Progression towards Written Calculations
A guide for parents.

Scotforth St. Paul's C. of E.

Primary and Nursery School

From Reception, children will develop mental strategies. These will be supported by informal jottings as the children deal with larger numbers. Children will always ask the question "Can I do it in my head?" before using a formal written method.

Progression Towards a Written Method for Addition

In developing a written method for addition, it is important that children understand the concept of addition, in that it is:

- Combining two or more groups to give a total or sum
- Increasing an amount

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of subtraction
- commutative i.e. 5 + 3 = 3 + 5
- associative i.e. 5 + 3 + 7 = 5 + (3 + 7)

The fact that it is commutative and associative means that calculations can be rearranged, e.g.
4 + 13 = 17 is the same as 13 + 4 = 17.

Reception

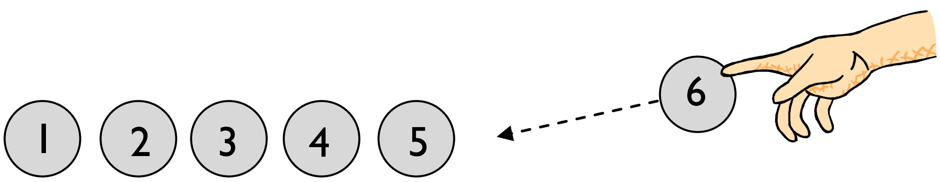
Children are encouraged to develop a mental picture of the number system in their heads in preparation for calculation.

- They will use the language of 'more' and 'fewer' to compare two sets of objects.
- They will know number pairs that make 10 e.g. 1+9=10, 2+8=10, etc.
- They will know which number is one more or less than a given number to 10.

The children will also experience problem solving activities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

Counting all method
Children will begin to develop their ability to add by using practical equipment to count out the correct amount for each number in the calculation and then combine them to find the total. For example, when calculating 4 + 2, they are encouraged to count out four counters and count out two counters.

To find how many altogether, touch and drag them into a line one at a time whilst counting.

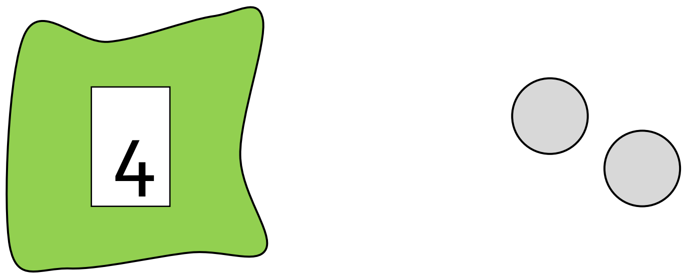


By touch counting and dragging in this way, it allows children to keep track of what they have already counted to ensure they don't count the same item twice.

Counting on method

To support children in moving from a counting all strategy to one involving counting on, children should still have two groups of objects but one should be covered so that it cannot be counted. For example, when calculating 4 + 2, count out the two groups of counters as before.

then cover up the larger group with a cloth.



For most children, it is beneficial to place the digit card on top of the cloth to remind the children of the number of counters underneath. They can then start their count at 4, and touch count 5 and 6 in the same way as before, rather than having to count all of the counters separately as before.

Those who are ready will begin to record number stories using number sentences.

<u>Year 1</u>

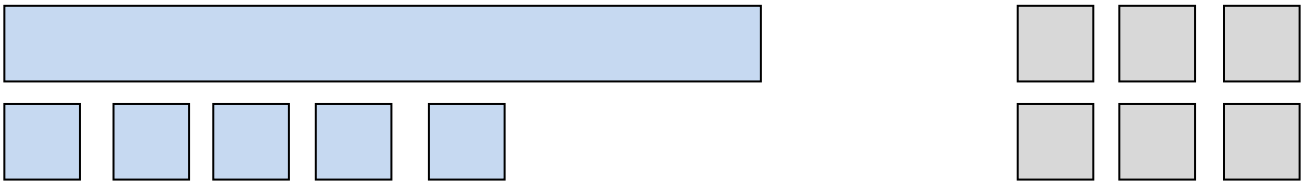
Children must learn by heart number bonds to 10, as this underpins future work.

Children will continue to use practical equipment, combining groups of objects to find the total by counting all or counting on using numbers up to 20. Using their developing understanding of place value, they will move on to be able to use Base 10 equipment to make teens numbers using separate tens and units.

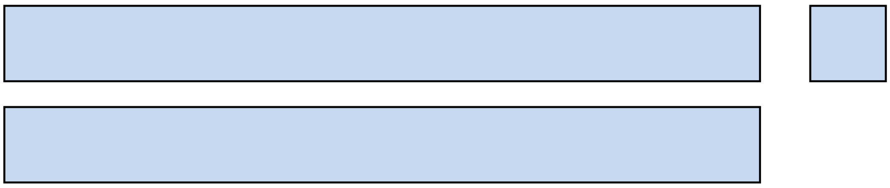
For example, when adding 11 and 5, they can make the 11 using a ten rod and a unit.

The units can then be combined to aid with seeing the final total, e.g.

so 11 + 5 = 16. If possible, they should use two different colours of base 10 equipment so that the initial amounts can still be seen.
Children are introduced to the concept of exchange and use practical equipment to practise this skill. For example, when adding 15 + 6, children make 15 with a ten stick and five units before 6 more units are added.



Children then exchange the ten single units for a ten stick leaving two ten sticks and one unit to show the total 21.



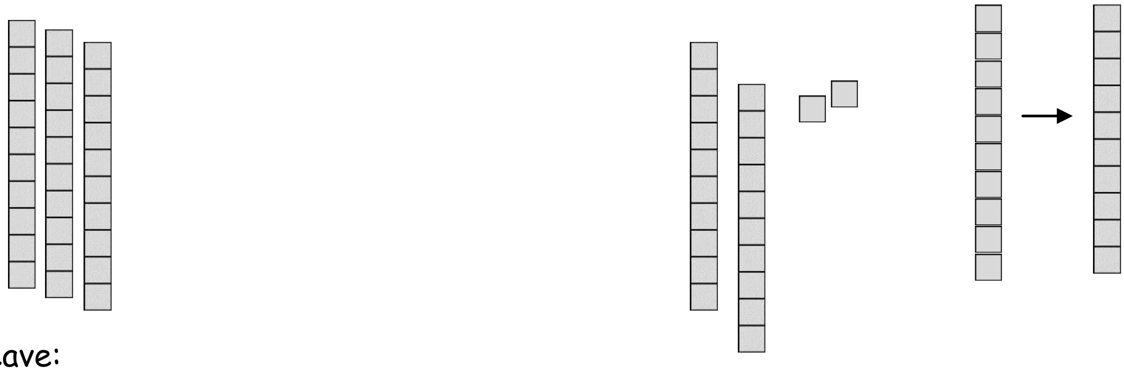
The concept of number and it's relative size plays a crucial role in children's everyday mathematical experiences in year one. Children must learn to subitise small numbers (see and know the number without having to count) and represent numbers in a variety of ways. Children will learn to solve one-step problems using concrete objects and pictorial representations. They also learn to solve missing number problems e.g. 17=Δ+9

Year 2

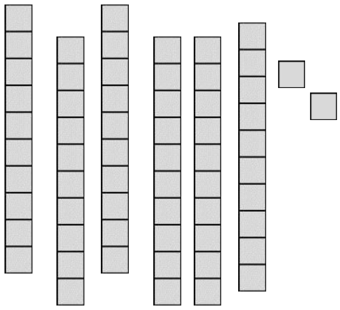
Children will continue to use the Base 10 equipment to support their calculations using numbers to 100. For example, to calculate 32 + 21, they can make the individual amounts, counting the tens first and then count on the units.

Children continue to use exchange when dealing with more than ten units - this is the start of children understanding 'carrying' in vertical addition. For example, when calculating 35 + 27, they can represent the amounts using Base 10 as shown:

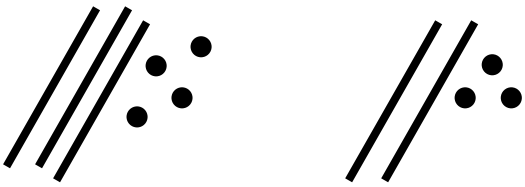
Then, identifying the fact that there are enough units/ones to exchange for a ten, they can carry out this exchange:



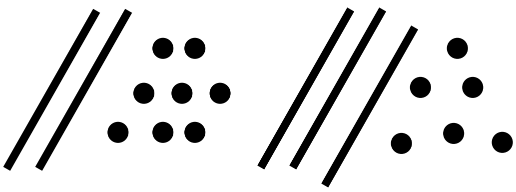
To leave:



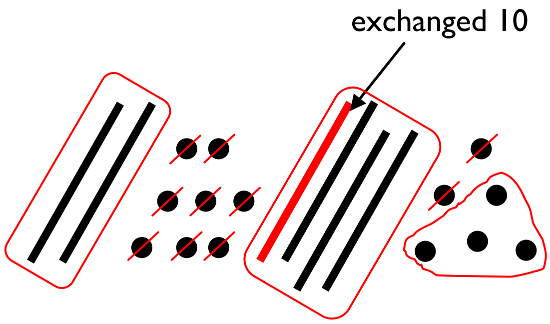
Children now learn to record the calculations using their own drawings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the unit blocks).
e.g. 34 + 23 =



With exchange:
e.g. 28 + 36 =



will become



so 28 + 36 = 64

It is important that children circle the remaining tens and units/ones after exchange to identify the amount remaining.
In Year 2 children learn to add 2-digit numbers to

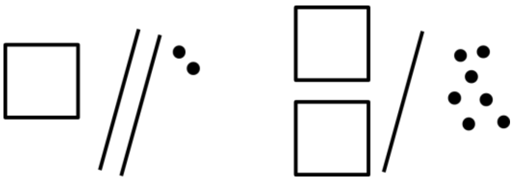
- ones/units e.g. 76+5=
- tens e.g. 32+30=
- 2-digit numbers e.g. 46+33=

They also learn to add three 1-digit numbers.

Year 3

Children will build on their knowledge of using Base 10 equipment from Y2 and continue to use the idea of exchange.

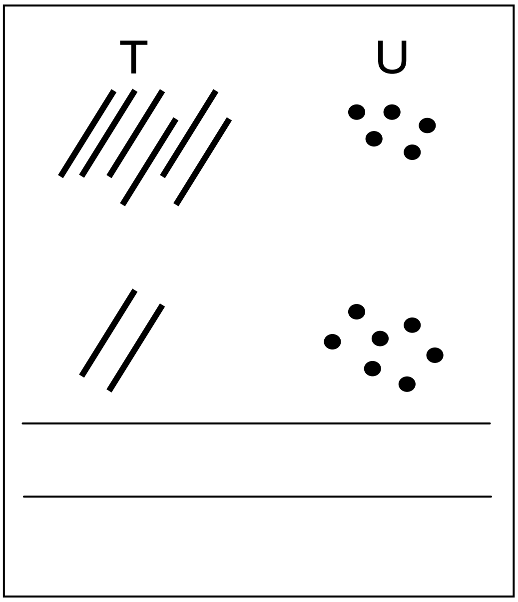
Continuing on from Y2 they represent numbers using rods and dots as jottings to aid calculations. When they are ready, a square is introduced to represent 100.
e.g. 122 + 217



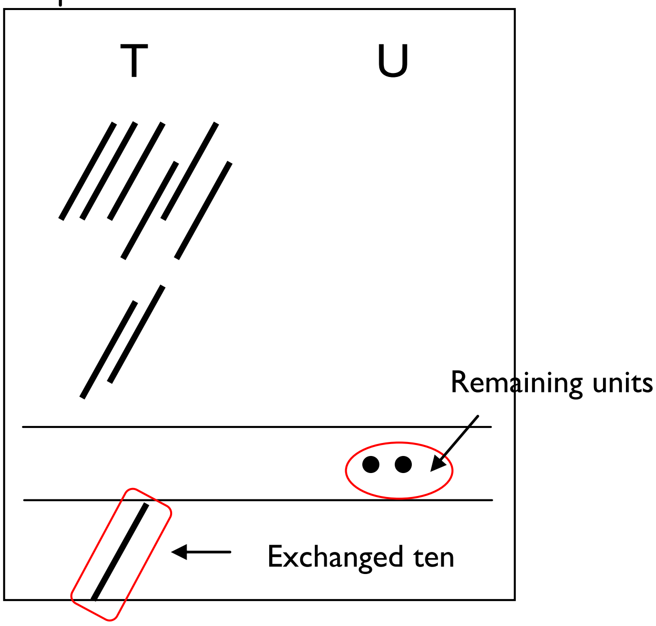
Children should add the **least significant digits** first (i.e. start with the units/ones), and in an identical method to that from year 2, should identify whether there are greater than ten units which can be exchanged for one ten.

They can use a place value grid to begin to set the calculation out vertically and to support their knowledge of exchange between columns (as in Step 1 in the diagram below).
For example, 65 + 27

Step 1



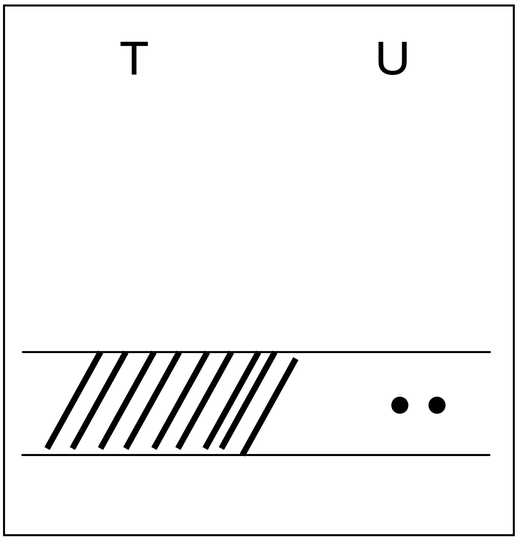
Step 2



Children would exchange ten units/ones for a ten, placing the exchanged ten below the equals sign. Any remaining units/ones that cannot be exchanged for a ten move into the equals sign as they are the units part of the answer (as in the diagram in Step 2 above).

If there are any tens that can be exchanged for a hundred, this can be done next. If not, the tens move into the equals sign as they are the tens part of the answer (as in the diagram in Step 3 below).

Step 3



Written method

Step 1

	T	U
	6	5
+	2	7

Step 2

	T	U
	6	5
+	2	7
		2
	1	

Step 3

	T	U
	6	5
+	2	7
	9	2
	1	

Children should utilise this practical method to link their understanding of exchange to how the column method is set out. Teachers will model the written method alongside this practical method initially until children can utilise just the written method.

In addition to the practical and written column methods of addition, children apply their knowledge of partitioning when adding pairs of two and three digit numbers. For example, when adding 29 and 46 children may choose to record their working as:

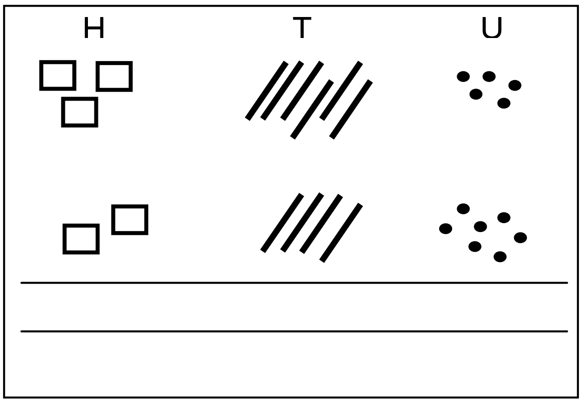
29 + 46

60 + 15 = 75

By the end of year 3, children should also extend this method for three digit numbers.

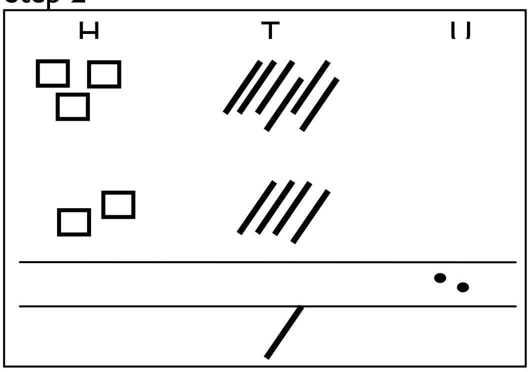
For example, 365 + 247

Step 1



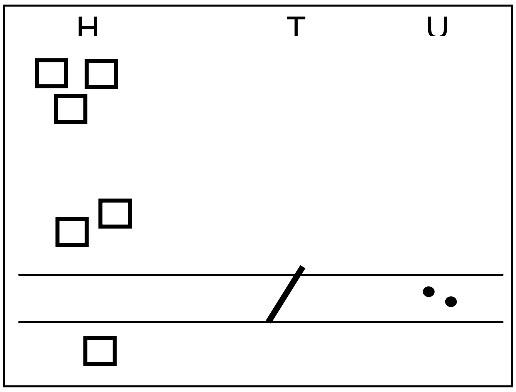
	H	T	U
	3	6	5
+	2	4	7

Step 2



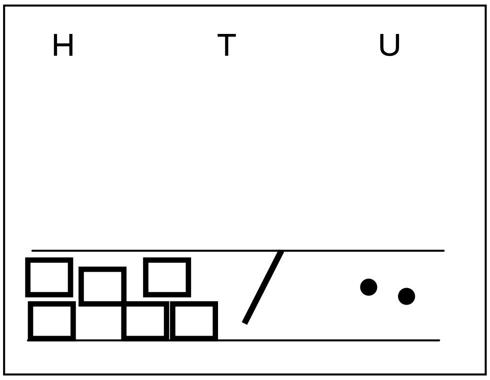
	H	T	U
	3	6	5
+	2	4	7
			2
		1	

Step 3



	H	T	U
	3	6	5
+	2	4	7
		1	2
	1	1	

Step 4



	H	T	U
	3	6	5
+	2	4	7
	6	1	2
	1	1	

In Year 3 children learn to add 3-digit numbers to

- ones/units e.g. 476+5=
- tens e.g. 932+30=
- hundreds e.g. 321+700=

Year 4

Children will move to year four using whichever method they were using as they transitioned from year 3.

Children will be expected to work with numbers with up to 4-digits. They will be introduced to sums with zero hundreds, tens or units and be expected to calculate solutions for these.

For example, when adding a 2 and a 3 digit number.

H	T	U
3	6	7
	8	5
4	5	2
1	1	

The concept of estimation is a fundamental part of any calculation and this is introduced more specifically in year four. Children are routinely expected to make a reasonable estimation before starting to solve the calculation.
For example, when adding 325 and 492, children would record the sum and an estimation bubble.

	325
+	492

300 + 500
= 800

Children use knowledge of rounding to estimate.

By the end of year 4, children should be using the written method confidently and with understanding. They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- decimals with one decimal place, knowing that the decimal points line up under one another.
- fractions with the same denominator.

Year 5

Children continue to use the carrying method to solve calculations using numbers with more than 4 digits such as:

	5	3	3	6	4
+		2	2	4	7
	5	5	6	1	1
			1	1	

	3	1	2	1
			3	7
+		1	4	8
	3	3	0	6
		1	1	

	3	.	5	6
+	2	.	4	7
	6	.	0	3
			1	

Children also continue to show their estimations prior to calculating sums and are asked to evaluate the accuracy of estimations through the use of a smiley face symbol.

They will also be adding:

- amounts of money and measures, including those where they have to initially convert from one unit to another, e.g. £ to pence or km to cm.
- fractions both with the same denominator and denominators that are multiples.

Year 6

Children should extend the carrying method and use it to add whole numbers and decimals with any number of digits.

				4	2
		6	4	3	2
			7	8	6
					3
+		4	6	8	1
	1	1	9	4	4
	1	1	2	1	

	4	0	1	.	2	0
		2	6	.	8	5
+			0	.	7	1
	4	2	8	.	7	6
			1			

When adding decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be adding decimals with up to three decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.
They will be expected to solve addition problems that involve multiple steps e.g.

"Three pandas are eating bamboo sticks. There are 51 altogether. They all eat an odd number of sticks. How many bamboo sticks did they each eat? How many different ways can you do it?"

Progression Towards a Written Method for Subtraction

In developing a written method for subtraction, it is important that children understand the concept of subtraction, in that it is:

- Removal of an amount from a larger group (take away)
- Comparison of two amounts (difference)

They also need to understand and work with certain principles, i.e. that it is:

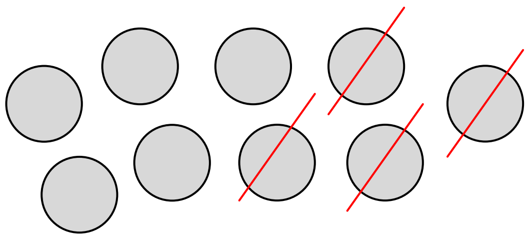
- the inverse of addition
- not commutative i.e. 5 - 3 is not the same as 3 - 5
- not associative i.e. 10 - 3 - 2 is not the same as 10 - (3 - 2)

Reception

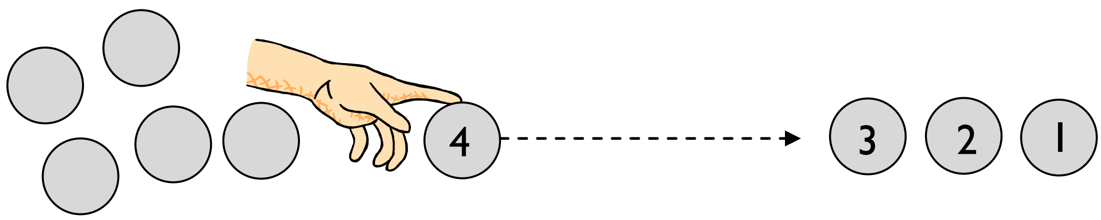
Children are encouraged to develop a mental picture of the number system in their heads to use for calculation.

Taking away

Children will begin to develop their ability to subtract by using practical equipment to count out the first number and then remove or take away the second number to find the solution by counting how many are left e.g. 9 – 4.



For illustration purposes, the amount being taken away are shown crossed out. Children would be encouraged to physically remove these using touch counting.



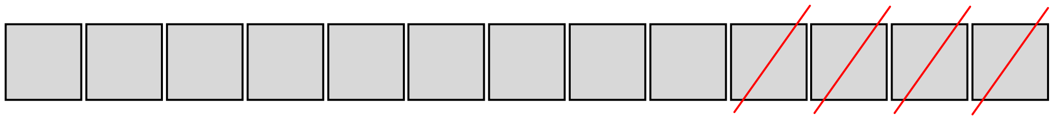
By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they don't have to keep recounting. They will then touch count the amount that are left to find the answer.

Those who are ready will begin to record number stories using number sentences.

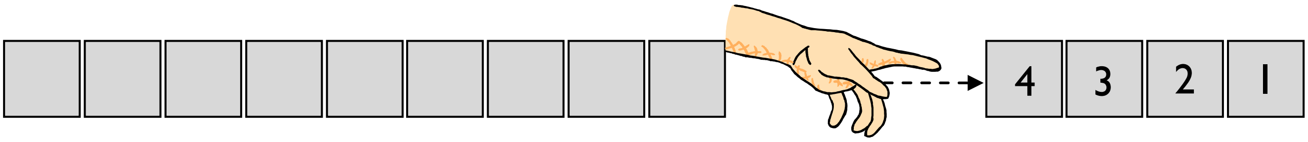
The children will also experience problem solving activities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

Year 1

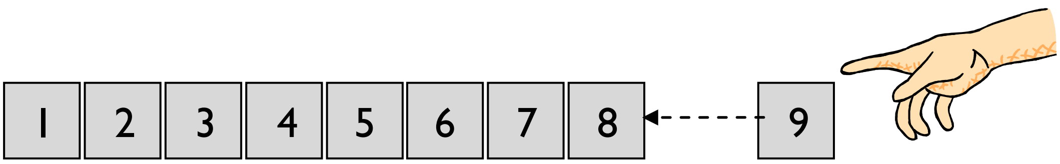
Children will continue to use their knowledge of number bonds to mentally subtract numbers up to 20 e.g. using 7+9 to recall 16-7 and 16-9.
Children will continue to use practical equipment and taking away strategies. To avoid the need to exchange for subtraction at this stage, it is advisable to continue to use equipment such as counters, cubes and the units from the Base 10 equipment, but not the tens, e.g. 13 - 4



Touch count and remove the number to be taken away, in this case 4.



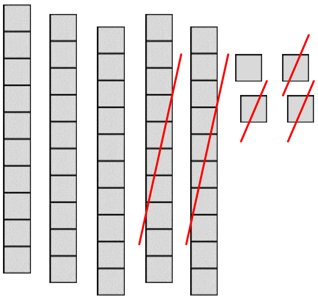
Touch count to find the number that remains.



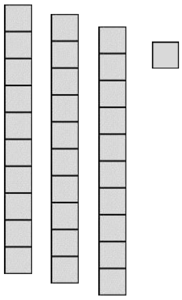
Children will learn to solve one-step problems using concrete objects and pictorial representations. They also learn to solve missing number problems e.g. 7=∆-9

Year 2

Children will begin to use the Base 10 equipment to support their calculations using numbers up to 100, still using a take away or removal method. They need to understand that the number being subtracted does not appear as an amount on its own, but rather as part of the larger amount. For example, to calculate 54 - 23, children would count out 54 using the Base 10 equipment (5 tens and 4 units). They need to consider whether there are enough units/ones to remove 3, in this case there are, so they would remove 3 units and then two tens, counting up the answer of 3 tens and 1 unit to give 31.

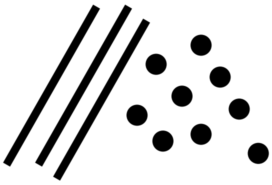


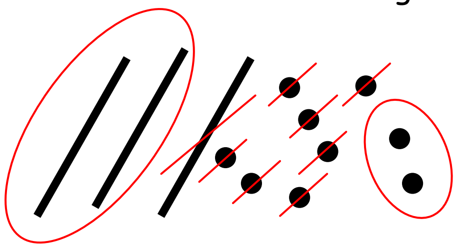
which leaves



so 54 – 23 = 31

Children can also record the calculations using their own drawings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the unit blocks), e.g. to calculate 39 - 17 children would draw 39 as 3 tens (lines) and 4 units (dots) and would cross out 7 units and then one ten, counting up the answer of 2 tens and 2 units to give 22.

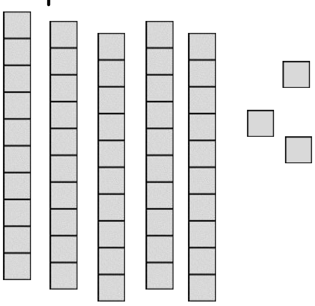




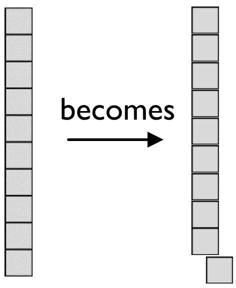
Circling the tens and units that remain will help children to identify how many remain.

When the amount of units to be subtracted is greater than the units in the original number, an exchange method is required. This relies on children's understanding of ten units being an equivalent amount to one ten. To calculate 53 - 26, by using practical equipment, they would count out 53 using the tens and units, as in Step 1. They need to consider whether there are enough units/ones to remove 6. In this case there are not so they need to exchange a ten into ten ones to make sure that there are enough, as in step 2.

Step 1

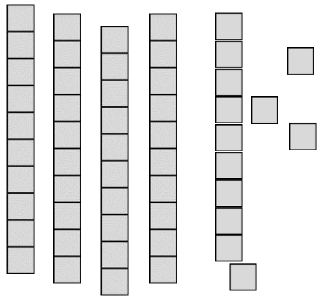


Step 2

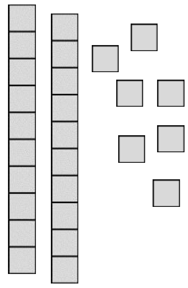


The children can now see the 53 represented as 40 and 13, still the same total, but partitioned in a different way, as in step 3 and can go on to take away the 26 from the calculation to leave 27 remaining, as in Step 4.

Step 3

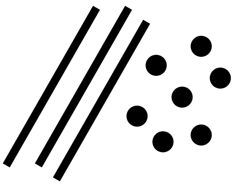


Step 4

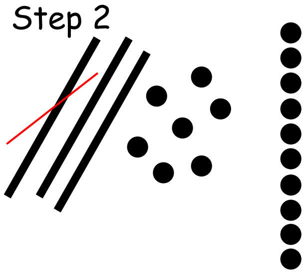


When recording their own drawings, when calculating 37 – 19, children would cross out a ten and exchange for ten units. Drawing them in a vertical line, as in Step 2, ensures that children create ten ones and do not get them confused with the units that were already in place.

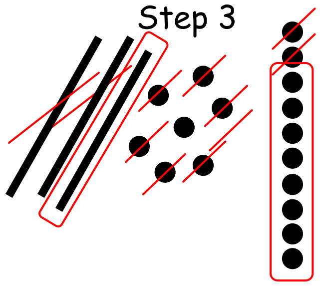
Step 1



Step 2



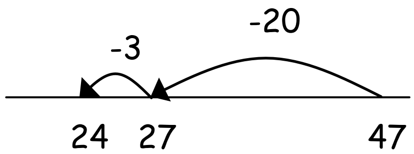
Step 3



Circling the tens and units that remain will help children to identify how many remain.

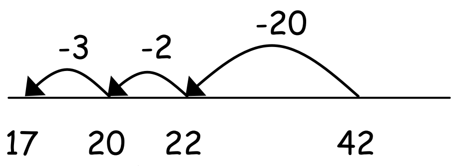
Children also learn to use a blank number line to represent their subtraction calculations. They learn to count on and back depending upon the numbers involved in the calculation. For example,

47 – 23 = 24



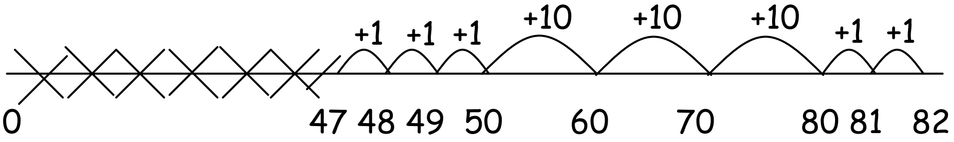
Initially children learn to count back one ten and one unit at a time. When confident children then learn to count back all of the units in one jump and all of the tens in one jump (as noted in this example).

Children also learn to bridge through ten to help them become more efficient. For example, 42 – 25 = 17



Children jump back to 20 as multiples of ten are easier to work with.

When the numbers involved in the calculation are close together or near to multiples of 10 or 100, children will use counting on to solve the calculation. For example, 82 - 47



The number line should still show 0 so children can cross out the section from 0 to the smallest number. They then associate this method with 'taking away'.

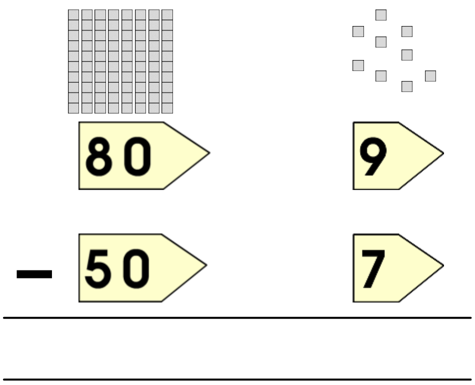
In Year 2 children learn to subtract from 2-digit numbers

- ones/units e.g. 76-5=
- tens e.g. 32-30=
- 2-digit numbers e.g. 46-33=

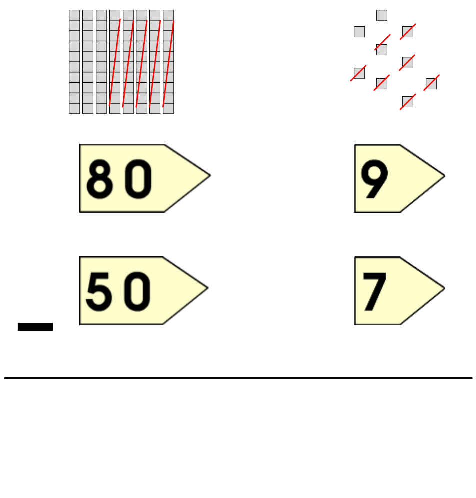
Year 3

Children continue to use number lines as jottings to aid their calculations. They will also build upon their knowledge of using Base 10 equipment from Y2 and continue to use the idea of exchange. This process should be demonstrated using arrow cards to show the partitioning and Base 10 materials to represent the first number, removing the units and tens as appropriate (as with the more informal method in Y2).

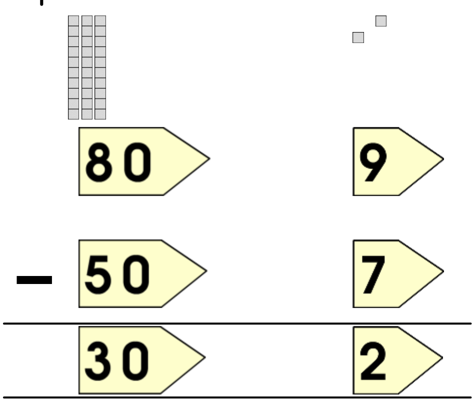
Step 1



Step 2



Step 3

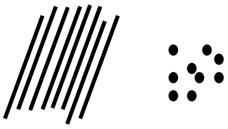


Emphasise that the second (bottom) number is being subtracted from the first (top) number rather than the lesser number from the greater.

This will be recorded by the children as:

	80	and	9		
-	50	and	7		
	30	and	2	=	32

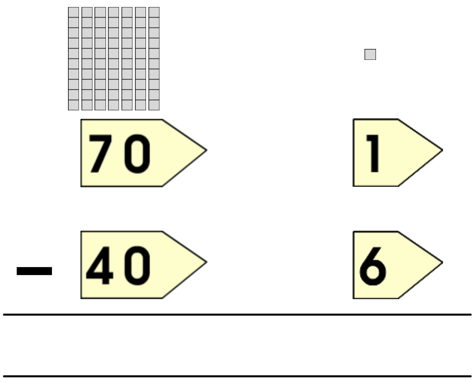
Children can also use jottings of the Base 10 materials (as in Year 2) to support with their calculation, as in the example below.



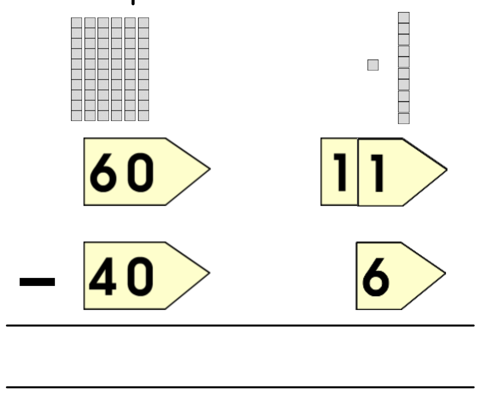
	80	and	9		
-	50	and	7		
	30	And	2	=	32

From this the children will begin to solve problems which involve exchange. Children need to consider whether there are enough units/ones to remove 6. In this case there are not (Step 1) so they need to exchange a ten into ten ones to make sure that there are enough, as they have been doing in the method for Year 2 (Step 2). They should be able to see that the number is just partitioned in a different way, but the amount remains the same (71 = 70 + 1 = 60 + 11).

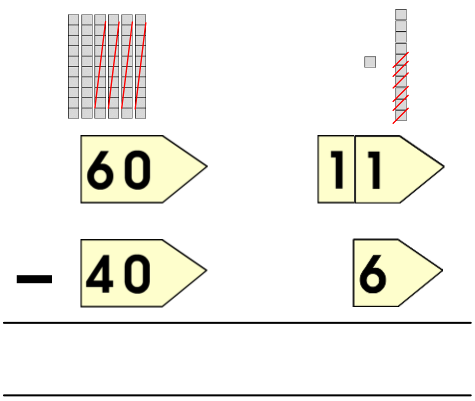
Step 1



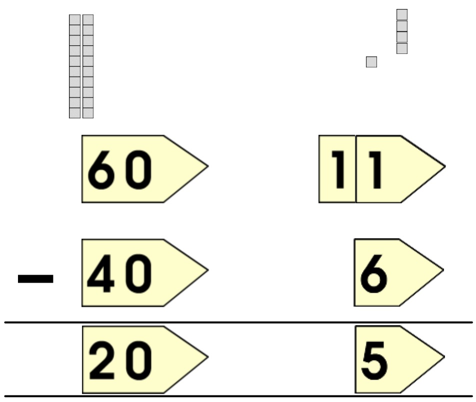
Step 2



Step 3



Step 4



This will be recorded by the children as:

	60				
	~~70~~	and	[1]1		
-	40	and	6		
	20	and	5	=	25

	60 and 11
-	40 and 5
	20 and 6

By the end of year 3, children should also extend this method for three digit numbers.

In Year 3 children learn to subtract from 3-digit numbers

- ones/units e.g. 476-5=
- tens e.g. 932-30=
- hundreds e.g. 321-200=

Year 4

Children will move to Y4 using whichever method they were using as they transitioned from Y3.

The concept of estimation is a fundamental part of any calculation and this is introduced more specifically in year four. Children are routinely expected to make a reasonable estimation before starting to solve the calculation.

754 – 286 =

Step 1

754 – 286 = → 750 – 300 = 450

Children use knowledge of rounding to estimate.

Step 2

	700	and	50	and	4		
-	200	and	80	and	6		

Step 3 (exchanging from tens to units)

			40		
	700	and	~~50~~	and	[1]4
-	200	and	80	and	6

Step 4 (exchanging from hundreds to tens)

	600		140		
	~~700~~	and	~~50~~	and	[1]4
-	200	and	80	and	6

Step 5

	600		140				
	~~700~~	and	~~50~~	and	[1]4		
-	200	and	80	and	6		
	400	and	60	and	8	=	468

This would be recorded by the children as:

	600		140					
	~~700~~	and	~~50~~	and	¹4			
-	200	and	80	and	6			
	400	and	60	and	8	=	468	

When children are ready, this leads on to the compact method of decomposition:

		6	14	
	4	~~7~~	~~5~~	¹4
-	3	2	8	6
	1	4	6	8

By the end of Y4, children should be using the written method confidently and with understanding. They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- fractions with the same denominator;
- decimals with one decimal place, knowing that the decimal points line up under one another.

Year 5

Children continue to use estimation to find an approximate answer for each calculation. The importance of selecting the most sensible method becomes more apparent.

For example:

- 783 – 98

Children should use compensation.
783 – 100 = 683
683+2 = 685

- 308 - 295

Children should use rounding.
308 → 300 (8 more)
300 → 295 (5 less)
8+5 = 13

Children should continue to use the decomposition method to solve calculations such as:

	6		6	
	~~7~~	¹0	~~7~~	¹2
-	3	2	2	6
	3	8	4	6

	2		13	
	~~3~~	.	~~4~~	¹2
-	1	.	7	6
	1	.	6	6

They will also be subtracting:

- amounts of money and measures, including those where they have to initially convert from one unit to another
- fractions both with the same denominator and denominators that are multiples.

Year 6

Children continue to estimate solutions before tackling the calculation. They also continue to select the most appropriate method for each calculation.
Children should extend the decomposition method and use it to subtract whole numbers and decimals with any number of digits.

```
     5  13
     6̶  4̶ ¹3  2
-    4  6  8  1
   -----------
     1  7  5  1
   -----------
```

```
   3     6     11
   4̶ ¹1  7̶  .  2̶ ¹0
-     3  4  .  7  1
   -----------------
   3  8  2  .  4  9
   -----------------
```

When subtracting decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be subtracting:

- decimals with up to three decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.
- They will be expected to solve subtraction problems that involve multiple steps e.g.

David has six white mice, three males and three females. Each of the three couples has 7 female baby mice. The each of these females has 8 babies. One night David's little sister Aisha leaves the mice cage open and 47 escape. How many mice does David have left?

Progression Towards a Written Method for Multiplication

In developing a written method for multiplication, it is important that children understand the concept of multiplication, in that it is:

- repeated addition

They should also be familiar with the fact that it can be represented as an array

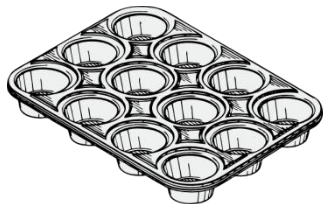
They also need to understand and work with certain principles, i.e. that it is:

- the inverse of division
- commutative i.e. 5 x 3 is the same as 3 x 5
- associative i.e. 2 x 3 x 5 is the same as 2 x (3 x 5)

Reception

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation.

Children will investigate doubling items practically putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays.



They may develop ways of recording calculations using pictures, etc.

A child's jotting showing the fingers on each hand as a double.

A child's jotting showing double three as three cookies on each plate.

The children will understand that doubling is adding the same number to itself.
They should know the doubles of numbers to 5. The children will also experience problem solving activities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

Year One

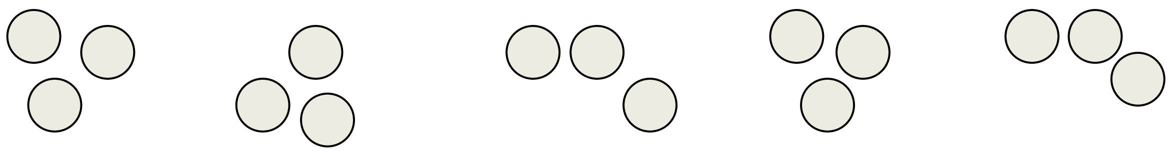
In Year one, children will continue to solve multiplication problems using practical equipment and jottings. They may use the equipment to make groups of objects. Children should see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc and use this in their learning, answering questions such as 'How many eggs would we need to fill the egg box? How do you know?'

Year Two

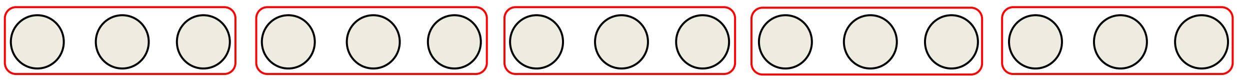
- Initially, children will continue to use arrays and number lines where appropriate linked to the multiplication tables that they know (2, 3, 5 and 10).
- Children will develop their understanding of multiplication and use jottings to support their calculations.
- Children learn that

  3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x 3

Children should understand and be able to calculate multiplication as repeated addition, supported by the use of practical apparatus such as counters or cubes. For example, 5 x 3 can be shown as five groups of three with counters, either grouped in a random pattern, as below:

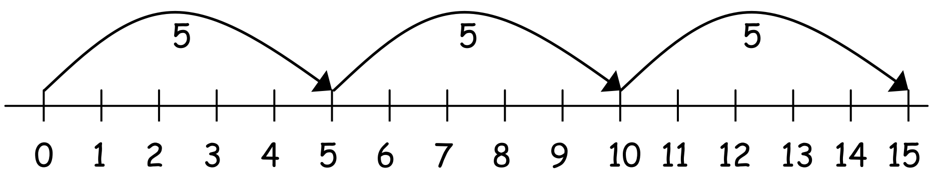


or in a more ordered pattern, with the groups of three indicated by the border outline:



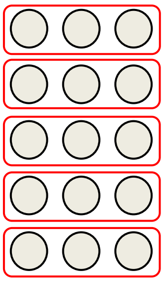
This repeated addition can be shown easily on a number line:

5 x 3 = 5 + 5 + 5

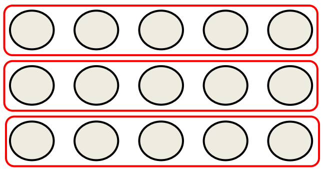


Children should then develop this knowledge to show how multiplication calculations can be represented by an array (this knowledge will support with the development of the grid method in the future). Again, children should be encouraged to use practical apparatus and jottings to support their understanding.

For example, 5 x 3 can be represented as an array in two forms (as it has commutativity):



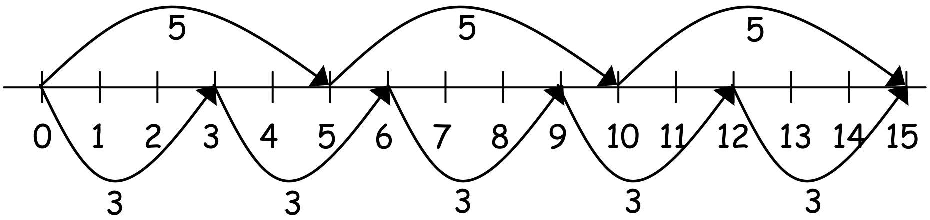
3 + 3 + 3 + 3 + 3 = 15



5 + 5 + 5 = 15

For mathematical accuracy 5 x 3 is represented by the second example, rather than the first as it is five, three times. However, because we use terms such as 'groups of' or 'lots of', children are more familiar with the initial notation. Once children understand the commutative order of multiplication the order is irrelevant.

This can also be shown on a number line as:

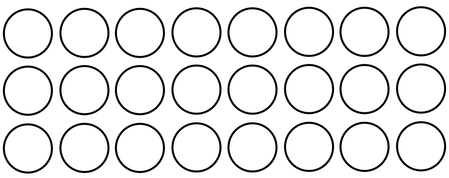


Year Three

Initially, children will continue to use arrays and number lines where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10).

For example, 3 x 8

Using practical equipment, children may show:



3 x 8 = 8 + 8 + 8 = 24

Using jottings on squared paper or in numeracy books, children may show:

	x	x	x	x	x	x	x	x	
	x	x	x	x	x	x	x	x	
	x	x	x	x	x	x	x	x	

3 x 8 = 8 + 8 + 8 = 24

As children progress to multiplying a two-digit number by a single digit number, children should use their knowledge of partitioning two digit numbers into tens and units/ones to help them.

For example, when calculating 14 x 8,

Children partition 14 into 10 + 4 then multiply the 10 by 8 and the 4 by 8.

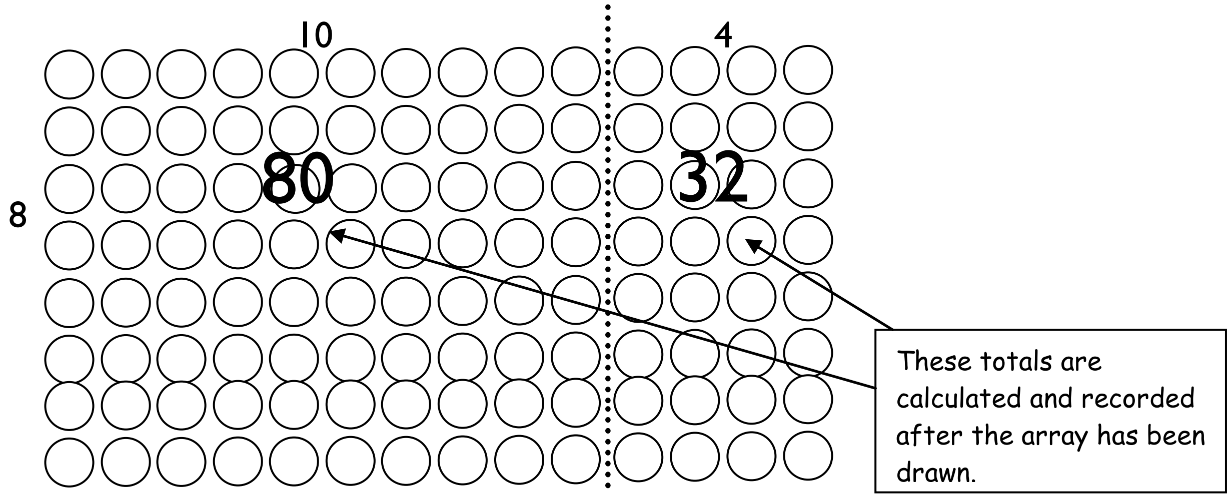
$$14 \times 8 = (10 \times 8) + (4 \times 8)$$

Finally add the two solutions to find the total.

$$= 80 + 32$$

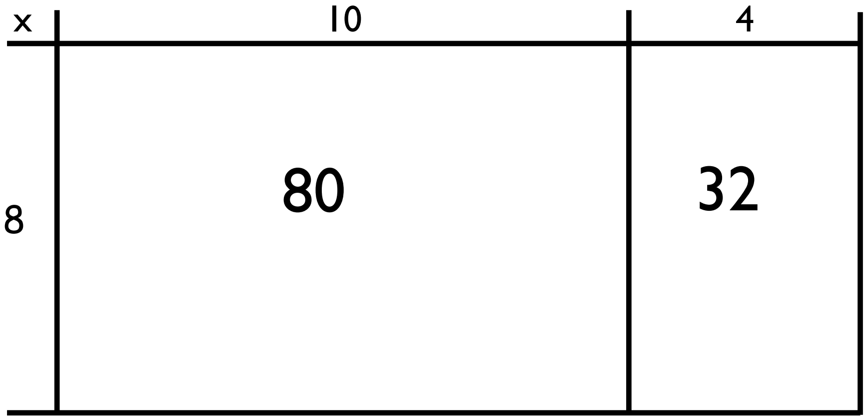
$$= 112$$

This method can also be demonstrated using the array.



This array forms the basis of the grid method, in which children calculate the number of counters or the amount on each side of the partition line. In this example, children would record 80 (8x10) and 32 (8x4) on their grid.

By placing a box around the array, as in the example below, and by removing the array, the grid method can be seen.



It is really important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording.

From this, children can use the grid method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20. Children should be encouraged to set out their addition in a column at the side to ensure the place value is maintained. When children are working with numbers where they can confidently and correctly calculate the addition mentally, they may do so.

13 x 8

x	10	3
8	80	24

$$\begin{array}{r} 80 \\ +\ 24 \\ \hline 104 \\ \hline \end{array}$$

When children are ready, they can then progress to using this method with other two-digit numbers.

37 x 4

x	30	7
4	120	28

$$\begin{array}{r} 120 \\ +\ 28 \\ \hline 148 \\ \hline \end{array}$$

- Children should also be using this method to solve problems and multiply numbers in the context of money or measures.
- Children also learn to manipulate the numbers to complete missing number calculations, for example, ☐ x 5 = 20 3 x △ = 24 ☐ x ◊ = 32

Year Four

By the end of Year 4 the children are expected to recall multiplication tables up to 12x12.

Children will move to Y4 using whichever method they were using as they transitioned from Y3. They will further develop their knowledge of the grid method to multiply any two-digit by any single-digit number, e.g.

79 x 8

x	70	9
8	560	72

$$\begin{array}{r} 560 \\ +\ 72 \\ \hline 632 \\ \hline \end{array}$$

To support the grid method, children should develop their understanding of place value and facts that are linked to their knowledge of tables. For example, in the calculation above, children should use their knowledge that 7 x 8 = 56 to know that 70 x 8 = 560.

By the end of the year, they will extend their use of the grid method to be able to multiply three-digit numbers by a single digit number, e.g.

346 x 8

x	300	40	6
8	2400	320	48

```
  2400
+  320
+   48
------
  2768
```

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Year Five

Children should continue to use the grid method and extend it to multiplying numbers with up to four digits by a single digit number, e.g.

4346 x 8

x	4000	300	40	6
8	32000	2400	320	48

```
  32000
+  2400
+   320
+    48
-------
  34768
```

The formal method of short multiplication will be introduced to the children when they are secure with the grid method for multiplying by a single digit number.

For example, 342 x 7 or 2741 x 6

```
  3 4 2          2 7 4 1
x     7        x       6
-------        ---------
2 3 9 4        1 6 4 4 6
  2 1            4 2
```

Children continue to use the grid method for all other multiplication calculations (TUxTU, HTUxTU and ThHTU xTU).

For example, 2693 x 24

x	2000	600	90	3
20	40000	12000	1800	60
4	8000	2400	360	12
=	48000	14400	2160	72

```
  48000
+ 14400
+  2160
+    72
  64632
  1 1
```

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Children will also be taught to:

- identify multiples and factors
- know and use prime numbers
- multiply whole numbers and decimals by 10, 100 and 1000
- use square and cube numbers
- multiply proper fractions and mixed numbers by whole numbers with the help of resources and diagrams.

Year Six

By the end of Y6, children should be able to use the grid method to multiply any number by a two-digit number. They should also develop the method to be able to multiply decimal numbers with up to two decimal places.

For example, 4.92 x 3

x	4	0.9	0.02
3	12	2.7	0.06

```
    12.00
+    2.70
+    0.06
    14.76
```

Use zero as a place holder for all decimal calculations.

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers, including those with decimals, in the context of money or measures, e.g. to calculate the cost of 7 items at £8.63 each, or the total length of six pieces of ribbon of 2.28m each.

The formal method of long multiplication will be introduced to the children when they are secure with the grid method.

For example, 24 x 16 or 124 x 26

```
     2
     2 4
x    1 6
-------
   2 4 0
   1 4 4
-------
   3 8 4
-------
```

```
     1 2
     1 2 4
x      2 6
---------
     7 4 4
   2 4 8 0
---------
   3 2 2 4
---------
   1   1
```

Children will also be taught to multiply simple pairs of proper fractions, writing the answer in its simplest form e.g. $\frac{1}{4} \times \frac{1}{2}$ = 1/8

Progression Towards a Written Method for Division

In developing a written method for division, it is important that children understand the concept of division, in that it is:

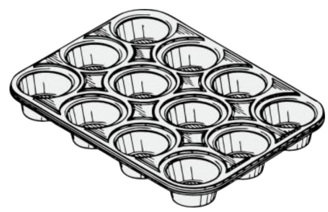
- repeated subtraction
- sharing into equal amounts

They also need to understand and work with certain principles, i.e. that it is:

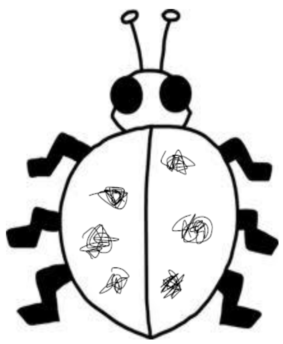
- the inverse of multiplication
- not commutative i.e. 15 ÷3 is not the same as 3 ÷ 15
- not associative i.e. 30 ÷ (5 ÷ 2) is not the same as (30 ÷ 5) ÷ 2

Reception

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. Children may also investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins which are arrays.



They may develop ways of recording calculations using pictures, etc.



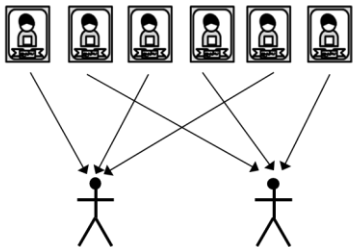
A child's jotting showing halving six spots between two sides of a ladybird.

A child's jotting showing how they shared the apples at snack time between two groups.

The children will understand that halving is sharing into two equal portions.
They should know the halves of numbers to 5. The children will also experience problem solving activities using a wide variety of practical equipment, including small world play, role play, counters, cubes etc.

Year One

In Year 1, children will continue to solve division problems using practical equipment and jottings. They should use the equipment to share objects and separate them into groups, answering questions such as 'If we share these six apples between the three of you, how many will you each have? How do you know?' or 'If six football stickers are shared between two people, how many do they each get?' They may solve both of these types of question by using a 'one for you, one for me' strategy until all of the objects have been given out.

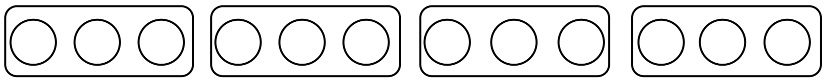


Children should be introduced to the concept of simple remainders in their calculations at this practical stage, being able to identify that the groups are not equal and should refer to the remainder as '… left over'.

Year Two

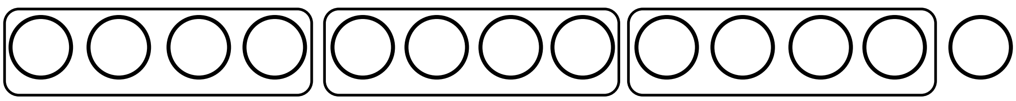
Children will be taught to recall division facts for the 2, 3, 5 and 10 multiplication tables. Children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculatione.g.

12 ÷ 3 =



Children need to understand that this calculation reads as 'How many groups of 3 are there in 12?'
They should also continue to develop their knowledge of division with remainders, e.g.

13 ÷ 4 =



13 ÷ 4 = 3 remainder 1

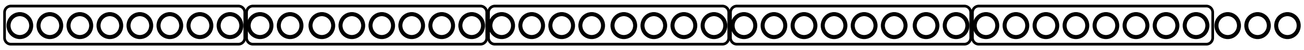
Children need to be able to make decisions about what to do with remainders after division and round up or down accordingly. In the calculation 13 ÷ 4, the answer is 3 remainder 1, but whether the answer should be rounded up to 4 or rounded down to 3 depends on the context, as in the examples below:
I have £13. Books are £4 each. How many can I buy?
Answer: 3 (the remaining £1 is not enough to buy another book)

Apples are packed into boxes of 4. There are 13 apples. How many boxes are needed?
Answer: 4 (the remaining 1 apple still needs to be placed into a box)

Year Three

Initially, children will continue to use division by grouping (including those with remainders), where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

43 ÷ 8 =



43 ÷ 8 = 5 remainder 3

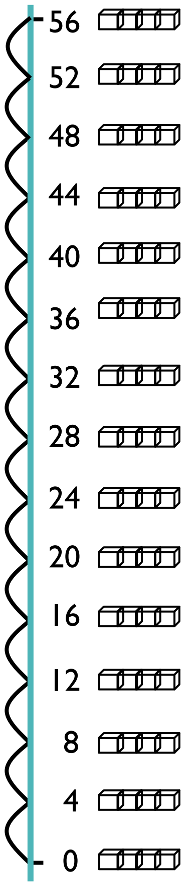
In preparation for developing the 'chunking' method of division, children should first use the repeated subtraction on a vertical number line alongside the continued use of practical equipment. There are two stages to this:

Stage 1 – repeatedly subtracting individual groups of the divisor
Stage 2 – subtracting multiples of the divisor (initially 10 groups and individual groups, then 10 groups and other multiples in line with tables knowledge)

After each group has been subtracted, children should consider how many are left to enable them to identify the amount remaining on the number line.

Stage 1

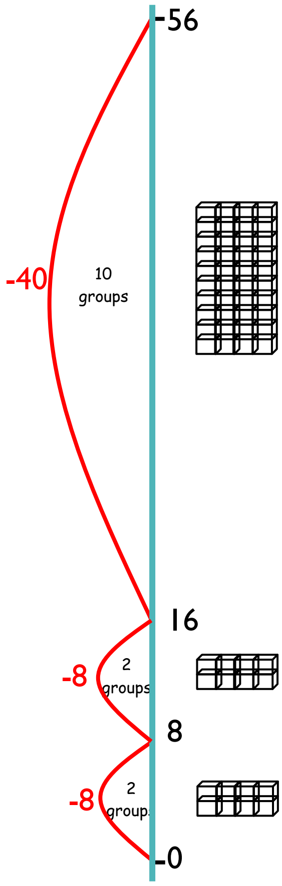
56 ÷ 4 = 14 (groups of 4)



Stage 2

56 ÷ 4 = 10(groups of 4) + 2(groups of 4) + 2(groups of 4)

= 14(groups of 4)

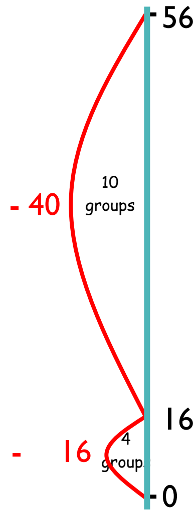


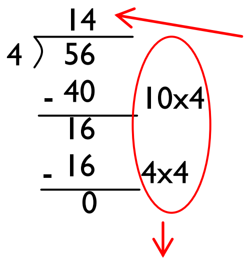
Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Year Four

Children will be taught to corresponding division facts for multiplication tables up to 12x12.
Children will continue to develop their use of grouping (repeated subtraction) to be able to subtract multiples of the divisor, moving on to the use of the 'chunking' method.

56 ÷ 4 =





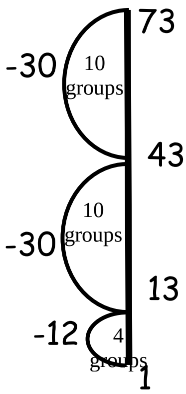
Children should write their answer above the calculation to make it easy for them and the teacher to distinguish.

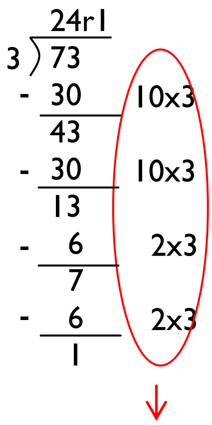
Answer: 14

The number line method used in year 3 can be linked to the chunking method to enable children to make links in their understanding.

When developing their understanding of 'chunking', children should utilise a 'key facts' box, as shown below. This enables an efficient recall of tables facts and will help them in identifying the largest group they can subtract in one chunk. Any remainders should be shown as integers, e.g.

73 ÷ 3





Answer: 24 r 1

Key facts box

1 x 3 = 3
2 x 3 = 6
3 x 3 = 9
4 x 3 = 12
5 x 3 = 15
10 x 3 = 30

By the end of year 4, children should be able to use the chunking method to divide a three digit number by a single digit number. To make this method more efficient, the key facts in the menu box should be extended to include 4x and 20x, e.g.

196 ÷ 6

```
     32r4
6 ) 196
 -  120   (20x6)
     76
 -   60   (10x6)
     16
 -   12   (2x6)
      4
```

Key facts box

- 1x6 = 6
- 2x6 =12
- 3x6 =18
- 4x6 = 24
- 5x6 = 30
- 10x6 = 60
- 20x6 =120

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Year Five

Children may continue to use the key facts box for as long as they find it useful. Using their knowledge of linked tables facts, children should be encouraged to use higher multiples of the divisor. Any remainders should be shown as integers, e.g.

523 ÷ 8

```
     65r3
8 ) 523
 -  320   (40x8)
    203
 -  160   (20x8)
     43
 -   40   (5x8)
      3
```

When children are secure with this method they are introduced to the formal method of short division. For example,

98 ÷ 7 = 14 and 432 ÷ 5 = 86 r 2

```
    1 4
7 ) 9^2 8
```

```
      8 6  r 2
5 ) 4 3^3 2
```

By the end of year 5, children should be able to use the chunking method to divide a four digit number by a single digit number. If children still need to use the key facts box, it can be extended to include 100x. For example,

2458 ÷ 7

```
     351r1
7 ) 2458
 -  2100   (300x7)
     358
 -   350   (50x7)
       8
 -     7   (1x7)
       1
```

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Year Six

To develop the chunking method further, it should be extended to include dividing a four-digit number by a two-digit number, e.g.

6367 ÷ 28 = 227 r 11

```
       227r11
  28 ) 6367
     - 5600     200x28
        767
     -  560      20x28
        207
     -  140       5x28
         67
     -   56       2x28
         11
```

Again when children are secure with this, they will use the formal method of short division. For example,

496 ÷ 11 = 45 r 1 or 45 $\frac{1}{11}$

```
       4 5  r 1
  11 ) 4 9 ⁵6
```

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

In addition, children should also be able to use the chunking method and solve calculations interpreting the remainder as a decimal up to two decimal places, e.g. 362 ÷ 17

362 ÷ 17

```
       21.29
  17 ) 362
     - 340      20x17
        22
     -  17       1x17
         5.0
     -   3.4     0.2x17
         1.60
     -   1.53    0.09x17
         0.07
```

To enable children to continue the calculation, they need to understand that 5 is the same as 5.0

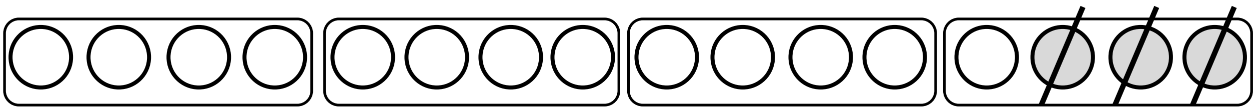
When recalling and deriving multiplication and division facts, children should also identify decimal equivalents of times tables,
e.g. if 2 x 17 = 34, I know that 0.2 x 17 = 3.4
if 9 x 17 = 153, 0.9 x 17 = 15.3
so 0.09 x 17 = 1.53

When children are secure with this they will be moved on to record their work as, for example, 432 ÷ 15

```
        2 8
15 ) 4 3 2
     3 0 0   (15 x 20)
     1 3 2
     1 2 0   (15 x 8)
       1 2
```

The answer in this example is 28 r 12. Children also learn to record their answers as $28\frac{4}{5}$ $\left(\frac{12}{15} = \frac{4}{5}\right)$ and 28.8 (using knowledge of equivalent fractions.)

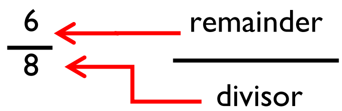
For simple fraction and decimal equivalents, this could also be demonstrated using a simple calculation such as 13 ÷ 4 to show the remainder initially as a fraction.



Using practical equipment, children can see that for 13 ÷ 4, the answer is 3 remainder 1, or put another way, there are three whole groups and a remainder of 1. This remainder is one part towards a full group of 4, so is $\frac{1}{4}$. To show the remainder as a fraction, it becomes the numerator where the denominator is the divisor (the number that you are dividing by in the calculation).

3574 ÷ 8

```
8 ) 3574
  - 3200    400x8
     374
     320     40x8
  -   54
      48      6x8
  -    6
```



So 3574 ÷ 8 is $446\frac{6}{8}$
(when the remainder is shown as a fraction)

To show the remainder as a decimal relies upon children's knowledge of decimal fraction equivalents. For decimals with no more than 2 decimal places, they should be able to identify:

Half: $\frac{1}{2}$ = 0.5
Quarters: $\frac{1}{4}$ = 0.25, $\frac{3}{4}$ = 0.75
Fifths: $\frac{1}{5}$ = 0.2, $\frac{2}{5}$ = 0.4, $\frac{3}{5}$ = 0.6, $\frac{4}{5}$ = 0.8
Tenths: $\frac{1}{10}$ = 0.1, $\frac{2}{10}$ = 0.2, $\frac{3}{10}$ = 0.3, $\frac{4}{10}$ = 0.4, $\frac{5}{10}$ = 0.5, $\frac{6}{10}$ = 0.6, $\frac{7}{10}$ = 0.7, $\frac{8}{10}$ = 0.8, $\frac{9}{10}$ = 0.9

and reduce other equivalent fractions to their lowest terms.

In the example above, 3574 ÷ 8, children should be able to identify that the remainder as a fraction of $\frac{6}{8}$ can be written as $\frac{3}{4}$ in its lowest terms. As $\frac{3}{4}$ is equivalent to 0.75, the answer can therefore be written as 446.75

Children will also be taught to divide proper fractions by whole numbers e.g. 1/3 ÷ 2 = 1/6

Updated December 2016