Year 11 → 12 transition

Quadratics

Starter

What does a quadratic look like?

How many solutions does it have?

At GCSE you learnt 3 techniques for solving quadratics, what were they?

HINT: a) _ A _ _ O _ I _ I _ _

                 b) _UA_ _A _ I _ _O_ _U_A

                 c) _O_ _ _ E_ I _ _ _ _ E _ _ UA _ E

Starter

Factorise

a $x^2 + 4x + 3$ **b** $x^2 + 7x + 10$ **c** $y^2 - 3y + 2$ **d** $x^2 - 6x + 9$

Factorise

a $2x^2 + 3x + 1$ **b** $2 + 7p + 3p^2$ **c** $2y^2 - 5y + 3$ **d** $2 - m - m^2$

Answers

Factorise

a $x^2 + 4x + 3$ **b** $x^2 + 7x + 10$ **c** $y^2 - 3y + 2$ **d** $x^2 - 6x + 9$

(x+1)(x+3)   **(x+2)(x+5)**   **(y-1)(y-2)**   **(x-3)(x-3)**

Factorise

a $2x^2 + 3x + 1$ **b** $2 + 7p + 3p^2$ **c** $2y^2 - 5y + 3$ **d** $2 - m - m^2$

(2x+1)(x+1)   **(3p+1)(p+2)**   **(2y-3)(y-1)**   **(m+2)(1-m)**

Example - Solving equations using factorisation

a $x^2 - 4x + 3 = 0$

Solving equations using factorisation

Using factorisation, solve each equation.

a $x^2 - 4x + 3 = 0$ **b** $x^2 + 6x + 8 = 0$ **c** $x^2 + 4x - 5 = 0$ **d** $x^2 - 7x = 8$

e $x^2 - 25 = 0$ **f** $x(x - 1) = 42$ **g** $x^2 = 3x$ **h** $27 + 12x + x^2 = 0$

Answers

a) $x^2 - 4x + 3 = 0$ $x \to 3$ $+ \to -4$

$(x - 1)(x - 3) = 0$

$x = 1$ or $x = 3$

b) $x^2 + 6x + 8 = 0$ $x \to 8$ $+ \to 6$

$(x + 2)(x + 4) = 0$

$x = -2$ or $x = -4$

c) $x^2 + 4x - 5 = 0$ $x \to -5$ $+ \to 4$

$(x + 5)(x - 1) = 0$

$x = 1$ or $x = -5$

d) $x^2 - 7x = 8$

$x^2 - 7x - 8 = 0$ $x \to -8$ $+ \to -7$

$(x - 8)(x + 1) = 0$

$x = 8$ or $x = -1$

e) $x^2 - 25 = 0$

$(x - 5)(x + 5) = 0$

$x = 5$ or $x = -5$

Answers

f) $x(x-1) = 42$

$x^2 - x - 42 = 0$ $x \to -42$
                      $+ \to -1$

$(x-7)(x+6) = 0$

$x = 7$ or $x = -6$

g) $x^2 = 3x$

$\Rightarrow x^2 - 3x = 0$

$\Rightarrow x(x-3) = 0$

$x = 0$ or $x = 3$

h) $27 + 12x + x^2 = 0$ $x \to 27$
                           $+ \to 12$

$(x+3)(x+9) = 0$

$x = -3$ or $x = -9$

Challenge : Solve these tricky quadratics

i $60 - 4x - x^2 = 0$

s $(2x + 5)^2 = 5 - x$

e $\dfrac{5}{x^2} + \dfrac{4}{x} - 1 = 0$

Answers

TRICKY

i) $60 - 4x - x^2 = 0$

$x^2 + 4x - 60 = 0$ $\times \rightarrow -60$
   $+ \rightarrow 4$

$(x + 10)(x - 6) = 0$

$x = 6$ or $x = -10$

s) $(2x + 5)^2 = 5 - x$

$(2x + 5)(2x + 5) = 5 - x$

$4x^2 + 20x + 25 = 5 - x$

$4x^2 + 21x + 20 = 0$ $\times \rightarrow 80$
   $+ \rightarrow 21$

$4x^2 + 5x + 16x + 20 = 0$

$x(4x + 5) + 4(4x + 5) = 0$

$(x + 4)(4x + 5) = 0$

$x = -4$ or $x = -\frac{5}{4} = -1.25$

e) $\frac{5}{x^2} + \frac{4}{x} - 1 = 0$ $\times x^2$

$5 + 4x - x^2 = 0$

$x^2 - 4x - 5 = 0$

$(x - 5)(x + 1) = 0$

$x = 5$ or $x = -1$

Simultaneous equations

Solve each pair of simultaneous equations.

a $y = 3x$

$y = 2x + 1$

b $y = x - 6$

$y = \frac{1}{2}x - 4$

c $y = 2x + 6$

$y = 3 - 4x$

Answers

b) $y = x - 6$

$y = \frac{1}{2}x - 4$

$x - 6 = \frac{1}{2}x - 4$

$\frac{1}{2}x - 2 = 0$

$\frac{1}{2}x = 2$

$x = 4, \quad y = -2$

c) $y = 2x + 6$

$y = 3 - 4x$

$2x + 6 = 3 - 4x$

$6x + 3 = 0$

$6x = -3$

$x = -\frac{1}{2}, \quad y = 5$

Solving simultaneous equations with quadratics

$$x^2 - y + 3 = 0$$

$$x - y + 5 = 0$$

Solving simultaneous equations with quadratics

Solve each pair of simultaneous equations.

a $x^2 - y + 3 = 0$

$x - y + 5 = 0$

b $2x^2 - y - 8x = 0$

$x + y + 3 = 0$

c $x^2 + y^2 = 25$

$2x - y = 5$

d $x^2 + 2xy + 15 = 0$

$2x - y + 10 = 0$

e $x^2 - 2xy - y^2 = 7$

$x + y = 1$

f $3x^2 - x - y^2 = 0$

$x + y - 1 = 0$

g $2x^2 + xy + y^2 = 22$

$x + y = 4$

h $x^2 - 4y - y^2 = 0$

$x - 2y = 0$

i $x^2 + xy = 4$

$3x + 2y = 6$

Answers

b) $2x^2 - y - 8x = 0$

$x + y + 3 = 0 \Rightarrow y = -x - 3$

$2x^2 - (-x - 3) - 8x = 0$

$\Rightarrow 2x^2 - 7x + 3 = 0 \qquad \begin{array}{l} \times \to 6 \\ + \to -7 \end{array}$

$2x^2 - 6x \mid -x + 3 = 0$

$2x(x - 3) \mid -1(x - 3) = 0$

$\Rightarrow (2x - 1)(x - 3) = 0$

$\Rightarrow$ if $x = 3$, $y = -6$

if $x = \frac{1}{2}$, $y = -\frac{7}{2}$

c) $x^2 + y^2 = 25$

$2x - y = 5 \Rightarrow y = 2x - 5$

$x^2 + (2x - 5)(2x - 5) - 25 = 0$

$x^2 + 4x^2 - 10x - 10x + 25 - 25 = 0$

$5x^2 - 20x = 0$

$5x(x - 4) = 0$

if $x = 0$, $y = -5$

if $x = 4$, $y = 3$

Answers

d) $x^2 + 2xy + 15 = 0$

$2x - y + 10 = 0 \implies y = 2x + 10$

$x^2 + 2x(2x + 10) + 15 = 0$

$x^2 + 4x^2 + 20x + 15 = 0$

$5x^2 + 20x + 15 = 0 \quad \div 5$

$x^2 + 4x + 3 = 0$

$(x + 3)(x + 1) = 0$

$x = -1 \quad \text{or} \quad x = -3$

$y = 8 \quad \text{or} \quad y = 4$

e) $x^2 - 2xy - y^2 = 7$

$x + y = 1 \implies y = 1 - x$

$x^2 - 2x(1 - x) - (1 - x)(1 - x) - 7 = 0$

$x^2 - 2x + 2x^2 - (1 - 2x + x^2) - 7 = 0$

$x^2 - 2x + 2x^2 - 1 + 2x - x^2 - 7 = 0$

$2x^2 - 8 = 0$

$2x^2 = 8$

$x^2 = 4$

$x = -2 \quad \text{or} \quad x = 2$

$y = 3 \quad \text{or} \quad y = -1$

Answers

f) $3x^2 - x - y^2 = 0$

$x + y - 1 = 0 \Rightarrow y = 1 - x$

$3x^2 - x - (1-x)(1-x) = 0$

$3x^2 - x - (1 - 2x + x^2) = 0$

$3x^2 - x - 1 + 2x - x^2 = 0$

$2x^2 + x - 1 = 0 \quad \begin{matrix} x \to -2 \\ + \to 1 \end{matrix}$

$2x^2 + 2x \mid - x - 1 = 0$

$2x(x+1) \mid -1(x+1) = 0$

$(2x-1)(x+1) = 0$

if $x = -1$ then $y = 2$

if $x = \frac{1}{2}$ then $y = \frac{1}{2}$

Extension 1

Solve each of the following equations.

a $x - 5 + \dfrac{4}{x} = 0$　　　　**b** $x - \dfrac{10}{x} = 3$　　　　**c** $2x^3 - x^2 - 3x = 0$

e $\dfrac{5}{x^2} + \dfrac{4}{x} - 1 = 0$　　　**f** $\dfrac{x-6}{x-4} = x$　　　　**g** $x + 5 = \dfrac{3}{x+3}$

Answers

a) x=4 or x=1
b) x=-2 or x=5
c) x=0 or x=3/2 or x=-1
e) x=-1 or x=5
f) x=2 or x=3
g) x=-2 or x=-6

Extension 2

Solve each pair of simultaneous equations.

a $x - \dfrac{1}{y} - 4y = 0$

   $x - 6y - 1 = 0$

b $xy = 6$

   $x - y = 5$

c $\dfrac{3}{x} - 2y + 4 = 0$

   $4x + y - 7 = 0$

Extension 2

Solve each pair of simultaneous equations.

a $x - \dfrac{1}{y} - 4y = 0$

    $x - 6y - 1 = 0$

b $xy = 6$

    $x - y = 5$

c $\dfrac{3}{x} - 2y + 4 = 0$

    $4x + y - 7 = 0$

x=-5, y=-1
x=4, y=1/2

x=-1, y=-6
x=6, y=1

x=1/2, y=5
x=3/4, y=4