



The Federation of St Elphege's and Regina Coeli Catholic Schools

Calculation Policy

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200
201	202	203	204	205	206	207	208	209	210
211	212	213	214	215	216	217	218	219	220
221	222	223	224	225	226	227	228	229	230
231	232	233	234	235	236	237	238	239	240
241	242	243	244	245	246	247	248	249	250
251	252	253	254	255	256	257	258	259	260
261	262	263	264	265	266	267	268	269	270
271	272	273	274	275	276	277	278	279	280
281	282	283	284	285	286	287	288	289	290
291	292	293	294	295	296	297	298	299	300

Counting:

Year R (Early learning goal)

- Children count reliably with numbers from one to 20

Year 1

- count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number
- count, read and write numbers to 100 in numerals, count in different multiples including 1s, 2s, 5s and 10s

Year 2

- count in steps of 2s, 3s, and 5s from 0, and count in 1s and 10s from any number, forward or backward

Year 3

- count forward and backward in multiples of 1s, 2s, 3s, 4s, 5s, 8s, 10s, 50s, and 100s;
- count up and down in 10ths, $\frac{1}{4}$ s and $\frac{1}{2}$ s - recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10

Year 4

- count forward and backward in multiples of 1-10s, 25s, 50s, 100s and 1000s;
- count backwards through zero to include negative numbers;
- count up and down in 10ths, 100ths, $\frac{1}{4}$ s, and $\frac{1}{2}$ s - recognise that hundredths arise when dividing an object by a hundred and dividing tenths by ten

Year 5

- count forward or backward in multiples of 1-10s, 25s, 50s, 100s, 250s, 1000s, 10 000s, 100 000s and into negative numbers;
- Count in 10ths, 100ths, $\frac{1}{4}$ s and $\frac{1}{2}$ s.

Year 6

- count forward or backward in multiples of 1-10s, 25s, 50s, 100s, 250s, 500s, 1000s, 10 000s, 100 000s and into negative numbers.
- Count in 10ths, 100ths, $\frac{1}{4}$ s, $\frac{1}{2}$ s and in decimals such as 0.1s, 0.01s, 0.2s, 0.25s, 0.5s.

Recall of times tables and its associated division facts:

Year 2: 2, 5 and 10

Year 3: 2, 3, 4, 5, 6, 8, 10

Year 4: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

Year 5: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

Year 6: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

Use the following 100 squares to provide visual aid as children get used to the position of numbers in relation to each other in our number system when counting.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0
6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0
7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0
8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0
9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10

10	20	30	40	50	60	70	80	90	100
110	120	130	140	150	160	170	180	190	200
210	220	230	240	250	260	270	280	290	300
310	320	330	340	350	360	370	380	390	400
410	420	430	440	450	460	470	480	490	500
510	520	530	540	550	560	570	580	590	600
610	620	630	640	650	660	670	680	690	700
710	720	730	740	750	760	770	780	790	800
810	820	830	840	850	860	870	880	890	900
910	920	930	940	950	960	970	980	990	1000

100	200	300	400	500	600	700	800	900	1000
1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
2100	2200	2300	2400	2500	2600	2700	2800	2900	3000
3100	3200	3300	3400	3500	3600	3700	3800	3900	4000
4100	4200	4300	4400	4500	4600	4700	4800	4900	5000
5100	5200	5300	5400	5500	5600	5700	5800	5900	6000
6100	6200	6300	6400	6500	6600	6700	6800	6900	7000
7100	7200	7300	7400	7500	7600	7700	7800	7900	8000
8100	8200	8300	8400	8500	8600	8700	8800	8900	9000
9100	9200	9300	9400	9500	9600	9700	9800	9900	10000

Provide children with opportunities to investigate and discover the patterns on a multiplication square. Allow them to realise the commutative nature of multiplication and how division facts can be derived from known multiplication facts.

x	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Addition:

Models and Images

Counting apparatus

Place value apparatus

Digi-flips

Place value cards

Number tracks

Numbered number lines

Marked but unnumbered number lines

Empty number lines

Hundred square

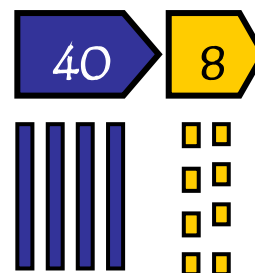
Counting stick

Bead string

Models and Images charts

ITPs – Number Facts, Ordering Numbers, Number Grid, Counting on and back in ones and tens

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



sum addition total
make
and + more
add plus
altogether increase

Reception

Early learning goal:

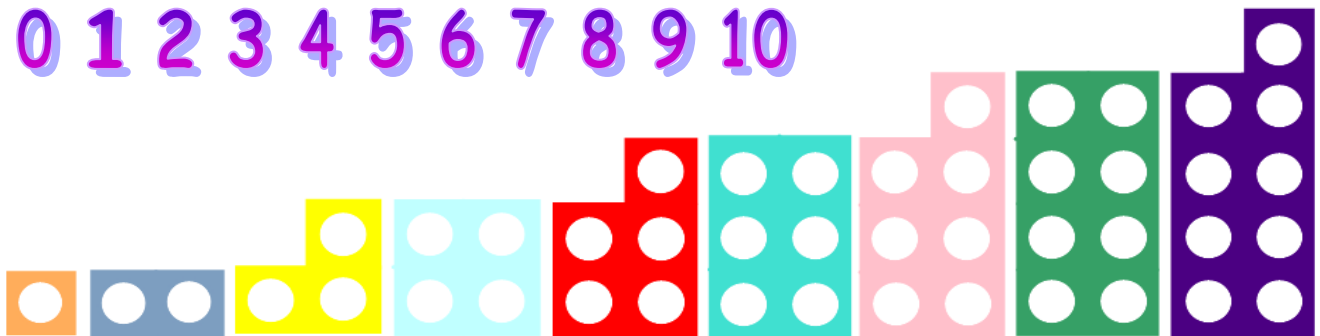
Children count reliably with numbers from 1 to 20, place them in order and say which number is one more than a given number.

Using quantities and objects, they add two single-digit numbers and count on to find the answer.

Recognise numerals 0 to 10.

Use Numicon activities to provide children with broad experiences of physically combining objects representing numbers with each other in order to make their calculating 'real'.

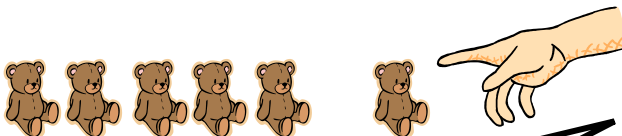
0 1 2 3 4 5 6 7 8 9 10



Count reliably up to 10 everyday objects.

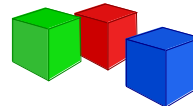


To find how many altogether, touch and drag them into a line one at a time whilst counting.



1, 2, 3, 4, 5, 6 ... there are 6 teddies altogether.

Find one more than a number.



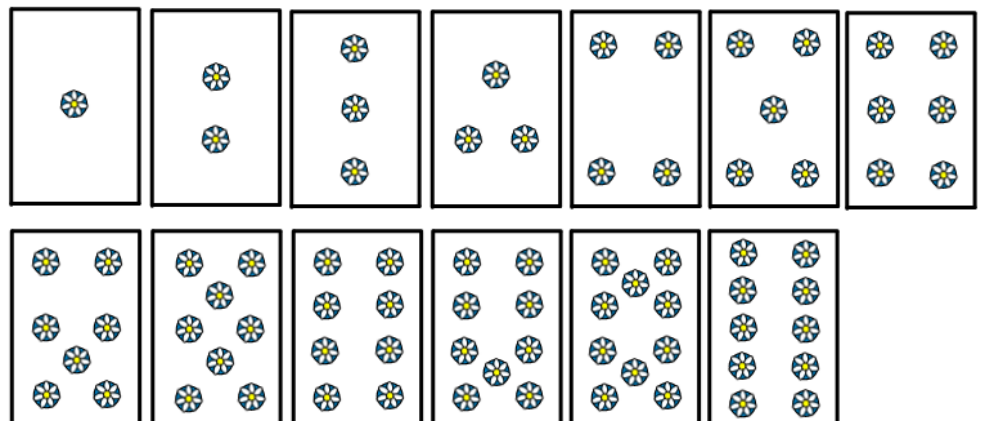
One more than three is four



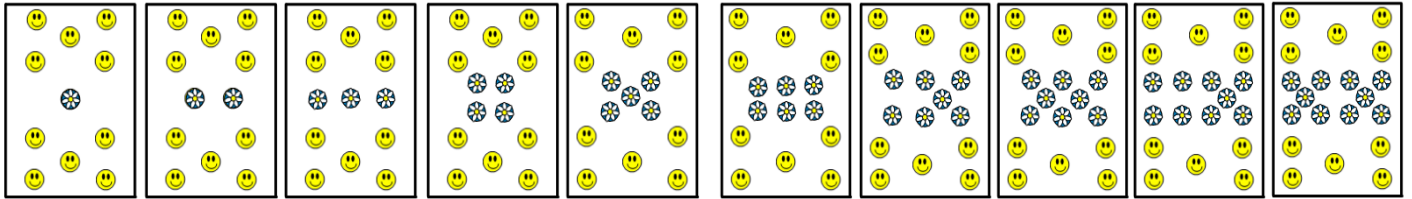
The number after three is four.

What is the number after.....?

Recognise numerals 0 to 10 and understand the meaning of each number by recognising and knowing their clusters

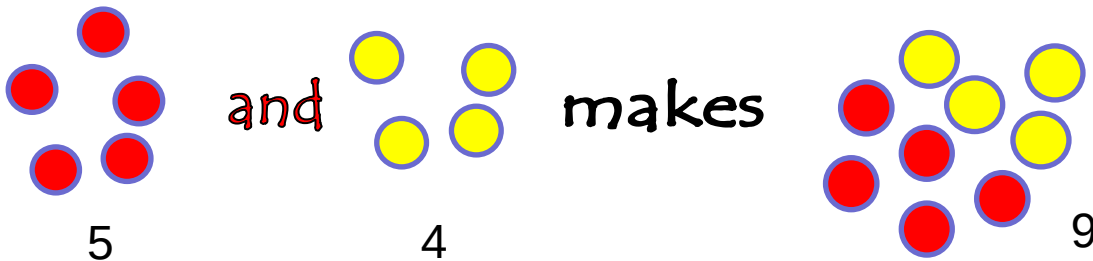


Recognise numerals up to 20 and understand the meaning of each number by recognising and knowing their clusters.

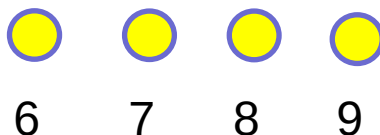


10 11 12 13 14 15 16 17 18 19 20

Begin to relate addition to combining two groups of objects and count all.

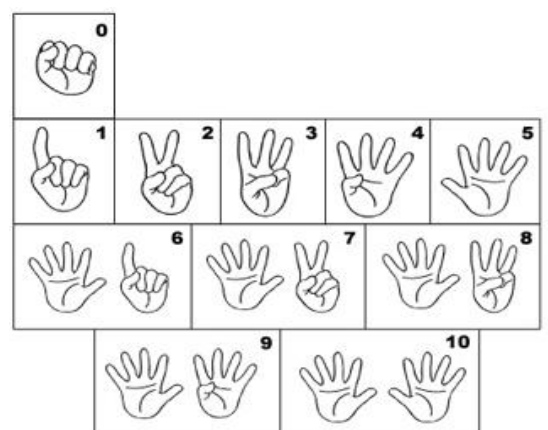


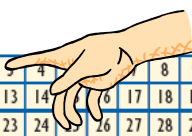
...then encourage counting on by covering up the larger group with a cloth or a number-card. Line up the second group and count from 5 by pointing at each one.



...five. six, seven, eight, **NINE!**
9 altogether.

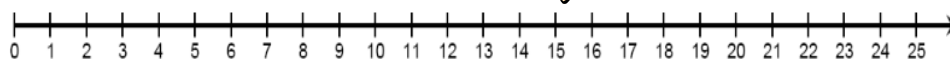
Children to begin to know by heart each of the number clusters on fingers in order to prepare them for mental addition.





1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Count in ones as they become more familiar with using numbers on a number line and 100-square.
Children are to point to each number as they count.



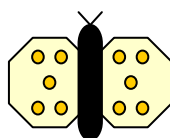
Begin to use the + and = signs to record mental calculations in a number sentence



$$6 + 4 = 10$$

Children begin to solve problems using doubling, halving, grouping and sharing.

$$5 + 5 = 10$$



Children show mental recall of number bonds to 10 and use these for problem solving. Some children show mental recall of number bonds to 20.

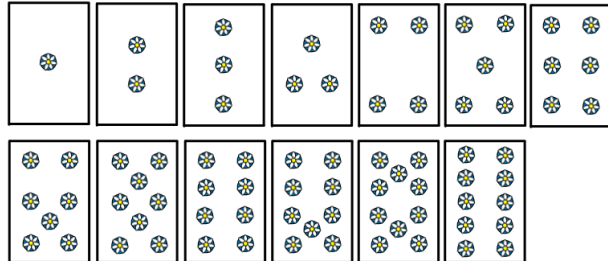
Year 1

Counting choir

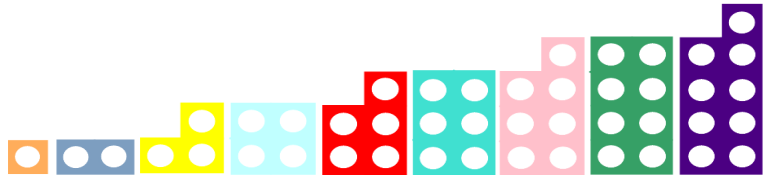
Count, read and write numbers to 100 in numerals, count in different multiples including ones, twos, fives and tens and recognise patterns with the help of a 100 square.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

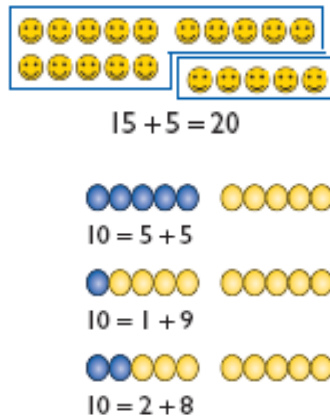
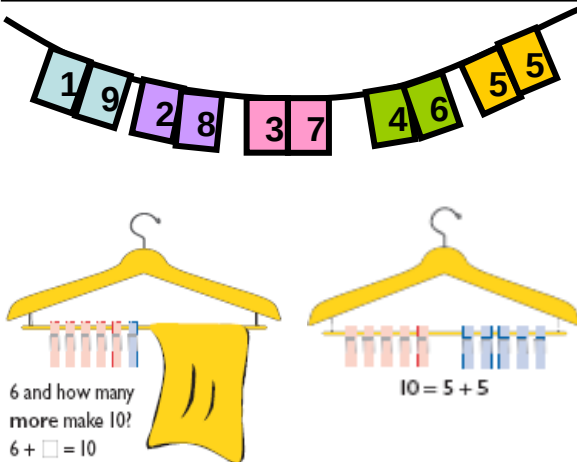
Continue with the use of cluster cards in order for children to develop the skill of decomposing and recomposing numbers mentally.



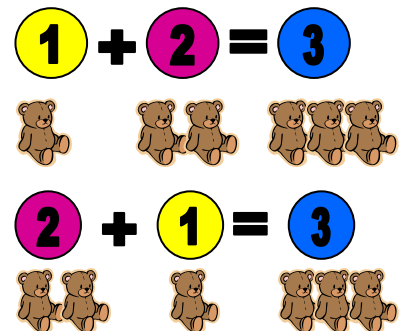
Continue with Numicon activities to embed knowledge of number bonds to 10.



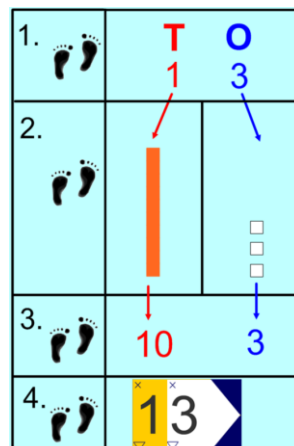
Know by heart all pairs of numbers with a total of 10 and 20



Know that addition can be done in any order.



Children begin to partition tens and ones using Denes and arrow cards.



Children begin to use Denes as a concrete apparatus to add tens and ones and teen numbers and ones.

$$10 + 3$$



They count on from ten in ones as they point to each ones block.

...ten, eleven, twelve, thirteen!
13 altogether.

$$12 + 3$$

They choose tens and ones to make 12 first. The ones are then combined to be able to count the final total.

Some children might pick up the pattern quickly and able to add the tens and ones without having to count these.

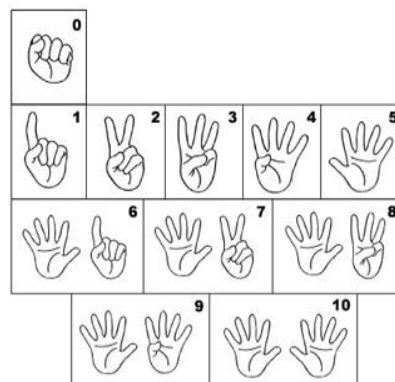
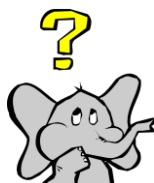
1.		1 2 + 3
2.		
3.		
4.		Count and write answer: _____

When adding mentally, put biggest number first and count on. Use actions such as pointing to head then use fingers to count on.

NOTE: Children must know by heart number clusters on fingers in order to do this securely.

$$3 + 5$$

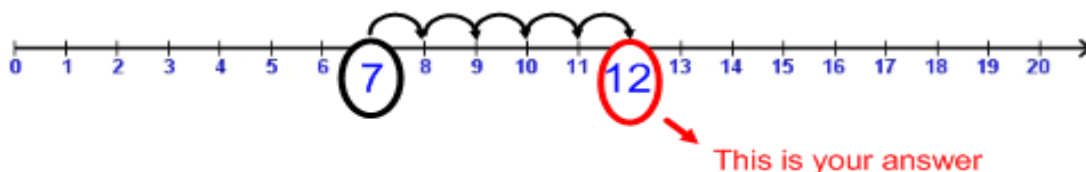
$$3 + 10$$



Progression in difficulty when adding on a number line

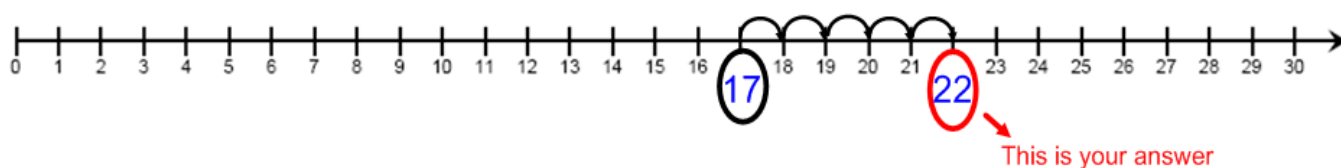
1. 1 digit + 1 digit

$$7 + 5 =$$



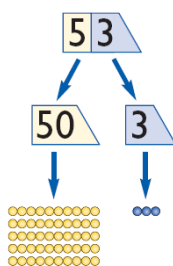
2. 2 digit + 1 digit

$$17 + 5 =$$



3. 2 digit + 2 digit

Begin to partition numbers in order to add two 2-digit numbers.



1.	T 2	O 3
2.		
3.	20	3

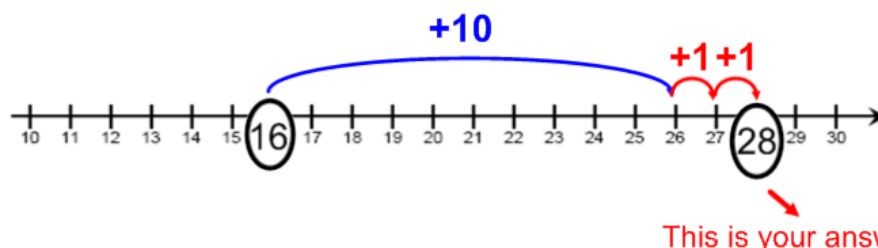
$$16 + 12 = 28$$

Partition your second number

$$16 + \begin{matrix} \text{T} & \text{O} \\ 1 & 2 \end{matrix} =$$

Below the equation, a blue arrow points from the '1' to 10, and a red arrow points from the '2' to 2.

Jump of tens first, then jumps of ones



Please Note:

For this method to become successful and embedded, children must be able to add 10 and one from any given number using their knowledge of place value and having had plenty of experience with jumping in ones and tens on a 100 square.

Year 2

Counting choir

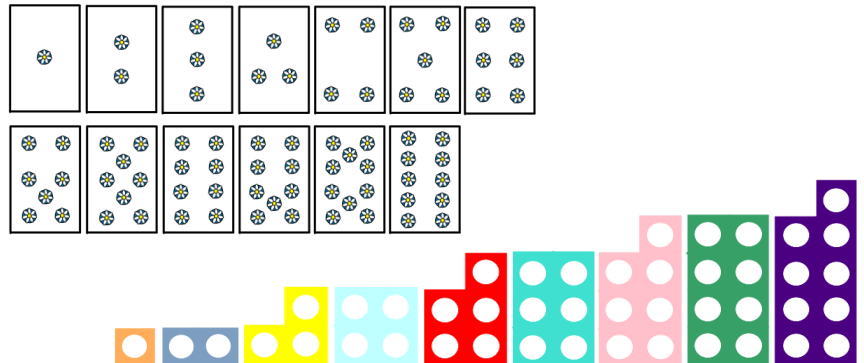
Count in steps of 2, 3, and 5 from 0 and from any of its multiples.

Count in tens and ones from any number, forward or backward.

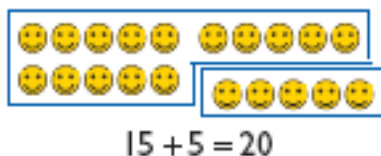
Alternate the above counting within one session to help with calculating on a number line.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Continue with the use of cluster cards and Numicon activities in order for children to develop the skill of decomposing and recomposing numbers mentally and to help with calculating on a number line.

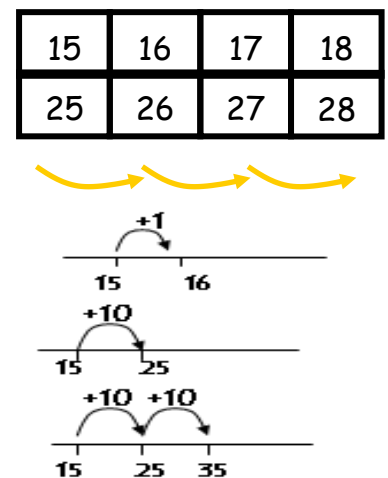


Continue to practice the mental recall of all pairs of number bonds with a total of 10 and 20

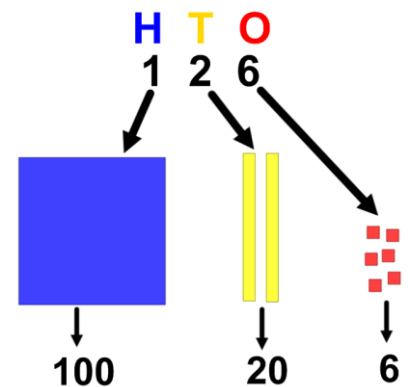
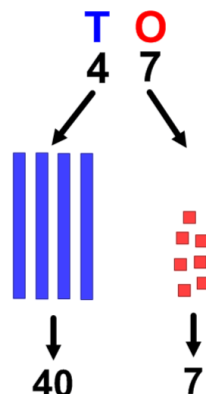


Know which digit changes when adding 1s or 10s to any number.

$$15 + 1 = 16 \quad 15 + 10 = 25$$

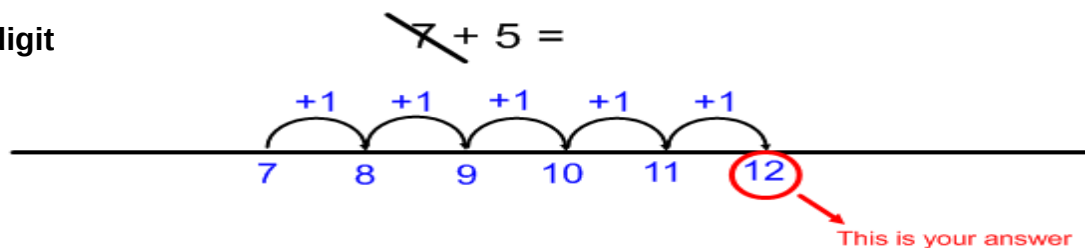


Continue with partitioning two digit numbers into their Tens and Ones and extend to partitioning three digit numbers into their H, T, O.

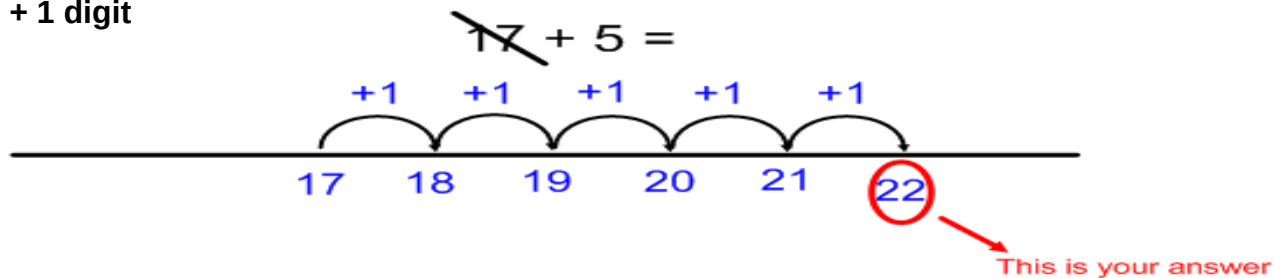


In Year 2 children learn to add using a blank number line.
Progression in difficulty when adding on a blank number line

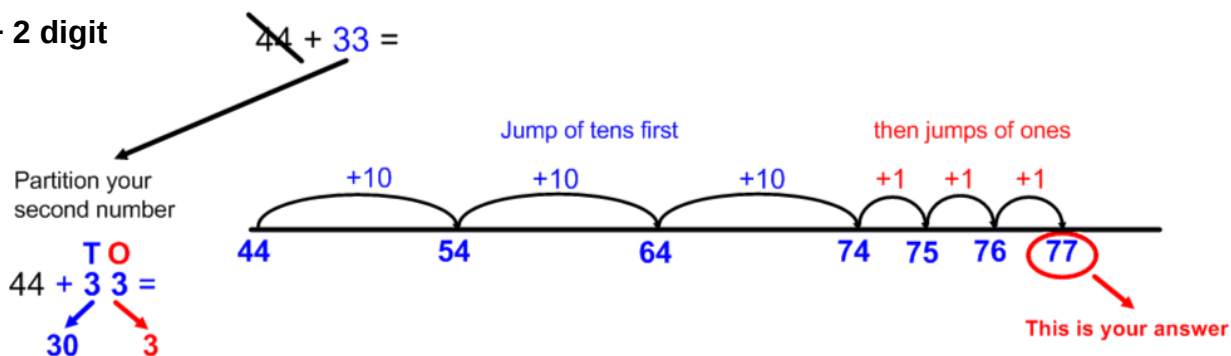
1. 1 digit + 1 digit



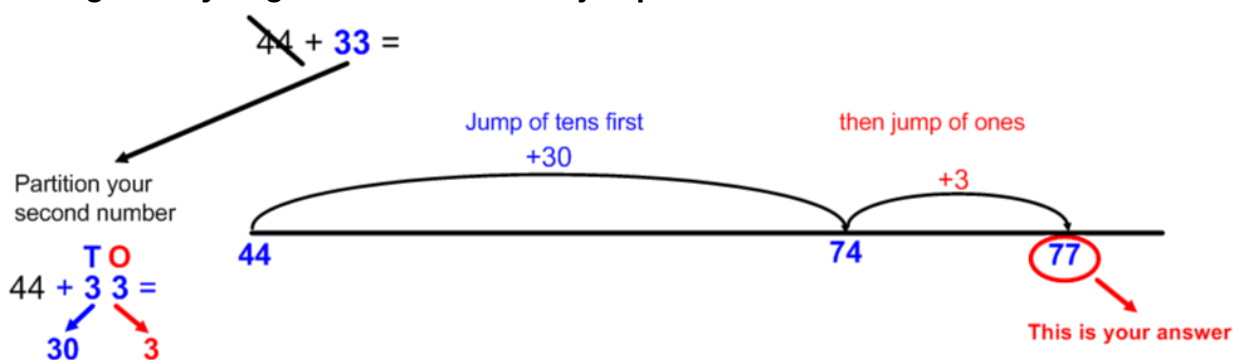
2. 2 digit + 1 digit



3. 2 digit + 2 digit

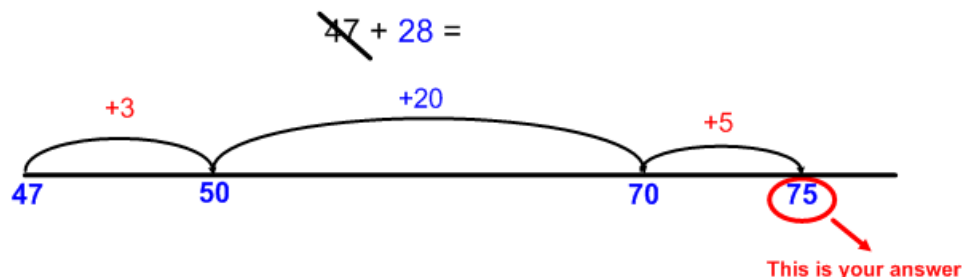


4. 2 digit + 2 digit: They begin to combine their jumps



5. Begin bridging through 10 then teach the formal method of column addition.

For children to be confident with this method they must be able to quickly recompose numbers. Suggested mental maths activity is the use of cluster cards and Numicon.



Year 2 Summer term, Year 3, 4, 5 and 6: Formal written method of addition

Begin with teaching this method without carrying.

Carried digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.

Later, extend to adding three-digit and two-digit numbers, two three-digit numbers and numbers with varied number of digits.

PLEASE NOTE THAT THE NUMBER LINE METHOD SHOULD STILL BE MODELLED AS PART OF A MENTAL MATHS STRATEGY!

$$\begin{array}{r} 42 \\ + 31 \\ \hline 73 \end{array}$$

$$\begin{array}{r} 47 \\ + 36 \\ \hline 73 \\ 1 \end{array}$$

$$\begin{array}{r} 47 \\ + 76 \\ \hline 123 \\ 11 \end{array}$$

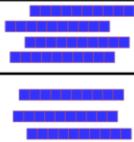
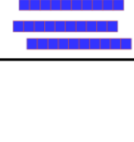
$$\begin{array}{r} 258 \\ + 87 \\ \hline 345 \\ 11 \end{array}$$

$$\begin{array}{r} 676 \\ + 558 \\ \hline 1234 \\ 11 \end{array}$$

Column addition should be introduced together with apparatus using a Place Value grid to provide full understanding. Procedural method should be taught alongside conceptual.

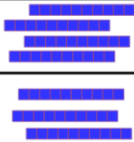
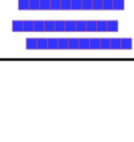
Children use the Place value grid to place Denes into following the same layout as the written calculation. Begin by adding the ones first.

Step 1.

Tens	Ones
	
	

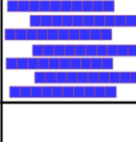
$$\begin{array}{r} \text{T O} \\ 42 \\ + 31 \\ \hline \end{array}$$

Step 2.

Tens	Ones
	
	

$$\begin{array}{r} \text{T O} \\ 42 \\ + 31 \\ \hline 3 \end{array}$$

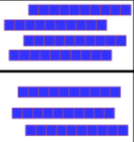
Step 3.

Tens	Ones
	

$$\begin{array}{r} \text{T O} \\ 42 \\ + 31 \\ \hline 73 \end{array}$$

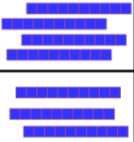
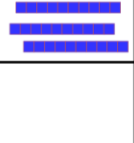
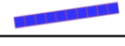
Children use Denes to exchange 10 ones into a ten in order to understand the rules of carrying. Begin by adding the ones first.

Step 1.

Tens	Ones
	
	

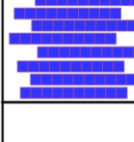
$$\begin{array}{r} \text{T O} \\ 47 \\ + 36 \\ \hline \end{array}$$

Step 2.

Tens	Ones
	
	
	

$$\begin{array}{r} \text{T O} \\ 47 \\ + 36 \\ \hline 3 \\ 1 \end{array}$$

Step 3.

Tens	Ones
	

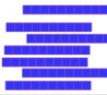
$$\begin{array}{r} \text{T O} \\ 47 \\ + 36 \\ \hline 83 \\ 1 \end{array}$$

Step 1.

Hundreds	Tens	Ones
		
		

$$\begin{array}{r} 47 \\ + 76 \\ \hline \end{array}$$

Step 2.

Hundreds	Tens	Ones
		
		
		
		

$$\begin{array}{r} 47 \\ + 76 \\ \hline 3 \\ 1 \end{array}$$

Children use Denes to exchange 10 ones into a ten, then use hundreds to exchange 10 tens into 1 hundred.

Step 3.

Hundreds	Tens	Ones
		
		

$$\begin{array}{r} 47 \\ + 76 \\ \hline 23 \\ 1 \end{array}$$

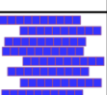
Step 4.

Hundreds	Tens	Ones
		

$$\begin{array}{r} 47 \\ + 76 \\ \hline 123 \end{array}$$

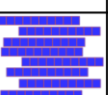
Adding 3 digit and a 2 digit number with carrying.

Step 1.

Hundreds	Tens	Ones
		
		

$$\begin{array}{r} 258 \\ + 87 \\ \hline \end{array}$$

Step 2.

Hundreds	Tens	Ones
		
		
		
		

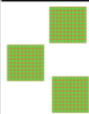
$$\begin{array}{r} 258 \\ + 87 \\ \hline 5 \\ 1 \end{array}$$

Step 3.

Hundreds	Tens	Ones
		
		
		

$$\begin{array}{r} 258 \\ + 87 \\ \hline 35 \\ 1 \end{array}$$

Step 4.

Hundreds	Tens	Ones
		

$$\begin{array}{r} 258 \\ + 87 \\ \hline 335 \end{array}$$

Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable. Children should also be taught how to add several numbers with different numbers of digits using the column method and understanding the place value.

$$\begin{array}{r}
 258 \\
 87 \\
 56138 \\
 4 \\
 + 9321 \\
 \hline
 65808 \\
 1 \quad \cancel{2} \quad \cancel{2}
 \end{array}$$

Children however need to be careful how they set out the numbers when calculating with decimals.

$$12.5 + 23.7$$

$$\begin{array}{r}
 12.5 \\
 + 23.7 \\
 \hline
 36.2 \\
 1
 \end{array}$$

$$123.5 + 24.6$$

$$\begin{array}{r}
 123.5 \\
 + 24.6 \\
 \hline
 148.1 \\
 1
 \end{array}$$

$$34.5 + 27.43$$

$$\begin{array}{r}
 34.5 \quad 0 \\
 + 27.43 \\
 \hline
 61.93 \\
 1
 \end{array}$$

Use a zero as
a place
holder.

$$34.5 + 7.43$$

$$\begin{array}{r}
 34.5 \quad 0 \\
 + 7.43 \\
 \hline
 41.93 \\
 1
 \end{array}$$

Use a zero as
a place
holder.

In these examples children need to understand that the decimal points are always written underneath each other when using column addition.

Subtraction

Mental Skills

Recognise the size and position of numbers

Count on or back in ones and tens

Know number facts for all numbers to 20

Subtract multiples of 10 from any number

Partition and recombine numbers (only partition the number to be subtracted)

Bridge through 10

Models and Images

Counting apparatus

Place value apparatus

Place value cards

Number tracks

Numbered number lines

Marked but unnumbered lines

Hundred square

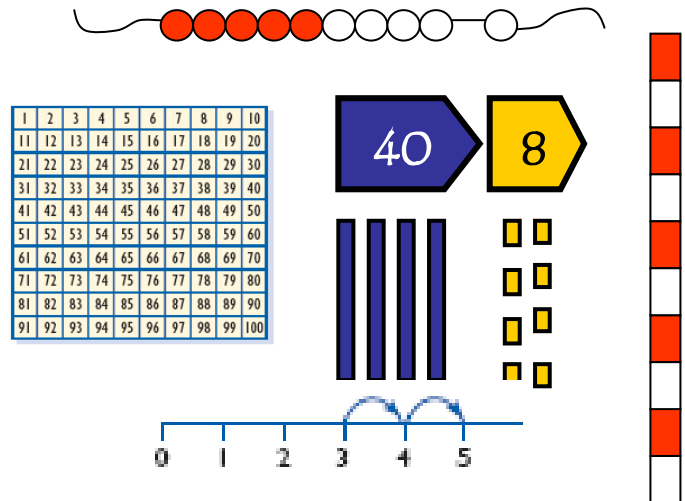
Empty number lines.

Counting stick

Bead strings

Models and Images Charts

ITPs – Number Facts, Counting on and back in ones and tens, Difference



subtract

count on count back

fewer — less

take away minus

 difference

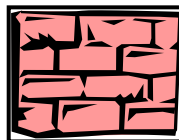
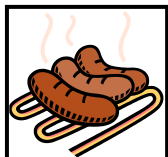
Reception

Early learning goal:

Children count reliably with numbers from 1 to 20, place them in order and say which number is one less than a given number. Using quantities and objects, they subtract two single-digit numbers and count back to find the answer.

Begin to count backwards in familiar contexts such as number rhymes or stories

Five fat sausages
frying in a pan ...



Ten green bottles
hanging on the
wall...

10, 9, 8, 7, ...

Continue the count back
in ones from any given
number

Begin to relate
subtraction to
'taking away'.
Children physically
take away teddies.

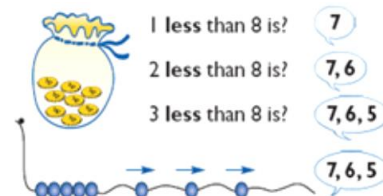


... 'Three teddies *take away*
two teddies leaves one
teddy'

Find one less than any number.

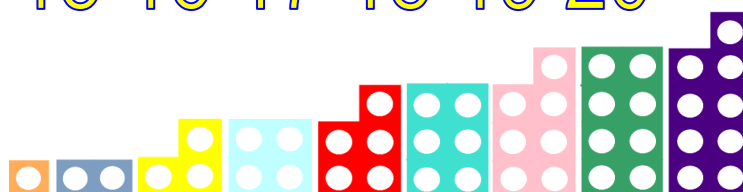
What is the number before 8?

1	2	3	4	5	6	7	8	9	10

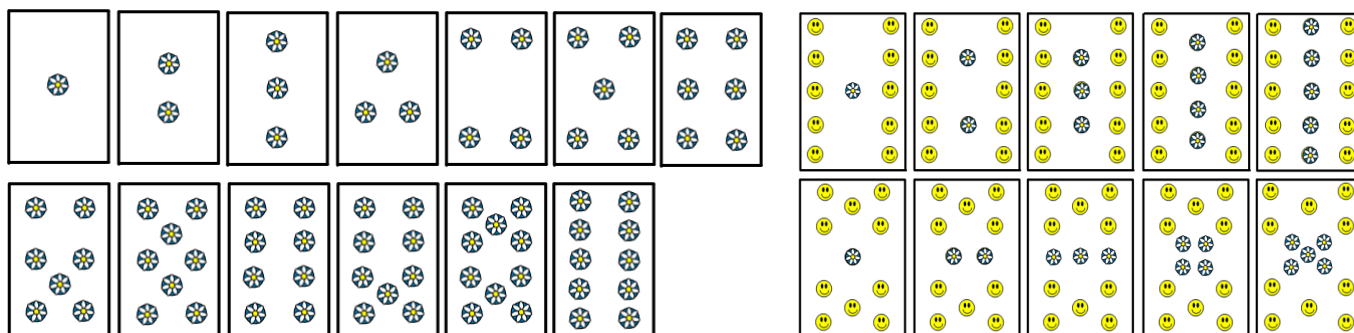


10 11 12 13 14 15 16 17 18 19 20

Use Numicon activities to embed
understanding of the concept,
subtraction.



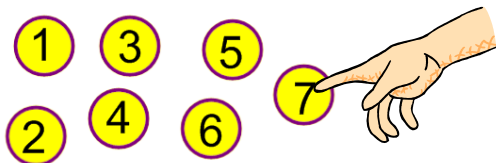
Use cluster cards to cover up (hide) some of the clusters within and relate this to subtraction.



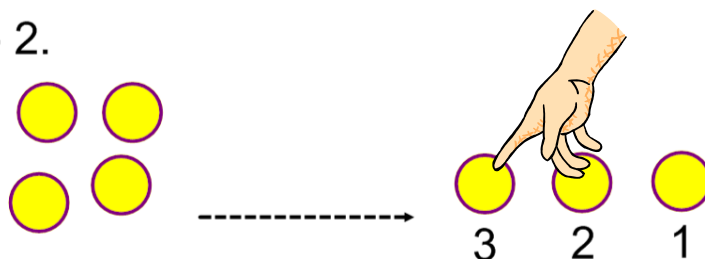
Children will begin to develop their ability to subtract by using practical equipment to count out the first number and then remove or take away the second number to find the solution by counting how many are left e.g. $7 - 3$. Those who are ready, may recall their calculations using a number sentence.

$$7 - 3$$

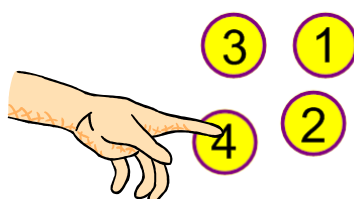
Step 1.



Step 2.



Step 3.

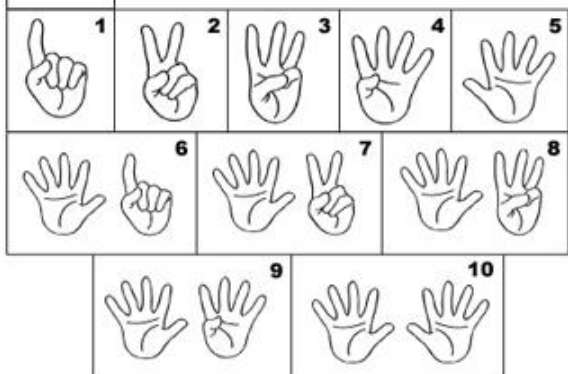


...one, two, three,
four!
7 take away 3 is 4!

Those who are familiar with the smaller number clusters, should be encouraged to say the number of counters left without having to count it.

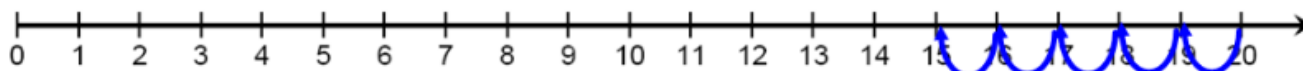
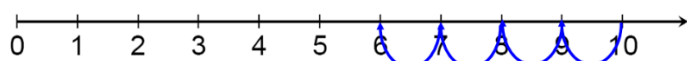


Use their knowledge of number clusters on fingers to take away.



**'five take away two
is three!'**

Count backwards along a number line to take away.



Some children show mental recall of number bonds to 10 and use these for problem solving.

Year 1

Begin to use the – and = signs to record mental calculations in a number sentence

Maria had six sweets and she ate four. How many did she have left?



Know by heart subtraction facts for numbers up to 10 and 20

$$6 - 4 = 2$$



$$6 + ? = 10$$

$$10 - 6 = ?$$

$$? + 6 = 10$$

$$10 - 4 = 6$$



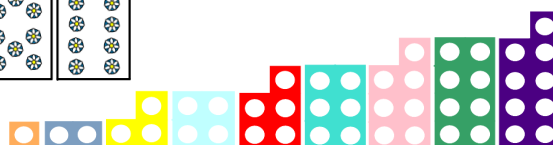
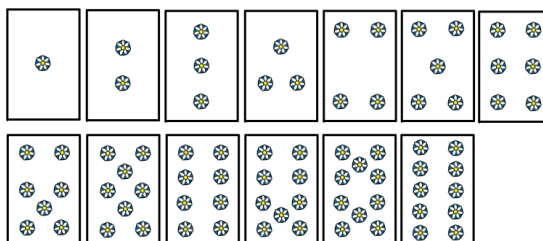
$$12 + 8 = 20$$

$$20 - 8 = 12$$

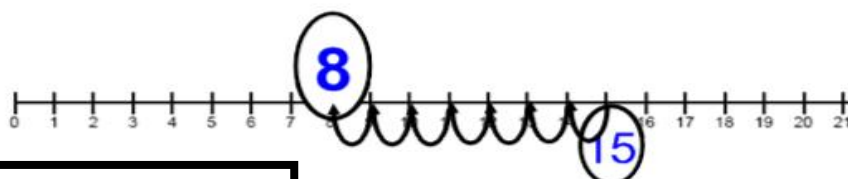
$$8 + 12 = 20$$

$$20 - 12 = 8$$

Continue with the use of cluster cards and Numicon activities in order for children to develop the skill of decomposing and recomposing numbers mentally and to help with calculating on a number line.



$$15 - 7 = 8$$



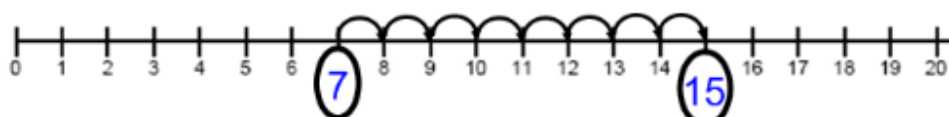
Subtract single digits by counting back under the number line in ones and solve problems using this method.

The answer is the number you land on $15 - 7 = 8$



The difference is?

Begin to find the difference by counting on from the smallest number. **This is to be taught in the summer term.**



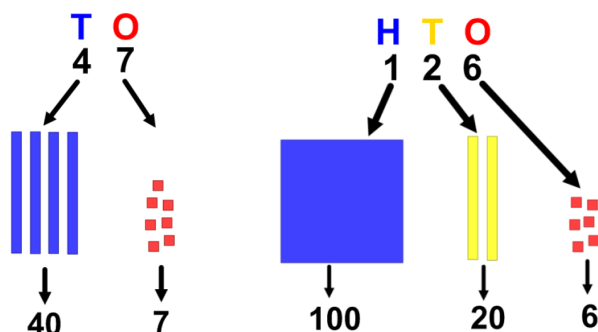
Extend by counting on in 10s, then ones.

...one, two, three, four, five, six, seven, eight. The answer is 8!

Circle both numbers then count on from the smaller to the larger number above the number line. The answer is the number of steps made
 $15 - 7 = 8$
 Children count as they make their jump.

Year 2

Partitioning two and three digit numbers.



Knowing which digit changes when adding 1s or 10s to any number.

15	16	17	18
25	26	27	28

$$15 + 1 = 16$$

$$15 + 10 = 25$$

Subtracting using a number – line.

The vocabulary to be used is FINDING THE DIFFERENCE. In other words, children are finding how many numbers there are in between two chosen numerals.

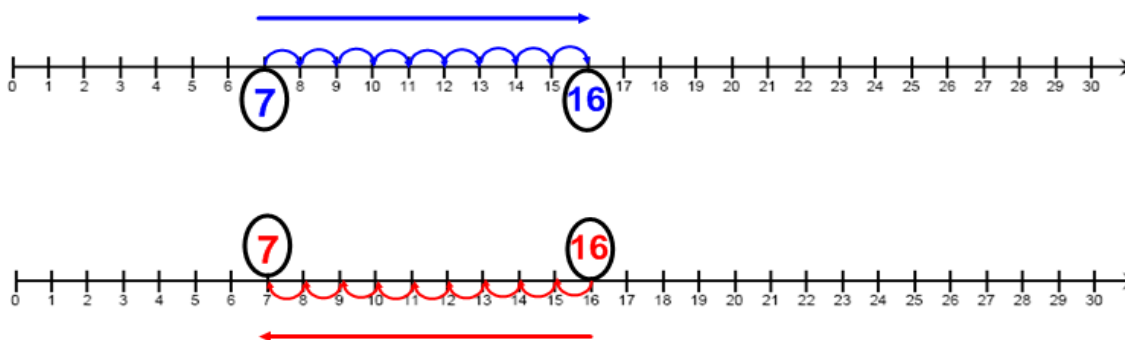
1) The steps are recorded by counting up from the smaller to the larger number to find the difference.

Children are taught to write the bigger number first when writing the number sentence. They then circle both numbers on the number line.

When this is first taught, teachers should model both, counting on and back on the number line so that children can see that the answer is the same.

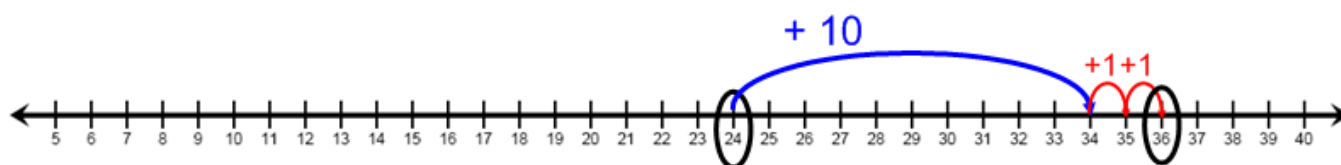
Starting with the smallest number, children count on. They count the number of jumps.

$$16 - 7 = 8$$



2) Finding the difference between two 2 digit numbers by counting on in tens first, then counting on in 1s using a marked number line.

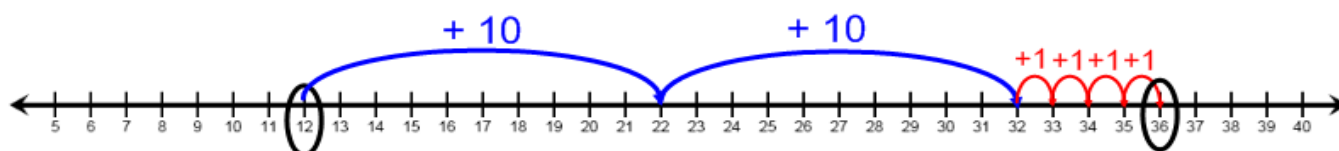
$$36 - 24 = 12$$



In order for children to be able to do this, they must have a secure knowledge of 'ten more than' any number. Provide children with laminated 100 squares to help them establish whether they can make a jump of ten.

3) Finding the difference between two 2 digit numbers by counting on in tens first, then counting on in 1s using a marked number line. The calculation becomes more complicated as children are having to make two jumps of ten.

$$36 - 12 = 24$$



4) Finding the difference between two 2-digit numbers by counting on using an empty number line.

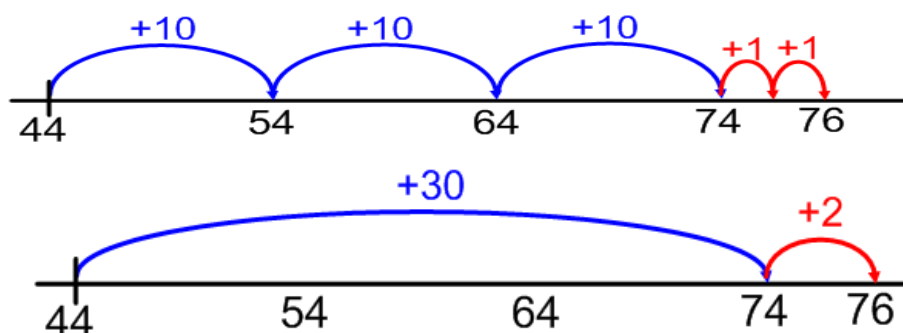
Children are taught to jump in tens and ones from the smaller number to get to the larger number. The jumps are added to get the answer.

$$76 - 44 = 32$$

Mental maths suggestion:

Counting choir and counting using meter stick.

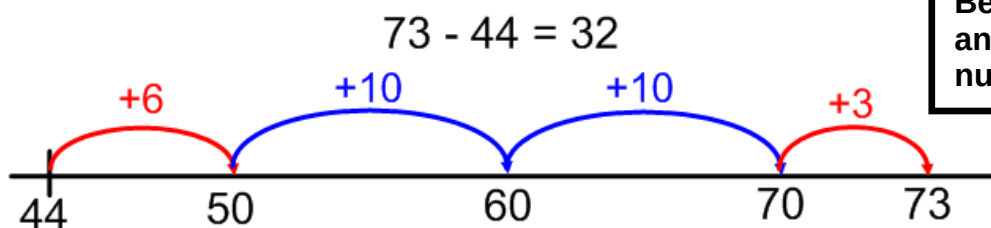
$$76 - 44 = 32$$



5) Finding the difference using their knowledge of number bonds to the next multiple of 10.

Essential knowledge:

Being able to decompose and recompose any number under 10.



Years 3, 4, 5 and 6

Formal written method of subtraction

1) $67 - 32 =$

$$\begin{array}{r} 67 \\ - 32 \\ \hline 35 \end{array}$$

2) $74 - 27 =$

$$\begin{array}{r} 6 4 \\ \cancel{7} 4 \\ - 27 \\ \hline 47 \end{array}$$

3) $741 - 367 =$

$$\begin{array}{r} 6 13 1 \\ \cancel{7} \cancel{4} 1 \\ - 367 \\ \hline 374 \end{array}$$

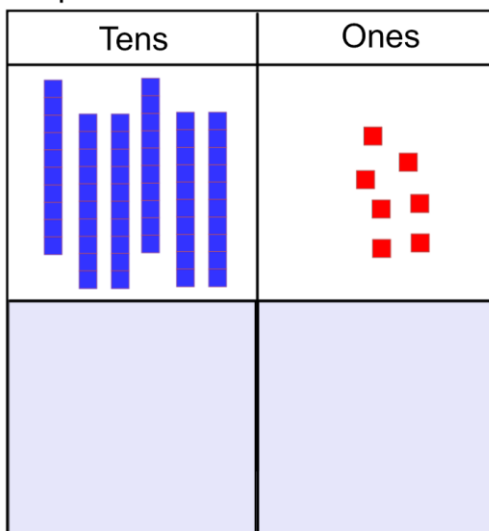
4) $501 - 278 =$

$$\begin{array}{r} 4 9 13 \\ \cancel{5} \cancel{0} 13 \\ - 278 \\ \hline 225 \end{array}$$

Column subtraction should be introduced together with apparatus using a Place Value grid to provide full understanding. Procedural method should be taught alongside conceptual.

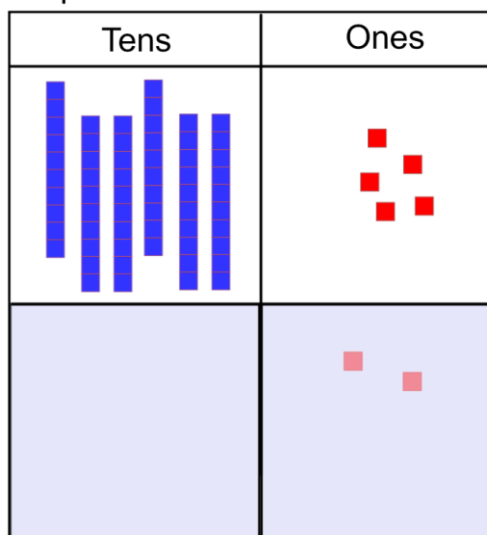
- 1) Children use the Place value grid to place Denes into the 'recycling bin' below as they take away Ones and Tens subsequently.

Step 1.



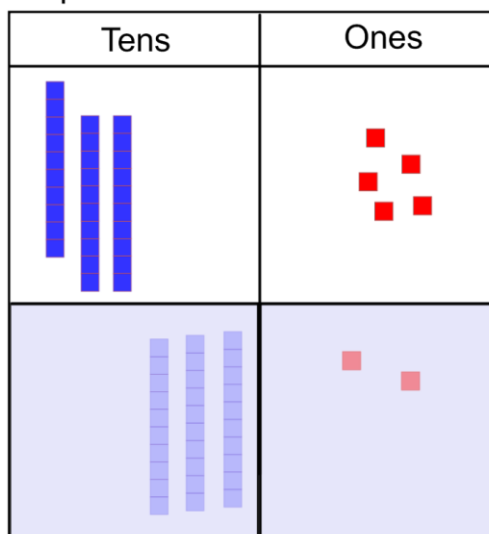
$$\begin{array}{r} 67 \\ - 32 \\ \hline \end{array}$$

Step 2.



$$\begin{array}{r} 67 \\ - 32 \\ \hline 5 \end{array}$$

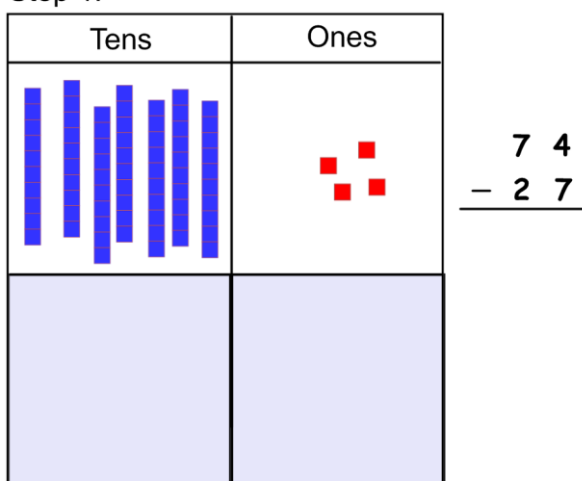
Step 3.



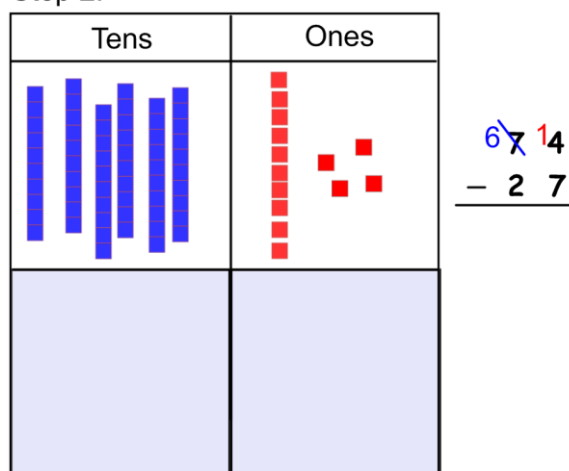
$$\begin{array}{r} 67 \\ - 32 \\ \hline 35 \end{array}$$

- 2) Children use Denes to take one Ten from the Tens column and exchange this into ten Ones. They can see that there are only 6 Tens left in Tens column and 14 ones in the Ones column. Subtraction happens by children dragging Denes into the 'recycling bin' Ones first, then Tens.

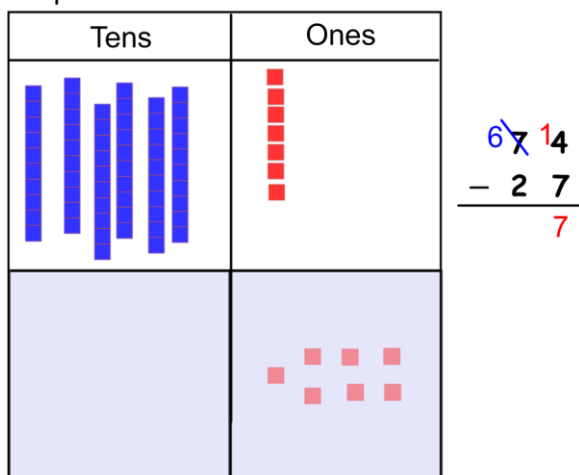
Step 1.



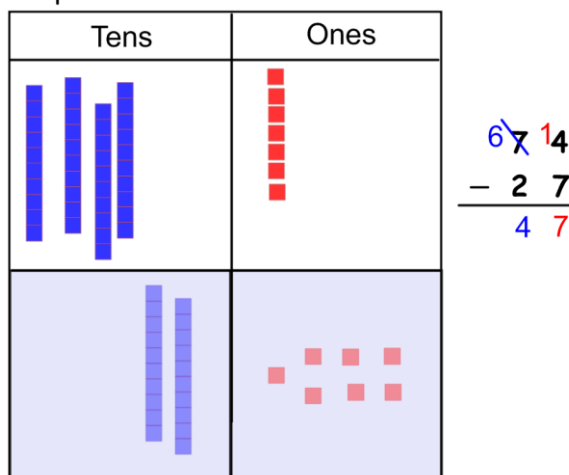
Step 2.



Step 3.

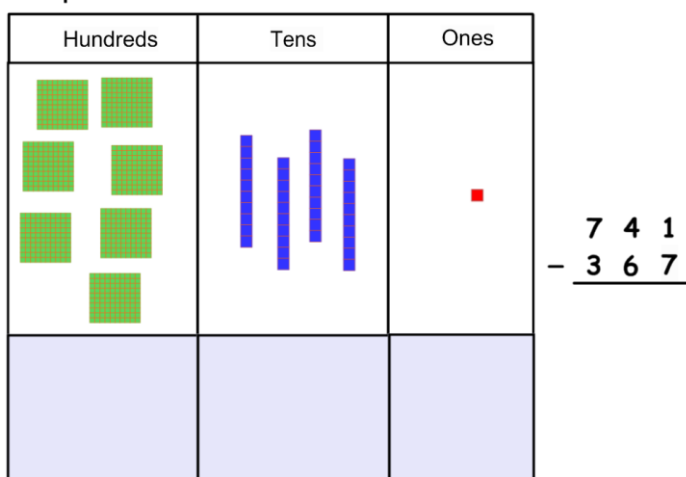


Step 4.

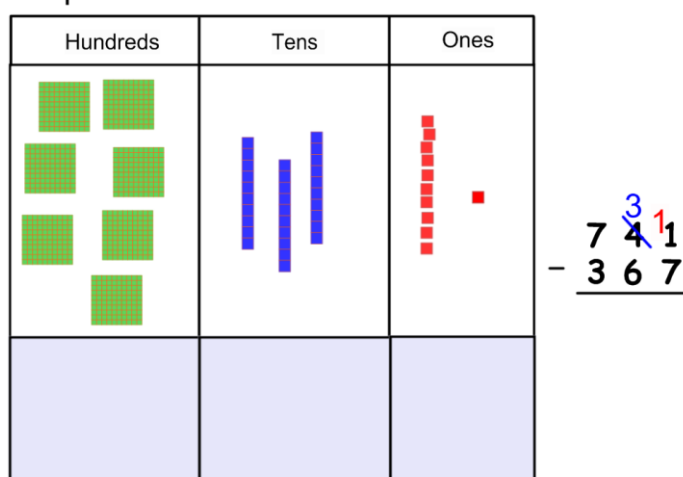


- 3) Children use the same method of exchange but this is now extended to the Hundreds column. Subtraction happens by children dragging Denes into the 'recycling bin' Ones first, then Tens.

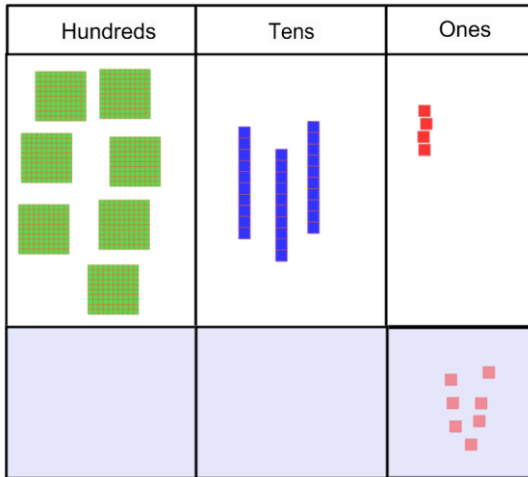
Step 1.



Step 2.

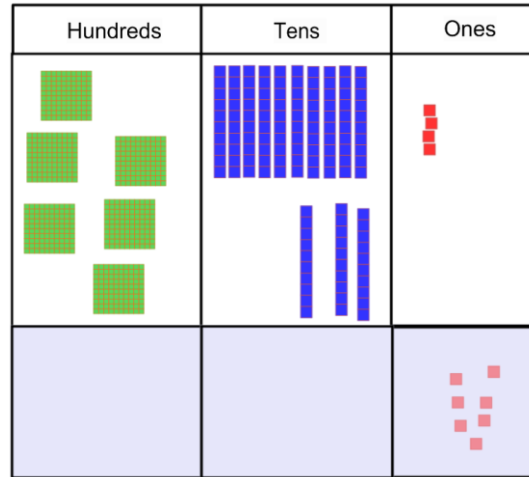


Step 3.



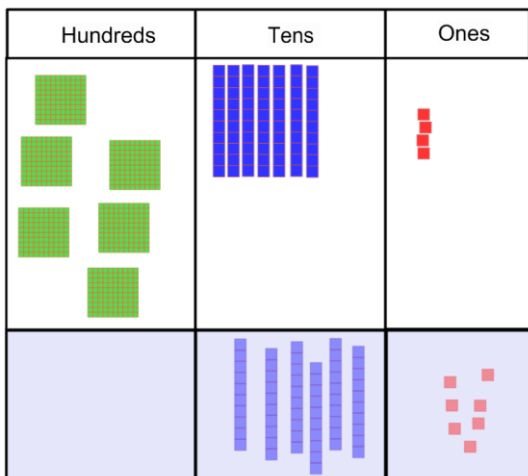
$$\begin{array}{r} 741 \\ - 367 \\ \hline 4 \end{array}$$

Step 4.



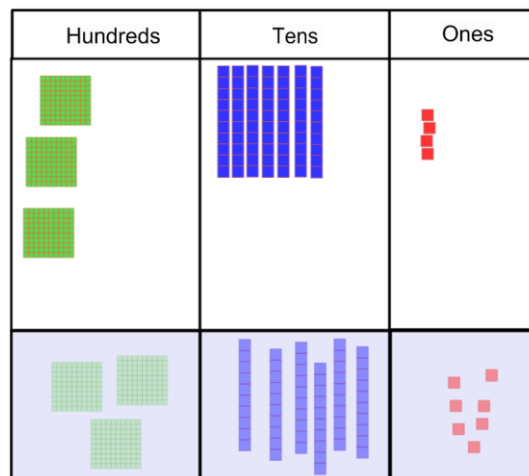
$$\begin{array}{r} 613 \\ - 367 \\ \hline 4 \end{array}$$

Step 5.



$$\begin{array}{r} 613 \\ - 367 \\ \hline 74 \end{array}$$

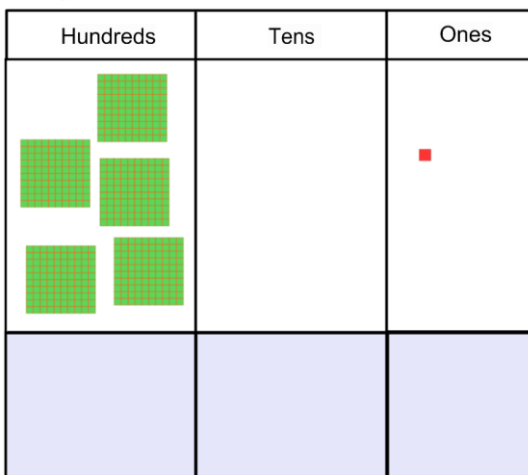
Step 6.



$$\begin{array}{r} 374 \\ - 367 \\ \hline 7 \end{array}$$

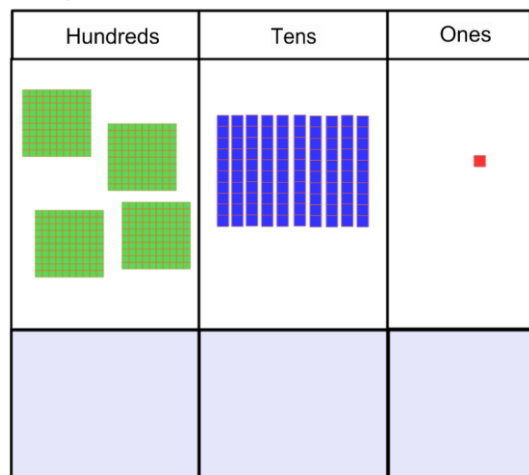
4) The method becomes further complicated when there are zero Tens or Ones.

Step 1.



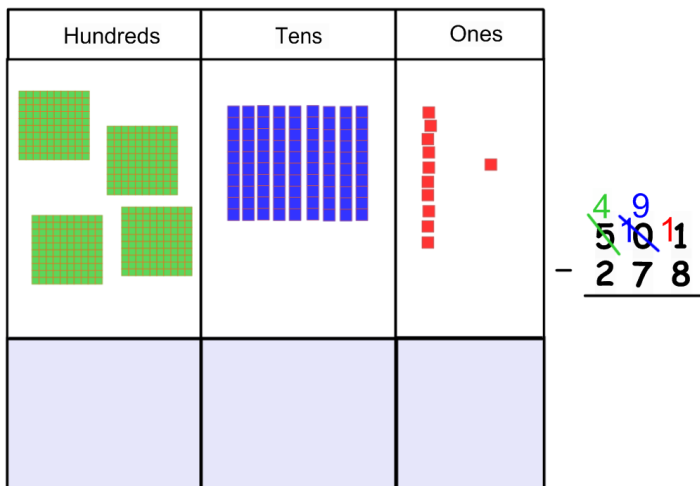
$$\begin{array}{r} 501 \\ - 278 \\ \hline \end{array}$$

Step 2.

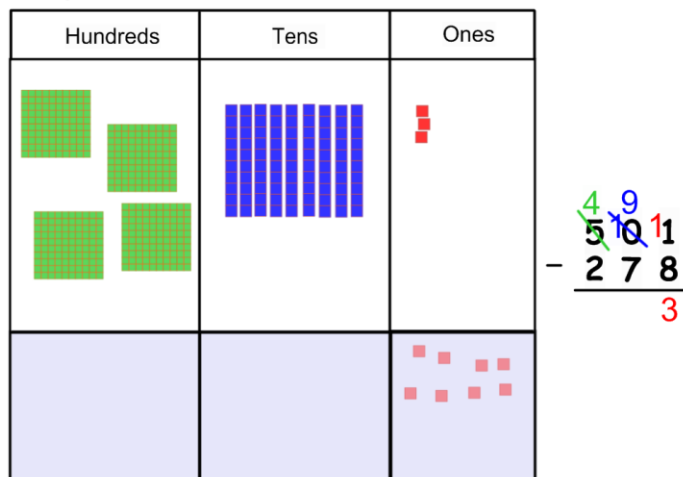


$$\begin{array}{r} 4101 \\ - 278 \\ \hline \end{array}$$

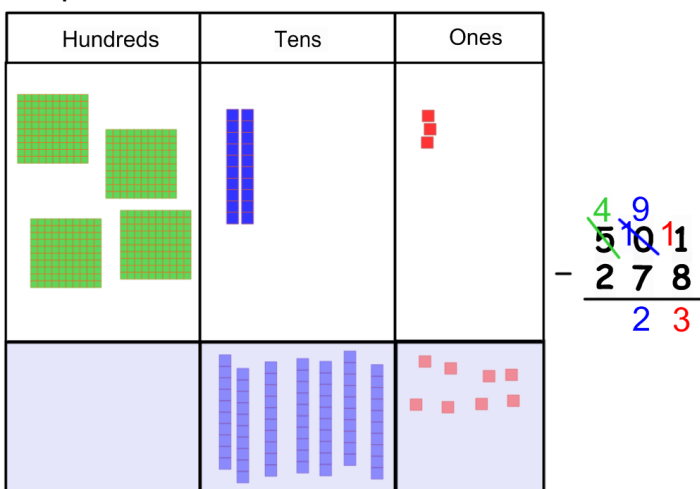
Step 3.



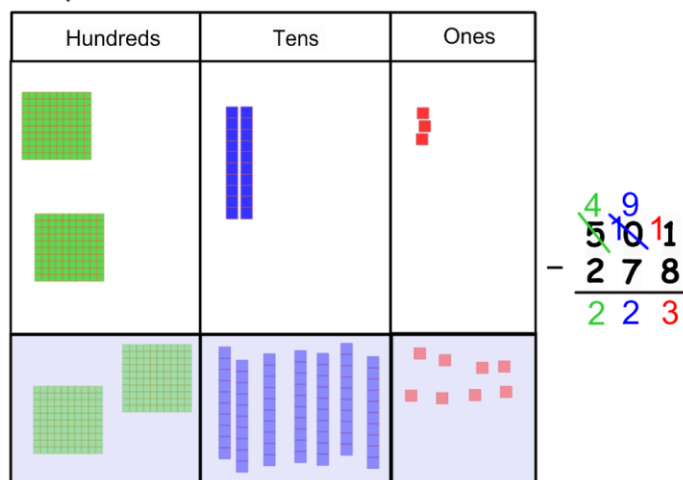
Step 4.



Step 5.



Step 6.



Children should be encouraged to use inverse operations to check if their answer is correct. This gives them opportunity to practise both operations (addition and subtraction) at the same time. Explicit teaching needs to point out that if they add the bottom number to the answer they should end up with the top number.

Calculation:

$$\begin{array}{r} 491 \\ - 278 \\ \hline 223 \end{array}$$

Checking using the inverse

$$\begin{array}{r} 223 \\ + 278 \\ \hline 501 \end{array}$$

Please note:

This is purely the written method of subtraction. When calculating mentally using smaller numbers, teachers should model counting on using an empty number line. When calculating with time and finding time differences, the number line method should be used EVERY TIME.

When subtracting decimal numbers, children must carefully align the numbers so that the decimal points are underneath each other use the same method as demonstrated above to subtract. It is assumed that children who are ready to subtract using decimals do not need to use Denes to aid them as they have a secure Place Value and number knowledge.

$$\begin{array}{r} 412.31 \\ - 07.84 \\ \hline 45.46 \end{array}$$

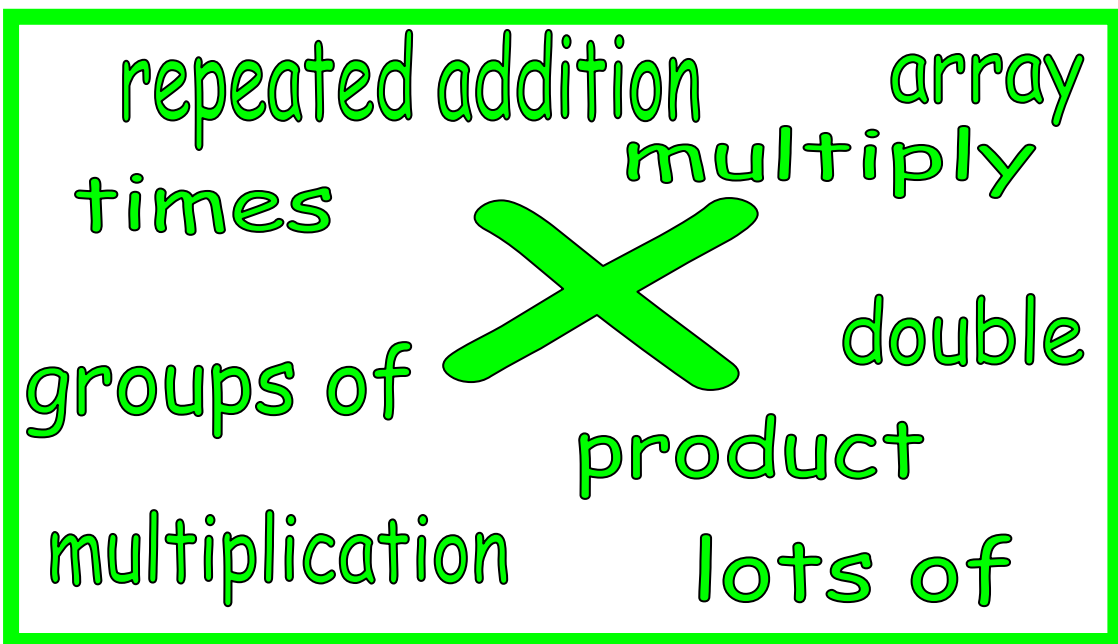
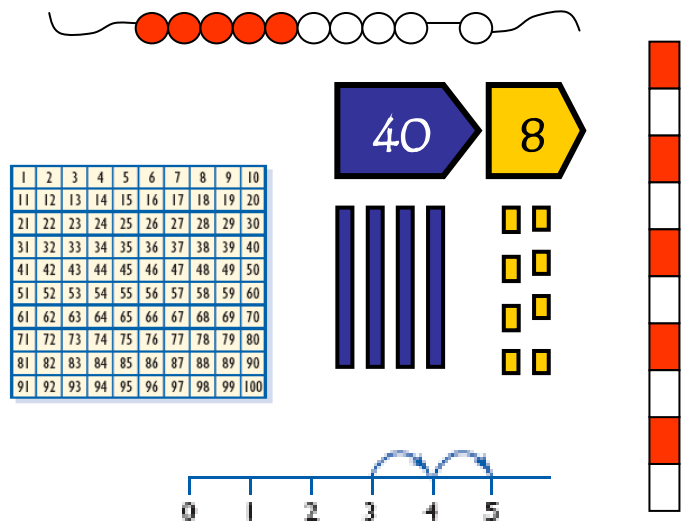
Multiplication

Mental Skills

Recognise the size and position of numbers
 Count on in different steps 2s, 5s, 10s
 Double numbers up to 10
 Recognise multiplication as repeated addition
 Quick recall of multiplication facts
 Use known facts to derive associated facts
 Multiplying by 10, 100, 1000 and understanding the effect
 Multiplying by multiples of 10

Models and Images

Counting apparatus
 Place value apparatus
 Arrays
 100 squares
 Number tracks
 Numbered number lines
 Marked but unnumbered lines
 Empty number lines.
 Multiplication squares
 Counting stick
 Bead strings
 Models and Images charts
 ITPs – Multiplication grid, Number Dials, Multiplication Facts



Reception

Early learning goal:

They solve problems, including doubling, halving and sharing.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical opportunities to double using a wide variety of equipment, counters, cubes, egg-boxes, ice-cube trays, baking tins etc.

Children develop understanding of doubles using their fingers.



Year 1

Counting choir

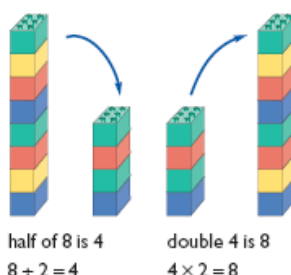
Count in steps of 1s, 2s, 5s and 10s forward and back from 0 and from any of its multiples using the 100 square taking the opportunity to discuss patterns that are recognised.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Other resources that aid counting

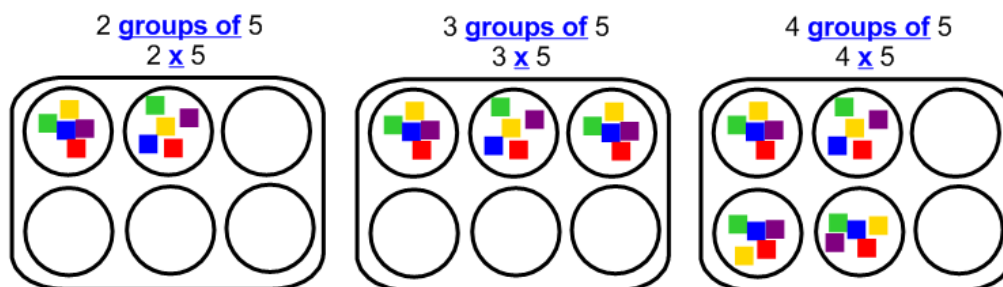


Know doubles and corresponding halves



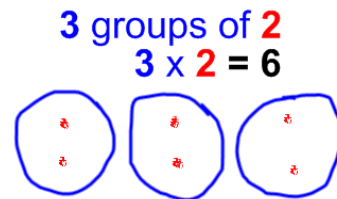
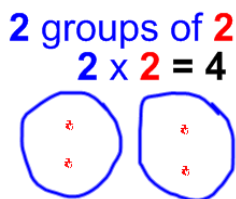
Use fingers to work out doubles up to double 5.

- Using laminated sheets with circles (groups) on them, children group objects using the correct mathematical vocabulary.



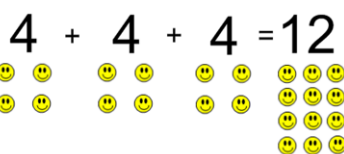
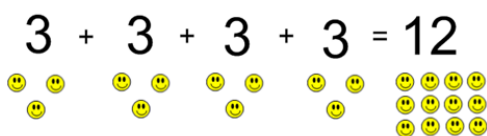
- Children begin using jottings of simple multiplication with the associated vocabulary.

They begin by drawing the number of groups, then draw the number of dots inside the circles. They count the number of dots they have altogether to get to the answer.

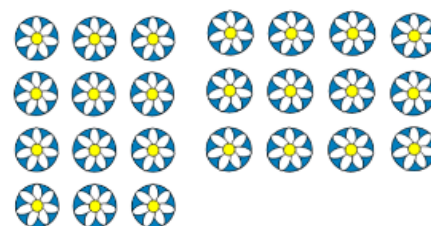


- Children are exposed to the different ways in which multiplication can be expressed using concrete materials and linking it to real life situations. They begin to understand that repeated addition can also be expressed as multiplication using concrete materials.

Expressing multiplication as repeated addition



Expressing multiplication as arrays



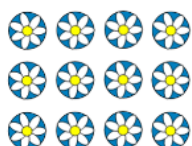
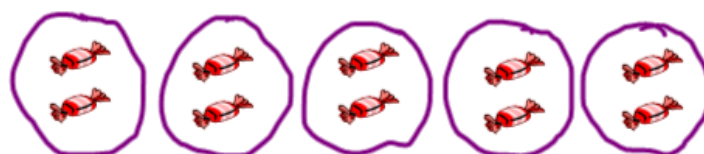
- Children begin to commit multiples of 2, 5, 10 to memory and use these facts to solve problems.



There are 10 spiders... how many legs do they have altogether?

$$8 \times 10 = 80$$

When Peter behaves well in school he gets 2 sweets at the end of the day. If he behaves well for 5 days, how many sweets will he get altogether?

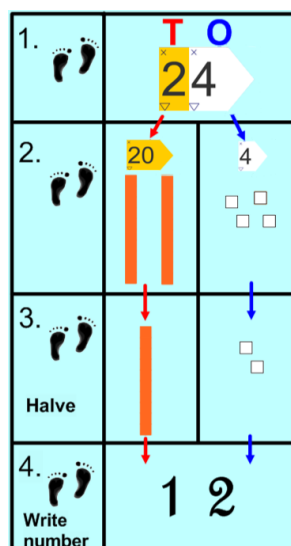
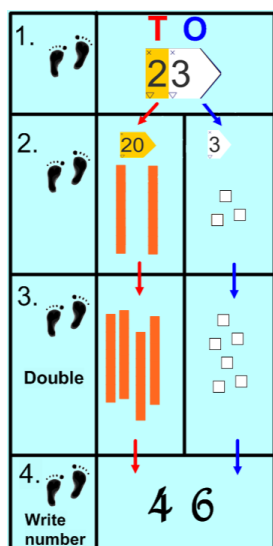


There are 4 flower beds in a garden. Each flower bed has 3 flowers. How many flowers are in the garden altogether?

Year 2

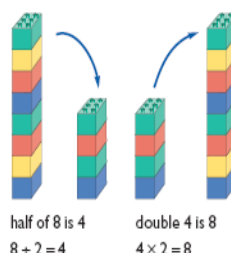
Counting choir

Count in steps of 2s, 3s, 5s, 10s and 20s forward and back from 0 and from any of its multiples using the 100 / 200 square and taking the opportunity to discuss patterns that are recognised.



Know doubles and corresponding halves and extend to partitioning numbers then double / partitioning numbers then halve.

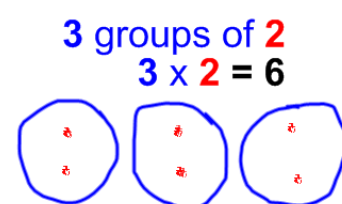
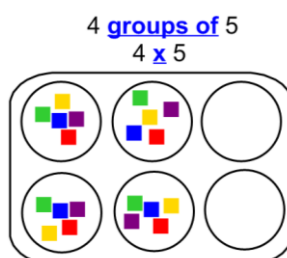
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
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61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200



Use fingers to work out doubles up to double 5.

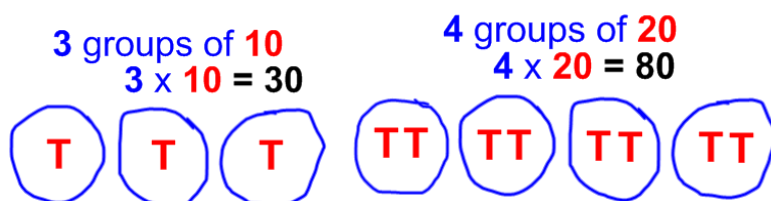
- Children continue using jottings of simple multiplication with the associated vocabulary and those who still find this difficult will use the laminated sheets with circles to group concrete objects.

They begin by drawing the number of groups, then draw the number of dots inside the circles. They count the number of dots they have altogether to get to the answer.



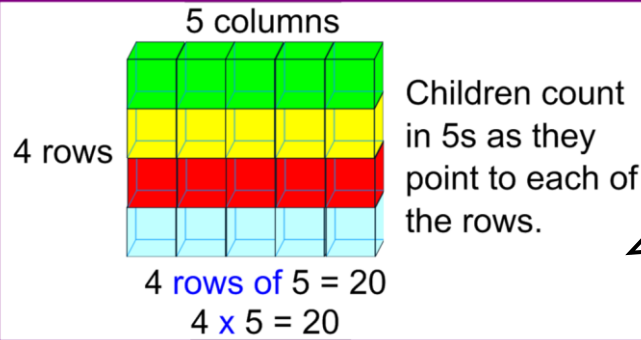
- Teach using jottings when multiplying multiples of ten by writing T in each of the groups.

They begin by drawing the number of groups, then write the number of T's inside the circles. They count the number of T's using their knowledge of counting in tens to obtain an answer.

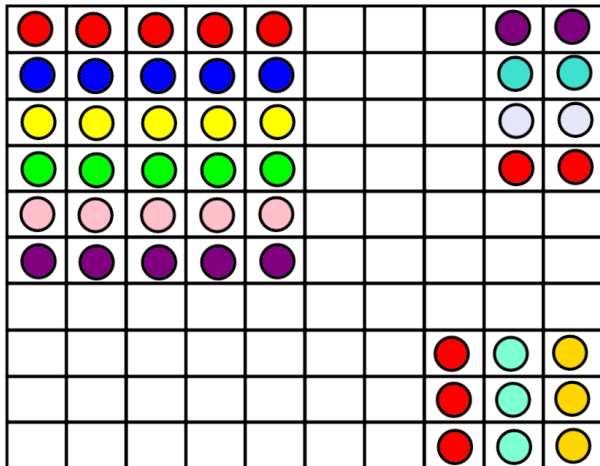


3. They further develop their skills of problem solving using multiplication and begin to relate it to the area of a rectangle / square.

Children investigate the number of multilink cubes needed to create a block with a given number of length and width.



... 'five, ten, fifteen, 20. Four times five is 20!'



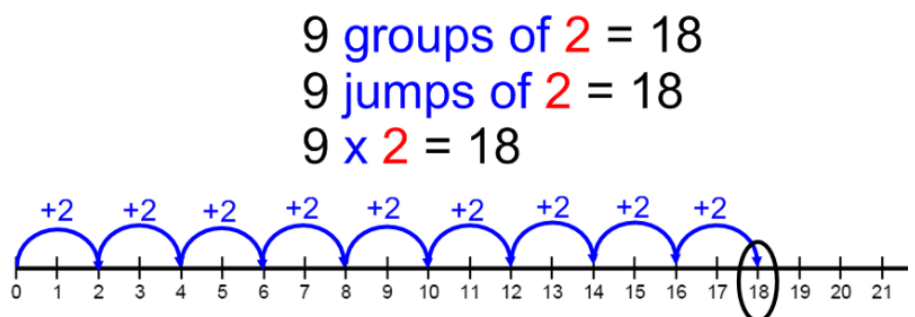
They might use counters in an empty grid to create arrays and they begin to relate this to the commutative nature of multiplication by saying and / or writing:

$$\begin{aligned} 6 \text{ rows of } 5 &= 5 \text{ columns of } 6 \\ 6 \times 5 &= 5 \times 6 \\ 30 &= 30 \end{aligned}$$

Those who are ready can begin to jot arrays in their books using the squares as a guide.

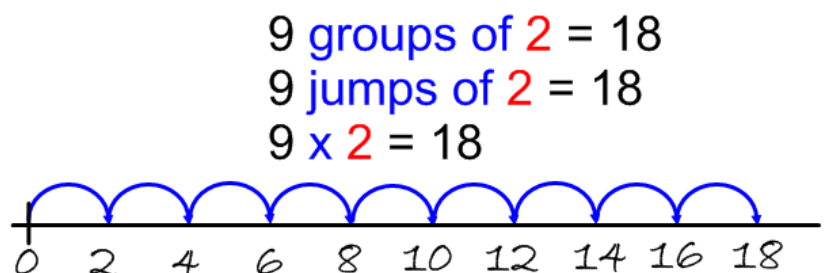
4. Teach jumping on a marked number line in multiples of 2, 3, 5, 10.

This method requires children to keep the jumps equal in size as they count the number of jumps. This is a challenging process, however it further embeds the understanding of repeated addition. The constant re-enforcement of vocabulary 'groups of' is very important.



5. As children become confident with counting in multiples of 2, 3, 5, 10 they begin to use the empty number line to solve multiplication problems.

In this method there are strong links with the activity of counting choir using 100 / 200 squares and the recognition of patters with each of the multiples. Children write their own number after each time they make a jump.



Year 3, 4, 5 and 6: Short and Long Multiplication

Because children have to get used to a new layout which does not necessarily provide understanding, it is important that the multiplication method is taught on split screen which shows the conceptual understanding alongside the procedural. Children must have secure times tables knowledge to 10 x 10 in order for them to see the benefits of this quick efficient method.

The carrying of digits further complicate the learning of this method, therefore the following progression in the teaching is recommended.

1) Begin with numbers where carrying is not involved. Start with 2 digit, then 3 digit numbers.
Examples:
 32×3 and 423×3

$$\begin{array}{r} 32 \\ \times 3 \\ \hline 96 \end{array}$$

$$\begin{array}{r} 423 \\ \times 3 \\ \hline 1269 \end{array}$$

2) Then move onto multiplying 3 digit numbers by 1 digit with carrying.
Example:
 324×3

$$\begin{array}{r} 324 \\ \times 3 \\ \hline 972 \end{array}$$

3) When knowledge is secure, higher numbers are used to introduce carrying.
Examples:
 643×4 643×8

$$\begin{array}{r} 643 \\ \times 4 \\ \hline 2572 \end{array}$$

$$\begin{array}{r} 643 \\ \times 8 \\ \hline 5144 \end{array}$$

4) Children will now be ready to move onto multiplying HTU x TU
Example:
 643×24

$$\begin{array}{r} 643 \\ \times 24 \\ \hline 2572 \\ + 32150 \\ \hline 34722 \end{array}$$

Always start multiplying by the unit number. So 3 is multiplied by 2 first, then 3 is multiplied by 3.

Again, begin by multiplying the units.

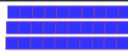
Carrying must be recorded as shown.

All children should be able to do this by the end of year 4.
Year 4 should move onto 2D x 2D or 3D x 2D in the summer term but only those children who are secure with their multiplication facts up to 10 x 10.

Begin by multiplying the unit with each of the digits. Children need to be taught that the 0 in the second row is written as a placeholder because we are now multiplying the tens with each digit.

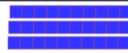
1) Multiplication without carrying. Children use Denes to create the given number or groups.

Step 1.

Tens	Ones
	

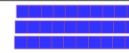
$$\begin{array}{r} 32 \\ \times 3 \\ \hline \end{array}$$

Step 2.

Tens	Ones
	

$$\begin{array}{r} 32 \\ \times 3 \\ \hline 6 \end{array}$$

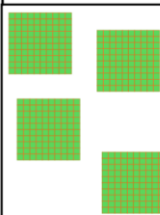
Step 3.

Tens	Ones
	

$$\begin{array}{r} 32 \\ \times 3 \\ \hline 96 \end{array}$$

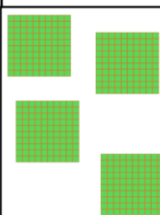
2) 3 digit number by 1 digit multiplication without carrying.

Step 1.

Hundreds	Tens	Ones
		

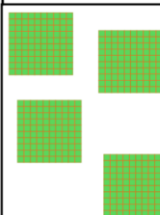
$$\begin{array}{r} 423 \\ \times 3 \\ \hline \end{array}$$

Step 2.

Hundreds	Tens	Ones
		

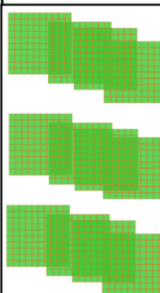
$$\begin{array}{r} 423 \\ \times 3 \\ \hline 9 \end{array}$$

Step 3.

Hundreds	Tens	Ones
		

$$\begin{array}{r} 423 \\ \times 3 \\ \hline 69 \end{array}$$

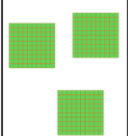
Step 4.

Hundreds	Tens	Ones
		

$$\begin{array}{r} 423 \\ \times 3 \\ \hline 1269 \end{array}$$

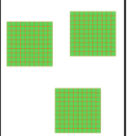
3) 3 digit number by 1 digit multiplication with carrying.

Step 1.

Hundreds	Tens	Ones
		

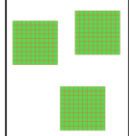
$$\begin{array}{r} 324 \\ \times 3 \\ \hline \end{array}$$

Step 2.

Hundreds	Tens	Ones
		

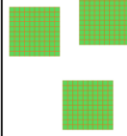
$$\begin{array}{r} 324 \\ \times 3 \\ \hline \end{array}$$

Step 3.

Hundreds	Tens	Ones
		

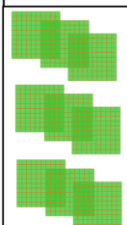
$$\begin{array}{r} 324 \\ \times 3 \\ \hline 2 \end{array}$$

Step 4.

Hundreds	Tens	Ones
		

$$\begin{array}{r} 324 \\ \times 3 \\ \hline 72 \end{array}$$

Step 5.

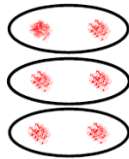
Hundreds	Tens	Ones
		

$$\begin{array}{r} 324 \\ \times 3 \\ \hline 972 \end{array}$$

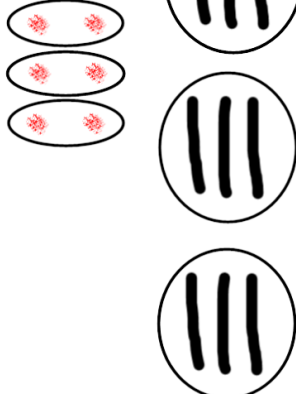
Once children understood the concept of carrying in the Tens column through practising with plenty examples, there is no need to carry on using Denes.

Some children might find it easier to use jottings when multiplying instead of Denes.

Step 1.

$$\begin{array}{r} 32 \\ \times 3 \\ \hline 6 \end{array}$$


Step 2.

$$\begin{array}{r} 32 \\ \times 3 \\ \hline 96 \end{array}$$


Multiplying with decimal numbers

Teach estimating the approximate answer to the multiplication using mental methods. In the below example children are encouraged to multiply the whole numbers of 6 and 5 to get the answer of 30. This will help them gauge whether the magnitude of the number they get as a result is right.

$$6.43 \times 5.4 = 34.722$$

$$\begin{array}{r} 643 \\ \times 54 \\ \hline 2572 \\ + 32150 \\ \hline 34722 \\ 1 \end{array}$$

Decimal points are taken out of both numbers and calculate multiplication just like whole numbers. Once an answer is obtained, the number of digits after the decimal point in both numbers are counted to indicate the number of digits after the decimal point in the answer.

Mental method of multiplying 2 digit numbers by 1 digit.

$$\begin{array}{l} 1 \quad 4 \times 3 = 42 \\ \swarrow \quad \searrow \\ 10 \quad 4 \end{array} \quad \begin{array}{l} 10 \times 3 = 30 \\ 4 \times 3 = 12 \\ 30 + 12 = 42 \end{array}$$

$$\begin{array}{l} 1 \quad 4 \times 3 \\ \swarrow \quad \searrow \\ 30 + 12 = 42 \end{array}$$

Although written method is applied to calculate 2 digit numbers by 1 digit to teach the process of written multiplication, it is important that children are taught sufficient mental strategies to calculate this as well.

Year 3, 4, 5 and 6: Ratio

1. Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

If 1 child gets 3 sweets, how many sweets will 2 children get?

Child	Sweets
	
	
5	15

Scaling factors: $\times 2$ (from 1 child to 2 children), $\times 5$ (from 3 sweets to 15 sweets).

If 12 sweets are shared equally between 4 children, how many will each child get?

Child	Sweets
4	12
1	3

Scaling factors: $\div 4$ (from 4 children to 1 child), $\div 4$ (from 12 sweets to 3 sweets).

Pre-requisite skill to scaling problems:

$$3 \times \square = 12$$

$$20 \div \square = 5$$

2. The comparison of measures includes simple scaling by integers (for example, a given quantity or measure is twice as long or five times as high) and this connects to multiplication.

Beatrice is baking a cake for 4 people. The ingredients are given for 2 people. How much flour will she need?

Recipe	People	Flour
 2 eggs 300g of flour 150g of sugar	2	300g
	4	600g

Scaling factor: $\times 2$ (from 2 people to 4 people).

3. Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts

Here are two cubes. The small one has a Volume of 8cm³ and the large has a Volume of 64cm³. What is the ratio of small to large cube?

Small	Large
8	64
1	8

Ratio: $1 : 8$

Scaling factors: $\div 8$ (from 8 to 1), $\div 8$ (from 64 to 8).

Peter is making a smoothie for 7 people. The ingredients given are for 4 people. Calculate the amount of pineapple juice he will need.

Smoothie for 4	People	Pineapple juice
 260ml pineapple juice 12 raspberries 14 grapes 2 bananas	4	260ml
	1	65ml
	7	455ml

Scaling factors: $\div 4$ (from 4 people to 1 person), $\div 4$ (from 260ml to 65ml), $\times 7$ (from 1 person to 7 people), $\times 7$ (from 65ml to 455ml).

Division

Mental Skills

Recognise the size and position of numbers

Count back in different steps 2s, 5s, 10s

Halve numbers to 20

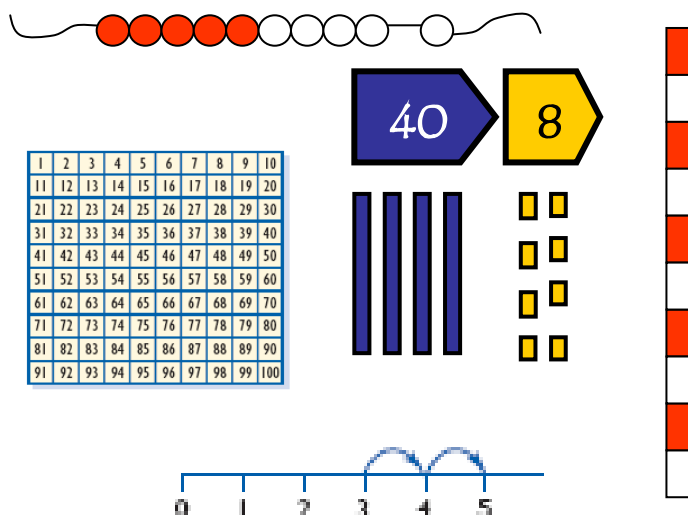
Recognise division as repeated subtraction

Quick recall of division facts

Use known facts to derive associated facts

Divide by 10, 100, 1000 and understanding the effect

Divide by multiples of 10



Models and Images

Counting apparatus

Arrays

100 squares

Number tracks

Numbered number lines

Marked but unnumbered lines

Empty number lines

Multiplication squares

Models and Images charts

ITPs – Multiplication remainders grid, Number Dials, Grouping

divided by group
 into lots of $\div$ into groups of
 divisible remainder
 half factor

Reception

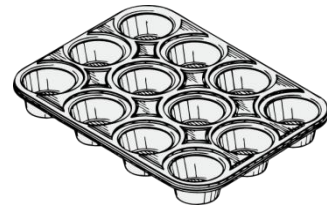
Early learning goal:

They solve problems, including doubling, halving and sharing.

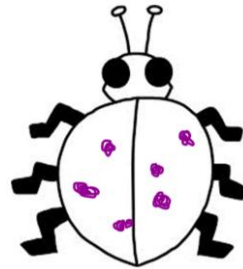
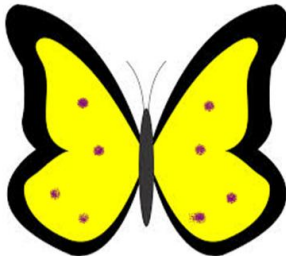
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical opportunities to share using a wide variety of equipment, counters, cubes, mini-bears etc.



Children may also investigate by putting items into groups and arrays using egg boxes, ice cube trays and baking tins etc.



They may develop ways of recording calculations using pictures such as two sides of a lady-bird, butterfly.

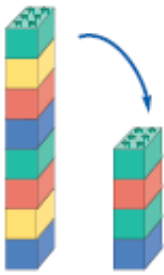


Year 1

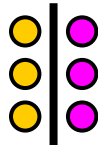
Counting choir

Count in steps of 2s, 5s and 10s forward and back from 0 and from any of its multiples using the 100 square and taking the opportunity to discuss patterns that are recognised.

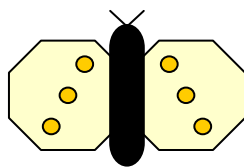
1	2	3	4	5	6	7	8	9	10
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51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



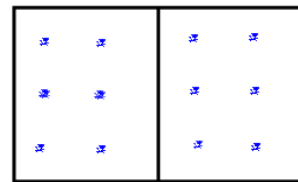
half of 8 is 4
 $8 \div 2 = 4$



Half of 6 is 3
 $\frac{1}{2} \text{ of } 6 = 3$



Half of 12. Share dots equally one by one.



1. Children learn to share objects practically.

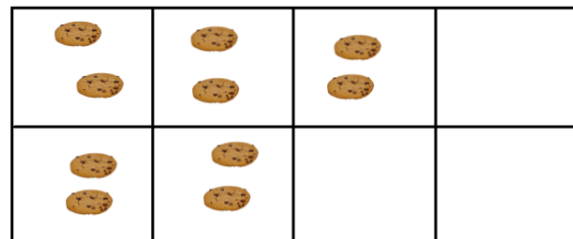


6 muffins shared between 3 people = 2 each
 $6 \div 2 = 3$

2. Children use grouping to solve problems involving division. With the help of laminated sheets children place the given number of objects into groups using the correct mathematical vocabulary. **Please note: to distinguish between grouping using multiplication and division a different type of grouping sheet is used as shown below.**

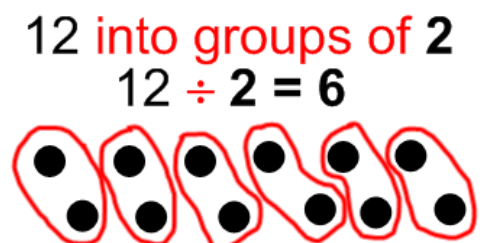
10 cookies into groups of 2
 How many groups?
 $10 \div 2 = 5$

I have 10 cookies. Put them into groups of 2. How many groups have we got altogether?



3. Grouping with the use of jottings. Children first draw the total number of items using dots, then put circles around the given number of dots. They count the number of groups to obtain an answer.

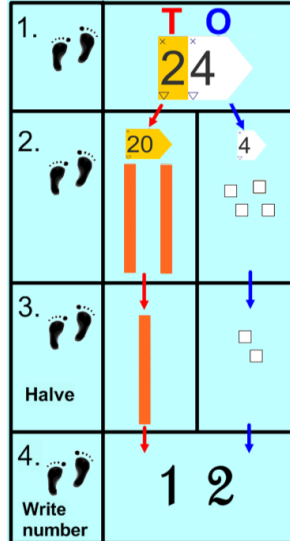
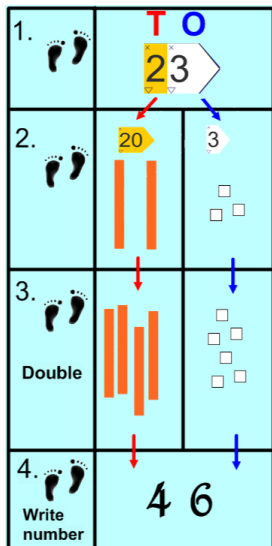
I have 12 multilink cubes. If I put them into groups of 2, how many groups have I got?



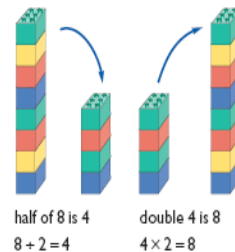
Year 2

Counting choir

Count in steps of 2s, 3s, 5s, 10s and 20s forward and back from 0 and from any of its multiples using the 100 / 200 square and taking the opportunity to discuss patterns that are recognised.



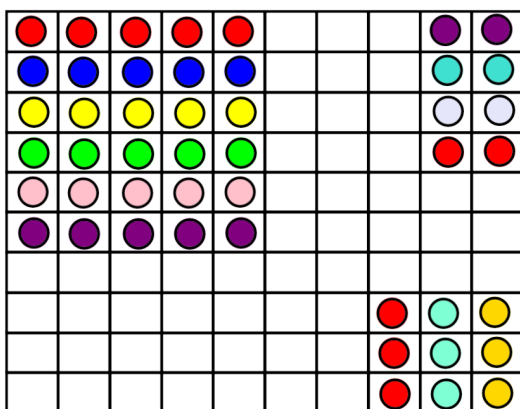
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200



Use fingers to work out doubles up to double 5.

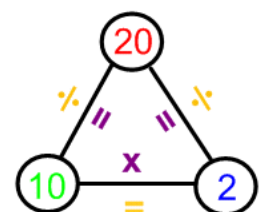
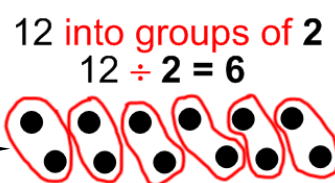
Know doubles and corresponding halves and extend to partitioning numbers then double / partitioning numbers then halve.

Use known multiplication facts to work out corresponding division facts



... 'If I know that 4 rows of two = eight, I also know that eight into rows of 4 is 2 or eight into groups of 2 is 4!'

... 'If I know that 12 into groups of two = six, I also know that six groups of 2 is 12!'



- Children continue to use concrete materials and physical resources to share objects equally with or without the help of the laminated grouping sheets.

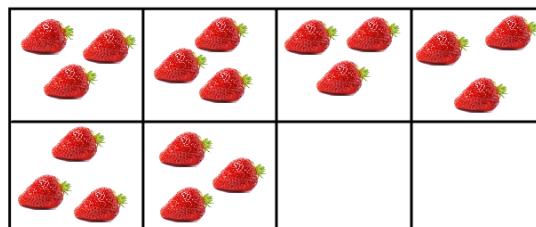
12 muffins shared
between 3 people = 4



I have 18 strawberries. Put them into groups of 3. How many groups have we got altogether?

18 strawberries into groups of 3
How many groups?

$$18 \div 3 = 6$$

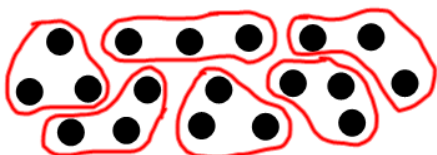


- Grouping with the use of jottings. Children first draw the total number of items using dots, then put circles around the given number of dots. They count the number of groups to obtain an answer. In year 2 children are exposed to grouping in all multiples between 2-9.

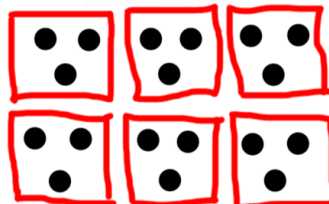
I have 18 multilink cubes. If I put them into groups of 3, how many groups have I got?

Children who have secure knowledge of number clusters should arrange their dots using these.

18 into groups of 3
 $18 \div 3 = 6$



18 into groups of 3
 $18 \div 3 = 6$

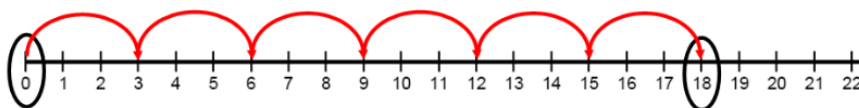


- Children begin to use a marked number line to solve division problems.

Please note: Although division is associated with repeated subtraction, we teach children to **COUNT ON** in jumps of the given number because when it comes to teaching division with remainders on a number line, this is the only way it will work. This eliminates confusion.

This method requires children to find out how many jumps of 3 can they make between 0 and 18. They circle 0 and 18 on the number line before they commence their equal jumps of 3. The constant re-enforcement of vocabulary 'into groups of' is very important.

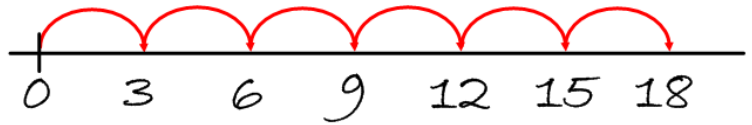
18 into groups of 3 = 6 groups
18 into jumps of 3 = 6 jumps
 $18 \div 3 = 6$



4. More confident children, who are able to reliably count in multiples of 2, 3, 5, 10 use an empty number line to make their jumps.

They write their own numbers underneath the number-line each time they complete a jump to keep track of where they are. The challenge in this process is to remember to stop once they got to the required number. In this case, 18.

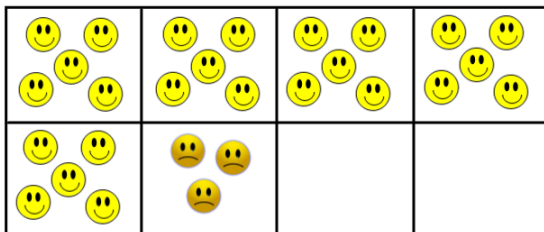
18 into **groups of 3** = 6 **groups**
 18 into **jumps of 3** = 6 **jumps**
 $18 \div 3 = 6$



5. Reinforce division as grouping through arrays and jottings and introduce remainders.

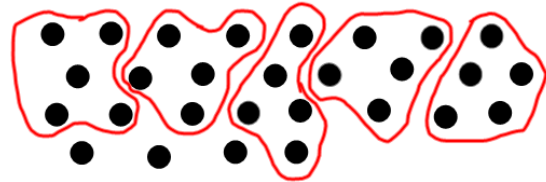
5.1 Practical using division mat.

28 children **into groups of 5**
 How many children left without a group?
 $28 \div 5 = 5 \text{ r } 3$



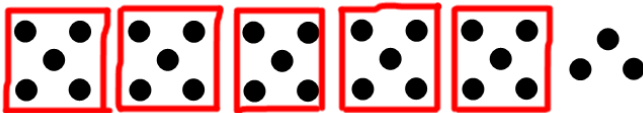
5.2 Jottings without the use of number clusters.

28 children **into groups of 5**
 How many children left without a group?
 $28 \div 5 = 5 \text{ r } 3$



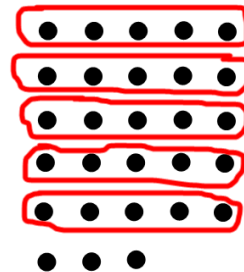
5.3 Jottings with the use of number clusters.

28 children **into groups of 5**
 How many children left without a group?
 $28 \div 5 = 5 \text{ r } 3$



5.4 Jottings using arrays.

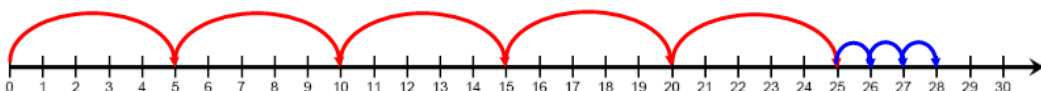
28 children **into groups of 5**
 How many children left without a group?
 $28 \div 5 = 5 \text{ r } 3$



6. Children begin to use a marked number line to solve division problems involving remainders.

The challenge in this process is to remember to not to carry on jumping in multiples of 5s after number 25 and to realise that the remaining jumps need to be made in jumps of ones to work out the remainder. Children must be able to count in multiples of 2, 3, 5 and 10 securely in order to use this method successfully.

28 children **into groups of 5**
 How many children left without a group?
 $28 \div 5 = 5 \text{ r } 3$



Year 3, 4, 5 and 6: Short division

Children must have secure division facts knowledge to 10 x 10 in order for them to see the benefits of this quick efficient method. Teaching must however follow the order of difficulty to overcome possible misconceptions.

Those who are not yet ready for this method should carry on with grouping through the use of arrays as the models in previous pages show.

Because children have to get used to a new layout which does not necessarily provide understanding, it is important that the division method is taught on split screen which shows the conceptual understanding alongside the procedural.

1) We begin teaching with a number into which the divisor goes into exactly. Digit by digit we divide the dividend by the divisor.	2) We then teach a sum that has a remainder in the middle. The remainder is written in small in front of the next dividend digit. Then we divide 16 by 2.	3) The next level of difficulty is to write 0 above the digit into which the divisor doesn't go into.	4) Make a point of teaching the following: when the divisor doesn't go into the last digit of the dividend we write the 0 but we will also write that number as a remainder.
$\begin{array}{r} 31 \\ 2 \overline{) 62} \\ \underline{62} \\ 0 \end{array}$ $\begin{array}{r} 432 \\ 2 \overline{) 864} \\ \underline{864} \\ 0 \end{array}$	$\begin{array}{r} 482 \\ 2 \overline{) 964} \\ \underline{964} \\ 0 \end{array}$	$\begin{array}{r} 098r2 \\ 4 \overline{) 394} \\ \underline{394} \\ 0 \end{array}$ $\begin{array}{r} 108r3 \\ 4 \overline{) 435} \\ \underline{435} \\ 0 \end{array}$	$\begin{array}{r} 130r3 \\ 4 \overline{) 5123} \\ \underline{5123} \\ 0 \end{array}$

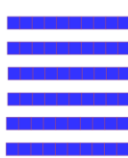
Suggested mental maths starter before teaching the division method with remainders is to find remainders when dividing numbers mentally.

Example: $27 \div 5 = 5 \text{ r}2$ or $38 \div 6 = 6 \text{ r}2$ or $82 \div 9 = 9 \text{ r}1$

All children should be able to calculate using this method by the end of year 3.

1) Division without remainder. Children use the Place Value grid to group Denes.

Step 1.

Tens	Ones
	

$$2 \overline{) 62}$$

Step 2.

Tens	Ones
	

$$\begin{array}{r} 3 \\ 2 \overline{) 62} \\ \underline{62} \\ 0 \end{array}$$

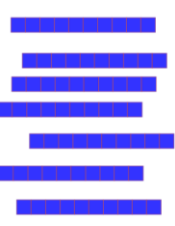
Step 3.

Tens	Ones
	

$$\begin{array}{r} 31 \\ 2 \overline{) 62} \\ \underline{62} \\ 0 \end{array}$$

2) Division with remainder. Children use Place Value grid to group Denes.

Step 1.

Tens	Ones
	

$$2 \overline{) 75}$$

Step 2.

Tens	Ones
	

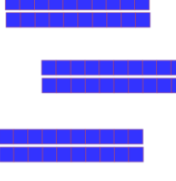
$$2 \overline{) 75} \quad \begin{matrix} 3 \\ \hline \end{matrix}$$

Step 3.

Tens	Ones
	

$$2 \overline{) 75} \quad \begin{matrix} 3 \\ \hline \end{matrix}$$

Step 4.

Tens	Ones
	

$$2 \overline{) 75} \quad \begin{matrix} 37 \\ \hline \end{matrix} r1$$

Once children got the idea of remainders using Tens and Ones, it is ineffective to carry on using Denes for more complicated calculation.

Year 5 and 6: Long division HTU ÷ TU

This method is followed on from the short division however uses a different format to make finding the remainder easier to calculate.

When we first begin teaching this, provide children with an already prepared fact box. Once more confident, get children to create their own.

Fact box:

$2 \times 24 = 48$
 $3 \times 24 = 72$
 $4 \times 24 = 96$
 $5 \times 24 = 120$
 $6 \times 24 = 144$
 $7 \times 24 = 168$
 $8 \times 24 = 192$
 $9 \times 24 = 216$
 $10 \times 24 = 240$

$$\begin{array}{r} 0 \quad 2 \quad 4 \quad r1 \quad 2 \\ 2 \quad 4 \overline{) 5 \quad 8 \quad 8} \\ \underline{- 4 \quad 8} \\ 1 \quad 0 \quad 8 \\ \underline{- 9 \quad 6} \\ 1 \quad 2 \end{array}$$

An alternative method to the long division is to write remainder just like in short division. Children must ensure that their digits are written with enough gap in between them in order to be able to fit in the remainder.

$$\begin{array}{r} 0 \quad 2 \quad 4 \quad r1 \quad 2 \\ 2 \quad 4 \overline{) 5 \quad 8 \quad 108} \end{array}$$

Children must be taught to express long division as decimals as well as a mixed number fraction.

To express remainders as a decimal number, we must carry on with the division by bringing down a zero until we have remainders.

Children should use their knowledge of place value and conversions between fractions and decimals to express the answer as a decimal as well as a mixed number fraction.

$$\begin{array}{r}
 0 \text{ } 2 \text{ } 4 \text{ } . \text{ } 5 \\
 2 \text{ } 4 \overline{) 5 \text{ } 8 \text{ } 8} \\
 \underline{- \text{ } 4 \text{ } 8} \\
 1 \text{ } 0 \text{ } 8 \\
 \underline{- 9 \text{ } 6} \\
 1 \text{ } 2 \text{ } 0 \\
 \underline{- 1 \text{ } 2 \text{ } 0} \\
 0 \text{ } 0 \text{ } 0
 \end{array}
 = 2 \text{ } 4 \frac{1}{2}$$

$$\begin{array}{r}
 0 \text{ } 2 \text{ } 4 \text{ } . \text{ } 5 \\
 2 \text{ } 4 \overline{) 5 \text{ } 8 \text{ } 8 \text{ } 0} \\
 10 \\
 12
 \end{array}
 = 2 \text{ } 4 \frac{1}{2}$$

In the alternative method the zero is written next to the last digit of the dividend and carry the remainder in front of that zero.

In both of the above methods children should check if their answer is correct using inverse operations by multiplying their answer by the divisor and adding the remainder to their answer.

Using and applying: Once confident with this method, provide children with plenty of opportunity to be able to use and apply their newly gained skills to solve problems that involves getting answers with remainders and decimals.