

Indices

1. Rules of Indices

INDICES

$$a^0 = 1$$

$$a^{-p} = \frac{1}{a^p}$$

$$a^p \times a^q = a^{p+q}$$

$$a^p \div a^q = a^{p-q}$$

$$(a^p)^q = a^{pq}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a} \quad (n \neq 0)$$

(a) Write down the value of $49^{\frac{1}{2}}$.

(b) Write down the value of 32^0 .

(c) Write down the value of $8^{-\frac{2}{3}}$.

(a) Write down the value of $49^{\frac{1}{2}}$.

Power $\frac{1}{2}$ means square root.
So the answer is 7

(b) Write down the value of 32^0 .

Anything to the power zero is 1

(c) Write down the value of $8^{-\frac{2}{3}}$.

Power $\frac{2}{3}$ means cube root and then square this.
So cube root of 8 is 2, and 2^2 is 4.
Negative power means 1 over
So the answer is $\frac{1}{4}$

Example 2

Without using a calculator, work out the following:

(a) $\left(\frac{100}{49}\right)^{\frac{1}{2}}$

(b) $\left(\frac{9}{4}\right)^{-\frac{3}{2}}$

Example 2

Without using a calculator, work out the following:

(a) $\left(\frac{100}{49}\right)^{\frac{1}{2}}$

Just square root the numerator and denominator to get $10/7$.
Note that this method does not work if there are mixed numbers.

(b) $\left(\frac{9}{4}\right)^{-\frac{3}{2}}$

Square root to get $3/2$.
Then cube the answer to get $27/8$.
Then reciprocal (because of the negative) to get $8/27$

2. Bingo

BINGO

Evaluate

$$49^{\frac{1}{2}}$$

<https://www.mathsbox.org.uk/topic/t/number/n4.php>

Fractional and Negative Indices			
Name	$\frac{1}{7}$	216	$\frac{1}{2}$
15	8	10	$\frac{1}{27}$
6	10	9	5
20	5	$\frac{1}{3}$	64
07			

We've had to miss this out for
our remote learning task!

3. Match them up!

$8^{\frac{2}{3}}$	$(-1)^{\frac{1}{3}}$	$9^{\frac{3}{2}}$	$16^{\frac{1}{4}}$
$\left(\frac{1}{2}\right)^{-3}$	$\left(\frac{2}{5}\right)^{-2}$	$\left(\frac{1}{9}\right)^{-\frac{1}{2}}$	$36^{-\frac{1}{2}}$
3^{-2}	$\left(\frac{16}{25}\right)^{\frac{3}{2}}$	3^{-1}	$4^{-\frac{3}{2}}$
$8^{-\frac{1}{3}}$	$(-1)^{-2}$	$\left(\frac{1}{5}\right)^{-1}$	$\left(\frac{4}{9}\right)^{\frac{1}{2}}$

$\frac{2}{3}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{64}{125}$
$\frac{1}{3}$	4	1	$\frac{1}{2}$
8	-1	$\frac{1}{9}$	3
5	$\frac{25}{4}$	27	2

$8^{\frac{2}{3}}$ A	$(-1)^{\frac{1}{3}}$ B	$9^{\frac{3}{2}}$ C	$16^{\frac{1}{4}}$ D
$\left(\frac{1}{2}\right)^{-3}$ A	$\left(\frac{2}{5}\right)^{-2}$ B	$\left(\frac{1}{9}\right)^{-\frac{1}{2}}$ C	$36^{-\frac{1}{2}}$ D
3^{-2} A	$\left(\frac{16}{25}\right)^{\frac{3}{2}}$ B	3^{-1} C	$4^{-\frac{3}{2}}$ D
$8^{-\frac{1}{3}}$ A	$(-1)^{-2}$ B	$\left(\frac{1}{5}\right)^{-1}$ C	$\left(\frac{4}{9}\right)^{\frac{1}{2}}$ D

$\frac{2}{3}$ D	$\frac{1}{8}$ D	$\frac{1}{6}$ D	$\frac{64}{125}$ B
$\frac{1}{3}$ C	4 A	1 B	$\frac{1}{2}$ A
8 A	-1 B	$\frac{1}{9}$ A	3 C
5 C	$\frac{25}{4}$ B	27 C	2 D

Write the following in index form (as a power)

(a) $\sqrt[4]{x}$

(b) $\frac{2}{x}$

(c) $\frac{1}{2x}$

(d) $\frac{5}{2\sqrt{x}}$

$$x^{-3}$$

$$x^{\frac{3}{2}}$$

$$2x^{-2}$$

$$\frac{1}{2}x^{-\frac{1}{2}}$$

$$\frac{1}{x}$$

$$\frac{1}{2x^2}$$

$$\frac{2}{\sqrt{x}}$$

$$\frac{\sqrt{x}}{x}$$

First column				Second column			
1	\sqrt{x}	11	$\frac{1}{x^2} \times \frac{1}{x^3}$	A	$x^{\frac{2}{3}}$	K	$x^{\frac{2}{3}}$
2	$\frac{1}{x}$	12	$(\sqrt[3]{x})^2$	B	$\frac{1}{2}x^{-3}$	L	$x^{\frac{2}{3}}$
3	$\frac{1}{\sqrt{x}}$	13	$\sqrt{x^7}$	C	$\frac{1}{2}x^{-1}$	M	x^{-2}
4	$\sqrt[3]{x}$	14	$\sqrt{\left(\frac{1}{x^8}\right)}$	D	x^{-4}	N	$x^{\frac{2}{3}}$
5	$\frac{1}{x^2}$	15	$\frac{1}{\sqrt{x^{-8}}}$	E	x^{-1}	O	$2x^{-1}$
6	$\frac{1}{2x^3}$	16	$x \times \sqrt{x \times x^6}$	F	$\frac{1}{2}x$	P	$x^{\frac{2}{3}}$
7	$\frac{2}{x^3}$	17	$\sqrt{\left(\frac{4}{x^2}\right)}$	G	x^4	Q	x^{-5}
8	$x\sqrt{x}$	18	$\sqrt{\left(\frac{1}{4x^2}\right)}$	H	x^2	R	$x^{\frac{2}{3}}$
9	$\frac{\sqrt{x}}{x^2}$	19	$\sqrt{\left(\frac{x^2}{4}\right)}$	I	$2x$	S	$2x^{-3}$
10	$\frac{1}{x^{-2}}$	20	$\sqrt{4x^2}$	J	$x^{\frac{1}{2}}$	T	$x^{\frac{2}{3}}$

Solutions

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
P	E	K	J	M	B	S	N	A	H	Q	R	T	D	G	L	O	C	F	I

4. Simplify Expressions

Simplify

a $2p^2 \times 4p^5$

b $x^2 \times x^3 \times x^5$

c $12n^7 \div 2n^2$

d $(y^3)^4$

e $(2b)^3 \div 4b^2$

f $p^3q \times pq^2$

g $x^4y^3 \div xy^2$

h $2r^2s \times 3s^2$

i $6x^5y^8 \div 3x^2y$

j $6a^4b^5 \times \frac{2}{3}ab^3$

k $(5rs^2)^3 \div (10rs)^2$

l $3p^4q^3 \div \frac{1}{5}pq^2$

Simplify

a $2p^2 \times 4p^5$

b $x^2 \times x^3 \times x^5$

c $12n^7 \div 2n^2$

d $(y^3)^4$

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j $6a^4b^5 \times \frac{2}{3}ab^3$

k $(5rs^2)^3 \div (10rs)^2$

l $3p^4q^3 \div \frac{1}{5}pq^2$

a $= 8p^7$

b $= x^{10}$

c $= 6n^5$

d $= y^{12}$

e $= 2b$

f $= p^4q^3$

g $= x^3y$

h $= 6r^2s^3$

i $= 2x^3y^7$

j $= 4a^5b^8$

k $= 125r^3s^6 \div 100r^2s^2$
 $= \frac{5}{4}rs^4$

l $= 15p^3q$

Simplify

a $x^8 \times x^{-6}$

b $y^{-2} \times y^{-4}$

c $6p^3 \div 2p^7$

d $(2x^{-4})^3$

e $y^3 \times y^{-\frac{1}{2}}$

f $2b^{\frac{2}{3}} \times 4b^{\frac{1}{4}}$

g $x^{\frac{3}{5}} \div x^{\frac{1}{3}}$

h $a^{\frac{1}{2}} \div a^{\frac{4}{3}}$

i $p^{\frac{1}{4}} \div p^{-\frac{1}{5}}$

j $(3x^{\frac{2}{5}})^2$

k $y \times y^{\frac{5}{6}} \times y^{-\frac{3}{2}}$

l $4t^{\frac{3}{2}} \div 12t^{\frac{1}{2}}$

m $\frac{b^2 \times b^{\frac{1}{4}}}{b^{\frac{1}{2}}}$

n $\frac{y^{\frac{1}{2}} \times y^{\frac{1}{3}}}{y}$

o $\frac{4x^{\frac{2}{3}} \times 3x^{-\frac{1}{6}}}{6x^{\frac{3}{4}}}$

p $\frac{2a \times a^{\frac{3}{4}}}{8a^{-\frac{1}{2}}}$

Simplify

a $x^8 \times x^{-6}$

b $y^{-2} \times y^{-4}$

c $6p^3 \div 2p^7$

d $(2x^{-4})^3$

e $y^3 \times y^{-\frac{1}{2}}$

f $2b^{\frac{2}{3}} \times 4b^{\frac{1}{4}}$

g $x^{\frac{3}{5}} \div x^{\frac{1}{3}}$

h $a^{\frac{1}{2}} \div a^{\frac{4}{3}}$

i $p^{\frac{1}{4}} \div p^{-\frac{1}{5}}$

j $(3x^{\frac{2}{3}})^2$

k $y \times y^{\frac{5}{6}} \times y^{-\frac{3}{2}}$

l $4t^{\frac{3}{2}} \div 12t^{\frac{1}{2}}$

m $\frac{b^2 \times b^{\frac{1}{4}}}{b^{\frac{1}{2}}}$

n $\frac{y^{\frac{1}{2}} \times y^{\frac{1}{3}}}{y}$

o $\frac{4x^{\frac{2}{3}} \times 3x^{-\frac{1}{6}}}{6x^{\frac{1}{4}}}$

p $\frac{2a \times a^{\frac{3}{4}}}{8a^{-\frac{1}{2}}}$

a $= x^2$

b $= y^{-6}$

c $= 3p^{-4}$

e $= y^{\frac{5}{2}}$

f $= 8b^{\frac{2}{3} + \frac{1}{4}} = 8b^{\frac{11}{12}}$

g $= x^{\frac{3}{5} - \frac{1}{3}} = x^{\frac{4}{15}}$

i $= p^{\frac{1}{4} - (-\frac{1}{5})} = p^{\frac{9}{20}}$

j $= 9x^{\frac{4}{5}}$

k $= y^{1 + \frac{5}{6} - \frac{3}{2}} = y^{\frac{1}{3}}$

m $= b^{2 + \frac{1}{4} - \frac{1}{2}} = b^{\frac{7}{4}}$

n $= y^{\frac{1}{2} + \frac{1}{3} - 1} = y^{-\frac{1}{6}}$

o $= 2x^{\frac{2}{3} + (-\frac{1}{6}) - \frac{3}{4}} = 2x^{-\frac{1}{4}}$

Expand and simplify

a $x(x^2 - x^{-1})$

b $2x^3(x^{-1} + 3)$

c $x^{-1}(3x - x^3)$

e $\frac{1}{2}x^2(6x + 4x^{-1})$

f $3x^{\frac{1}{2}}(x^{-\frac{1}{2}} - x^{\frac{3}{2}})$

g $x^{-\frac{3}{2}}(5x^2 + x^{\frac{7}{2}})$

i $(x^2 + 1)(x^4 - 3)$

j $(2x^5 + x)(x^4 + 3)$

k $(x^2 - 2x^{-1})(x - x^{-2})$

Expand and simplify

a $x(x^2 - x^{-1})$

b $2x^3(x^{-1} + 3)$

c $x^{-1}(3x - x^3)$

e $\frac{1}{2}x^2(6x + 4x^{-1})$

f $3x^{\frac{1}{2}}(x^{-\frac{1}{2}} - x^{\frac{3}{2}})$

g $x^{-\frac{3}{2}}(5x^2 + x^{\frac{7}{2}})$

i $(x^2 + 1)(x^4 - 3)$

j $(2x^5 + x)(x^4 + 3)$

k $(x^2 - 2x^{-1})(x - x^{-2})$

a $= x^3 - 1$

b $= 2x^2 + 6x^3$

c $= 3 - x^2$

e $= 3x^3 + 2x$

f $= 3 - 3x^2$

g $= 5x^{\frac{1}{2}} + x^2$

i $= x^6 + x^4 - 3x^2 - 3$

j $= 2x^9 + 6x^5 + x^5 + 3x$

k $= x^3 - 1 - 2 + 2x^{-3}$

$= 2x^9 + 7x^5 + 3x$

$= x^3 - 3 + 2x^{-3}$

Simplify

a $\frac{x^3 + 2x}{x}$

b $\frac{4t^5 - 6t^3}{2t^2}$

e $\frac{p + p^{\frac{3}{2}}}{p^{\frac{3}{4}}}$

f $\frac{8w - 2w^{\frac{1}{2}}}{4w^{-\frac{1}{2}}}$

Simplify

a $\frac{x^3 + 2x}{x}$

b $\frac{4t^5 - 6t^3}{2t^2}$

e $\frac{p + p^{\frac{3}{2}}}{p^{\frac{3}{4}}}$

f $\frac{8w - 2w^{\frac{1}{2}}}{4w^{-\frac{1}{2}}}$

a $= x^2 + 2$

b $= 2t^3 - 3t$

e $= p^{\frac{1}{4}} + p^{\frac{3}{4}}$

f $= 2w^{\frac{3}{2}} - \frac{1}{2}w$

5. Solving Equations

Find the value of x such that

a $2^x = 64$

b $5^{x-1} = 125$

c $3^{x+4} - 27 = 0$

d $8^x - 2 = 0$

e $3^{2x-1} = 9$

f $16 - 4^{3x-2} = 0$

g $9^{x-2} = 27$

h $8^{2x+1} = 16$

Find the value of x such that

a $2^x = 64$

b $5^{x-1} = 125$

c $3^{x+4} - 27 = 0$

d $8^x - 2 = 0$

e $3^{2x-1} = 9$

f $16 - 4^{3x-2} = 0$

g $9^{x-2} = 27$

h $8^{2x+1} = 16$

a $2^x = 2^6$
 $x = 6$

b $5^{x-1} = 5^3$
 $x - 1 = 3$
 $x = 4$

c $3^{x+4} = 27 = 3^3$
 $x + 4 = 3$
 $x = -1$

d $(2^3)^x = 2^{3x} = 2$
 $3x = 1$
 $x = \frac{1}{3}$

e $3^{2x-1} = 3^2$
 $2x - 1 = 2$
 $x = \frac{3}{2}$

f $16 = 4^2 = 4^{3x-2}$
 $2 = 3x - 2$
 $x = \frac{4}{3}$

g $(3^2)^{x-2} = 3^{2x-4} = 3^3$
 $2x - 4 = 3$
 $x = \frac{7}{2}$

h $(2^3)^{2x+1} = 2^{6x+3} = 2^4$
 $6x + 3 = 4$
 $x = \frac{1}{6}$

Solve each equation.

a $2^{x+3} = 4^x$

b $5^{3x} = 25^{x+1}$

c $9^{2x} = 3^{x-3}$

d $16^x = 4^{1-x}$

e $4^{x+2} = 8^x$

f $27^{2x} = 9^{3-x}$

g $6^{3x-1} = 36^{x+2}$

h $8^x = 16^{2x-1}$

Solve each equation.

a $2^{x+3} = 4^x$

b $5^{3x} = 25^{x+1}$

c $9^{2x} = 3^{x-3}$

d $16^x = 4^{1-x}$

e $4^{x+2} = 8^x$

f $27^{2x} = 9^{3-x}$

g $6^{3x-1} = 36^{x+2}$

h $8^x = 16^{2x-1}$

a $2^{x+3} = (2^2)^x = 2^{2x}$
 $x + 3 = 2x$
 $x = 3$

b $5^{3x} = (5^2)^{x+1} = 5^{2x+2}$
 $3x = 2x + 2$
 $x = 2$

c $(3^2)^{2x} = 3^{4x} = 3^{x-3}$
 $4x = x - 3$
 $x = -1$

d $(4^2)^x = 4^{2x} = 4^{1-x}$
 $2x = 1 - x$
 $x = \frac{1}{3}$

e $(2^2)^{x+2} = (2^3)^x$
 $2^{2x+4} = 2^{3x}$
 $2x + 4 = 3x$
 $x = 4$

f $(3^3)^{2x} = (3^2)^{3-x}$
 $3^{6x} = 3^{6-2x}$
 $6x = 6 - 2x$
 $x = \frac{3}{4}$

g $6^{3x-1} = (6^2)^{x+2}$
 $6^{3x-1} = 6^{2x+4}$
 $3x - 1 = 2x + 4$
 $x = 5$

h $(2^3)^x = (2^4)^{2x-1}$
 $2^{3x} = 2^{8x-4}$
 $3x = 8x - 4$
 $x = \frac{4}{5}$

6. Challenges!

Challenge 1

Simplify the following

(a) $2 \times \frac{x}{y}$

(b) $\frac{3}{x} + \frac{2}{x}$

(c) $\frac{1}{5x} + \frac{1}{x}$

(d) $\frac{1}{x^2} \times \sqrt{x} \times 3x^4 \div 2\sqrt[3]{x}$

Challenge 2

Simplify the following:

(a) $\frac{x^2 \times (x^3)^5}{x^9}$

(b) $\frac{\sqrt[3]{zx^2y}}{z^2\sqrt{x}} \div \frac{x^0z^3}{(y^2)^3}$

7. Task for September

See 'Homework Questions for September' word/pdf document

4m, 7a, 12d, 6a, 16a, c4

4 Simplify

a $\sqrt{12}$

g $\sqrt{45}$

m $\sqrt{216}$

7 Simplify

a $(\sqrt{5} + 1)(2\sqrt{5} + 3)$

d $(3\sqrt{2} - 1)(2\sqrt{2} + 5)$

12 Express each of the following as simply as possible with a rational denominator.

a $\frac{1}{\sqrt{2}+1}$

b $\frac{4}{\sqrt{3}-1}$

c $\frac{1}{\sqrt{6}-2}$

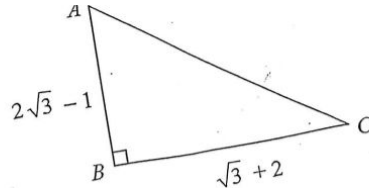
d $\frac{3}{2+\sqrt{3}}$

e $\frac{1}{2+\sqrt{5}}$

f $\frac{\sqrt{2}}{\sqrt{5}-2}$

a 6

16



In triangle ABC , $AB = 2\sqrt{3} - 1$, $BC = \sqrt{3} + 2$ and $\angle ABC = 90^\circ$.

a Find the exact area of triangle ABC in its simplest form.

b Show that $AC = 2\sqrt{5}$.

$\therefore \angle ACB = 5\sqrt{3} - 8$.

6 Showing your method clearly,
 a express $\sqrt{37.5}$ in the form $a\sqrt{6}$,
 b express $\sqrt{9\frac{3}{5}} - \sqrt{6\frac{2}{3}}$ in the form $b\sqrt{15}$.

C4 Express

$3^7 + 9^4 + 15 \times 27^2$ as a power of 3