all of the coordinate pairs and join the points with a curve (freehand)

Interpret other graphs

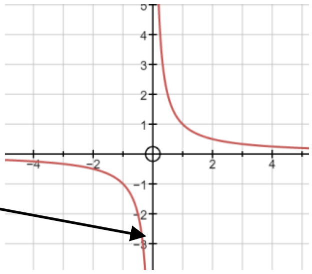
Cubic Graphs

$y = x^3 + 2x^2 - 2x + 1$

If x^3 is the highest power in your equation then you have a cubic graph

Reciprocal Graphs

$y = \frac{1}{x}$

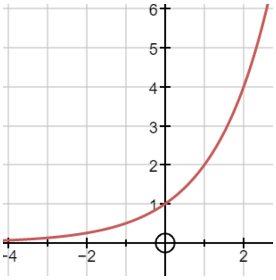


Reciprocal graphs never touch the y axis.
This is because x cannot be 0
This is an asymptote

Exponential Graphs

$y = 2^x$

Exponential graphs have a power of x

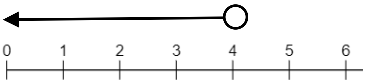


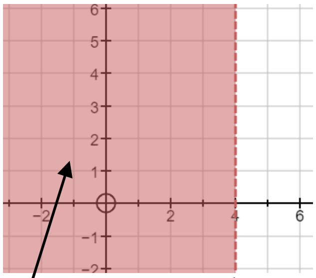
Represent Inequalities

Multiple methods of representing inequalities

$x < 4$

All values are less than 4



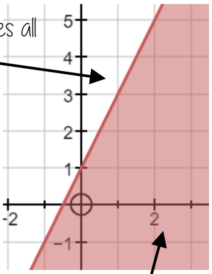


The shaded area indicates all possible values of x

The dotted line shows that the inequality does not include these points

The solid line shows that the inequality includes all the points on this line

$y \geq 2x + 1$



The shaded area indicates all possible solutions to this inequality

YEAR 9 – REASONING WITH GEOMETRY…
Solving Ratio & Proportion Problems

What do I need to be able to do?

By the end of this unit you should be able to:
- Solve problems with direct proportion
- Use conversion graphs
- Solve problems with inverse proportion
- Solve ratio problems
- Solve 'best buy' problems

Keywords

Proportion: a comparison between two numbers
Ratio: a ratio shows the relative size of two variables
Direct proportion: as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.
Inverse proportion:: as one variable is multiplied by a scale factor the other is divided by the same scale factor.

Direct Proportion

As one variable changes the other changes at the same rate.

This is a multiplicative change

4 cans of pop = £2.40

4 cans of pop = £2.40
× 0.5 ↓ 2 cans of pop = £1.20 ↓ × 0.5

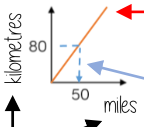
This multiplier is the same In the same way that this would be for ratio

4 cans of pop = £2.40
× 3 ↓ 12 cans of pop = £7.20 ↓ × 3

Sometimes this is easiest if you work out how much one unit is worth first e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables



This is always a straight line because as one variable increases so does the other at the same rate

Labelling of both axes is vital

To make conversions between units you need to find the point to compare – then find the associated point by using your graph.
Using a ruler helps for accuracy
Showing your conversion lines help as a "check" for solutions

Inverse Proportion

As one variable is multiplied by a scale factor the other is divided by the same scale factor

Examples of inversely proportional relationships

Time taken to fill a pool and the number of taps running

Time taken to paint a room and the number of workers

T is inversely proportional to G. When T=2 then G=20

÷ 2 × 4

T	1	2	8
G	40	20	5

× 2 ÷ 4

Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts

	Shop A	**Shop B**
	4 cans for £1.20	3 cans for 93p
	↓ £1.20 ÷ 4	↓ £0.93 ÷ 3
Cost per item	1 can is £0.30 Or 30p	1 can is £0.31 Or 31p

Shop A is the best value as it is 1p cheaper per can of pop

	Shop A	
	4 cans for £1.20	3 cans for 93p
	↓ 4 ÷ £1.20	↓ 3 ÷ £0.93
Cost per pound	£1 buys 3.333 cans of pop	£1 buys 3.23 cans of pop

Shop A is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound

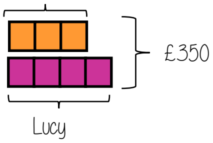
Best value is the most product for the lowest price per unit

Sharing a whole into a given ratio

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy
3 : 4



Find the value of one part

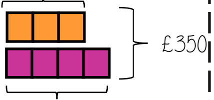
Whole: £350
7 parts to share between
(3 James, 4 Lucy)

£350 ÷ 7 = £50

☐ = one part = £50

Put back into the question

James: Lucy
× 50 3 : 4 × 50
£150:£200



James = 3 × £50 = £150
Lucy = 4 × £50 = £200

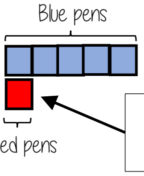
Finding a value given 1:n (or n:1)

Inside a box are blue and red pens in the ratio 5:1
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red
5 : 1

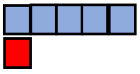
☐ = one part = 10 pens



One unit = 10 pens

Put back into the question

Blue : Red
× 10 5 : 1 × 10
50: 10



Blue pens = 5 × 10 = 50 pens
Red pens = 1 × 10 = 10 pens

There are 50 Blue Pens

YEAR 9 — REPRESENTATIONS...

Probability

What do I need to be able to do?

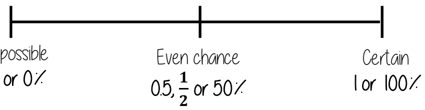
By the end of this unit you should be able to:

- Find single event probability
- Find relative frequency
- Find expected outcomes
- Find independent events
- Use diagrams to work out probabilities

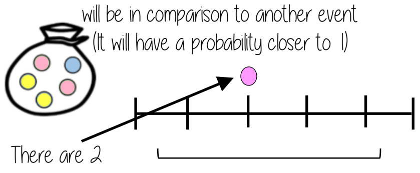
Keywords

Probability: the chance that something will happen
Relative Frequency: how often something happens divided by the outcomes
Independent: an event that is not effected by any other events.
Chance: the likelihood of a particular outcome.
Event: the outcome of a probability – a set of possible outcomes.
Biased: a built in error that makes all values wrong by a certain amount.

The probability scale



The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability

There are 5 possible outcomes So 5 intervals on this scale, each interval value is $\frac{1}{5}$

Single event probability

Probability is always a value between 0 and 1

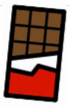
The probability of getting a blue ball is $\frac{1}{5}$

∴The probability of NOT getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	



P(white chocolate) = 1 – 0.15 – 0.35
= 0.5

Relative Frequency

$$\frac{\text{Frequency of event}}{\text{Total number of outcomes}}$$

Remember to calculate or identify the overall number of outcomes!

Colour	Frequency	Relative Frequency
Green	6	0.3
Yellow	12	0.6
Blue	2	0.1
	20	

Relative frequency can be used to find expected outcomes

e.g Use the relative probability to find the expected outcome for green if there are 100 selections.

Relative frequency x Number of times

$$0.3 \times 100 = 30$$

Expected outcomes

Expected outcomes are estimations. It is a long term average rather than a prediction.

Dark	Milk	White
0.15	0.35	0.5

The sum of the probabilities is 1

An experiment is carried out 400 times.
Show that dark chocolate is expected to be selected 60 times

$$0.15 \times 400 = 60$$

Independent events

The rolling of one dice has no impact on the rolling of the other. The individual probabilities should be calculated separately.

Probability of event 1 × Probability of event 2

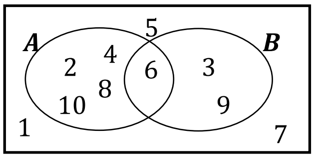
Find the probability of getting a 5 and a red

$$P(5) = \frac{1}{6} \qquad P(R) = \frac{1}{4}$$

$$P(5 \; and \; R) = \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$$

Using diagrams

Recap Venn diagrams, Sample space diagrams and Two-way tables



	Car	Bus	Walk	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

The possible outcomes from rolling a dice

The possible outcomes from tossing a coin

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

YEAR 9 — REASONING WITH GEOMETRY

Trigonometry

What do I need to be able to do?

By the end of this unit you should be able to:

- Work fluently with hypotenuse, opposite and adjacent sides
- Use the tan, sine and cosine ratio to find missing side lengths
- Use the tan, sine and cosine ratio to find missing angles
- Calculate sides using Pythagoras' Theorem

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)

Scale Factor: the multiplier of enlargement

Constant: a value that remains the same

Cosine ratio: the ratio of the length of the adjacent side to that of the hypotenuse. The sine of the complement.

Sine ratio: the ratio of the length of the opposite side to that of the hypotenuse.

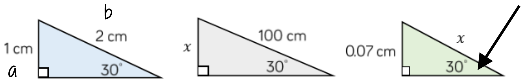
Tangent ratio: the ratio of the length of the opposite side to that of the adjacent side.

Inverse: function that has the opposite effect.

Hypotenuse: longest side of a right-angled triangle. It is the side opposite the right-angle.

Ratio in right-angled triangles

When the angle is the same the ratio of sides a and b will also remain the same



a : b
1 : 2

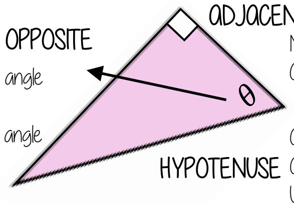
$\frac{a}{b} = \frac{1}{2}$

a : b
x : 100
50 : 100

a : b
0.07 : x
0.07 : 0.14

Hypotenuse, adjacent and opposite

ONLY right-angled triangles are labelled in this way



OPPOSITE: Always opposite an acute angle. Useful to label second. Position depend upon the angle in use for the question

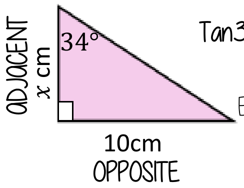
ADJACENT: Next to the angle in question. Often labelled last

HYPOTENUSE: Always the longest side. Always opposite the right angle. Useful to label this first

Tangent ratio: side lengths

$\text{Tan}\theta = \frac{\text{opposite side}}{\text{adjacent side}}$

Substitute the values into the tangent formula



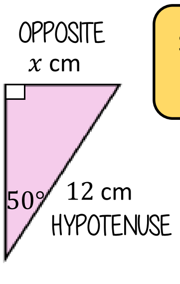
$\text{Tan}34 = \frac{10}{x}$

Equations might need rearranging to solve

$x \times \text{Tan}34 = 10$

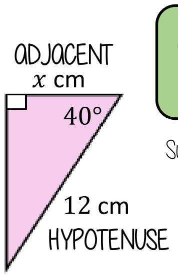
$x = \frac{10}{\text{Tan}34} = 14.8\text{cm}$

Sin and Cos ratio: side lengths



$\text{Sin}\theta = \frac{\text{opposite side}}{\text{hypotenuse side}}$

NOTE
The Sin(x) ratio is the same as the Cos(90-x) ratio

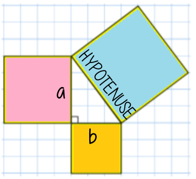


$\text{Cos}\theta = \frac{\text{adjacent side}}{\text{hypotenuse side}}$

Substitute the values into the ratio formula

Equations might need rearranging to solve

Pythagoras theorem



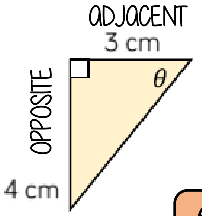
$\text{Hypotenuse}^2 = a^2 + b^2$

This is commutative — the square of the hypotenuse is equal to the sum of the squares of the two shorter sides

Places to look out for Pythagoras
- Perpendicular heights in isosceles triangles
- Diagonals on right angled shapes
- Distance between coordinates
- Any length made from a right angles

Sin, Cos, Tan: Angles

Inverse trigonometric functions



Label your triangle and choose your trigonometric ratio

Substitute values into the ratio formula

$\text{Tan}\theta = \frac{3}{4}$

$\theta = \text{Tan}^{-1}\ \frac{3}{4}$

$\theta = 36.9°$

$\theta = \text{Tan}^{-1}\ \frac{\text{opposite side}}{\text{adjacent side}}$

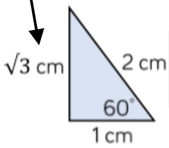
$\theta = \text{Sin}^{-1}\ \frac{\text{opposite side}}{\text{hypotenuse side}}$

$\theta = \text{Cos}^{-1}\ \frac{\text{adjacent side}}{\text{hypotenuse side}}$

Key angles

Because trig ratios remain the same for similar shapes you can generalise from the following statements.

This side could be calculated using Pythagoras



$\text{Tan}30 = \frac{1}{\sqrt{3}}$

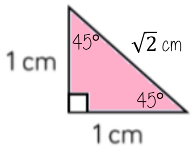
$\text{Tan}60 = \sqrt{3}$

$\text{Cos}30 = \frac{\sqrt{3}}{2}$

$\text{Cos}60 = \frac{1}{2}$

$\text{Sin}30 = \frac{1}{2}$

$\text{Sin}60 = \frac{\sqrt{3}}{2}$



$\text{Tan}45 = 1$

$\text{Cos}45 = \frac{1}{\sqrt{2}}$

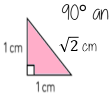
$\text{Sin}45 = \frac{1}{\sqrt{2}}$

Key angles 0° and 90°

$\text{Tan}0 = 0$

~~Tan90~~

This value cannot be defined — it is impossible as you cannot have two 90° angles in a triangle



$\text{Sin}0 = 0$

$\text{Sin}90 = 1$

$\text{Cos}0 = 1$

$\text{Cos}90 = 0$

YEAR 9 — REASONING WITH GEOMETRY

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Vectors

What do I need to be able to do?

By the end of this unit you should be able to:

- Understand and represent vectors
- Use and read vector notation
- Draw and understand vectors multiplied by a scalar
- Draw and understand addition of vectors
- Draw and understand addition and subtraction of vectors

Keywords

Direction: the line our course something is going
Magnitude: the magnitude of a vector is its length
Scalar: a single number used to represent the multiplier when working with vectors
Column vector: a matrix of one column describing the movement from a point
Resultant: the vector that is the sum of two or more other vectors
Parallel: straight lines that never meet

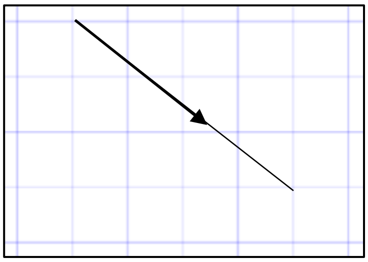
Understand and represent vectors

Column vectors have been seen in translations to describe the movement of one image onto another

Movement along the x-axis →

Movement along the y-axis →

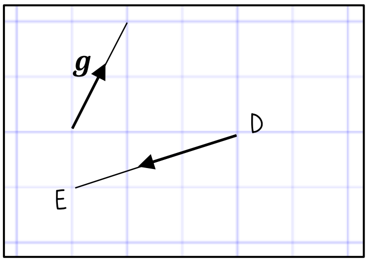
$\begin{pmatrix}4\\-3\end{pmatrix}$ →



Vectors show both direction and magnitude

The arrow is pointing in the direction from starting point to end point of the vector.	The direction is important to correctly write the vector
The magnitude is the length of the vector (This is calculated using Pythagoras theorem and forming a right-angled triangle with auxiliary lines)	The magnitude stays the same even if the direction changes

Understand and represent vectors



Vector notation $\overrightarrow{DE}$ is another way to represent the vector joining the point D to the point E.

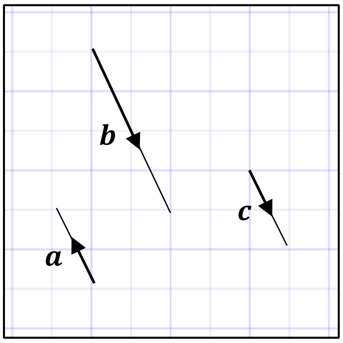
$\overrightarrow{DE} = \begin{pmatrix}-3\\-1\end{pmatrix}$

The arrow also indicates the direction from point D to point E

Vectors can also be written in bold lower case so $\boldsymbol{g}$ represents the vector $\boldsymbol{g} = \begin{pmatrix}1\\2\end{pmatrix}$

Vectors multiplied by a scalar

Parallel vectors are scalar multiples of each other



$\boldsymbol{a} = \begin{pmatrix}-1\\2\end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix}2\\-4\end{pmatrix}$ $\boldsymbol{c} = \begin{pmatrix}1\\-2\end{pmatrix}$

$\boldsymbol{b} = 2 \times \boldsymbol{c} = 2\boldsymbol{c}$

Multiply $\boldsymbol{c}$ by 2 this becomes $\boldsymbol{b}$.
The two lines are parallel

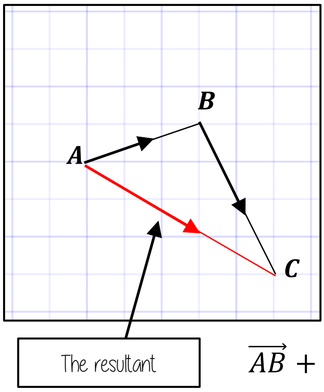
$\boldsymbol{a} = -1 \times \boldsymbol{c} = -\boldsymbol{c}$

The vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ are also parallel. A negative scalar causes the vector to reverse direction.

$\boldsymbol{b} = -2 \times \boldsymbol{a} = -2\boldsymbol{a}$

Addition of vectors

$\overrightarrow{AB} = \begin{pmatrix}3\\1\end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix}2\\-4\end{pmatrix}$



$\overrightarrow{AB} + \overrightarrow{BC}$

$= \begin{pmatrix}3\\1\end{pmatrix} + \begin{pmatrix}2\\-4\end{pmatrix}$

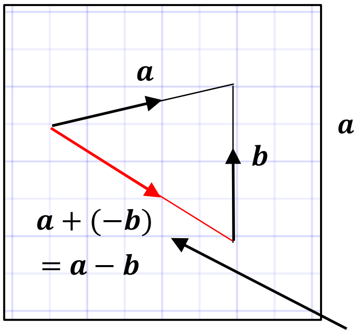
$= \begin{pmatrix}3+2\\1+-4\end{pmatrix}$

$\overrightarrow{AC} = \begin{pmatrix}5\\-3\end{pmatrix}$

Look how this addition compares to the vector $\overrightarrow{AC}$

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} = \begin{pmatrix}5\\-3\end{pmatrix}$

Addition and subtraction of vectors



$\boldsymbol{a} = \begin{pmatrix}5\\1\end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix}0\\4\end{pmatrix}$

$\boldsymbol{a} + (-\boldsymbol{b}) = \begin{pmatrix}5+ & -0\\1+ & -4\end{pmatrix} = \begin{pmatrix}5\\-4\end{pmatrix}$

The resultant is $\boldsymbol{a} - \boldsymbol{b}$ because the vector is in the opposite direction to b which needs a scalar of -1