Year 11 Transition Lesson 1

Surds

Need to know facts: Surds

SURDS

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{a} \times \sqrt{a} = a$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

INDICES

$$\sqrt{a} = a^{1/2}$$

$$\frac{1}{a} = a^{-1}$$

RATIONALISING THE DENOMINATOR

$$\frac{1}{\sqrt{a}} \quad \text{multiply by.....} \quad \frac{\sqrt{a}}{\sqrt{a}}$$

$$\frac{1}{b + \sqrt{a}} \quad \text{multiply by.....} \quad \frac{b - \sqrt{a}}{b - \sqrt{a}}$$

Examples (if needed)

1) Simplify: $\sqrt{48}$
2) Simplify: $\sqrt{48} + \sqrt{27}$
3) Simplify: $(\sqrt{5} + 2)(3\sqrt{5} - 1)$

4) Rationalise the denominator of:

a) $\dfrac{3}{\sqrt{5}}$ b) $\dfrac{4}{2 - \sqrt{5}}$ c) $\dfrac{4 + \sqrt{5}}{2 - \sqrt{5}}$

Further Examples (if needed)

5) Write $2\sqrt{2}$ as a power of 2

6) Simplify a) $\sqrt{13.5}$ b) $\sqrt{0.25}$

7) Find the area of a right angle triangle with base $\sqrt{3} + 2$ and height $\sqrt{5} - 1$

SURDS EXAMPLES

① $\sqrt{48} = \sqrt{16 \times 3}$
$= \sqrt{16} \times \sqrt{3}$
$= 4\sqrt{3}$

$\sqrt{4 \times 12}$
$= \sqrt{4} \times \sqrt{12}$
$= 2\sqrt{12}$
$= 2\sqrt{4 \times 3}$
$= 2\sqrt{4} \times \sqrt{3}$
$= 2 \times 2 \times \sqrt{3}$
$= 4\sqrt{3}$

② $\sqrt{48} + \sqrt{27}$
$= 4\sqrt{3} + \sqrt{9 \times 3}$
$= 4\sqrt{3} + \sqrt{9} \times \sqrt{3}$
$= 4\sqrt{3} + 3\sqrt{3}$
$= 7\sqrt{3}$

③ $(\sqrt{5} + 2)(3\sqrt{5} - 1)$
$= \sqrt{5} \times 3\sqrt{5} - 2 + 2 \times 3\sqrt{5} - \sqrt{5}$
$= \sqrt{5} \times \sqrt{5} \times 3 - 2 + 6\sqrt{5} - \sqrt{5}$
$= 5 \times 3 - 2 + 5\sqrt{5}$
$= 13 + 5\sqrt{5}$

1
4
9
16
25
36
49
64
81
100
121
144

(4) RATIONALISE THE DENOMINATOR

a) $\frac{3}{\sqrt{5}} \times \left(\frac{\sqrt{5}}{\sqrt{5}}\right)$ ("1") $= \frac{3\sqrt{5}}{\sqrt{5}\times\sqrt{5}} = \frac{3\sqrt{5}}{5}$

b) $\frac{4}{2-\sqrt{5}} \times \left(\frac{2+\sqrt{5}}{2+\sqrt{5}}\right) = \frac{4(2+\sqrt{5})}{(2-\sqrt{5})(2+\sqrt{5})} = \frac{8+4\sqrt{5}}{4-\sqrt{5}\times\sqrt{5}-2\sqrt{5}+2\sqrt{5}}$

$= \frac{8+4\sqrt{5}}{4-5-2\sqrt{5}+2\sqrt{5}} = \frac{8+4\sqrt{5}}{-1} = -8-4\sqrt{5}$

c) $\frac{4+\sqrt{5}}{2-\sqrt{5}} \times \left(\frac{2+\sqrt{5}}{2+\sqrt{5}}\right)$ (1) $= \frac{(4+\sqrt{5})(2+\sqrt{5})}{-1}$

$= \frac{8+\sqrt{5}\times\sqrt{5}+2\sqrt{5}+4\sqrt{5}}{-1} = \frac{8+5+6\sqrt{5}}{-1}$

$= \frac{13+6\sqrt{5}}{-1} = -13-6\sqrt{5}$

FURTHER EXAMPLES

(5) $2\sqrt{2}$ as a power of 2

$2\sqrt{2} = 2^{1} \times 2^{1/2}$

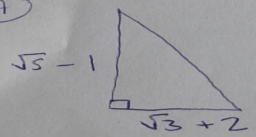
$= 2^{1+1/2} = 2^{3/2}$

(6) a) Simplify $\sqrt{13.5} = \sqrt{\frac{27}{2}} = \frac{\sqrt{27}}{\sqrt{2}} = \frac{\sqrt{9\times 3}}{\sqrt{2}}$

$= \frac{\sqrt{9}\times\sqrt{3}}{\sqrt{2}} = \frac{3\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{3}\times\sqrt{2}}{2} = \frac{3\sqrt{6}}{2}$

b) Simplify $\sqrt{0.25} = \sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \pm\frac{1}{2}$

(7)



Area of right triangle = base x height ÷ 2

$\frac{(\sqrt{5}-1)(\sqrt{3}+2)}{2} = \frac{\sqrt{5}\times\sqrt{3} - 2 - \sqrt{3} + 2\sqrt{5}}{2}$

$= \frac{\sqrt{15} - 2 - \sqrt{3} + 2\sqrt{5}}{2}$ cm²

STRIDE TRANSITION WORK!

4 Simplify

a $\sqrt{12}$ b $\sqrt{28}$ c $\sqrt{80}$ d $\sqrt{27}$ e $\sqrt{24}$ f $\sqrt{128}$

g $\sqrt{45}$ h $\sqrt{40}$ i $\sqrt{75}$ j $\sqrt{112}$ k $\sqrt{99}$ l $\sqrt{147}$

m $\sqrt{216}$ n $\sqrt{800}$ o $\sqrt{180}$ p $\sqrt{60}$ q $\sqrt{363}$ r $\sqrt{208}$

5 Simplify

a $\sqrt{18}+\sqrt{50}$ b $\sqrt{48}-\sqrt{27}$ c $2\sqrt{8}+\sqrt{72}$

d $\sqrt{360}-2\sqrt{40}$ e $2\sqrt{5}-\sqrt{45}+3\sqrt{20}$ f $\sqrt{24}+\sqrt{150}-2\sqrt{96}$

6 Showing your method clearly,

a express $\sqrt{375}$ in the form $a\sqrt{6}$,

b express $\sqrt{\frac{2}{3}}-\sqrt{\frac{6}{4}}$ in the form $b\sqrt{15}$.

7 Simplify

a $(\sqrt{5}+1)(2\sqrt{5}+3)$ b $(1-\sqrt{2})(4\sqrt{2}-3)$ c $(2\sqrt{7}+3)^2$

d $(3\sqrt{2}-1)(2\sqrt{2}+5)$ e $(\sqrt{5}-\sqrt{2})(\sqrt{5}+2\sqrt{2})$ f $(3-\sqrt{8})(4+\sqrt{2})$

8 Express each of the following as simply as possible with a rational denominator.

a $\frac{1}{\sqrt{5}}$ b $\frac{2}{\sqrt{3}}$ c $\frac{1}{\sqrt{8}}$ d $\frac{14}{\sqrt{7}}$ e $\frac{3\sqrt{2}}{\sqrt{3}}$ f $\frac{\sqrt{5}}{\sqrt{15}}$

g $\frac{1}{3\sqrt{7}}$ h $\frac{12}{\sqrt{72}}$ i $\frac{1}{\sqrt{80}}$ j $\frac{3}{2\sqrt{54}}$ k $\frac{4\sqrt{20}}{3\sqrt{18}}$ l $\frac{3\sqrt{175}}{2\sqrt{27}}$

11 a Simplify $(2-\sqrt{3})(2+\sqrt{3})$.

b Express $\frac{2}{2-\sqrt{3}}$ in the form $a+b\sqrt{3}$.

12 Express each of the following as simply as possible with a rational denominator.

a $\frac{1}{\sqrt{2}+1}$ b $\frac{4}{\sqrt{3}-1}$ c $\frac{1}{\sqrt{6}+2}$ d $\frac{3}{2+\sqrt{3}}$

e $\frac{1}{2+\sqrt{5}}$ f $\frac{\sqrt{2}}{\sqrt{2}-1}$ g $\frac{6}{\sqrt{7}+3}$ h $\frac{1}{3+2\sqrt{2}}$

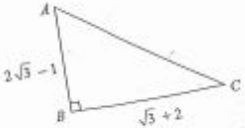
14 Express each of the following in the form $a+b\sqrt{5}$, where a and b are integers.

a $\sqrt{20}(\sqrt{5}-3)$

b $(1-\sqrt{5})(3+2\sqrt{5})$

c $\frac{1+\sqrt{5}}{\sqrt{5}-2}$

16



In triangle ABC, $AB=2\sqrt{3}-1$, $BC=\sqrt{3}+2$ and $\angle ABC=90°$.

a Find the exact area of triangle ABC in its simplest form.

b Show that $AC=2\sqrt{5}$.

c Show that $\tan(\angle ACB)=5\sqrt{3}-8$.

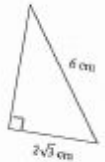


Diagram NOT accurately drawn

The diagram shows a right-angled triangle.
The length of the base of the triangle is $2\sqrt{3}$ cm.
The length of the hypotenuse of the triangle is 6 cm.
The area of the triangle is A cm^2.
Show that $A=k\sqrt{2}$ giving the value of k.

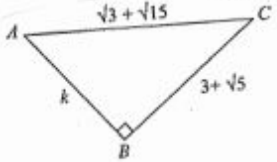


Diagram NOT accurately drawn

All measurements on the triangle are in centimetres.
ABC is a right-angled triangle. k is a positive integer.
Find the value of k.

The diagram shows a solid cylinder.

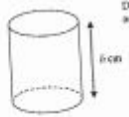


Diagram NOT accurately drawn

The cylinder has radius $4\sqrt{3}$ cm and height h cm.
The total surface area of the cylinder is $56\pi\sqrt{6}$ cm^2.
Find the exact value of h.
Give your answer in the form $a\sqrt{2}+b\sqrt{3}$ where a and b are integers.
Show your working clearly.

ABD is a right-angled triangle.

All measurements are given in centimetres.
C is the point on BD such that $CD=\frac{\sqrt{3}}{3}$
$AD=BD=\frac{\sqrt{2}}{2}$
Work out the exact area, in cm^2, of the shaded region.

B3 Write $\sqrt{8}$ as a power of 2	B4 Express $3\sqrt{3}$ as a power of 9

C1 Express $\frac{1}{81}$ as a single power of 3	C2 Express $\frac{1}{\sqrt{3}}$ as a single power of 9	C3 Express $\frac{1}{4\sqrt{2}}$ as a single power of 2	C4 Express $3^3+9^4+15\times27^2$ as a power of 3

Bring answers to the following for first lesson in September!

(4)m

(7)a

(12)d

(6)a

(16)a

With working out!!!

Challenge: C4

4m, 7a, 12d, 6a, 16a, c4

4 Simplify

a $\sqrt{12}$

g $\sqrt{45}$

m $\sqrt{216}$

7 Simplify

a $(\sqrt{5}+1)(2\sqrt{5}+3)$

d $(3\sqrt{2}-1)(2\sqrt{2}+5)$

12 Express each of the following as simply as possible with a rational denominator.

a $\dfrac{1}{\sqrt{2}+1}$ **b** $\dfrac{4}{\sqrt{3}-1}$ **c** $\dfrac{1}{\sqrt{6}\div 2}$ **d** $\dfrac{3}{2+\sqrt{3}}$

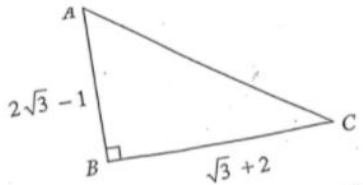
e $\dfrac{1}{2+\sqrt{5}}$ **f** $\dfrac{\sqrt{2}}{\sqrt{\ }}$ **g** 6

6 Showing your method clearly,

a express $\sqrt{37.5}$ in the form $a\sqrt{6}$,

b express $\sqrt{9\frac{3}{5}}-\sqrt{6\frac{2}{3}}$ in the form $b\sqrt{15}$.

16



In triangle ABC, $AB = 2\sqrt{3}-1$, $BC = \sqrt{3}+2$ and $\angle ABC = 90°$.

a Find the exact area of triangle ABC in its simplest form.

b Show that $AC = 2\sqrt{5}$.

$(ACB) = 5\sqrt{3}-8$.

C4 Express

$3^7+9^4+15\times 27^2$ as a power of 3

STRIDE TRANSITION WORK! ANSWERS.

4 Simplify

a $\sqrt{12}$ $2\sqrt{3}$ b $\sqrt{28}$ $2\sqrt{7}$ c $\sqrt{80}$ $4\sqrt{5}$ d $\sqrt{27}$ $3\sqrt{3}$ e $\sqrt{24}$ $2\sqrt{6}$ f $\sqrt{128}$ $8\sqrt{2}$

g $\sqrt{45}$ $3\sqrt{2}$ h $\sqrt{40}$ $2\sqrt{10}$ i $\sqrt{75}$ $5\sqrt{3}$ j $\sqrt{112}$ $4\sqrt{7}$ k $\sqrt{99}$ $3\sqrt{11}$ l $\sqrt{147}$ $7\sqrt{3}$

[redacted] $\sqrt{800}$ $20\sqrt{2}$ o $\sqrt{180}$ $6\sqrt{5}$ p $\sqrt{60}$ $2\sqrt{15}$ q $\sqrt{363}$ $11\sqrt{3}$ r $\sqrt{208}$ $4\sqrt{13}$

5 Simplify

a $\sqrt{18}+\sqrt{50}$ $8\sqrt{2}$ b $\sqrt{48}-\sqrt{27}$ $\sqrt{3}$ c $2\sqrt{8}+\sqrt{72}$ $10\sqrt{2}$

d $\sqrt{360}-2\sqrt{40}$ $2\sqrt{10}$ e $2\sqrt{5}-\sqrt{45}+3\sqrt{20}$ $5\sqrt{5}$ f $\sqrt{24}+\sqrt{150}-2\sqrt{96}$ $-\sqrt{6}$

7 Simplify

a $(\sqrt{5}+1)(2\sqrt{5}+3)$ $7\sqrt{2}-11$ b $(1-\sqrt{2})(4\sqrt{2}-3)$ c $(2\sqrt{7}+3)^2$ $37+12\sqrt{7}$

d $(3\sqrt{2}-1)(2\sqrt{2}+5)$ $7+13\sqrt{2}$ e $(\sqrt{5}-\sqrt{2})(\sqrt{5}+2\sqrt{2})$ $1+\sqrt{10}$ f $(3-\sqrt{8})(4+\sqrt{2})$ $8-5\sqrt{2}$

8 Express each of the following as simply as possible with a rational denominator.

a $\frac{1}{\sqrt{5}}$ $\frac{1}{5}\sqrt{5}$ b $\frac{2}{\sqrt{3}}$ $\frac{2}{3}\sqrt{3}$ c $\frac{1}{\sqrt{8}}$ $\frac{1}{4}\sqrt{2}$ d $\frac{14}{\sqrt{7}}$ $2\sqrt{7}$ e $\frac{3\sqrt{2}}{\sqrt{3}}$ $\sqrt{6}$ f $\frac{\sqrt{5}}{\sqrt{15}}$ $\frac{1}{3}\sqrt{3}$

g $\frac{1}{3\sqrt{7}}$ $\frac{1}{21}\sqrt{7}$ h $\frac{12}{\sqrt{72}}$ $\sqrt{2}$ i $\frac{1}{\sqrt{80}}$ $\frac{1}{20}\sqrt{5}$ j $\frac{3}{2\sqrt{54}}$ $\frac{1}{12}\sqrt{6}$ k $\frac{4\sqrt{20}}{3\sqrt{18}}$ $\frac{4}{9}\sqrt{10}$ l $\frac{3\sqrt{175}}{2\sqrt{27}}$ $\frac{5}{6}\sqrt{21}$

11 a Simplify $(2-\sqrt{3})(2+\sqrt{3})$. $= 1$

b Express $\frac{2}{2-\sqrt{3}}$ in the form $a+b\sqrt{3}$. $= 4+2\sqrt{3}$

$2+\sqrt{3}$

12 Express each of the following as simply as possible with a rational denominator.

a $\frac{1}{\sqrt{2}+1}$ $\sqrt{2}-1$ b $\frac{4}{\sqrt{3}-1}$ $2(\sqrt{3}+1)$ c $\frac{1}{\sqrt{6}-2}$ $\frac{1}{2}\sqrt{6}+1$ d $\frac{3}{2+\sqrt{3}}$ [redacted]

e $\frac{1}{2+\sqrt{5}}$ $\sqrt{5}-2$ f $\frac{\sqrt{2}}{\sqrt{2}-1}$ $2+\sqrt{2}$ g $\frac{6}{\sqrt{7}+3}$ $3(3-\sqrt{7})$ h $\frac{1}{3+2\sqrt{2}}$ $3-2\sqrt{2}$

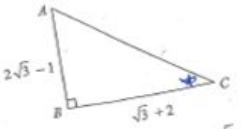
14 Express each of the following in the form $a+b\sqrt{5}$, where a and b are integers.

a $\sqrt{20}(\sqrt{5}-3)$ $= 10-6\sqrt{5}$

b $(1-\sqrt{5})(3+2\sqrt{5})$ $= -7-\sqrt{5}$

c $\frac{1+\sqrt{5}}{\sqrt{5}-2}$ $= 7+3\sqrt{5}$

16



In triangle ABC, $AB = 2\sqrt{3}-1$, $BC = \sqrt{3}+2$ and $\angle ABC = 90°$.

a Find the exact area of triangle ABC in its simplest form. [redacted]

b Show that $AC = 2\sqrt{5}$. $AC^2 = 20$

c Show that $\tan(\angle ACB) = 5\sqrt{3}-8$. *

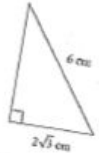


Diagram NOT accurately drawn

$k = 6$

The diagram shows a right-angled triangle.
The length of the base of the triangle is $2\sqrt{3}$ cm.
The length of the hypotenuse of the triangle is 6 cm.
The area of the triangle is A cm^2.
Show that $A = k\sqrt{2}$ giving the value of k.

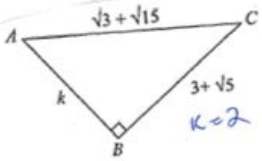
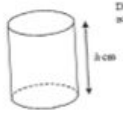


Diagram NOT accurately drawn

$k = 2$

All measurements on the triangle are in centimetres.
ABC is a right-angled triangle. k is a positive integer.
Find the value of k.

The diagram shows a solid cylinder.



$7\sqrt{2}-4\sqrt{3}$

The cylinder has radius $4\sqrt{3}$ cm and height h cm.
The total surface area of the cylinder is $56\pi\sqrt{6}$ cm^2.
Find the exact value of h.
Give your answer in the form $a\sqrt{2}+b\sqrt{3}$ where a and b are integers.
Show your working clearly.

ABD is a right-angled triangle.

$\frac{1}{4}-\frac{\sqrt{6}}{12}$

All measurements are given in centimetres.
C is the point on BD such that $CD = \frac{\sqrt{3}}{3}$
$AD = BD = \frac{\sqrt{2}}{2}$
Work out the exact area, in cm^2, of the shaded region.

6 Showing your method clearly,

a express $\sqrt{37.5}$ in the form $a\sqrt{6}$,

b express $\sqrt{9\frac{3}{5}}-\sqrt{6\frac{2}{3}}$ in the form $b\sqrt{15}$.

$= \frac{2}{15}\sqrt{15}$

B3 Write $\sqrt{8}$ as a power of 2	B4 Express $3\sqrt{3}$ as a power of 9
$2^{3/2}$	$9^{3/4}$

C1 Express $\frac{1}{81}$ as a single power of 3	C2 Express $\frac{1}{\sqrt{3}}$ as a single power of 9	C3 Express $\frac{1}{4\sqrt{2}}$ as a single power of 2	C4 Express $3^7+9^4+15\times27^2$ as a power of 3
3^{-4}	$9^{-1/4}$	$2^{-5/2}$	[redacted]

Questions or Problems

Email lucy.tyler@svf.org.uk if any further issues.