Summer Task 2023

Topic	Question	Score	RAG
Surds	1	/4	
	2	/1	
	3	/1	
	4	/1	
Indices	1	/11	
	2	/7	
	3	/12	
	4	/4	
Straight Lines	1	/2	
	2	/1	
	3	/1	
	4	/1	
Manipulating Algebraic Expressions	1	/4	
	2	/5	
	3	/6	
	4	/6	
	5	/5	
Quadratics	1	/5	
	2	/5	
	3	/3	
	4	/1	
	5	/3	
	6	/6	
Circles	1	/1	
Simultaneous Equations	1	/2	
	2	/1	
	3	/1	

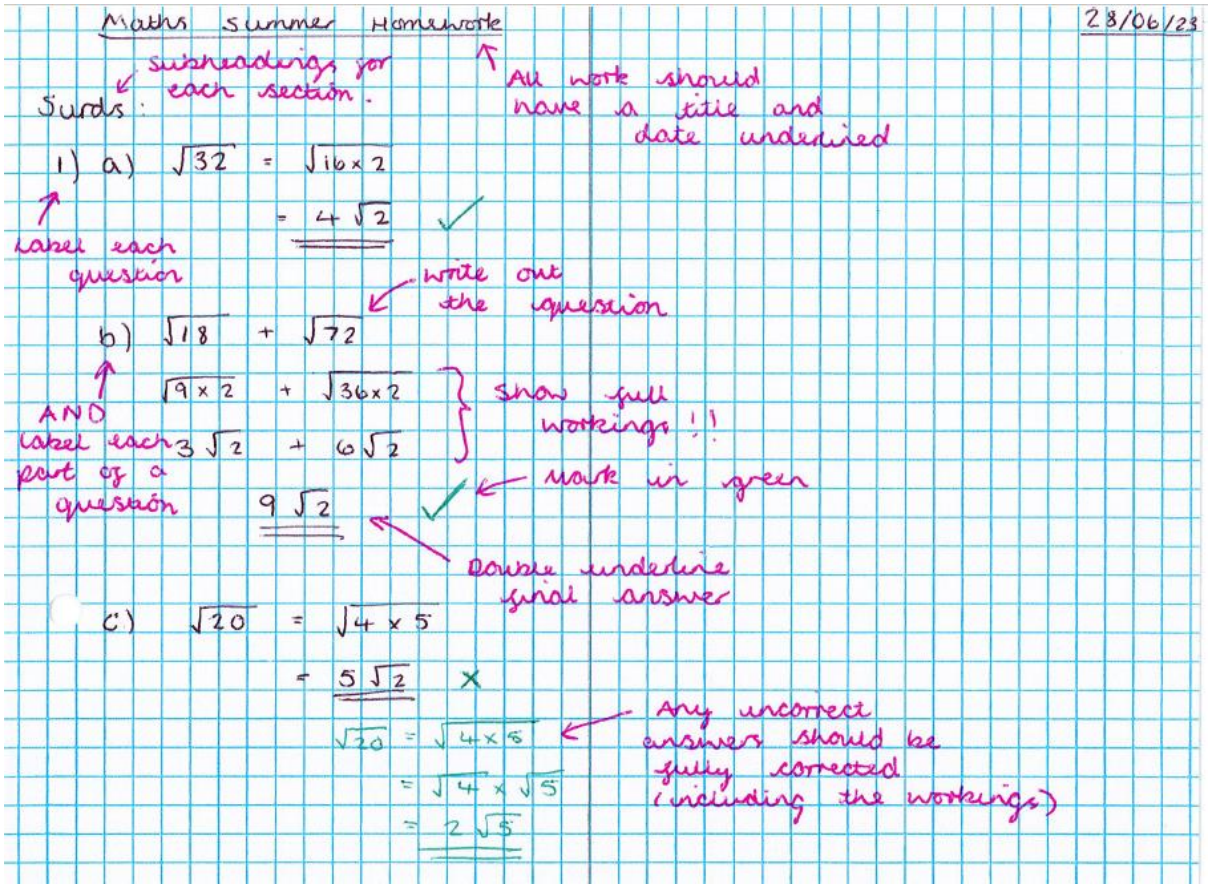
Expectations

You need to complete every question to the best of your ability. We have provided answers so that you can mark your work (in green pen) and correct any mistakes that you have made. On the front page of the task, there is a table for you to record your scores. From this, you should then use the RAG (Red, Amber, Green) column to identify which topics you need to focus your revision on before you start year 12.

Once you have identified these areas of weakness, you can then use the following resource https://www.mathsgenie.co.uk/gcse.html to practice GCSE style questions from these particular topics. There are also videos that you can watch if you are struggling to remember the method.

By the time you come back in September, you should be confident in attempting questions similar to those in the Summer Task. You will have a short assessment on your return to ensure that you are ready for A-Level Maths.

Your work should be presented as follows:



Surds

1) Simplify the following:
   a) $\sqrt{75}$
   b) $\sqrt{80}$
   c) $\sqrt{54} - \sqrt{24}$
   d) $2\sqrt{12} + 6\sqrt{108}$
2) Express $\frac{6}{\sqrt{2}}$ in the form $a\sqrt{b}$, where *a* and *b* are positive integers.
3) Expand and simplify $(\sqrt{3} + \sqrt{15})^2$ give your answer in the form $n + m\sqrt{5}$, where *n* and *m* are integers.
4) Rationalise $\frac{(6-\sqrt{2})}{(10+\sqrt{2})}$.

Indices

1) Simplify:
   a) $a^5 \times a^{12}$
   b) $(m^2)^3$
   c) $b^4 \times b \times b^{12}$
   d) $(a^3)^3 \times (a^4)^2$
   e) $7^5 \div 7^0$
   f) $4a \times 5a^2$
   g) $40b^5 \div 5b^3$
   h) $(a^8)^2 \div (a^5)^3$
   i) $(2b^2)^3 \times (3b^4)^2$
   j) $(4b^4)^2 \div (2b^3)^3$
   k) $(ab)^3 \times (ab)^2$
2) Which of the following expressions are square?
   a) $2^6 \times 2^5$
   b) $(6^5)^2$
   c) $4^9 \times 4^9$
   d) 36
   e) 3^6
   f) $(7^3)^3$
   g) $(14^{12})^4$

3) Showing ALL STEP BY STEP WORKING, calculate:
   a) 4^{-2}
   b) $81^{\frac{1}{2}}$
   c) $32^{\frac{2}{5}}$
   d) $27^{\frac{2}{3}}$
   e) 15^{-2}
   f) $2.25^{\frac{1}{2}}$
   g) $\left(\frac{9}{4}\right)^{\frac{1}{2}}$
   h) $16^{-\frac{3}{2}}$
   i) $64^{-\frac{5}{6}}$
   j) $49^{-\frac{3}{2}}$
   k) $\left(\frac{8}{27}\right)^{-\frac{1}{3}}$
   l) $\sqrt{125} = 5^{x}, so\ x =$
4) Simplify
   a) $3x^{-7}$
   b) $\frac{1}{3}x^{-2}$
   c) $(4x)^{-2}$
   d) $\frac{1}{2x^{-3}}$

Straight Lines

1) Find the equation of the line passing through:
   a) (5, 9) & (8, 3)
   b) (4,7) & (8, 12)
2) Find the equation of the line parallel to $x + y = 5$ and passing through the point (3,2).
3) Find the equation of the line perpendicular to $x - 3y = 15$ and passing through the point (5,-2).
4) Find the equation for the perpendicular bisector between (7, 10) and (4, 11).

Manipulating Algebraic Expressions

1) Simplify:
   a) $\frac{8x^2}{2x^2}$

b) $\frac{8ab^2}{12ab}$

c) $\frac{5ab}{10b}$

d) $\frac{12a^2b}{4ab^2}$

2) Write as a single fraction and simplify:

a) $\frac{3x}{2} \times \frac{2a}{3x}$

b) $\frac{3y^2}{3} \times \frac{2x}{9y}$

c) $\frac{x^2}{x^2+2x} \div \frac{x}{x+2}$

d) $\frac{a^2}{5} \div \frac{a}{10}$

e) $\frac{x}{5} \times \frac{y^2}{x^2}$

3) Simplify:

a) $\frac{7a^2b}{35ab^2}$

b) $\frac{(2a)^2}{4a}$

c) $\frac{7yx}{8xy}$

d) $\frac{5x+2x^2}{3x}$

e) $\frac{4a+5a^3}{5a}$

f) $\frac{5ab}{15a+10a^2}$

4) Simplify:

a) $\frac{18a-3ab}{6a^2}$

b) $\frac{4ab+8a^2}{2ab}$

c) $\frac{x^2+2x}{x^2-3x}$

d) $\frac{x^2-4x-21}{x^2-5x-14}$

e) $\frac{x^2+6x+5}{x^2-x-2}$

f) $\frac{x^2+7x+10}{x^2-4}$

5) Simplify:

a) $\frac{x-1}{3} + \frac{x+2}{4}$

b) $\frac{x+1}{3} - \frac{2x+1}{4}$

c) $\frac{3}{x-2} + \frac{4}{x}$

d) $\frac{3}{x-2} - \frac{4}{x+1}$

e) $\frac{x-6}{x+2} + \frac{2x}{x-3}$

Quadratics

1) Solve the following quadratics using the quadratic formula (which we use when we cannot complete the square or factorise).
   a) $3x^2 - 7x - 20 = 0$
   b) $6x^2 - 11x - 7 = 0$
   c) $2y^2 - 5y + 1 = 0$
   d) $\frac{1}{2}y^2 + 3y + 1 = 0$
   e) $2x^2 + 11x + 5 = 0$
2) Put the following in the form $(x + a)^2 + b$ where a, b are integers:
   a) $x^2 + 4x + 1$
   b) $x^2 - 6x + 9$
   c) $x^2 + 2x - 15$
   d) $x^2 + 3x$
   e) $x^2 + 16x + 5$
3) Solve the following by completing the square:
   a) $x^2 + 4x - 3 = 0$
   b) $x^2 - 3x - 2 = 0$
   c) $x^2 + 12x = 1$
4) Find the turning point of $x^2 + 6x + 10 = 0$. What does the turning point tell you about the solutions to $x^2 + 6x + 10 = 0$?
5) $f(x) = x^2 + 4x + 7$

   a) Give the equation for the line of symmetry through $f(x)$

   b) What is the coordinate of the minimum point of $f(x)$?

   c) What is the GREATEST (maximum) value of: $\frac{1}{x^2+4x+7}$?

6) Solve these equations by factorising (there is one that cannot be factorised hidden among them – can you find it?):

   a) $x^2 + 5x - 14 = 0$

   b) $x^2 = 6 - x$

   c) $x + 3 = \frac{14}{x}$

   d) $2x + 2 = \frac{13}{x} - 23$

   e) $2p^2 - 32 = 0$

f) $\frac{2}{x-2} + \frac{4}{x+1} = 3$

Circles

1) The line *l* is a tangent to the circle $x^2 + y^2 = 68$ at the point P (2, 8). Find the equation of the line and hence find the x and y intercepts of *l* .

Simultaneous Equations

1) Solve the following simultaneous equations
   a) $3x + y = -4$

      $3x - 4y = 6$

   b) $5x + y = 21$

      $x - 3y = 9$

2) Solve the following simultaneous equation

   $x^2 + y^2 = 25$

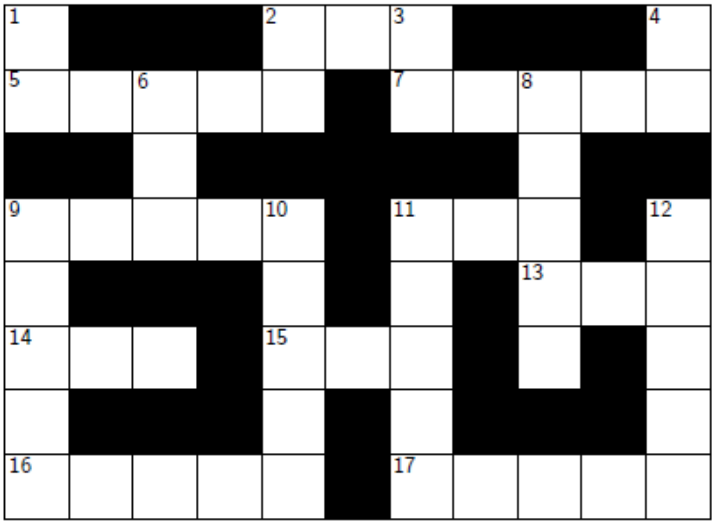
   $y - 3x = 13$

3) Show algebraically that the line $x - 2y = 10$ is a tangent to the circle with equation $x^2 + y^2 = 20$.

Crossnumber

Rules

Although many of the clues have multiple answers, there is only one solution to the completed crossnumber. As usual, no numbers begin with 0. You should be able to complete this mini cross-number using only a pen and paper.



Across

2 Why is 6 afraid of 7? **(3)**

5 None of the digits of this number are 1. The highest common factor of each digit and its neighbours is 1. **(5)**

7 The product of 23 and the first five prime numbers. **(5)**

9 A multiple of 1111. **(5)**

11 Each digit of this number is half the previous digit. **(3)**

13 One less than a square number. **(3)**

14 The sum of the digits of 16A multiplied by the sum of the digits of 10D. **(3)**

15 The number of days in the year 2792. **(3)**

16 A factor of 10D. **(5)**

17 Each digit of this number starts with the last letter of the previous digit. **(5)**

Down

1 The coefficient of x^3 when $(x+1)^7$ is expanded. **(2)**

2 The smallest number that has five syllables in its name. **(2)**

3 A multiple of 5. **(2)**

4 In this number's name, the letters appear in alphabetical order. **(2)**

6 Start with 1. Add one and square the result. Then add one and square the result again. Then add one and square the result again again. **(3)**

8 Equal to another answer in this crossnumber. **(5)**

9 An anagram of 17A. **(5)**

10 The only five digit factorial!* **(5)**

11 One more than a multiple of 10D. **(5)**

12 The mean of 7A, 16A, 9D, 11D and 12D. **(5)**

*The nth factorial, written $n!$, is equal to $n \times (n-1) \times \ldots \times 2 \times 1$.

Answers

Surds:

1) (a) $5\sqrt{3}$
   (b) $4\sqrt{5}$
   (c) $\sqrt{6}$
   (d) $40\sqrt{3}$
2) $3\sqrt{2}$
3) 2) $18 + 6\sqrt{5}$
4) 3) $\frac{31-8\sqrt{2}}{49}$

Indices:

1) (a) a^{17} (b) m^6 (c) b^{17} (d) a^{17} (e) 7^5 (f) $20a^3$ (g) $8b^2$ (h) a (i) $72b^{14}$ (j) $2b^{-1} = \frac{2}{b}$
   (k) a^5b^5
2) $(6^5)^2$, $4^9 \times 4^9$, 36, 3^6, $(14^{12})^4$
3) (a) $\frac{1}{16}$ (b) 9 (c) 4 (d) 9 (e) $\frac{1}{225}$ (f) 1.5 (g) $\frac{3}{2}$ (h) $\frac{1}{64}$ (i) $\frac{1}{32}$ (j) $\frac{1}{343}$ (k) $\frac{3}{2}$ (l) $x = \frac{3}{2}$
4) (a) $\frac{3}{x^7}$ (b) $\frac{1}{3x^2}$ (c) $\frac{1}{16x^2}$ (d) $\frac{x^3}{2}$

Straight Lines:

1) (a) $y = -2x + 19$ (b) $y = \frac{5}{4}x + 2$
2) $y = -x + 5$
3) $y = -3x - 1$
4) $y = 3x - 6$

Manipulating Algebraic Expressions:

1) (a) 4 (b) $\frac{2}{3}b$ (c) $\frac{1}{2}a = \frac{a}{2}$ (d) $\frac{3a}{b}$
2) (a) a (b) $\frac{2xy}{9}$ (c) 1 (d) $2a$ (e) $\frac{y^2}{5x}$
3) (a) $\frac{a}{5b}$ (b) a (c) $\frac{7}{8}$ (d) $\frac{5+2x}{3}$ (e) $\frac{4+5a^2}{5} = \frac{4}{5} + a^2$ (f) $\frac{b}{3+2a}$
4) (a) $\frac{6-b}{2a}$ (b) $2 + \frac{4a}{b}$ (c) $\frac{x+2}{x-3}$ (d) $\frac{x+3}{x+2}$ (e) $\frac{x+5}{x-2}$ (f) $\frac{x+5}{x-2}$
5) (a) $\frac{7x+2}{12}$ (b) $\frac{1-2x}{12}$ (c) $\frac{7x-8}{x^2-2x}$ (d) $\frac{11-x}{x^2-x-2}$ (e) $\frac{3x^2-5x+18}{x^2-x-6}$

Quadratics:

1) (a) x= 4, -5/3 (b) x= 7/3, -1/2 (c) x= $\frac{5}{4} \pm \frac{\sqrt{17}}{4}$ (d) x= $-3 \pm \sqrt{7}$ (e) x= -1/2 or -5
2) (a) $(x+2)^2 - 3$ (b) $(x-3)^2$ (c) $(x+1)^2 - 16$ (d) $\left(x + \frac{3}{2}\right)^2 - \frac{9}{4}$ (e) $(x+8)^2 - 59$
3) (a) $\pm\sqrt{7} - 2$ (b) $\frac{3}{2} \pm \frac{\sqrt{17}}{2}$ (c) $\pm\sqrt{37} - 6$
4) TP = (-3, 1). It is a minimum point, so all values are above 1. Therefore no real roots as does not cross x axis.

5) $f(x) = (x+2)^2 + 3$ (a) x= - 2 (b) (-2, 3) (c) As we divide by something smaller our answer gets bigger. So the greatest value will be when we divide by the smallest possible value (the minimum value). So $\mathbf{\frac{1}{3}}$.

6) (a) x = -7, x = 2 (b) x = -3, x = 2 (c) DOES NOT FACTORISE (d) x = ½, x = -13 (e) x = ± 4 (f) x= 0 , x = 3

Circles:

1) $y = -\frac{1}{4}x + 8\frac{1}{2}$ so (0, 8.5) and (34, 0)

Simultaneous Equations:

1) (a) $x = -\frac{2}{3}, y = -2$ (b) $x = 4.5, y = -1.5$
2) $x = -\frac{24}{5}, y = -\frac{7}{5}$ and $x = -3, y = 4$
3) $5y^2 + 40y + 80 = 0$ so only solution is $y = -2, x = 4$. As there is only one point of intersection, the line must be a tangent to the circle.

Crossnumber:

Mini Crossnumber