

Autumn  
Scheme of learning  
**Year 6**



#MathsEveryoneCan

# The White Rose Maths schemes of learning

## Teaching for mastery

Our research-based schemes of learning are designed to support a mastery approach to teaching and learning and are consistent with the aims and objectives of the National Curriculum.

### Putting number first

Our schemes have number at their heart. A significant amount of time is spent reinforcing number in order to build competency and ensure children can confidently access the rest of the curriculum.

### Depth before breadth

Our easy-to-follow schemes support teachers to stay within the required key stage so that children acquire depth of knowledge in each topic. Opportunities to revisit previously learned skills are built into later blocks.

### Working together

Children can progress through the schemes as a whole group, encouraging students of all abilities to support each other in their learning.

### Fluency, reasoning and problem solving

Our schemes develop all three key areas of the National Curriculum, giving children the knowledge and skills they need to become confident mathematicians.

## Concrete – Pictorial – Abstract (CPA)

Research shows that all children, when introduced to a new concept, should have the opportunity to build competency by following the CPA approach. This features throughout our schemes of learning.

### Concrete

Children should have the opportunity to work with physical objects/concrete resources, in order to bring the maths to life and to build understanding of what they are doing.



### Pictorial

Alongside concrete resources, children should work with pictorial representations, making links to the concrete. Visualising a problem in this way can help children to reason and to solve problems.



### Abstract

With the support of both the concrete and pictorial representations, children can develop their understanding of abstract methods.

An abstract representation of the addition problem 5 + 7. The equation is written inside a yellow rectangular box with a slight 3D effect.

If you have questions about this approach and would like to consider appropriate CPD, please visit [www.whiterosemaths.com](http://www.whiterosemaths.com) to find a course that's right for you.

# Teacher guidance

Every block in our schemes of learning is broken down into manageable small steps, and we provide comprehensive teacher guidance for each one. Here are the features included in each step.

**Notes and guidance** that provide an overview of the content of the step and ideas for teaching, along with advice on progression and where a topic fits within the curriculum.

**Things to look out for**, which highlights common mistakes, misconceptions and areas that may require additional support.

Year 5 | Autumn Term | Block 1 – Place Value | Step 1

## Roman numerals to 1,000

**Notes and guidance**

In Year 4, children learned about Roman numerals to 100. In this small step, they explore Roman numerals to 1,000, and the symbols D (500) and M (1,000) are introduced. Children explore further the similarities and differences between the Roman number system and our number system, learning that the Roman system does not have a zero and does not use placeholders. Children use their knowledge of M and D to recognise years using Roman numerals. Asking children to write the date in Roman numerals is one way to reinforce the concept daily.

**Things to look out for**

- Children may mix up which letter stands for which number.
- Children may add the individual values together instead of interpreting the values based on their position, for example interpreting CD as 600 instead of 400
- It is often more difficult to convert numbers that require large strings of Roman numerals.
- Children may think that numbers such as 990 can be written as XM instead of CMXC.

**Key questions**

- What patterns can you see in the Roman number system?
- What rules do we use when converting numbers to Roman numerals?
- What letters are used in the Roman number system? What does each letter represent?
- How do you know what order to write the letters when using Roman numerals?
- What is the same and what is different about representing the number “five hundred and three” in the Roman number system and in our number system?

**Possible sentence stems**

- The letter \_\_\_\_\_ represents the number \_\_\_\_\_
- I know \_\_\_\_\_ is greater than \_\_\_\_\_ because \_\_\_\_\_

**National Curriculum links**

- Read Roman numerals to 1,000 (M) and recognise years written in Roman numerals

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**Key questions** that can be posed to children to develop their mathematical vocabulary and reasoning skills, digging deeper into the content.

**Possible sentence stems** to further support children’s mathematical language and to develop their reasoning skills.

**National Curriculum links** to indicate the objective(s) being addressed by the step.




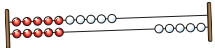
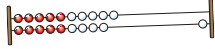









# Teacher guidance

A **Key learning** section, which provides plenty of exemplar questions that can be used when teaching the topic.

Year 2 | Autumn Term | Block 1 - Place Value | Step 1

## Numbers to 20

**Key learning**

- Complete the number tracks.
  - 0 1 2
  - 10 11 12
  - 7 8 13
- What numbers are shown?
  -   
  - Give your answers in numerals and words.
- What number is shown on each Rekenrek?
  - 
  - 
  - Give your answers in numerals and words.
- What numbers are shown?
  -    
  - Give your answers in numerals and words.
- Use words to complete the sentences.
  - The number after four is \_\_\_\_\_
  - The number before eight is \_\_\_\_\_
  - The number after nine is \_\_\_\_\_
- Make each number in three different ways.
  -     

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Activity symbols that indicate an idea can be explored practically

**Reasoning and problem-solving** activities and questions that can be used in class to provide further challenge and to encourage deeper understanding of each topic.

Year 3 | Autumn Term | Block 1 - Place Value | Step 4

## Hundreds

**Reasoning and problem solving**

 I am going to count in 100s from zero.

Dora

Write two numbers that Dora will say.

any two multiples of 100

No

Mo is counting in hundreds.

 ... 8 hundred, 9 hundred, 10 hundred

Mo should have said 1 thousand, 10 hundreds is equal to 1 thousand.

How should Mo have said the last number?


 Dora will say the number 160

Tiny

Is Tiny correct? How do you know?

Balloons come in bags of 10

Rosie has 300 balloons.



Rosie has 30 bags of balloons.

How many bags does she have?

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Answers provided where appropriate

# Activities and symbols

## Key Stage 1 activities

Key Stage 1 includes more hands-on activities alongside questions.

An activity to be led by the teacher



Use a Rekenrek in the ready position.

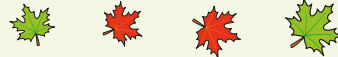


Ask children to show a number on their Rekenrek.

An outside activity or one that uses resources from nature



Find some seeds and leaves to represent Autumn.



Ask children to sort the objects in three different ways and then compare their answers with a partner.

An activity introduced by a reading from an appropriate fiction or non-fiction book



Read *The Button Box* by M Reid.

Give children a selection of buttons and ask them to sort the buttons in as many different ways as they can.

Encourage them to think about size, shape, colour and number of holes.

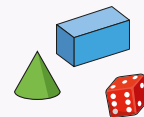


An investigation



Give children a selection of 3D shapes.

Ask children to sort the objects into two groups and then challenge a partner to say how the objects have been sorted.



## Key Stage 1 and 2 symbols

The following symbols are used to indicate:



concrete resources might be useful to help answer the question



a bar model might be useful to help answer the question



drawing a picture might help children to answer the question



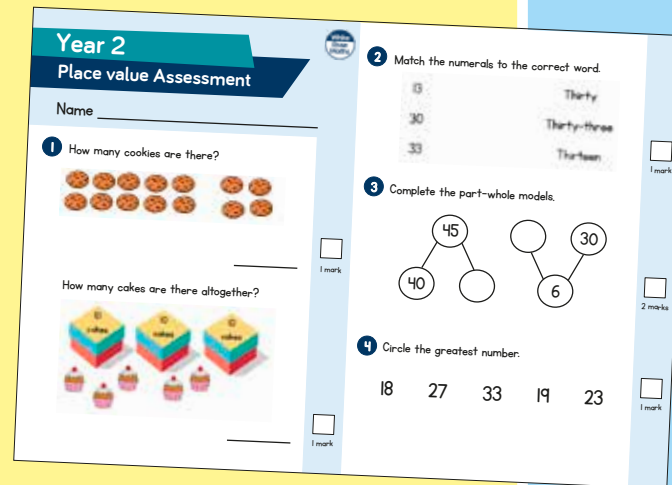
children talk about and compare their answers and reasoning



a question that should really make children think. The question may be structured differently or require a different approach from others and/or tease out common misconceptions.


# Free supporting materials


**End-of-block assessments** to check progress and identify gaps in knowledge and understanding.



**Year 2**  
**Place value Assessment**

Name \_\_\_\_\_

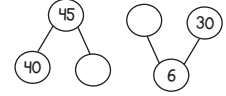
1 How many cookies are there?  
  
\_\_\_\_\_ 1 mark

How many cakes are there altogether?  
  
\_\_\_\_\_ 1 mark

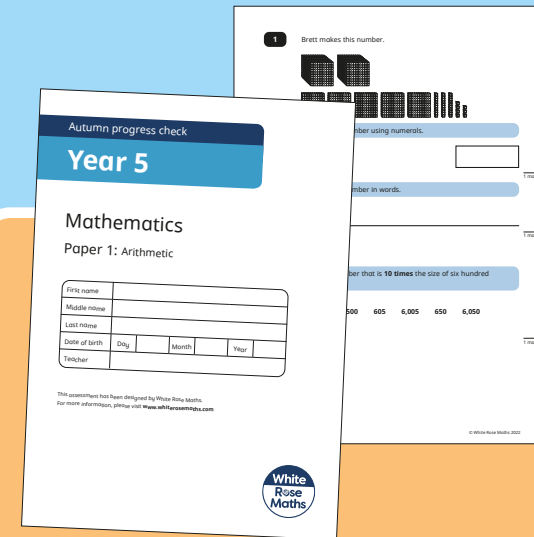
2 Match the numerals to the correct word.  

13	Thirty
30	Thirty-three
33	Thirteen

  
\_\_\_\_\_ 1 mark

3 Complete the part-whole models.  
  
\_\_\_\_\_ 2 marks

4 Circle the greatest number.  
18   27   33   19   23  
\_\_\_\_\_ 1 mark



Autumn progress check  
**Year 5**  
**Mathematics**  
Paper 1: Arithmetic

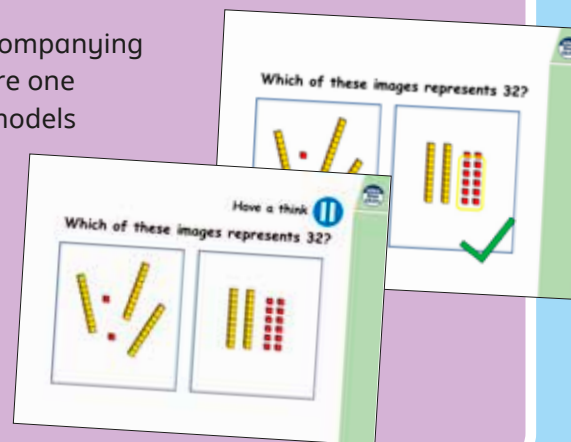
First name			
Middle name			
Last name			
Date of birth	Day	Month	Year
Teacher			


This assessment has been designed by White Rose Maths.  
For more information, please visit [www.whiterosemaths.com](http://www.whiterosemaths.com)


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**End-of-term assessments** for a more summative view of where children are succeeding and where they may need more support.

Each small step has an accompanying **home learning video** where one of our team of specialists models the learning in the step. These can also be used to support students who are absent or who need to catch up content from earlier blocks or years.



Which of these images represents 32?  


Have a think  
Which of these images represents 32?  


# Free supporting materials

Primary Progression – Place Value						
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
<b>Place Value: Counting</b>	<ul style="list-style-type: none"> <li>count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number</li> <li>Count numbers to 100 in numerals; count in multiples of twos, fives and tens</li> </ul> <p>Autumn 1 Autumn 4 Spring 2 Summer 4</p>	<ul style="list-style-type: none"> <li>count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward</li> </ul> <p>Autumn 1</p>	<ul style="list-style-type: none"> <li>count from 0 in multiples of 4, 8, 50 and 100, find 10 or 100 more or less than a given number</li> </ul> <p>Autumn 1 Autumn 3</p>	<ul style="list-style-type: none"> <li>count in multiples of 6, 7, 9, 25 and 1000</li> <li>count backwards through zero to include negative numbers</li> </ul> <p>Autumn 1 Autumn 4</p>	<ul style="list-style-type: none"> <li>count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000</li> <li>count forwards and backwards with positive and negative whole numbers, including through zero</li> </ul> <p>Autumn 1</p>	

**National Curriculum progression** to indicate how the schemes of learning fit into the wider picture and how learning progresses within and between year groups.

**Skill: Add three 1-digit numbers**

**Year: 2**

When adding three 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently.

This supports children in their understanding of commutativity.

Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers.

$7 + 6 + 3 = 16$

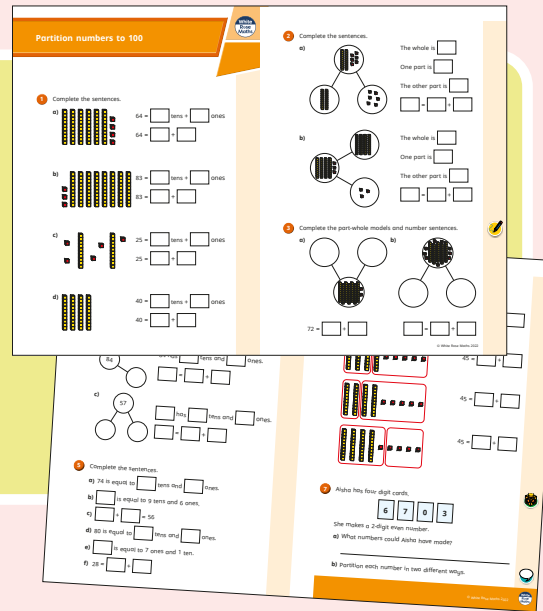
**Calculation policies** that show how key approaches develop from Year 1 to Year 6.

Ready to Progress – Number Facts Year 3			
	3NF-1	3NF-2	3NF-3
<b>RTP Criteria</b>	Secure fluency in addition and subtraction facts that bridge 10, through continued practice.	Recall multiplication facts, and corresponding division facts, in the 10, 5, 2, 4 and 8 multiplication tables, and recognise products in these multiplication tables as multiples of the corresponding number.	Apply place-value knowledge to know additive and multiplicative number facts (scaling facts by 10).
<b>White Rose Maths Small Steps</b>	<b>Autumn 2 Addition and Subtraction</b> <ul style="list-style-type: none"> <li>Add 3-digit and 1-digit numbers - crossing 10</li> <li>Subtract a 1-digit number from a 3-digit number - crossing 10</li> <li>Add 3-digit and 2-digit numbers - crossing 100</li> <li>Subtract a 2-digit number from a 3-digit number - crossing 100</li> </ul>	<b>Autumn 3 Multiplication and Division</b> <ul style="list-style-type: none"> <li>2 times-table</li> <li>5 times-table</li> <li>Divide by 2</li> <li>Divide by 5</li> <li>Divide by 10</li> <li>Multiply by 4</li> <li>Divide by 4</li> <li>The 4 times-table</li> <li>Multiply by 8</li> <li>Divide by 8</li> <li>The 8 times-table</li> </ul>	<b>Spring 1 Multiplication and Division</b> <ul style="list-style-type: none"> <li>Related calculations</li> <li>Scaling</li> </ul> <b>Spring 4 Measurement: Length and Perimeter</b> <ul style="list-style-type: none"> <li>Equivalent lengths (m and cm)</li> <li>Equivalent lengths (mm and cm)</li> </ul>

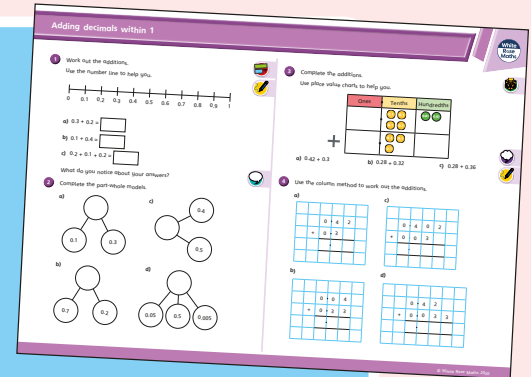
**Ready to progress** mapping that shows how the schemes of learning link to curriculum prioritisation.

# Premium supporting materials

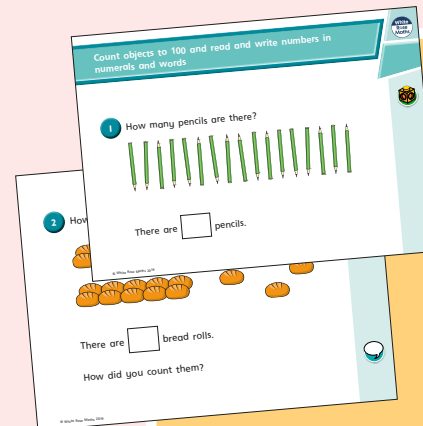
**Worksheets** to accompany every small step, providing relevant practice questions for each topic that will reinforce learning at every stage.



**Display** versions of the worksheet questions for front of class/whole class teaching.

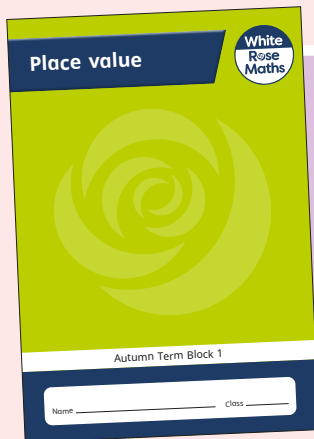


**PowerPoint™** versions of the worksheet questions to incorporate them into lesson planning.



**Answers** to all the worksheet questions.

Question	Answer
1	There are 17 pencils.
2	There are 12 bread rolls. Children may have counted 3 tens and 3 rolls.
3	twenty-eight
4	sixty-two
5	4 tens and 5 ones
6	a) seventeen b) twenty-one c) thirty-five d) eighty-two
7	a) 12 b) 80 c) 100 d) 9 e) 27 f) 14
8	79, 80, 81, 82, 83, 85 70, 79, 66, 64, 63
9	Eric has 20 sweets. Ed's friend gives her 7 sweets.

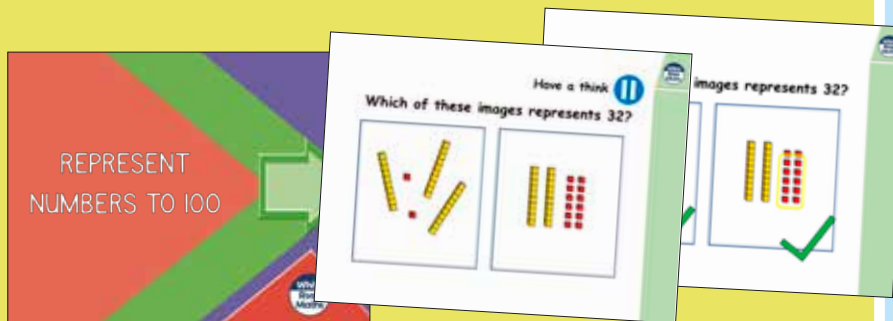


Also available as printed **workbooks**, per block.

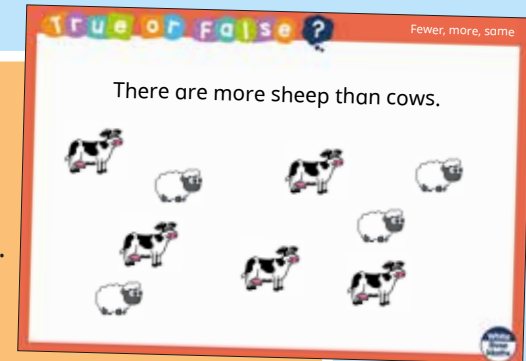


# Premium supporting materials

**Teaching slides** that mirror the content of our home learning videos for each step. These are fully animated and editable, so can be adapted to the needs of any class.



A **true or false** question for every small step in the scheme of learning. These can be used to support new learning or as another tool for revisiting knowledge at a later date.



**Flashback 4** starter activities to improve retention. Q1 is from the last lesson; Q2 is from last week; Q3 is from 2 to 3 weeks ago; Q4 is from last term/year. There is also a bonus question on each one to recap topics such as telling the time, times-tables and Roman numerals.

**Flashback 4** Year 4 | Week 5 | Day 1

- 1) Round 6,495 to the nearest 10, 100 and 1,000  $5 \times 2$   
6,500 6,500 6,000
- 2) Round 38 to the nearest 10 40
- 3) Complete the part-whole model.
- 4) Multiply 38 by 4 152



## Topic-based CPD videos

As part of our on-demand CPD package, our maths specialists provide helpful hints and guidance on teaching topics for every block in our schemes of learning.

## Meet the characters

Our class of characters bring the schemes to life, and will be sure to engage learners of all ages and abilities. Follow the children and their class pet, Tiny the tortoise, as they explore new mathematical concepts and ideas.

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# Yearly overview

The yearly overview provides suggested timings for each block of learning, which can be adapted to suit different term dates or other requirements.

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Number <b>Place value</b>		Number <b>Addition, subtraction, multiplication and division</b>				Number <b>Fractions A</b>		Number <b>Fractions B</b>		Measurement <b>Converting units</b>	
Spring	<b>Ratio</b>		<b>Algebra</b>		Number <b>Decimals</b>		Number <b>Fractions, decimals and percentages</b>		Measurement <b>Area, perimeter and volume</b>		<b>Statistics</b>	
Summer	Geometry <b>Shape</b>			Geometry <b>Position and direction</b>	Themed projects, consolidation and problem solving							

Autumn Block 1

# Place value

## Small steps

Step 1

Numbers to 1,000,000

Step 2

Numbers to 10,000,000

Step 3

Read and write numbers to 10,000,000

Step 4

Powers of 10

Step 5

Number line to 10,000,000

Step 6

Compare and order any integers

Step 7

Round any integer

Step 8

Negative numbers

# Numbers to 1,000,000

## Notes and guidance

In preparation for the next step (Numbers to 10,000,000), children recap their Year 5 learning by exploring numbers up to 1,000,000

Understanding that place value columns follow consistent patterns – ones, tens, hundreds, then (one) thousands, ten thousands, hundred thousands, before reaching millions – is key. Place value charts, Gattegno charts and place value counters can be used to support understanding of the relationships between columns and the construction of numbers.

Children also revise partitioning, exploring both standard and non-standard ways of composing numbers.

Writing numbers in words follows in Step 3

## Things to look out for

- Children may find it difficult to conceptualise such large numbers, as they cannot easily be represented concretely and lie outside their experience.
- Children may think that place value columns go in the order ones, tens, hundreds, thousands, millions.
- Children may find numbers with several placeholders (for example, 500,020) difficult.

## Key questions

- Where do the commas go when you write one million in figures?
- If 1,000,000 is the whole, what could the parts be?
- How else can you partition the number?
- What is the value of each digit in the number?
- Which columns will change if you add/subtract 10, 100, 1,000, ... to/from the number?
- When do you use placeholders in numbers?

## Possible sentence stems

- The value of the \_\_\_\_\_ in \_\_\_\_\_ is \_\_\_\_\_
- The column before/after the \_\_\_\_\_ column is the \_\_\_\_\_ column.

## National Curriculum links

- Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit
- Solve number and practical problems that involve the above

# Numbers to 1,000,000

## Key learning

- What is the value of the digit 4 in each of the numbers in the place value chart?

Thousands			Ones		
H	T	O	H	T	O
		4	3	2	7
	3	5	4	0	2
2	4	7	1	9	8
8	1	2	5	4	3

- Complete the number sentences.
  - ▶  $604,821 = 600,000 + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + 20 + 1$
  - ▶  $\underline{\hspace{2cm}} = 300,000 + 4,000 + 700 + 4$
  - ▶  $2,000 + 8 + 60,000 + 500 + 700,000 = \underline{\hspace{2cm}}$
- Count up in 10,000s from 74,000 to 204,000  
 Count down in 100,000s from 1,000,000 to zero.  
 Count down in 100s from 9,312 to 7,812

- What number is shown in the place value chart?

Thousands			Ones		
H	T	O	H	T	O
●●	●●●●	●	●●●●	●●	●●●●
	●●●●		●●●●	●	●●●●
	●●		●●●●		●
			●●		

What will the number be if you add four counters to the:

- tens column
  - ten-thousands column
  - hundreds column?
- Annie is using place value counters.  
 She has 4 ten-thousands counters, 12 thousands counters, 8 hundreds counters, 3 tens counters and 25 ones counters.  
 What is the greatest number she can make?
  - Fill in the missing numbers.  
 $1 \text{ million} = 900,000 + \underline{\hspace{2cm}} = 990,000 + \underline{\hspace{2cm}} = \underline{\hspace{2cm}} + 999,000$

# Numbers to 1,000,000

## Reasoning and problem solving

100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

What number is shown in the Gattegno chart?

Decrease the number shown by 30,000

Increase the number shown by 100,500

Challenge a partner to find other increases and decreases of the number.



463,528      433,528      564,028

Are the statements true or false?

Adding ten thousand to a number only ever changes the digits in exactly one column.

False

The number consisting of 70 thousands and 400 ones is 700,400

False

3 ten-thousands is the same as 30 thousands.

True

400 hundreds is the same as 4 ten-thousands.

True

A large number added to a large number is always a large number.

True

A large number subtracted from a large number is always a large number.

False



# Numbers to 10,000,000

## Notes and guidance

Children build on the previous step to explore numbers up to 10,000,000. They need to understand that the million can be considered a unit in the same way as the thousand. Numbers do not all have to be over 1,000,000 in this step; children should continue to experience smaller numbers alongside 7-digit numbers. The placement of commas and other separators should be discussed.

Familiar manipulatives and models, such as place value charts and counters, Gattegno charts and part-whole models, are used to represent numbers. Children partition the numbers in both standard and non-standard ways.

### Things to look out for

- Children may struggle with where to position the commas in large numbers.
- Children may not recognise large numbers written with no commas.
- Unless they are confident with previous learning, children may think that place value columns go in the order ones, tens, hundreds, thousands, millions.
- Children may find numbers with several placeholders (for example, 1,006,020) difficult.

## Key questions

- Where do the commas go when writing 7-digit numbers? How does this connect to place value charts?
- How does the place value chart help you to represent large numbers?
- What is the value of each digit in the number?
- Are 7-digit numbers always greater than 1,000,000?
- When do you use placeholders in numbers?
- What is the same and what is different about counting in 1,000s and counting in 1,000,000s?

## Possible sentence stems

- The value of the \_\_\_\_\_ in \_\_\_\_\_ is \_\_\_\_\_
- The column before/after the \_\_\_\_\_ column is the \_\_\_\_\_ column.

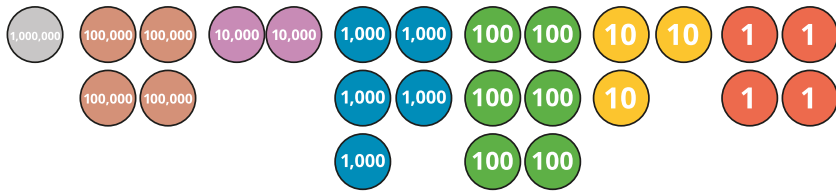
## National Curriculum links

- Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit
- Solve number and practical problems that involve the above

# Numbers to 10,000,000

## Key learning

- Count in 1,000,000s from zero to 10,000,000
- What number is represented?

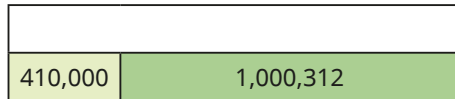


- Match the numbers to the representations.

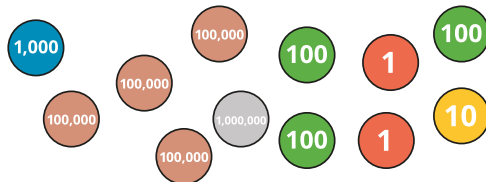
1,401,312

M	HTh	TTh	Th	H	T	O
●		●●●●	●	●●●	●	●●

1,041,312



1,410,312



- The meter shows the number of kilometres a car has travelled.



Ron writes the number as 3,678,42

Explain Ron's mistake.

- Here is a number in a place value chart.

Millions	Thousands			Ones		
O	H	T	O	H	T	O
4	2	8	7	2	9	5

What number is 300,000 greater than the number shown?

What number is 20,000 greater than the number shown?

- Count up in 10,000s from 463,500 to 1,000,500
- Count down in 10,000s from 463,500 to 3,500
- Count down in 1,000s from 463,500 to 433,500

# Numbers to 10,000,000

## Reasoning and problem solving

Jack has got some place value counters.

Some of my  
counters have a value  
of 1,000,000, some  
have a value of 10,000  
and some have a  
value of 1



Jack picks four counters.

What different numbers greater than  
1,000,000 could he make?

Jack wants to make a number greater  
than 5,000,000

What is the fewest number of counters  
he needs?

4,000,000

3,010,000

3,000,001

2,020,000

2,010,001

2,000,002

1,020,001

1,030,000

1,010,002

1,000,003

6 counters

Fill in the missing numbers.

$$824,309 = 800,000 + \underline{\hspace{2cm}} + 4,000 + 300 + 9$$

$$6,413,085 = \underline{\hspace{2cm}} + 80$$

$$58,904 = 50,000 + \underline{\hspace{2cm}} + 4$$

$$947,812 - 400,000 = \underline{\hspace{2cm}}$$

$$947,812 - 4,000 = \underline{\hspace{2cm}}$$

$$947,812 - 400 = \underline{\hspace{2cm}}$$

$$5,198,264 - \underline{\hspace{2cm}} = 5,098,264$$

$$5,198,264 - \underline{\hspace{2cm}} = 5,191,264$$

20,000

6,413,005

8,900

547,812

943,812

947,412

100,000

7,000

# Read and write numbers to 10,000,000

## Notes and guidance

Children should now be secure with the place value of numbers to 10,000,000. This small step develops their skill at reading and writing large numbers in words.

The focus of this step is learning the structure of how numbers are said and written in words, for example 4,378 as “four thousand, three hundred and seventy-eight” rather than just “four-three-seven-eight”. Using a comma as a separator helps children to read and write large numbers by tackling them in sections. This can be supported visually/concretely with place value charts, part-whole models or Gattegno charts.

Children should also be able to write numbers such as “half a million” in both words and numerals.

## Things to look out for

- Children who find the “teen” numbers difficult may have problems with numbers such as 5,317,418
- Children may find reading and writing numbers with placeholders (for example, 5,208,001) difficult.

## Key questions

- When a number is written with two commas, what does that tell you about the size of the number?
- What do the numbers before this comma represent?
- How do you write “one million” in words and numerals?
- How do you write “half a million” in words and numerals?
- When do we use “and” when reading or writing a number?

## Possible sentence stems

- The digit before the first/second comma is \_\_\_\_\_  
This part of the number is said/written as \_\_\_\_\_
- The digit after the first/second comma is \_\_\_\_\_  
This part of the number is said/written as \_\_\_\_\_
- The whole of the number is said/written as \_\_\_\_\_

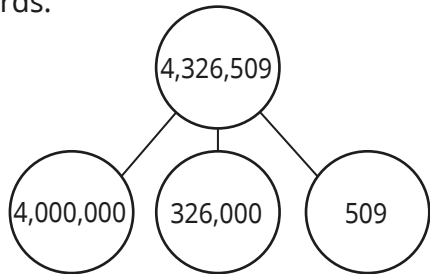
## National Curriculum links

- Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit
- Solve number and practical problems that involve the above

# Read and write numbers to 10,000,000

## Key learning

- Alex is using a part-whole model to help write the number 4,326,509 in words.

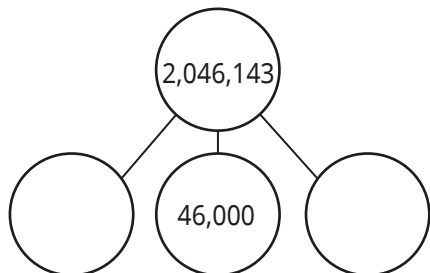


forty million and three hundred and twenty-six thousand and five hundred and nine

What mistakes has Alex made?

Write 4,326,509 correctly in words.

- Complete the part-whole model to show the number 2,046,143



Write the number 2,046,143 in words.

- Here is a number shown in a place value chart.

Millions	Thousands			Ones		
O	H	T	O	H	T	O
3	6	7	1	9	4	2

Write the number in words.

- A number is made up of 5 millions, 3 hundred-thousands, 7 tens and 9 ones.

Show the number on a place value chart.

Write the number in words and numerals.

- Write the numbers in numerals.

two million, eighty-three thousand and twelve

two million, eight hundred and three thousand and twenty

two million, eight hundred and twenty-three thousand and twelve

- Write 500,000 in words.
- Write the number "three and a half million" in numerals.

# Read and write numbers to 10,000,000

## Reasoning and problem solving

Use some of the digit cards and the clues to work out the number.



- The ten-thousands and hundreds columns have the same digit.
- The hundred-thousands digit is double the tens digit.
- The number has six digits.
- The number is less than six hundred and fifty-five thousand.

Find as many possible solutions, giving your answers in words and numerals.

Compare answers with a partner.



multiple possible answers, e.g.

650,533 – six hundred and fifty thousand, five hundred and thirty-three

Here is a number shown on a Gattegno chart.

1,000,000	2,000,000	3,000,000	4,000,000	5,000,000	6,000,000	7,000,000	8,000,000	9,000,000
100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Write in words the number that is:

- 80 greater than this number
- 80 less than this number
- 80,000 greater than this number
- 80,000 less than this number.

\_\_\_\_\_ six million, thirty thousand, five hundred and eighty-four

\_\_\_\_\_ six million, thirty thousand, four hundred and twenty-four

\_\_\_\_\_ six million, one hundred and ten thousand, five hundred and four

\_\_\_\_\_ five million, nine hundred and fifty thousand, five hundred and four

# Powers of 10

## Notes and guidance

Children should be confident with multiplying and dividing by 10, 100 and 1,000 from their learning in Year 5. In this small step, they use their place value knowledge to identify integers that are 10, 100, 1,000 times the size, or one-tenth, one-hundredth, one-thousandth the size of other integers. These relationships with decimal numbers are covered next term.

Children need to be aware that a value increases or decreases by a power of 10 between adjacent columns on a place value chart. They also need to realise that multiplying or dividing by 10 twice has the same effect as multiplying or dividing by 100 and that multiplying or dividing by 10 three times has the same effect as multiplying or dividing by 1,000

Place value charts and Gattegno charts are useful for modelling the effects of repeated multiplication and division by powers of 10

## Things to look out for

- Children may think that the overall effect of, for example,  $\times 10$  followed by  $\times 10$  is  $\times 20$
- The fact that numbers increase and decrease by a factor of 10 horizontally on a place value chart, but vertically on a Gattegno chart, may be confusing for children.

## Key questions

- How can you tell if a number is a power of 10?
- Is this number a multiple of a power of 10? How can you tell?
- If you move a digit one/two places to the left in a place value chart, how many times greater is the value of the digit?
- How can you use a Gattegno chart to find a number 10 times/one-tenth the size of a given number?

## Possible sentence stems

- \_\_\_\_\_ is 10 times the size of \_\_\_\_\_, so \_\_\_\_\_ is one-tenth the size of \_\_\_\_\_
- \_\_\_\_\_ is 100 times the size of \_\_\_\_\_, so \_\_\_\_\_ is one-hundredth the size of \_\_\_\_\_
- Multiplying/dividing by 10 twice/three times is the same as multiplying/dividing by \_\_\_\_\_

## National Curriculum links

- Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit
- Solve number and practical problems that involve the above

# Powers of 10

## Key learning

- What number is shown in the place value chart?

HTh	TTh	Th	H	T	O
		●● ●● ●	●● ●● ●● ●●	●●	●● ●● ●●

Multiply the number by 10 and show the answer in a place value chart.

What is the same and what is different?

Multiply the number by 100 and show the answer in a place value chart.

What is the same and what is different?

- Complete the statements.

\_\_\_\_\_ cm is the same length as 5,600 m.

\_\_\_\_\_ cm is the same length as 5,600 mm.

\_\_\_\_\_ m is the same length as 56,000 cm.

\_\_\_\_\_ m is the same length as 56,000 mm.

- What number is shown on the Gattegno chart?

1,000,000	2,000,000	3,000,000	4,000,000	5,000,000	6,000,000	7,000,000	8,000,000	9,000,000
100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Use the chart to make the number one hundred times the size of the number shown.

Use the chart to make the number one-hundredth the size of the number shown.

- Huan thinks that the number a thousand times the size of 2,500 is two and a half million.

Do you agree with Huan? Explain your answer.

- Which calculations have the same answers?

$460 \times 10$

$46,000 \div 1,000$

$46 \times 10 \times 10$

$46 \times 100 \times 100$

$460 \times 10 \div 100$

$4,600 \div 10 \times 1,000$



# Powers of 10

## Reasoning and problem solving

The Gattegno chart shows the answer to a calculation using powers of 10

1,000,000	2,000,000	3,000,000	4,000,000	5,000,000	6,000,000	7,000,000	8,000,000	9,000,000
100,000	200,000	300,000	400,000	500,000	600,000	700,000	800,000	900,000
10,000	20,000	30,000	40,000	50,000	60,000	70,000	80,000	90,000
1,000	2,000	3,000	4,000	5,000	6,000	7,000	8,000	9,000
100	200	300	400	500	600	700	800	900
10	20	30	40	50	60	70	80	90
1	2	3	4	5	6	7	8	9

Find two integer calculations using powers of 10 that give this answer.

Give your answers as calculations, for example:

\_\_\_\_\_ × (or ÷) \_\_\_\_\_ = \_\_\_\_\_ and sentences such as "\_\_\_\_\_ is 10 times (or one-tenth) the size of \_\_\_\_\_".

Compare answers with a partner.



various possible answers, e.g.

$$6,830 \times 10 = 68,300 \quad 68,300 \text{ is 10 times the size of } 6,830$$

$$6,830,000 \div 100 = 68,300$$

68,300 is one-hundredth the size of 6,830,000

Annie is thinking of a number.



1,000 more  
than my number  
is 4,700



Annie

What number is 1,000 times the size of Annie's number?

3,700,000

Tommy is thinking of a number.



Tommy

The number  
one-hundredth the  
size of my number  
is 38,746

What number is 100 less than Tommy's number?

3,874,500

# Number line to 10,000,000

## Notes and guidance

Children explore the number line to 10,000,000 using the unit of a million, making links to the familiar number lines to 10 and 10,000. They label partially filled number lines, identify points labelled on number lines and mark where a given number would lie on a number line.

Children should understand that half a million is equal to 500,000 and know that the midpoints between divisions on the number line to 10,000,000 can be written as, for example, “three and a half million” or “3,500,000”. This links to splitting different numbers and number lines into two, four, five and ten parts, which is also covered in this step.

## Things to look out for

- Where number lines have more than one set of divisions, children may mix up the intervals between large divisions and smaller divisions.
- Children may confuse the number of intervals and the number of divisions.
- Children may not use the correct multiples when looking at midpoints, for example thinking the midpoint between 1,000,000 and 2,000,000 is 1,000,005

## Key questions

- What are the values of the start and the end of the number line?
- What is each interval worth?
- How many small divisions are there between each of the large divisions on the number line? What is each small interval worth?
- What is the same and what is different about a number line that goes from 0 to 10,000 and a number line that goes from 0 to 10,000,000?
- What is the midpoint between \_\_\_\_\_ and \_\_\_\_\_?
- What is each interval worth if one million is split into two/four/five/ten equal parts?

## Possible sentence stems

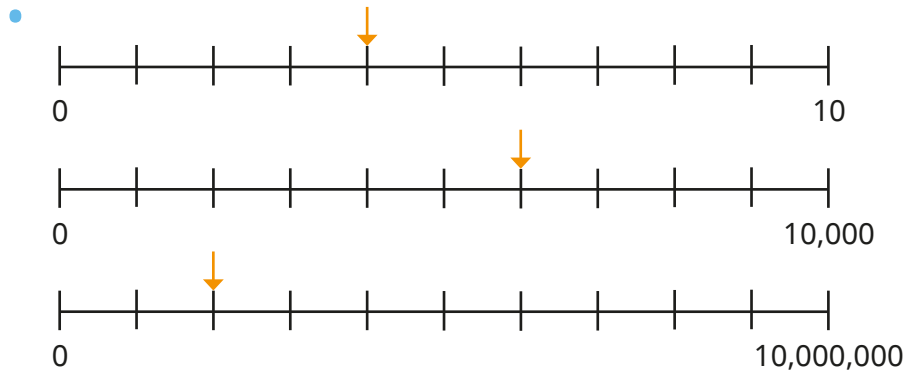
- The previous multiple of \_\_\_\_\_ is \_\_\_\_\_
- The next multiple of \_\_\_\_\_ is \_\_\_\_\_

## National Curriculum links

- Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit
- Solve number and practical problems that involve the above

# Number line to 10,000,000

## Key learning

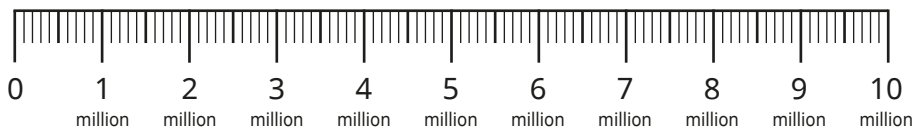


Label each division on the number lines.

What numbers are the arrows pointing to?

What is the same and what is different about the number lines?

- Here is a number line.

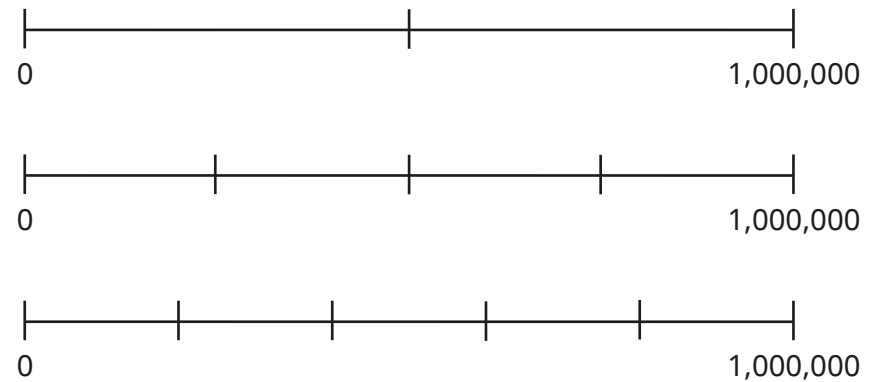


Draw arrows to show the positions of these numbers on the number line.

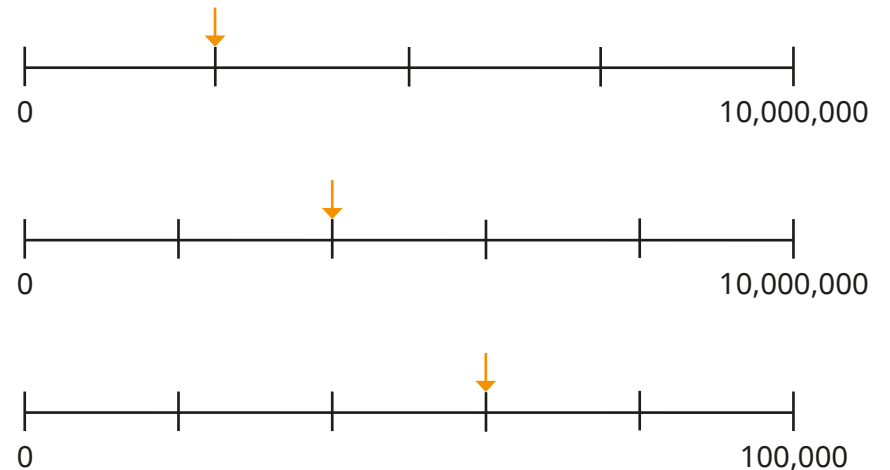
- |           |                         |           |           |
|-----------|-------------------------|-----------|-----------|
| 1,500,000 | five and a half million | 6,200,000 | 8,950,000 |
|-----------|-------------------------|-----------|-----------|

Which numbers can you place more accurately than others?

- Label the divisions on each number line.



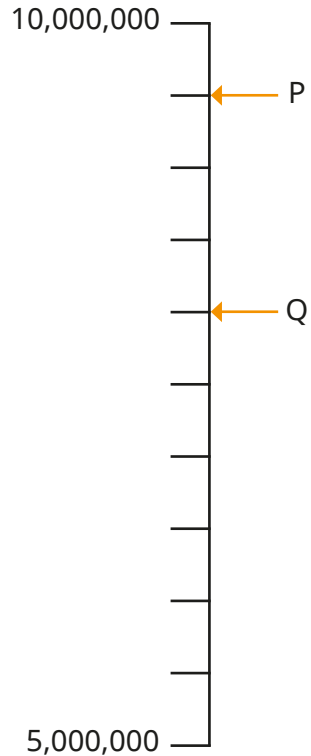
- What numbers are the arrows pointing to?



# Number line to 10,000,000

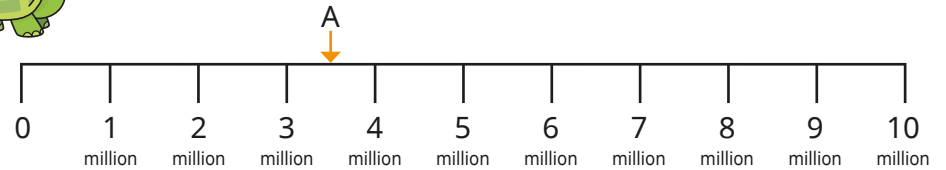
## Reasoning and problem solving

Find the difference between P and Q.



1,500,000

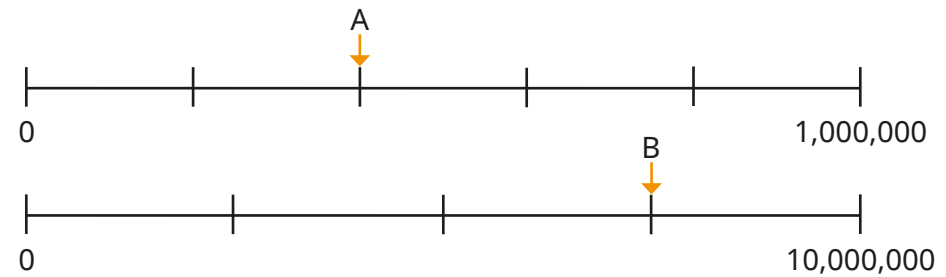
Compare methods with a partner.



Tiny says A is pointing to 3,050,000

Explain the mistake that Tiny has made.

Tiny has incorrectly found the midpoint of 3 and 4 million.



Work out  $B - A$ .

7,100,000

# Compare and order any integers

## Notes and guidance

In Year 5, children learned how to compare and order integers up to 1,000,000. This small step extends their learning to integers up to 10,000,000

Children compare numbers with the same number of digits, and with different numbers of digits, using their knowledge of place value columns. They present numbers in a variety of forms and use these different representations to aid their understanding when comparing and ordering.

Encourage the use of inequality symbols and precise mathematical language such as “greater than” and “less than”.

## Things to look out for

- Children may just look at the size of the leading digits and not consider the place value of the digits within the numbers.
- Children may need to be reminded of the meanings of the words “ascending” and “descending”.
- Children may need to be reminded about inequality symbols and their meanings.

## Key questions

- What is the value of each digit in the number?
- Which digit in each number has the greatest value? What is the value of these digits?
- When comparing two numbers with the same number of digits, what do you look at first?
- What is the difference between ascending and descending order?
- What is different about comparing numbers with the same number of digits and comparing numbers with different numbers of digits?

## Possible sentence stems

- The value of the first digit in the number \_\_\_\_\_ is \_\_\_\_\_
- \_\_\_\_\_ is less than/greater than \_\_\_\_\_

## National Curriculum links

- Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit
- Solve number and practical problems that involve the above

# Compare and order any integers

## Key learning

- Which is the greater number in each pair?

▶ 62,800	▶ 60,820
▶ 247,612	▶ 247,162
▶ 8,642,371	▶ 8,643,271

Explain how you know.

- Complete the statements to make them true.

M	HTh	TTh	Th	H	T	O	○	M	HTh	TTh	Th	H	T	O	
●●	●●	●●	●	●●	●	●●		●●	●	●●	●●	●●	●	●●	
M	HTh	TTh	Th	H	T	O	>	M	HTh	TTh	Th	H	T	O	
●		●●●	●	●●	●●	●●									

- Write the numbers in ascending order.

6,503,102      651,300      6,550,021      690,210

- Which calculation has the greater answer?

$600,000 + 50,000 + 7,000$	$400,000 + 256,000$
----------------------------	---------------------

- Write  $<$ ,  $>$  or  $=$  to make the statements correct.

62,520 ○ 602,250

3,218,000 ○ 399,875

426,000 ○ forty-four thousand

990,099 ○ one million

- Here are three numbers ordered from the greatest to the smallest, but one number has been covered up.

three hundred and thirteen thousand and thirty-three

✖

What might the covered number be?

# Compare and order any integers

## Reasoning and problem solving

Eva has put eight 6-digit numbers in ascending order.



- The first number in her list is 345,900
- The last number in her list is 347,000
- All the other numbers in her list have a digit sum of 20
- None of the numbers in her list have any repeated digits.

Find the other six numbers in Eva's list and write them in ascending order.

346,025  
346,052  
346,205  
346,250  
346,502  
346,520

$$\underline{\hspace{2cm}} + 80,000 < \text{half a million}$$

Complete the sentences.

The missing number could be \_\_\_\_\_

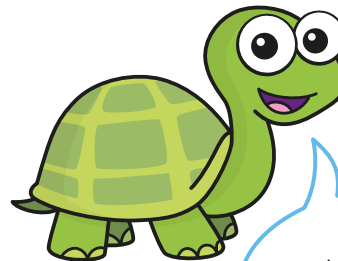
The missing number cannot be \_\_\_\_\_

The missing number must be \_\_\_\_\_

any number less than 420,000, e.g. 10,000

any number greater than or equal to 420,000, e.g. 600,000

multiple possible answers, e.g. less than 420,000



56,700 is greater than 201,000 because 5 is greater than 2

Explain the mistake that Tiny has made.

Tiny hasn't considered the place value of the digits.

# Round any integer

## Notes and guidance

In Year 5, children learned to round any number up to 1,000,000 to any power of 10 up to 100,000. This small step reviews and builds on this concept so that children also learn to round to the nearest million.

Children need to be confident with identifying the previous and next multiples of the appropriate power of 10 of the number, and finding the midpoints of those multiples. Number lines are useful as support here, as children can identify which multiple the number is closer to.

Children may need reminding that when a number is exactly halfway between two successive multiples the convention is to round to the greater multiple.

## Things to look out for

- Children may be confused by the language “round down”/“round up” and round 428,513 to 328,513 (or 300,000) to the nearest 100,000
- Children may look at the digit of the rounding rather than the next digit, for example, looking at the thousands column rather than the hundreds when rounding to the nearest thousand.

## Key questions

- Which multiples of 1,000,000 does the number lie between?
- How can you represent the rounding of this number on a number line?
- Which division on the number line is the number closer to?
- What is the number rounded to the nearest million?
- What is the most appropriate way of rounding this number?
- Which place value column should you look at to round the number to the nearest ten/hundred/thousand/ten thousand/hundred thousand/million?

## Possible sentence stems

- The previous multiple of \_\_\_\_\_ is \_\_\_\_\_
- The next multiple of \_\_\_\_\_ is \_\_\_\_\_
- \_\_\_\_\_ rounded to the nearest \_\_\_\_\_ is \_\_\_\_\_

## National Curriculum links

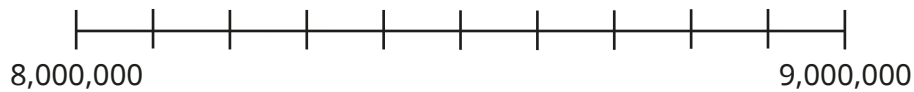
- Round any whole number to a required degree of accuracy
- Solve number and practical problems that involve the above



# Round any integer

## Key learning

- 



Draw an arrow to show the approximate position of 8,640,000 on the number line.

Round 8,640,000 to the nearest million.

- The population of London is 8,982,604  
Between which two multiples of 1,000,000 does this number lie?  
Round the population of London to the nearest million.

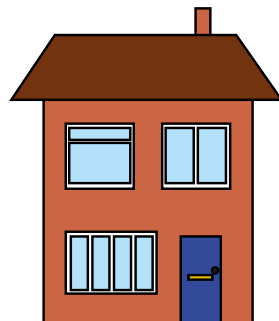
- In April 2021, the average price of a house in England was £273,486

Round this price to the nearest £100,000

Round this price to the nearest £10,000

Round this price to the nearest £1,000

Which do you think is the most appropriate number to round the price to?



- 

HTh	TTh	Th	H	T	O
●● ●● ●		●● ●● ●● ●	●● ●● ●● ●● ●	●● ●● ●●	●● ●● ●● ●● ●

Round the number in the place value chart to:

- the nearest ten thousand
- the nearest hundred thousand
- the nearest million.

- 



My number rounds to 38,000 to the nearest thousand.


What is the greatest possible value of Dexter's number?

What is the smallest possible value of Dexter's number?

# Round any integer

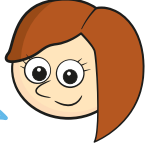
## Reasoning and problem solving

Mo and Rosie are each thinking of a number.



My number is 1,350,000 when rounded to the nearest ten-thousand.

Mo



My number is 1,000,000 when rounded to the nearest million.

Rosie

Both numbers are whole numbers.

What is the greatest possible difference between the two numbers?

854,999  
(if Mo's number is 1,354,999 and Rosie's number is 500,000)

Four children each have one of these cards.

15,987	15,813
15,101	16,101

Each child gives a clue about the number on their card.

Filip says, "My number rounds to 16,000 to the nearest thousand."

Esther says, "My number has 1 hundred."

Jack says, "My number is 15,990 when rounded to the nearest ten."

Dora says, "My number is 15,000 when rounded to the nearest thousand."

Match the cards to the children.

Filip: 15,813  
Esther: 16,101  
Jack: 15,987  
Dora: 15,101

# Negative numbers

## Notes and guidance

Children encountered negative numbers in Year 5. The focus of this small step is using negative numbers in real-life contexts while reinforcing children's understanding of the number line extending beyond zero.

Both horizontal and vertical number lines should be used, with the vertical line linking to reading temperatures on a thermometer. As well as adding and subtracting from positive and negative numbers, children learn to find the difference between numbers, including calculating intervals across zero. At this stage, children do not need to subtract negative numbers, so there is no need to cover calculations of the form  $7 - -5$ .

A recap of the Year 5 steps relating to this topic may be useful.

### Things to look out for

- When calculating intervals, children may count the divisions rather than the number of intervals.
- Children may have heard “rules” such as “two minuses make a plus” and mistakenly think that, for example,  $-3 - 2 = +5$
- Because 5 is greater than 3, children may think that  $-5$  is greater than  $-3$

## Key questions

- What is the same and what is different about the numbers 2 and  $-2$  (negative two)?
- How far is  $-5$  from zero? How far is  $-5$  from 1?
- Which is the greater temperature,  $-1$  degrees or  $-2$  degrees?
- How do you find the difference between two negative numbers?
- How do you find the difference between a positive number and a negative number?
- What is the same and what is different about counting forwards/backwards along a number line beyond zero?

## Possible sentence stems

- To find the number \_\_\_\_\_ greater/less than \_\_\_\_\_, I count \_\_\_\_\_ on the number line.
- \_\_\_\_\_ is \_\_\_\_\_ away from zero.

## National Curriculum links

- Use negative numbers in context, and calculate intervals across zero
- Solve number and practical problems that involve the above

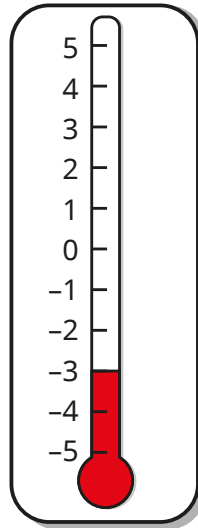
# Negative numbers

## Key learning

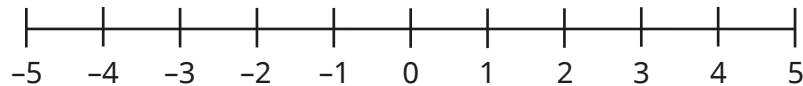
- What temperature does the thermometer show?

If the temperature drops by  $1^{\circ}\text{C}$ , what temperature will the thermometer show?

What temperature is  $5^{\circ}\text{C}$  warmer than the temperature shown on the thermometer?



- Use the number line to answer the questions.



What is 6 less than 4?

What is 5 more than  $-2$ ?

What is the difference between 3 and  $-3$ ?

- The table shows the temperatures in four places on a day in January.

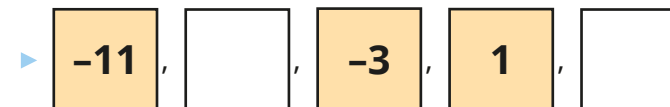
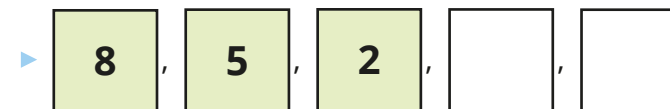
Bradford	$2^{\circ}\text{C}$
Harlow	$-3^{\circ}\text{C}$
Aberdeen	$-7^{\circ}\text{C}$
Southampton	$4^{\circ}\text{C}$

Which place has the lowest temperature?

Work out the difference between the temperature in Harlow and the temperature in Southampton.

The next day the temperature in Bradford dropped by  $6^{\circ}\text{C}$ . Work out the new temperature in Bradford.

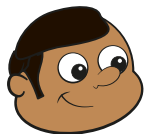
- Complete the number sequences.



# Negative numbers

## Reasoning and problem solving

A company has plans to construct a building with floors above and below ground.



If we build from floor -10 to floor 10, we will have 20 floors in total.

Do you agree? Explain your answer.

No  
There will be 21 floors as you need to include floor zero.

Find different ways of completing the calculation.

$$\underline{\quad} + \underline{\quad} = -2$$

multiple possible answers, e.g.  
 $-6 + 4$      $-80 + 78$   
 $-5 + 3$      $-2 + 0$

Is each statement always true, sometimes true or never true?

When you count forwards in tens from a positive 1-digit number, the final digits of all the numbers are the same.

When you count backwards in tens from a positive 1-digit number, the final digits of all the numbers are the same.

Give examples to support your answers.

What patterns can you see?

The first statement is always true (e.g. 8, 18, 28, 38 ...). Adding tens does not affect the ones column.

The second statement is sometimes true. It is true when we start at 5 (5, -5, -15, -25 ...), but false from every other number (e.g. 8, -2, -12, -22 ... or 7, -3, -13, -23 ...).



Autumn Block 2

**Addition, subtraction,  
multiplication and division**

## Small steps

Step 1

Add and subtract integers

Step 2

Common factors

Step 3

Common multiples

Step 4

Rules of divisibility

Step 5

Primes to 100

Step 6

Square and cube numbers

Step 7

Multiply up to a 4-digit number by a 2-digit number

Step 8

Solve problems with multiplication

## Small steps

Step 9

Short division

Step 10

Division using factors

Step 11

Introduction to long division

Step 12

Long division with remainders

Step 13

Solve problems with division

Step 14

Solve multi-step problems

Step 15

Order of operations

Step 16

Mental calculations and estimation



## Small steps

Step 17

Reason from known facts

# Add and subtract integers

## Notes and guidance

This small step reviews and extends children's learning of how to add and subtract integers with any number of digits.

Children use the formal column method for numbers with the same and different numbers of digits. They also practise mental strategies with both large and small numbers, using their understanding of place value.

Children solve multi-step problems, choosing which operations and methods to use based on the context of the problem and the types of numbers involved.

The use of concrete manipulatives can support children's understanding, especially where exchanges are required.

## Things to look out for

- Children may not line the numbers up correctly when setting out an addition or a subtraction.
- Children may try to use formal methods when mental strategies would be more appropriate, for example adding 999 is more easily done by adding 1,000 and then subtracting 1
- When solving multi-step problems, children may need support to choose the type and order of operations needed.

## Key questions

- What is the greatest digit you can have in a place value column?
- How do you exchange when adding?
- How do you exchange when subtracting?
- Which columns are affected by the exchange?
- How do you know whether to add or subtract the numbers?
- How can you check your answer to the calculation?

## Possible sentence stems

- In column addition/subtraction, we start with the \_\_\_\_\_ place value column.
- The \_\_\_\_\_ is in the \_\_\_\_\_ column. It represents \_\_\_\_\_

## National Curriculum links

- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving addition, subtraction, multiplication and division
- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

# Add and subtract integers

## Key learning

- Work out the additions.

	6	2	3		
+	3	5	8		
<hr/>					

	5	6	4	7	
+		8	6	1	
<hr/>					

	3	4	6	0	8		
+	2	9	0	8	7		
<hr/>							

- Work out the subtractions.

	7	5	2		
-	3	1	5		
<hr/>					

	8	1	6		
-	5	3	9		
<hr/>					

	3	4	6	0	8		
-	1	2	7	2	7		
<hr/>							

- Find the answers to the calculations.

	3	4	6	2	1		
+	2	5	7	3	4		
<hr/>							

	4	7	6	1	3	2	5
-		9	3	8	0	5	2
<hr/>							

- Which calculations would you work out mentally, and which would you work out using the column method?

$67,832 + 5,258$
------------------

$834,501 - 299,999$
---------------------

$450,000 + 201,000$
---------------------

8 million subtract $3\frac{1}{2}$ million
---

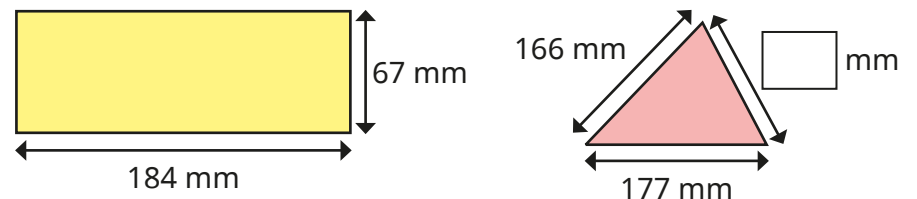
$604,000 - 25,000$
--------------------

Work out the answers to the calculations.

- Find the missing digits.

	5	2	2	4	7		
+	3		5	9	0	4	
<hr/>							
	9	0		3		2	

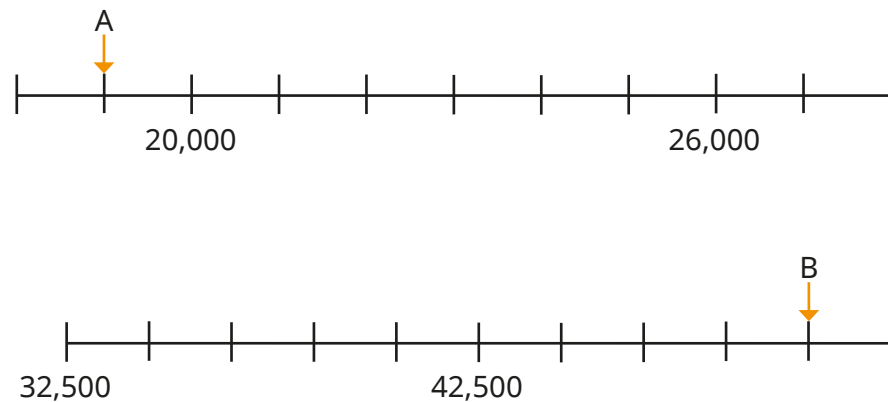
- The perimeter of the triangle is equal to the perimeter of the rectangle. Work out the unknown length of the triangle.



# Add and subtract integers

## Reasoning and problem solving

Find the difference between A and B.

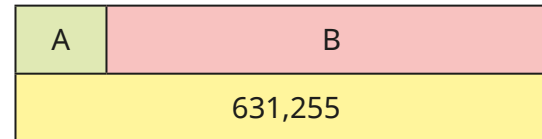


Explain your method to a partner.



31,500

Here is a bar model.



- A is an odd integer that rounds to 100,000 to the nearest 10,000
- The sum of the digits of A is 30
- B is an even integer that rounds to 500,000 to the nearest 100,000
- The sum of the digits of B is 10
- A and B are both multiples of 5

What could be the values of A and B?

Explain your reasoning to a partner.



multiple possible answers, e.g.

A = 99,255

B = 532,000

## Common factors

### Notes and guidance

This small step reinforces children's understanding of factors and common factors, introduced in Years 4 and 5 respectively.

Some children may still choose to use arrays and other representations, but knowledge of times-tables and the use of familiar rules of divisibility are to be encouraged. The rules of divisibility will be reviewed again later in the block.

Children work systematically to find the complete list of factors of a number, and learn to use their knowledge that factors usually come in pairs to spot missing factors.

Children are not required to formally identify the highest common factor of two or more numbers, but can be extended to consider this idea.

### Things to look out for

- Children may confuse the ideas of factors and multiples.
- Children may not be familiar with the use of the word "common" in this context.
- Errors may be made with times-tables, resulting in incorrect factors.
- Children may forget 1 and the number itself when listing factors.

### Key questions

- What are the factors of \_\_\_\_\_?
- What factors do \_\_\_\_\_ and \_\_\_\_\_ have in common?
- How can you easily tell if 2/5/10 is a factor of a number?
- If you know one factor of a number, how can you use it to find another factor of the number?
- Is 1 a factor of all numbers?
- How can you work systematically to find all the factors of a number?

### Possible sentence stems

- \_\_\_\_\_ is a factor of all numbers.
- The largest factor of a number is always \_\_\_\_\_
- \_\_\_\_\_ is a factor of \_\_\_\_\_ because \_\_\_\_\_ is in the \_\_\_\_\_ times-table.

### National Curriculum links

- Identify common factors, common multiples and prime numbers
- Solve problems involving addition, subtraction, multiplication and division

# Common factors

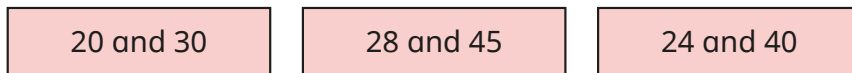
## Key learning

- List the factors of 24

List the factors of 36

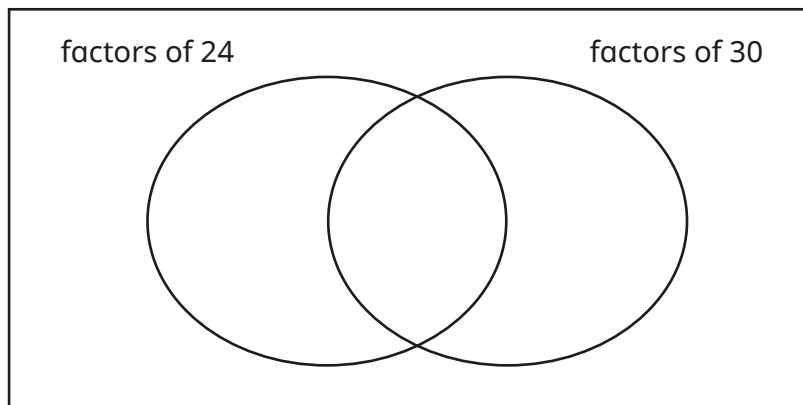
What are the common factors of 24 and 36?

- Find the common factors of each pair of numbers.



- Write the numbers in the sorting diagram.

1 2 3 4 5 6 8 10 12 15 24 30



List the common factors of 24 and 30

- Decide if each statement is true or false.

5 is a factor of both 95 and 75

3 is a common factor of 45 and 54

4 is not a common factor of 56 and 80

- Here is a table for sorting numbers.

Write one number in each box.

	Factor of 6	Not a factor of 6
Factor of 9		
Not a factor of 9		

Compare answers with a partner.

- Find the common factors of 300, 400 and 500
- The common factors of two numbers are 1, 3 and 5  
What could the two numbers be?

## Common factors

### Reasoning and problem solving

A fruit stall has 49 pears and 56 oranges.



The pieces of fruit are put into boxes with an equal number of pears or oranges in each box.

Tiny



There will be 8 pieces of fruit in each box.

There will be 7 pieces of fruit in each box.



Jack

Who is correct, Tiny or Jack?

Explain how you know.



Jack

Brett has two pieces of string.



One is 160 cm long and the other is 200 cm long.

He cuts them both into smaller pieces.

All the pieces are the same length.

What are the possible lengths of the smaller pieces of string?

1 cm, 2 cm, 4 cm,  
5 cm, 8 cm, 10 cm,  
20 cm, 40 cm

Dani has 54 red sweets and 45 green sweets.



She puts them into bags so that each bag has an equal number of red sweets and an equal number of green sweets.

What is the greatest number of bags she can make?

How many sweets of each colour will there be in each bag?

9 bags, each with  
6 red sweets and  
5 green sweets

# Common multiples

## Notes and guidance

Children are familiar with the idea of multiples of numbers from earlier study of times-tables. Building on this knowledge, they now find common multiples of two or more numbers.

As with factors, arrays and other representations may still be used as support, but knowledge of times-tables is key. Some multiples can be recognised using the rules of divisibility, which are explored in detail in the next small step.

Encourage children to work systematically to find lists of multiples rather than just finding the product of the given numbers, as this may miss some multiples.

Children do not need to be able to formally identify the lowest common multiple of two or more numbers, but can be challenged to consider the first common multiple of a pair of numbers.

## Things to look out for

- Children may confuse the ideas of factors and multiples.
- Errors may be made with times-tables, resulting in incorrect factors.
- A common misconception is that the only common multiple of a pair of numbers is the product of the numbers.

## Key questions

- How do you find the multiples of a number?
- What multiples do \_\_\_\_\_ and \_\_\_\_\_ have in common?
- What is the difference between a multiple and a factor?
- Can a number be both a factor and a multiple of another number?
- How can you tell if a number is a multiple of 2/5/10?
- When do numbers have common multiples that are less than their product?

## Possible sentence stems

- The first multiple of a number is always \_\_\_\_\_
- \_\_\_\_\_ is a multiple of \_\_\_\_\_ because \_\_\_\_\_  $\times$  \_\_\_\_\_ = \_\_\_\_\_
- \_\_\_\_\_ is a common multiple of \_\_\_\_\_ and \_\_\_\_\_

## National Curriculum links

- Identify common factors, common multiples and prime numbers
- Solve problems involving addition, subtraction, multiplication and division



# Common multiples

## Key learning

- Here is a hundred square.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Shade the multiples of 6

Circle the multiples of 5

What common multiples of 5 and 6 do you find?

Use these numbers to find other common multiples of 5 and 6

- Find the first three common multiples of each pair of numbers.

4 and 5

5 and 6

4 and 8

6 and 8

- Find five common multiples of 4 and 3

- Here is a table for sorting numbers.

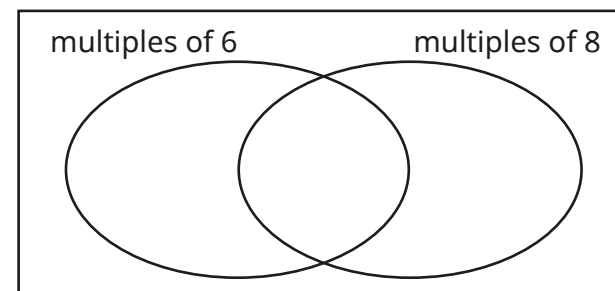
Write one number in each box.

	Multiple of 8	Not a multiple of 8
Multiple of 5		
Not a multiple of 5		

Compare answers with a partner.

- Write the numbers in the sorting diagram.

12 18 40 6 48 24 16 42 56 54 30



- Nijah plays football every 4 days and Kim plays football every 6 days.

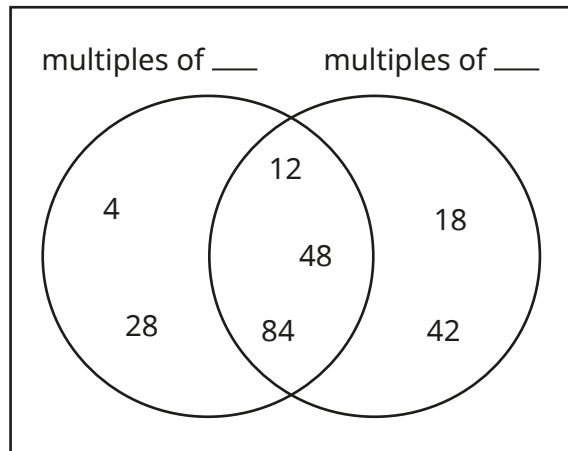
They both played football today.

In how many days will they next both play football on the same day?

# Common multiples

## Reasoning and problem solving

Complete the labels of the sorting diagram.



Write another number in each section.  
 Find a square number that will go in the middle section.  
 Compare answers with a partner.



various possible answers, e.g. multiples of 4, multiples of 6

multiple possible answers, e.g. 40, 72, 66

36, 144

Ms Fisher's age is double her sister's age.



They are both older than 20 but younger than 50

Their ages are both multiples of 7

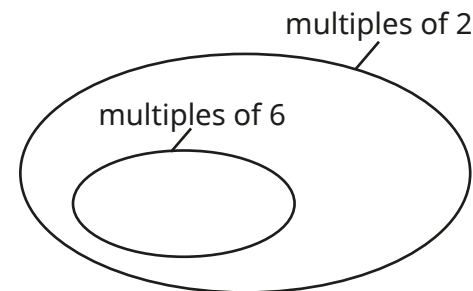
What are their ages?

Ms Fisher is 42 and her sister is 21

Write the numbers in the sorting diagram.



- 10
- 12
- 14
- 16
- 18
- 20



multiples of 2:  
10, 12, 14, 16, 18, 20  
 multiples of 6:  
12, 18

# Rules of divisibility

## Notes and guidance

Children should be familiar with most rules of divisibility from looking at patterns in times-tables in their earlier learning and the previous two steps.

Children recognise divisibility by 2, 5 or 10 by looking at the ones digits of a number. They know a number is divisible by 4 if halving the number gives an even result and the corresponding rule for divisibility by 8. They know that numbers are divisible by 3 if the sum of their digits is divisible by 3, and divisible by 9 if the sum of their digits is divisible by 9

Children now learn to combine these rules to deal with other potential factors, for example to be divisible by 6 a number must be divisible by both 2 and 3

Children should recognise that a 2-digit number is divisible by 11 if the digits are the same.

## Things to look out for

- Children may over-generalise rules, for example incorrectly applying the digit-sum rule for 3 and 9 or the final-digit rule for 5 to other numbers.
- Children may need support in understanding the combining of rules such as “a number is divisible by 12 if it is divisible by both 3 and 4”

## Key questions

- How does the ones digit help you to decide if a number is divisible by 2, 5 or 10?
- How can you use the rule for divisibility by 2 to find out if a number is divisible by 4/8?
- What two other numbers must a number be divisible by if the number is divisible by 6/12?
- How can you tell if a 2-digit number is divisible by 11?
- Which divisibility rules are based on the sum of the digits of a number?

## Possible sentence stems

- If a number is divisible by \_\_\_\_\_ and \_\_\_\_\_, then the number must also be divisible by \_\_\_\_\_
- If the sum of the digits is divisible by \_\_\_\_\_, then the number is divisible by \_\_\_\_\_
- A number is divisible by \_\_\_\_\_ if its ones digit is \_\_\_\_\_

## National Curriculum links

- Solve problems involving addition, subtraction, multiplication and division

# Rules of divisibility

## Key learning

- Which of the numbers are divisible by 2?

62	901	5,462
10,308	111,111	224,528

Which of the numbers are also divisible by 4? How can you tell?

- Use the digit sums to decide which numbers are divisible by 3 and which are also divisible by 9

78	801	5,460
12,307	555,222	48,117

- Find a number that matches each description.

a 3-digit number that is divisible by 5

a 6-digit number that is divisible by 10

a 4-digit number that is divisible by 5 and 3

a 5-digit number that is divisible by 3 but not divisible by 5

- Scott is packing cakes into boxes.

He puts an equal number of cakes into each box with no cakes left over.

He has 1,032 cakes to pack.

How many cakes can go in each box?



- Use ticks and crosses to complete the table.

	Is the number divisible by ...?				
	3	4	6	9	11
87					
96					
99					
216					
702					

- The children at a school all have lunch at the same time.

There are 672 children and an equal number of them sit at each table.

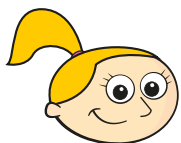
No more than 12 children sit at a table.

How many tables could there be?

# Rules of divisibility

## Reasoning and problem solving

The year number of a leap year is divisible by 4



If the final two digits of a number are divisible by 4, then the number itself is divisible by 4

Use Eva's rule to find out which of these years were, or will be, leap years.

1536

1674

1928

1992

2024

2050

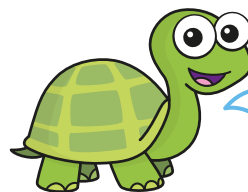
2062

2956

Why does this rule work?

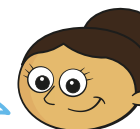
1536, 1928, 1992, 2024, 2956

Tiny and Dora are talking about rules for division.



Tiny

If a number is divisible by 10, then it must also be divisible by 5



Dora

If a number is divisible by 5, then it must also be divisible by 10

Tiny is correct.  
Dora is incorrect.

Do you agree with Tiny and Dora?

Explain your answer.

# Primes to 100

## Notes and guidance

Children first encountered prime numbers and composite numbers in Year 5. This small step reviews that learning and develops children's knowledge of factors so that they can deepen their understanding of prime numbers.

Children recognise that a number is prime when it has exactly two factors: 1 and itself. They also look at identifying the prime factors of a given number.

By the end of this step, children should be able to identify all the primes less than 100 and recall at least the primes to 19

Children should be familiar with square and cube numbers from earlier years, so this is something that can be revisited here, but is also covered in detail in the next small step.

### Things to look out for

- A common misconception is that 1 is a prime number.
- Children may think that all prime numbers are odd and not realise that 2 is a prime number.
- Numbers that are outside times-tables knowledge (e.g. 51) may be mistakenly thought of as prime. Encourage children to use divisibility rules from the previous step to check these.

## Key questions

- What is a prime number?
- What is a composite number?
- How many factors does a prime number have?
- Why is 1 not a prime number?
- How can you find the prime factors of a number?
- Are the multiples of prime numbers also prime?

## Possible sentence stems

- The factors of \_\_\_\_\_ are \_\_\_\_\_  
The prime factors of \_\_\_\_\_ are \_\_\_\_\_
- \_\_\_\_\_ is prime because it has exactly \_\_\_\_\_ factors.
- \_\_\_\_\_ is a composite number because \_\_\_\_\_ = \_\_\_\_\_ × \_\_\_\_\_

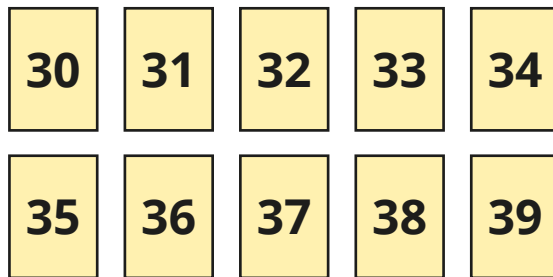
## National Curriculum links

- Identify common factors, common multiples and prime numbers
- Solve problems involving addition, subtraction, multiplication and division

# Primes to 100

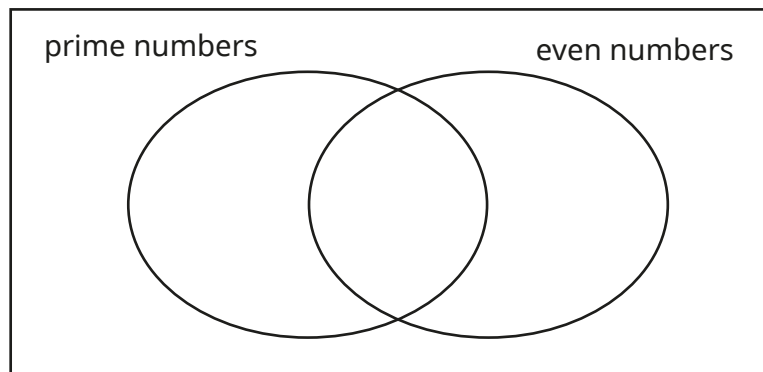
## Key learning

- List all the prime numbers that are less than 20
- Which of these numbers are prime and which are composite?



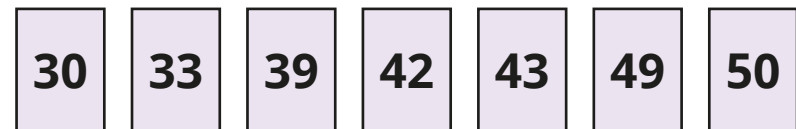
- Explain how you know 51 is a composite number.
- Write the numbers in the sorting diagram.

10 13 2 12 11 6 7



- List the factors of 20  
Which factors of 20 are prime?


- Find the prime factors of the numbers.




- The sum of two prime numbers is 36  
What might the numbers be?  
How many different answers can you find?
- Write the three prime numbers that multiply to make 105  
 $\text{_____} \times \text{_____} \times \text{_____} = 105$
- List the numbers from 40 to 49  
Which of the numbers are prime?  
Which of the numbers are square?  
Which of the numbers are composite?

# Primes to 100

## Reasoning and problem solving


Ron is thinking of a number. 

 I am thinking of a number greater than 10


Use the clues to work out Ron's number.


- It is a composite number.
- It has two prime factors.
- It is an odd number.
- It is a factor of 60

15

Shade the multiples of 6 on a hundred square. 

What do you notice about all the numbers either side of the multiples of 6?

 I think that there is always a prime number next to a multiple of 6

Is Whitney correct?  
Explain your reasoning. 

All the numbers next to a multiple of 6 are odd.

---

Yes



# Square and cube numbers

## Notes and guidance

Children encountered square and cube numbers in Year 5, and this small step revisits that learning and the notation for squared ( $^2$ ) and cubed ( $^3$ ).

The concept of square and cube numbers can be supported by making links to area and volume (the formula for the volume of a cuboid will be covered next term).

Children explore the factors of square and cube numbers, noticing that square numbers always have an odd number of factors, but cube numbers can have an odd or even number of factors.

The vocabulary of earlier small steps in this block, such as “factor”, “multiple” and “prime” can also be reinforced at this stage.

## Things to look out for

- Children may confuse the idea of squaring/cubing with multiplying by  $2/3$
- Children may not realise that 1 is both a square number and a cube number.

## Key questions

- How do you square a number?
- How do you cube a number?
- Are the squares of even/odd numbers even or odd?
- Are the cubes of even/odd numbers even or odd?
- Can a number be both a square number and a cube number?
- How can you use a square number to help find a cube number?

## Possible sentence stems

- To square a number, you multiply the number by \_\_\_\_\_
- To cube a number, you multiply the number by \_\_\_\_\_ and then by \_\_\_\_\_ again.
- I know \_\_\_\_\_ is a square/cube number because ...

## National Curriculum links

- Solve problems involving addition, subtraction, multiplication and division

# Square and cube numbers

## Key learning

- The table shows some square numbers and cube numbers.

Complete the table and describe any patterns and connections you notice. The first row has been done for you.

$1^2$	$1 \times 1$	1	$1^3$	$1 \times 1 \times 1$	1
					8
	$3 \times 3$		$3^3$		27
	$4 \times 4$			$4 \times 4 \times 4$	
		25	$5^3$		
				$6 \times 6 \times 6$	
$8^2$					

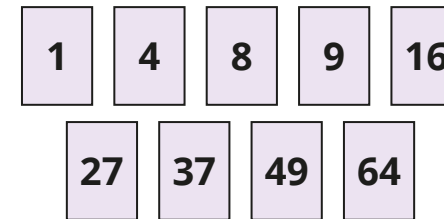
- Write  $>$ ,  $<$  or  $=$  to make the statements correct.

$$3^3 \bigcirc 4^2$$

$$8^2 \bigcirc 4^3$$

$$11^2 \bigcirc 5^3$$

- Here are some number cards.



Which numbers are square?

Which numbers are cube?

Which numbers are both square and cube?

Which numbers are prime?

- List the factors of the first five square numbers.

How many factors do they each have?

What do you notice about the number of factors a square number has?

Is the same true for cube numbers?

- $\bullet + \blacktriangle = 38$

$\bullet$  is a cube number.

$\blacktriangle$  is a prime number.

Find pairs of values for  $\bullet$  and  $\blacktriangle$ .

## Square and cube numbers

## Reasoning and problem solving

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Shade all the square numbers.

Use a different colour to shade the multiples of 4

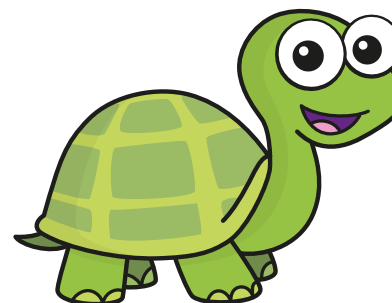
What do you notice?



Square numbers are always a multiple of 4 or one greater than a multiple of 4



Square numbers only end in 1, 4, 5, 6 or 9, but cube numbers can end in any number.



Do you agree with Tiny?

Tiny is correct about cube numbers, but square numbers can also end in zero, for example  $10^2 = 100$

# Multiply up to a 4-digit number by a 2-digit number

## Notes and guidance

Building on their learning from previous years, children use long multiplication to multiply numbers with up to four digits by 2-digit numbers.

Children should already be aware that multiplication is commutative, so answers to calculations such as  $56 \times 1,234$  can be found by rewriting as  $1,234 \times 56$  and using the standard format.

Children also solve word problems and/or multi-step problems. This will be revisited in the next step, where alternative strategies are also explored, for example for multiplying by 9 or 99

Children who require additional support may benefit from revising multiplication of 2- or 3-digit numbers by a single digit before moving on to multiplication by a 2-digit number.

## Things to look out for

- Children may omit the zero needed in the second line of a long multiplication.
- Children need to be secure with their times-tables, or have strategies for deriving them.
- When regrouping, children may misapply the procedure, particularly when a large number of digits are involved in the calculation.

## Key questions

- How do you set out a long multiplication?
- Which number do you multiply by first?
- What is important to remember when you begin to multiply by the tens digit?
- When do you need to make an exchange? How do you do this?
- What happens if there is an exchange needed in the last step of the calculation?

## Possible sentence stems

- To multiply by a 2-digit number, first multiply by the \_\_\_\_\_, then multiply by the \_\_\_\_\_ and then find the \_\_\_\_\_
- Multiplying by \_\_\_\_\_ is the same as multiplying by \_\_\_\_\_ and then multiplying the answer by \_\_\_\_\_

## National Curriculum links

- Multiply multi-digit numbers up to four digits by a 2-digit whole number using the formal written method of long multiplication
- Solve problems involving addition, subtraction, multiplication and division

# Multiply up to a 4-digit number by a 2-digit number

## Key learning

- Work out  $43 \times 6$   
Use your answer to find the answer to  $43 \times 60$

- Complete the calculations.

			2	3	
	×		6	4	
			9	2	
	+			0	

( $23 \times 4$ )  
( $23 \times 60$ )

			3	1	2
	×			2	3
			9	3	6
	+				

( $312 \times 3$ )  
( $312 \times 20$ )

- Work out the multiplications.

			4	2	6	7
	×				3	4

			3	0	4	6
	×				7	3

- 2,465 people buy tickets for a festival.  
Each ticket costs £48  
How much is spent altogether on the tickets?



- Work out the multiplications.

$$17 \times 562$$

$$23 \times 3,164$$

$$41 \times 5,312$$

- Huan receives a new comic book every month.  
Each book has 36 pages.  
He reads a comic book once a month for 6 years.  
How many pages does Huan read altogether?
- There are 27 classes in a school.  
There are 32 children in each class.  
Can all the children in the school sit in a cinema with 1,000 seats?  
If yes, how many spare seats will there be?  
If no, how many more seats are needed?

## Multiply up to a 4-digit number by a 2-digit number

## Reasoning and problem solving

The product of a 4-digit number and a 2-digit number will always have at least six digits.



No

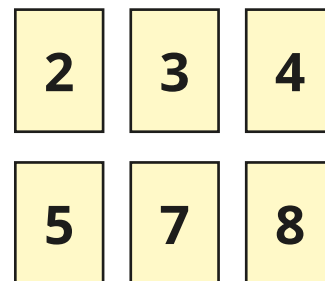
Do you agree with Dexter?  
Explain your answer.



What is the product of the greatest 4-digit number and the greatest 2-digit number?

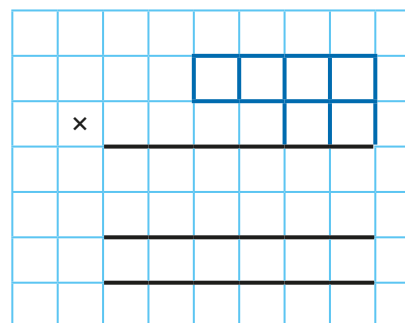


989,901



Write the digits in the boxes to find the greatest product.

You can use each digit once only.



$$8,432 \times 75 = 632,400$$

# Solve problems with multiplication

## Notes and guidance

In this small step, children use the column method for multiplication and explore alternative strategies for solving multiplication problems, including word problems.

Children use their knowledge of multiplying by powers of 10 and adjust calculations: for example, instead of multiplying a number by 99, they multiply the number by 100 and then subtract the number from the product.

Children explore using factors to find the answers to multiplication problems, such as multiplying by 5 and then by 7 as an alternative to multiplying by 35. This is a useful strategy for children who have good times-table knowledge but make errors with the algorithm for long multiplication.

### Things to look out for

- Children may try to use formal methods when alternative strategies would be more appropriate.
- Children may need support to identify the most efficient method, for example  $\times 100$  subtract  $\times 1$  may be better than  $\times 90$  add  $\times 9$
- When using the factorisation method, children may forget to multiply the first product by the second factor.

## Key questions

- What is the quickest way of multiplying whole numbers by 10/100/1,000?
- What number is 99 close to? How does this help you to multiply by 99?
- If you double a number and then double it again, what is the overall effect on the original number?
- What factor pairs have a product of \_\_\_\_\_? How does this help you to multiply by \_\_\_\_\_? Which factor pair is easiest to use?

## Possible sentence stems

- To multiply by \_\_\_\_\_, I can multiply by \_\_\_\_\_ and add/subtract \_\_\_\_\_ to/from the product.
- \_\_\_\_\_ = \_\_\_\_\_  $\times$  \_\_\_\_\_, so to multiply by \_\_\_\_\_ I can multiply by \_\_\_\_\_ and then multiply the product by \_\_\_\_\_

## National Curriculum links

- Perform mental calculations, including with mixed operations and large numbers
- Solve problems involving addition, subtraction, multiplication and division

# Solve problems with multiplication

## Key learning

- Work out the multiplications.

$78 \times 10$

$63 \times 100$

$56 \times 1,000$

Use your answers to work out these multiplications.

$78 \times 9$

$63 \times 99$

$56 \times 999$

- Office chairs cost £99

A company buys 38 chairs for its offices.

How much does the company pay altogether?

In a sale, the price of the chairs is reduced to £79

How much do 38 chairs cost at the sale price? How can you use your first answer to help you?

- Here is a strategy for multiplying numbers by 5

Multiply the number by 10 and find half of the answer.

Use the strategy to work out the multiplications.

$84 \times 5$

$628 \times 5$

$8,206 \times 5$

$3,512 \times 5$

Why does the strategy work?

- Explain why  $83 \times 4 = 83 \times 2 \times 2$

Find the missing numbers.

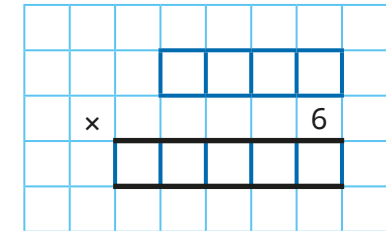
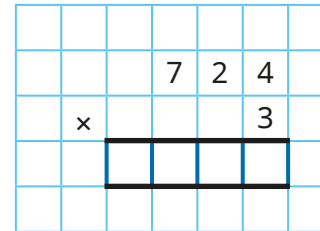
$37 \times 14 = 37 \times 2 \times \underline{\quad}$

$812 \times 25 = 812 \times 5 \times \underline{\quad}$

$256 \times 15 = 256 \times \underline{\quad} \times \underline{\quad}$

$902 \times 56 = \underline{\quad} \times \underline{\quad} \times 8$

- Complete the calculations to work out  $724 \times 18$



Find a different way to work out  $724 \times 18$

- Find the missing numbers.

$63 \times 24 = 63 \times 4 \times \underline{\quad}$

$63 \times 24 = 63 \times 3 \times \underline{\quad}$

Use both factorisations to work out  $63 \times 24$

Which strategy did you find easier?

Use similar strategies to work out the multiplications.

$84 \times 15$

$326 \times 45$

$612 \times 42$

$3,592 \times 32$



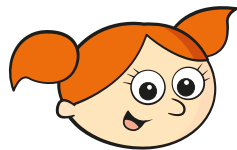
# Solve problems with multiplication

## Reasoning and problem solving

Alex is working out  $6,412 \times 16$



I'm going to keep doubling 6,412 until I have found  $6,412 \times 16$



How many calculations will Alex have to do?

Use Alex's method to find  $6,412 \times 16$

How else could Alex multiply by 16?

Talk about it with a partner.



four calculations

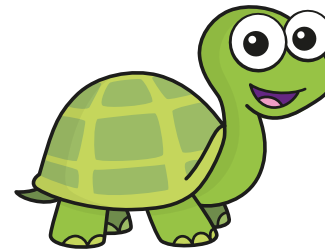
$$6,412 \times 2 = 12,824$$

$$6,412 \times 4 = 25,648$$

$$6,412 \times 8 = 51,296$$

$$6,412 \times 16 = 102,592$$

$35 = 1 \times 35$ ,  
so I can work out  
 $832 \times 35$  by multiplying by 1  
and then by multiplying  
by 35



Explain why Tiny's strategy is not a good one.

Use a different factor pair of 35 to work out  $832 \times 35$

Tiny's strategy is not good because you still have the same calculation of  $832 \times 35$  after multiplying by 1

$$35 = 5 \times 7$$

$$832 \times 5 = 4,160 \text{ and } 4,160 \times 7 = 29,120$$

or

$$832 \times 7 = 5,824 \text{ and } 5,824 \times 5 = 29,120$$

# Short division

## Notes and guidance

In Year 5, children built on earlier learning of short division and learned to divide numbers with up to four digits by single-digit numbers. This small step reinforces all this earlier learning in preparation for the upcoming steps on long division.

Children perform short divisions both with integer answers and where there is a remainder. They interpret the remainder in context, for example knowing that “4 remainder 1” could mean 4 complete boxes with 1 left over so 5 boxes will be needed.

Children may need to list multiples of the number they are dividing by to help them if their times-table knowledge is not secure.

### Things to look out for

- Children need to be confident with their times-tables “both ways”, i.e. knowing division facts as well as multiplication facts.
- Children may not recognise sharing and/or grouping division problems when presented in words.
- Numbers with placeholders (e.g. 80,320) may cause difficulty for children.
- Children may not be able to interpret the remainder.

## Key questions

- How many groups of 4 \_\_\_\_\_ are there in 40/400/4,000?
- How many groups of 4 \_\_\_\_\_ are there in 80/800/8,000?
- What do you do with any remaining ones at the end of a division?
- If you cannot make a group in a column, what do you do?
- What does the remainder mean in this question?

## Possible sentence stems

- \_\_\_\_\_ thousands divided by \_\_\_\_\_ is equal to \_\_\_\_\_ thousands with a remainder of \_\_\_\_\_  
The remainder is exchanged into \_\_\_\_\_ hundreds.
- \_\_\_\_\_ hundreds divided by \_\_\_\_\_ is equal to \_\_\_\_\_ hundreds with a remainder of \_\_\_\_\_  
The remainder is exchanged into \_\_\_\_\_ tens.

## National Curriculum links

- Solve problems involving addition, subtraction, multiplication and division
- Divide numbers up to four digits by a 2-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

# Short division

## Key learning

- Work out the divisions mentally.

$8 \div 2$        $80 \div 2$        $800 \div 2$        $8,000 \div 2$   
 $12 \div 4$        $120 \div 4$        $1,200 \div 4$        $1,200 \div 3$

- Complete the short divisions.

	3	9	6				

	3	6	4	2			

	3	5	1	2	7		

- Here is  $8,524 \div 4$  shown using place value counters and short division.

Th	H	T	O

		2	1	3	1		
	4	8	5	2	4		

Use this method to work out the divisions.

$5,520 \div 4$	$6,432 \div 3$	$2,665 \div 5$
----------------	----------------	----------------

- Complete the short divisions.

	3	8	6				

	5	6	7	3			

	4	5	3	2	2		

- 1,480 pencils are grouped into packets of 5  
How many groups of 5 pencils are there?



- 650 children from a school go to a theme park.  
On the first ride, each car seats 4 children.

How many cars are needed for the whole school to go on the first ride?

On the second ride, each car seats 6 children.

How many cars are needed for the whole school to go on the second ride?

- Tickets to see the school play cost £9

How many tickets can be bought with £100?

How many tickets can be bought with £350?

# Short division

## Reasoning and problem solving

Here are three bar models. They are not drawn to scale.

Work out the value of C.

A = 1,650  
B = 550  
C = 110

Work out the missing digits.

Work out the divisions.

275 ÷ 11      3,366 ÷ 11

6,036 ÷ 12      2,356 ÷ 12

Compare methods with a partner.

25  
306  
503  
196 r4

# Division using factors

## Notes and guidance

In this small step, children build on their understanding of using factors in multiplication and learn to divide by a 2-digit number using repeated division.

Children start with the familiar strategy that to divide by 4 they can halve and halve again. They move on to dividing by multiples of 10 before looking at slightly more complex divisions using two single-digit factors. It may be worth revising what factor pairs are and practising finding factor pairs of 2-digit numbers. Children need to be aware that the divisions can be carried out in any order. This means they can choose to divide first by the factor they find it easier to work with, and then by the factor they find more difficult.

## Things to look out for

- Children may partition the number they are dividing by into tens and ones instead of using factors.
- Children may factorise the number they are dividing by incorrectly.
- Children may need support identifying the most efficient pair of factors to use.
- Children may identify 1 and the number itself as a pair of factors and should recognise that this does not simplify the calculation.

## Key questions

- What does the word “factor” mean?
- What are the factors of the number you are dividing by?
- What numbers do you find it easy to divide by?
- How can you check your answer?
- Which factor are you going to divide by first/second? Why?

## Possible sentence stems

- Dividing by 4 is the same as dividing by \_\_\_\_\_ and \_\_\_\_\_ again.
- The factor pairs of \_\_\_\_\_ are \_\_\_\_\_
- To divide by \_\_\_\_\_, I can first divide by \_\_\_\_\_ and then divide the answer by \_\_\_\_\_
- \_\_\_\_\_ = \_\_\_\_\_ × \_\_\_\_\_, so to divide by \_\_\_\_\_ I can divide by \_\_\_\_\_ and then divide the answer by \_\_\_\_\_

## National Curriculum links

- Solve problems involving addition, subtraction, multiplication and division

# Division using factors

## Key learning

- Take 20 counters and share them into two equal groups.

Share each of these groups into two equal groups.

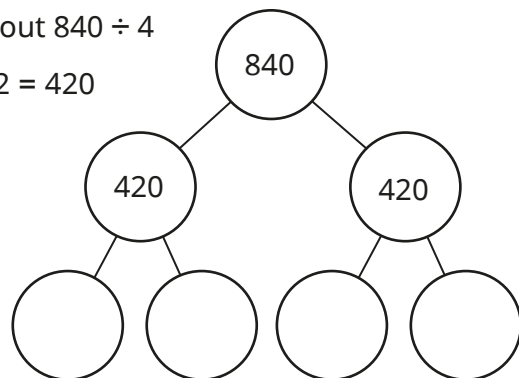
How many groups have you got now?

Complete the calculation.

$$20 \div 2 \div 2 = 20 \div \underline{\quad} = \underline{\quad}$$

- Esther is working out  $840 \div 4$

She knows  $840 \div 2 = 420$



How can Esther use this fact to help find  $840 \div 4$ ?

- 80 counters are divided into 10 equal groups.

How many counters are there in each group?

The counters are then shared into 2 equal groups.

How many counters are there in each group now?

- Complete the calculations.

▶  $600 \div 30 = 600 \div 10 \div \underline{\quad} = 60 \div \underline{\quad} = \underline{\quad}$

▶  $600 \div 20 = 600 \div 10 \div \underline{\quad} = 60 \div \underline{\quad} = \underline{\quad}$

▶  $600 \div 40 = 600 \div 10 \div \underline{\quad} = 60 \div \underline{\quad} = \underline{\quad}$

- Work out the divisions.

$900 \div 30$	$640 \div 40$	$650 \div 50$	$540 \div 20$
---------------	---------------	---------------	---------------

- Find  $720 \div 15$  by firstly dividing 720 by 5 and then dividing the result by 3

Why does dividing a number by 5 and then dividing by 3 give you the same answer as dividing the number by 15?


Use this strategy to work out the divisions.

$570 \div 15$	$560 \div 14$	$720 \div 18$
$725 \div 25$	$560 \div 14$	$1,176 \div 24$

Can any of the divisions be done in more than one way?

# Division using factors

## Reasoning and problem solving



To calculate  $4,320 \div 15$ , I will first divide 4,320 by 5 and then divide the answer by 10

Explain why Tommy is wrong.

Tommy has partitioned 15 into  $5 + 10$  instead of using the factor pair  $3 \times 5 = 15$

Dividing by 5 and then dividing by 10 is the same as dividing by 50

Use factor pairs to work out the divisions.

$1,248 \div 48$


$1,248 \div 24$

$1,248 \div 12$

What do you notice about your answers?

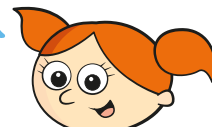
26, 52, 104

When the number you are dividing by is halved, the answer is doubled.




I'm going to work out  $4,632 \div 12$  by dividing 4,632 by 3 and then dividing the result by another number.

Annie



I'm going to work out  $4,632 \div 12$  by dividing 4,632 by 2 and then dividing the result by another number.

Alex



I'm going to work out  $4,632 \div 12$  using short division.

Amir

Compare the children's methods.

Children should compare the methods while also recognising that each child gets the same answer.

# Introduction to long division

## Notes and guidance

In this small step, children are introduced to long division as a different method for dividing by a 2-digit number, now including numbers that cannot be factorised into single-digit numbers.

Children divide 3-digit numbers without remainders, using an expanded method that shows the multiples, before progressing to a more formal long division method. They divide 4-digit numbers, still without remainders, using their knowledge of multiplying by 10 and 100. When dividing by composite numbers, it may be worth comparing the long division method with the method of division using factors covered in the previous small step.

Long division with remainders is covered in the next small step.

## Things to look out for

- Children may need support in setting out the long divisions, for example by providing the questions on pre-prepared squared grids with the questions already formatted.
- When dividing by prime numbers or large numbers, children may need support in working out the multiples of the number they are dividing by.

## Key questions

- How can you use multiples to divide by a 2-digit number?
- Why do we subtract as we go along?
- What does the arrow represent in the long division?
- Can this division be done using factors instead? Why or why not?
- What is the first step when performing a long division?

## Possible sentence stems

- \_\_\_\_\_ hundreds divided by \_\_\_\_\_ is equal to \_\_\_\_\_ hundreds with a remainder of \_\_\_\_\_  
The remainder is exchanged into \_\_\_\_\_ tens.
- \_\_\_\_\_ tens divided by \_\_\_\_\_ is equal to \_\_\_\_\_ with a remainder of \_\_\_\_\_  
The remainder is exchanged into \_\_\_\_\_ ones.

## National Curriculum links

- Divide numbers up to four digits by a 2-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- Solve problems involving addition, subtraction, multiplication and division



# Introduction to long division

## Key learning

- Here is  $360 \div 12$  using the long division method.

		0	3	6	
12		4	3	2	
		3	6	0	
			7	2	
			7	2	
				0	

(12 × 30)

(12 × 6)

**Multiples of 12:**  $12 \times 1 = 12$

$12 \times 2 = 24$

$12 \times 3 = 36$

$12 \times 4 = 48$

$12 \times 5 = 60$

$12 \times 6 = 72$

Use this method to work out the divisions.

750 ÷ 15	765 ÷ 17	702 ÷ 18
----------	----------	----------

- Here is a different way of setting out a long division.

		0	3	6	
12		4	3	2	
		3	6		
			7	2	
			7	2	
				0	

Use this method to work out the divisions.

836 ÷ 11	798 ÷ 14	608 ÷ 19
----------	----------	----------

- Here is  $7,355 \div 15$  using the long division method.

		0	4	8	9
15		7	3	3	5
		6	0	0	0
			1	3	3
			1	2	0
				1	3
				1	3
					0

(15 × 400)

(15 × 80)

(15 × 9)

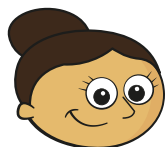
Use this method to work out the divisions.

2,208 ÷ 16	1,755 ÷ 45	1,536 ÷ 16
------------	------------	------------

- There are 1,989 players in a football tournament. Each team has 11 players and 2 reserves. How many teams are playing in the tournament?
- A farmer packs 8,280 eggs into cartons of 24. Use long division to find the number of cartons needed. Check your answer by dividing by factors.

# Introduction to long division

## Reasoning and problem solving



Dora

I'm going to work out  $6,756 \div 12$  by dividing 6,756 by 3 and then dividing the result by 4



Mo

I'm going to work out  $6,756 \div 12$  using long division.



Jack

I'm going to work out  $6,756 \div 12$  using short division.

Compare the children's methods and talk about your favourite with a partner.

Children should recognise that each child gets the same answer despite using different methods.

$$6,120 \div 17 = 360$$

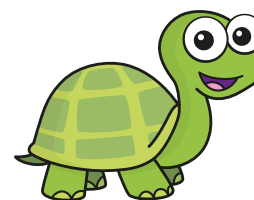


Use the given calculation to work out the missing number.

$$6,480 \div \underline{\quad} = 360$$

18

$1,950 \div 13$  is greater than  $1,950 \div 15$



Tiny is correct.

Find how much greater  $1,950 \div 13$  is than  $1,950 \div 15$

$1,950 \div 13$  is 20 greater than  $1,950 \div 15$

# Long division with remainders

## Notes and guidance

Now that children have learned to use the algorithm for long division with integer answers, they move on to long divisions with remainders.

This small step includes context questions where children interpret the remainder and/or adjust the number they are dividing. For example, when thinking about packing items into boxes, they consider the number of full boxes or the total number of boxes needed.

Children should always check that the remainder is less than the number they are dividing by. They can use estimation as a sense-check for their answers, for example  $834 \div 18$  is close to  $800 \div 20$  so the answer should be in the region of 40

## Things to look out for

- Children may need support in setting out the long divisions, for example by providing the questions on pre-prepared squared grids with the questions already formatted.
- When dividing by prime numbers or large numbers, children may need support in working out the multiples of the number they are dividing by.

## Key questions

- Why do we subtract as we go along?
- In a long division, what happens after the subtractions if you cannot divide exactly?
- What is the first step when performing a long division?

## Possible sentence stems

- \_\_\_\_\_ hundreds divided by \_\_\_\_\_ is equal to \_\_\_\_\_ hundreds with a remainder of \_\_\_\_\_  
The remainder is exchanged for \_\_\_\_\_ tens.
- \_\_\_\_\_ cannot be divided by \_\_\_\_\_, so there is a \_\_\_\_\_ of \_\_\_\_\_

## National Curriculum links

- Divide numbers up to four digits by a 2-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- Solve problems involving addition, subtraction, multiplication and division

# Long division with remainders

## Key learning

- Filip uses multiples to help divide 372 by 15

		0	2	4	r	12
15		3	7	2		
		3	0	0		
			7	2		
			6	0		
			1	2		

**Multiples of 15:**  $15 \times 1 = 15$   
 $15 \times 2 = 30$   
 $15 \times 3 = 45$   
 $15 \times 4 = 60$

(15 × 20)  
 (15 × 4)

Use Filip's method to work out the divisions.

$271 \div 17$	$623 \div 21$	$842 \div 32$
---------------	---------------	---------------

- Here is Aisha's method for finding 1,426 divided by 13

		0	1	0	9	r	9
13		1	4	2	6		
		1	3	0			
			1	2	6		
			1	1	7		
					9		

Use Aisha's method to work out the divisions.

$2,637 \div 16$	$4,453 \div 22$	$4,203 \div 18$
-----------------	-----------------	-----------------

- Mrs Hall needs 380 cupcakes for a party.



Cupcakes are sold in boxes of 15

How many boxes of cupcakes does she need to buy?

Will she have any cupcakes spare?

How do you know?

- One day, a bakery produces 7,849 biscuits.



The biscuits are packed into boxes of 64 biscuits.

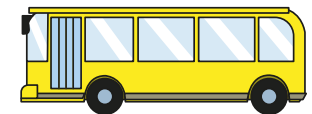
How many full boxes can be packed?

- 576 children and 32 adults need transport for a school trip.

A coach has seats for 55 people.

How many coaches are needed?

How many spare seats will there be?



- A portion of rice is 65 g.

How many portions can be served from an 8 kg bag of rice?

Will there be any rice left over?

If yes, how much?



# Long division with remainders

## Reasoning and problem solving

Which calculations will definitely have a remainder?



A  $8,164 \div 20$

B  $7,836 \div 15$

C  $4,678 \div 18$

D  $6,751 \div 12$

How do you know?



All the calculations will have a remainder.

Two digits are missing from the division.



					r	14	
18	6						

The missing digits are equal.

What must they be?

What could the digits be if they were not equal?

4 and 4

2 and 6

6 and 2

8 and 0

9 and 8

$835 \div 17 = 48 \text{ r}19$

Explain why the calculation cannot be correct.

The remainder cannot be greater than 17

# Solve problems with division

## Notes and guidance

In this small step, children explore division problems, looking at the most appropriate strategy for finding a solution.

As well as providing an opportunity to revisit the learning of the last few steps, children look at alternative methods such as partitioning the number into appropriate multiples of the number they are dividing by. They also use counting up in multiples, for example for calculations such as  $1,400 \div 200$ , and compare this with other strategies.

Encourage children to think about the numbers in a division question and to consider alternative strategies before they launch into a formal method.

Later in this block, children explore using known division facts to find other division or multiplication facts.

## Things to look out for

- Children may try to use formal methods when alternative strategies would be more appropriate.
- Children may try to apply strategies that work for multiplication to division situations where they do not work.
- Interpreting remainders in a given context can be challenging for children.

## Key questions

- What is the most useful way of partitioning the number?
- Would you use short division or long division? Why?
- If you double a number and then double it again, what is the overall effect on the original number?
- What factor pairs have a product of \_\_\_\_\_? How does this help you to divide by \_\_\_\_\_? Which factor pair is easiest to use?

## Possible sentence stems

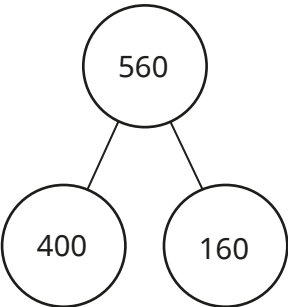
- I will partition the number into \_\_\_\_\_ and \_\_\_\_\_ because both \_\_\_\_\_ and \_\_\_\_\_ are divisible by \_\_\_\_\_
- \_\_\_\_\_ = \_\_\_\_\_  $\times$  \_\_\_\_\_, so to divide by \_\_\_\_\_ I can divide by \_\_\_\_\_ and then divide the quotient by \_\_\_\_\_

## National Curriculum links

- Perform mental calculations, including with mixed operations and large numbers
- Solve problems involving addition, subtraction, multiplication and division

# Solve problems with division

## Key learning

- 

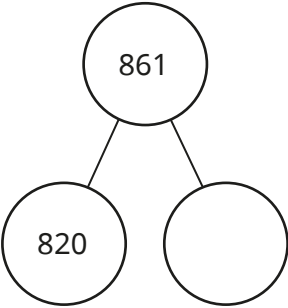
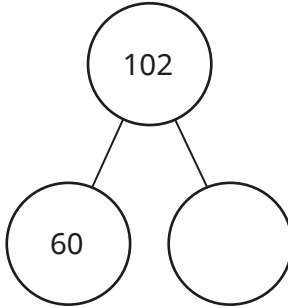
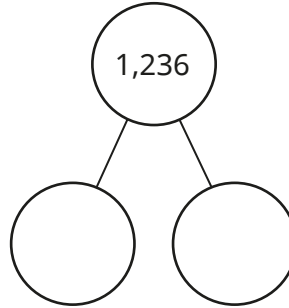
Complete the workings for  $560 \div 4$

$$400 \div 4 = \underline{\quad}$$

$$160 \div 4 = \underline{\quad}$$

So  $560 \div 4 = \underline{\quad} + \underline{\quad} = \underline{\quad}$

- Use partitioning to work out the divisions.

$861 \div 41$	$102 \div 6$	$1,236 \div 12$
		

- Which of the divisions can you work out mentally?

$340 \div 10$	$608 \div 2$	$500 \div 20$
$631 \div 1$	$2,100 \div 700$	$432 \div 18$

- Use your preferred method to work out the divisions.

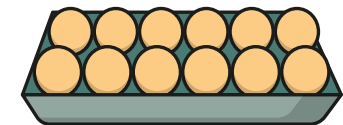
$780 \div 30$	$824 \div 4$	$900 \div 30$
$1,197 \div 21$	$4,200 \div 21$	$1,110 \div 15$

Did you use the same method for each question?

- Tom has saved £8 in 20p coins.  
How many 20p coins does Tom have?



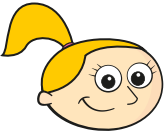
- Eggs are packed in trays of 12  
The trays are packed into boxes.  
Each box contains 480 eggs.  
How many trays are in each box?




- A builder needs 8,600 bricks to build a wall.  
There are 800 bricks in a load.  
How many loads must the builder buy?

# Solve problems with division

## Reasoning and problem solving




Eva



To divide a number by 5, I can divide the number by 10 and then halve the answer.

Ron



To divide a number by 5, I can divide the number by 10 and then double the answer.

Who is correct?  
Why is the other person incorrect?  
Use the correct strategy to work out the divisions.

$2,000 \div 5$	$3,600 \div 5$
$310 \div 5$	$100,000 \div 5$


Ron

---

Eva's strategy will give the result for the number divided by 20

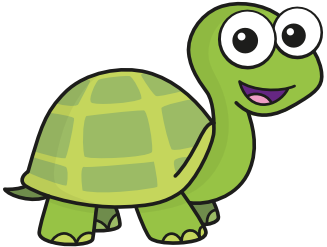
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400, 720  
62, 20,000



Tiny is trying to divide by 9

$10 - 1 = 9$ , so to divide by 9, I need to divide by 10 and subtract the number again.



Explain why Tiny is wrong.

Tiny is confusing strategies for multiplication and division.



# Solve multi-step problems

## Notes and guidance

In this small step, children apply the skills they have developed so far in this block to solving problems in real-life contexts.

The problems involve more than one calculation and children must decide which operations they need to perform and in what order to perform them; this will need careful modelling. As the focus of the step is making the correct choice of operation, calculators can be provided or the numbers simplified if necessary. Children should be encouraged to think about the best way to perform any of the calculations and use the most appropriate written, informal or mental method. For example, this might include using a number line to work out a subtraction after a long multiplication.

## Things to look out for

- In longer problems, children may find the number of words overwhelming and need encouragement to split the problem down into smaller parts.
- Children may find choosing the correct operation difficult.
- Children may need support to set out solutions with several parts clearly.

## Key questions

- What can you work out first?
- Is this step an addition, a subtraction, a multiplication or a division? How can you tell?
- Could you draw a diagram to represent the problem?
- Can you work out the answer to this part of the problem mentally or do you need another method?
- What can you do next?

## Possible sentence stems

- First, I need to work out \_\_\_\_\_  
The calculation I need to do is \_\_\_\_\_
- Next, I need to work out \_\_\_\_\_  
The calculation I need to do is \_\_\_\_\_

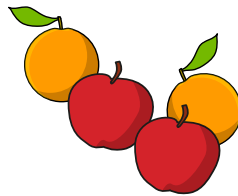
## National Curriculum links

- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving addition, subtraction, multiplication and division

## Solve multi-step problems

### Key learning

- The total mass of apples in a box is 25 kg.  
The total mass of oranges in a box is 24 kg.
  - ▶ There are 32 boxes of apples and 25 boxes of oranges in a supermarket.  
What is the total mass of apples and oranges?
  - ▶ A customer orders 300 kg of apples and 600 kg of oranges.  
How many boxes of fruit will the customer receive?



- There are 80 g of pasta in one portion.  
How much pasta is needed for 12 portions?  
How many portions can be made from a 16 kg bag of pasta?

- At a parade, there are 25 rows of people with 8 people in each row.  
Each person holds 2 flags.  
How many flags are needed for the parade?



- A coach has 55 seats and a minibus has 17 seats.  
431 people from a school go on a trip.  
The school books 6 coaches and 8 minibuses.  
How many spare seats will there be?

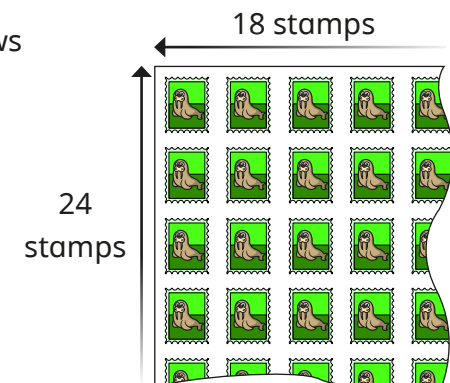
- Five boxes of toy trains cost £120  
Each box contains 6 trains.  
How much does each train cost?



- Dr Patel can type 40 words a minute.  
How many words can she type in an hour?  
How long does it take Dr Patel to type 1,000 words?

- A headteacher has £2,000 to spend on new furniture.  
He wants to buy 15 desks for £79 each and 30 chairs for £29 each.  
Does he have enough money?

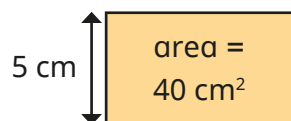
- A sheet of stamps has 24 rows and 18 columns of stamps.  
How many stamps are there altogether on 35 sheets?



# Solve multi-step problems

## Reasoning and problem solving

The area of a rectangular tile is  $40 \text{ cm}^2$   
The width of the tile is 5 cm.



A strip of tiles is made by laying tiles end-to-end.



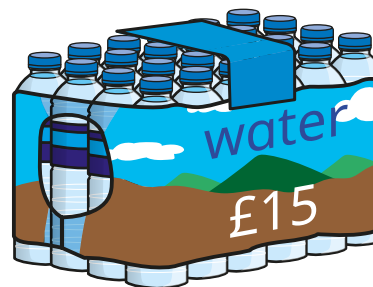
How long is a strip with 15 tiles?  
How many tiles are needed to make a strip 280 cm long?  
How many tiles are needed to make a strip 4 m long?

120 cm

35 tiles

50 tiles

24 bottles of water cost £15



How many bottles of water can you buy for £30?  
How many bottles of water can you buy for £300?  
How many bottles of water can you buy for £525?  
How much will 600 bottles of water cost?

48 bottles

480 bottles

840 bottles

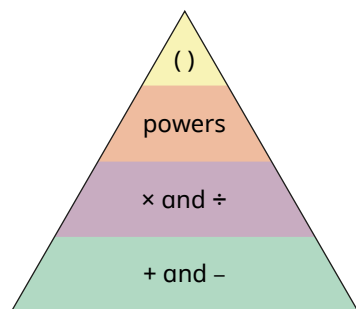
£375

# Order of operations

## Notes and guidance

In this small step, children learn the order of priority for operations in a calculation: that calculations in brackets should always be done first, and that multiplication and division have equal priority and should be performed before additions and subtractions.

This image may be useful when teaching the order of operations.



## Things to look out for

- If children have heard acronyms such as BIDMAS or BODMAS, they may mistakenly think that addition should be done before subtraction and incorrectly work out, for example,  $10 - 3 + 4$  as  $10 - 7 = 3$
- Similarly, children may not be aware that multiplication and division are of equal priority.

## Key questions

- Does it make a difference if you perform the operations in a different order?
- What do brackets in a calculation mean? What would happen if you did not use the brackets?
- Which operation has greater priority, addition or multiplication?
- How many pairs of operations do you know that have equal priority?
- How do you find the square of a number?

## Possible sentence stems

- \_\_\_\_\_ has greater priority than \_\_\_\_\_, so the first part of the calculation I need to do is \_\_\_\_\_

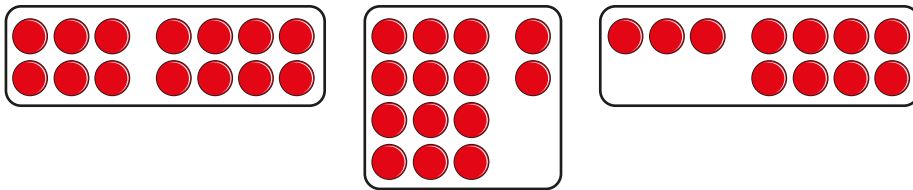
## National Curriculum links

- Perform mental calculations, including with mixed operations and large numbers
- Use their knowledge of the order of operations to carry out calculations involving the four operations

# Order of operations

## Key learning

- Match the counters to the calculations.



$3 + 4 \times 2$	$3 \times 4 + 2$	$(3 + 4) \times 2$
------------------	------------------	--------------------

- Draw counters to represent each calculation.

$4 + 1 \times 3$	$(4 + 1) \times 3$
------------------	--------------------

Work out the answers.

- Work out the calculations.

$(5 + 2) \times 3$	$6 + 4 \div 2$	$10 - 4 \div 2$
$5 + 2 \times 3$	$(6 + 4) \div 2$	$(10 - 4) \div 2$

- Add brackets to make the calculations correct.

▶ $6 + 4 \times 3 = 30$	▶ $20 - 20 \times 2 = 0$
▶ $12 \times 3 - 1 = 24$	▶ $10 \div 2 + 3 = 2$

- Work out the calculations.

$6 \times 4 + 5 \times 2$	$6 \times 4 - 5 \times 2$	$6 \times (4 + 5) \times 2$
---------------------------	---------------------------	-----------------------------

- Dani has 7 bags with 5 sweets in each bag. She adds one more sweet to each bag.

Which calculation shows how many sweets there are in total?

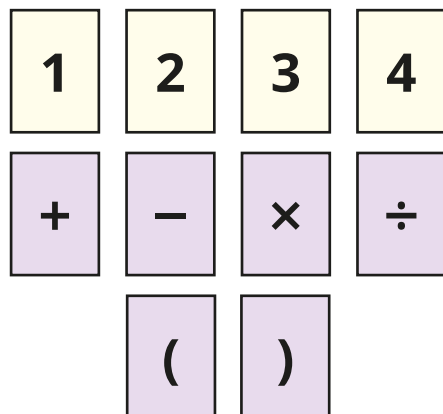
$7 \times (5 + 1)$	$7 \times 5 + 1$
--------------------	------------------

- Work out the calculations.

$6^2 - 3 \times 4$	$6^2 \div (4 + 5)$	$(7 - 4)^2$
--------------------	--------------------	-------------

## Order of operations

## Reasoning and problem solving



Use the digits and symbols to write as many calculations as you can that give different answers.

Is it possible to make every number from zero to 20?

multiple possible answers, e.g.

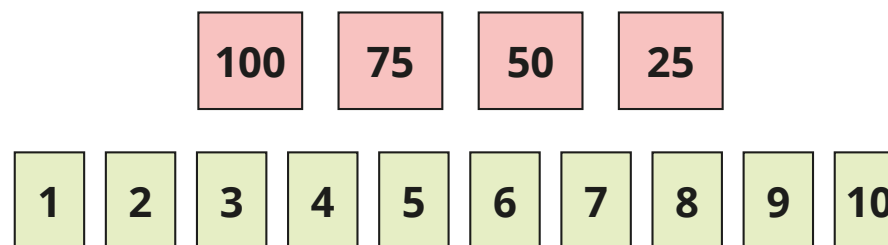
$$1 \times 2 \times 3 + 4 = 10$$

$$(1 + 2) \times 3 + 4 = 13$$

$$(1 + 2) \times (3 + 4) = 21$$

$$(1 + 2 + 3) \times 4 = 24$$

Here are some number cards.



Pick **one** large number from the top row.

Pick **five** smaller numbers from the bottom row.

Use a calculator or computer to generate a 3-digit target number.

Use your numbers, the four operations and brackets to find a number as close as possible to the target number.

Compare answers as a class.

# Mental calculations and estimation

## Notes and guidance

Children should use mental strategies and estimation whenever appropriate, and several examples have been included throughout the block. This small step reminds children of the importance of mental strategies and estimation, and gives them an opportunity to revisit and extend their learning from this block and previous years.

Children should be aware that estimating the answer of a calculation serves as a sense-check on whether their answer is correct, and this can be done either before or after a calculation. The numbers they choose when performing estimates should be simple enough for this to be done mentally.

Links should be made back to previous learning on rounding when simplifying numbers within a calculation.

## Things to look out for

- Children may try to use formal methods when alternative strategies would be more appropriate.
- Children may not round numbers to an appropriate degree of accuracy. For example, 4-digit numbers should usually be rounded to the nearest 1,000 and not to the nearest 100 or nearest 10

## Key questions

- Should you round the number to the nearest 10/100/1,000? Why?
- Are any of the numbers multiples of powers of 10? How does this help you to add/subtract/multiply/divide the numbers?
- What number is (for example) 99 close to? How does this help with the calculation? What adjustment do you need to make?
- How would partitioning/reordering the number(s) help?
- Why are estimates to the answers of calculations useful?

## Possible sentence stems

- The previous multiple of \_\_\_\_\_ is \_\_\_\_\_
- The next multiple of \_\_\_\_\_ is \_\_\_\_\_
- \_\_\_\_\_ rounded to the nearest \_\_\_\_\_ is \_\_\_\_\_

## National Curriculum links

- Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- Perform mental calculations, including with mixed operations and large numbers

# Mental calculations and estimation

## Key learning

- Use rounding to estimate the answer to each calculation.

$6,941 + 4,099$

$6,941 - 4,099$

$6,941 \times 18$

$6,941 \div 11$

Compare answers with a partner.

- What strategies would you use to find the exact answers to the calculations?

$480 + 20$

$480 - 20$

$480 \times 20$

$480 \div 20$

Compare answers with a partner.

- How could you change the order of the numbers in each of the calculations to make them easier to do mentally?

$97 + 58 + 43$

$68 + 57 - 28$

$12 \times 9 \times 5$

$50 \times 16 \times 2$

$4 \times 17 \times 25$

Work out the answers to the calculations.

- It is 816 km from Mr Trent's house to Glasgow.  
He drives 583 km of the way.



Approximately how much further does he have to drive?

- A textbook costs £19.99  
Approximately how many textbooks can be bought for £300?

- Work out the calculations.

$736 + 99$

$12,000 - 3$

$8,567 - 999$

$56 \times 9$

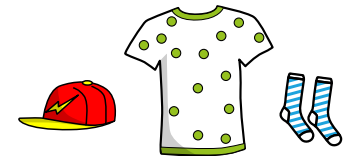
$6,999 + 8,500$

$34 \times 20$

$8,000 \div 20$

$8,204 - 6,899$

- Mo wants to buy a T-shirt for £9.99,  
a pair of socks for £2.49  
and a cap for £8.99



He has £22 in his wallet.

How can he quickly check whether he has enough money?



# Mental calculations and estimation

## Reasoning and problem solving

Here is a number line.



Estimate the number shown by arrow B for these values of A and C:

- A = 0 and C = 1,000
- A = 30 and C = 230
- A = 7 and C = 33
- A = 1 and C = 2
- A = 1,000 and C = 100,000

B is approximately nine-tenths of the way from A to C, so answers should be around:

- 900
- 210
- 30
- 1.9
- 90,000

$$2,000 - 1,287$$

Here are three strategies for working out the subtraction.



Whitney

I will use the column method.



Dexter

I will use number bonds from 87 to 100, then from 1,300 to 2,000



Teddy

I will subtract one from each number and then use the column method.

Whose strategy is most efficient?

Children can choose any strategy with the correct justification.

## Reason from known facts

### Notes and guidance

In this small step, children work out other facts from a given fact using their knowledge of place value, inverse operations, commutativity and the mental strategies practised in this block, particularly in the previous small step. Using diagrams, including area models and number lines, can help children to see the links between the different calculations. They need to be confident in multiplying and dividing by powers of 10. Children also explore the idea of doubling and halving.

It is important that children can not only work out an answer of a related fact, but also explain the connections between calculations that helped them arrive at this answer.

This small step will focus on integers, and decimal calculations will be covered in Spring Block 3

### Things to look out for

- Children may try to calculate the answers instead of looking at the relationships between the calculations and using reasoning.
- Children may over-generalise and try to use multiplication strategies that do not work for other operations.
- Children may need support to see the connections between the given fact and the adjusted calculation.

### Key questions

- What is an inverse operation?
- How can you use an inverse operation to find related facts?
- What is the same and what is different about the numbers in the given calculation and the numbers in the calculation you want to work out?
- How will the answer change if you increase/decrease/multiply/divide one/both of the numbers by \_\_\_\_\_?

### Possible sentence stems

- If I add/subtract \_\_\_\_\_ to/from one of the numbers in the calculation, then the answer will change by \_\_\_\_\_
- If I multiply/divide \_\_\_\_\_ one of the numbers in the calculation by \_\_\_\_\_, then the answer will change by \_\_\_\_\_

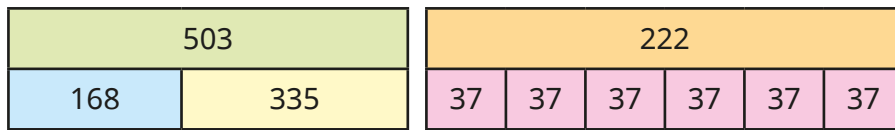
### National Curriculum links

- Perform mental calculations, including with mixed operations and large numbers
- Solve problems involving addition, subtraction, multiplication and division

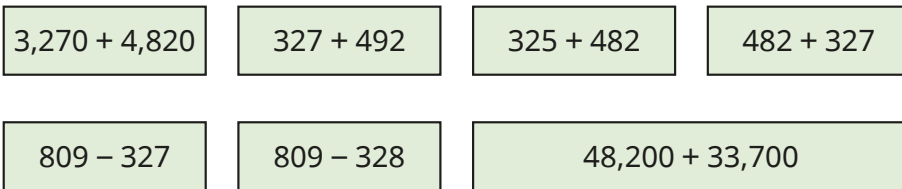
# Reason from known facts

## Key learning

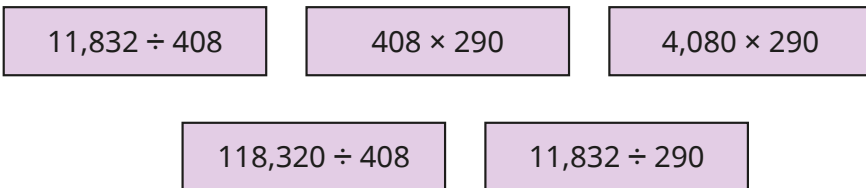
- Write four facts shown by each bar model.



- Use the fact that  $327 + 482 = 809$  to work out the answers to the calculations.



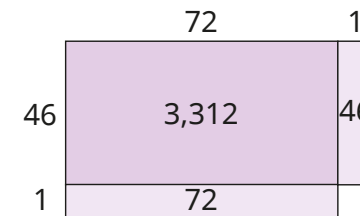
- Use the fact that  $11,832 \div 29 = 408$  to work out the answers to the calculations.



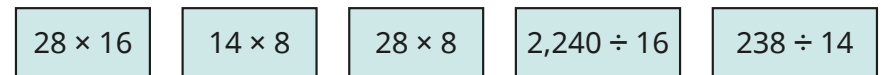
- Use the fact that  $46 \times 72 = 3,312$  to work out the multiplications.



You can use the area model to help you.



- Use the fact that  $5,138 \div 14 = 367$  to work out  $15 \times 367$
- Use the fact that  $14 \times 16 = 224$  to work out the calculations.



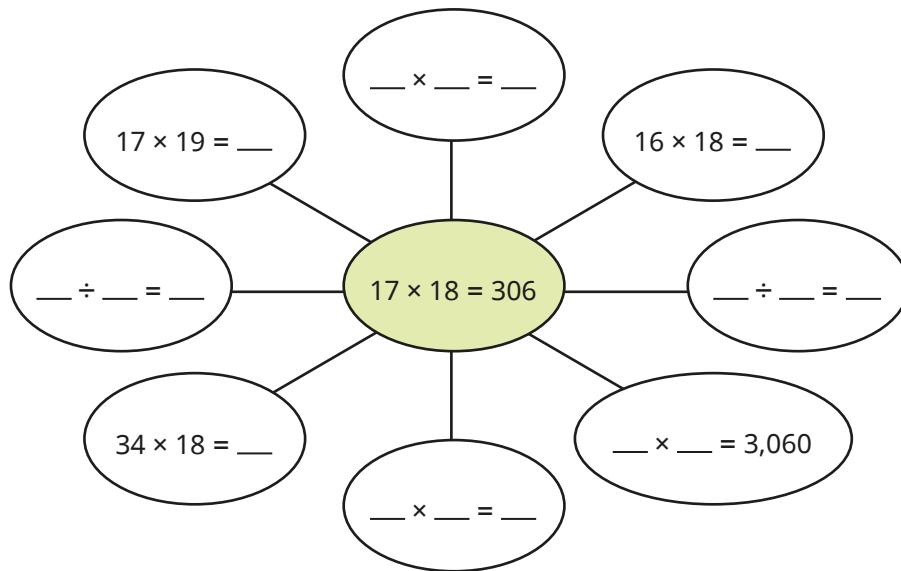
- Work out the missing numbers.

- ▶  $537 + 464 = 470 + \underline{\hspace{2cm}}$
- ▶  $25 \times 30 = 50 \times \underline{\hspace{2cm}}$
- ▶  $942 - 199 = \underline{\hspace{2cm}} - 200$
- ▶  $980 \div 20 = 1,000 \div 20 - \underline{\hspace{2cm}}$
- ▶  $38 \times 80 = 160 \times \underline{\hspace{2cm}}$
- ▶  $45 \times 79 = 45 \times \underline{\hspace{2cm}} - 45$

# Reason from known facts

## Reasoning and problem solving

Complete the spider diagram.



Compare methods with a partner.



$17 \times 19 = 323$

$34 \times 18 = 612$

$16 \times 18 = 288$

$170 \times 18 \text{ or } 17 \times 180 = 3,060$

Without working them out, which calculation has the greater answer?

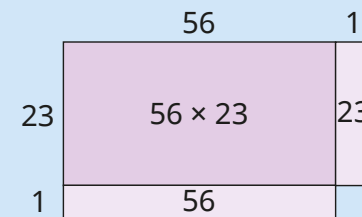


$57 \times 23$

$56 \times 24$

Draw a diagram to explain how you know.

Compare both calculations to  $56 \times 23$



$56 \times 24$  is 56 greater than  $56 \times 23$

$57 \times 23$  is only 23 greater than  $56 \times 23$

So  $56 \times 24$  is greater.

Autumn Block 3

# Fractions A

## Small steps

Step 1

Equivalent fractions and simplifying

Step 2

Equivalent fractions on a number line

Step 3

Compare and order (denominator)

Step 4

Compare and order (numerator)

Step 5

Add and subtract simple fractions

Step 6

Add and subtract any two fractions

Step 7

Add mixed numbers

Step 8

Subtract mixed numbers

## Small steps

Step 9

Multi-step problems

# Equivalent fractions and simplifying

## Notes and guidance

In this small step, children build on prior knowledge of equivalent fractions to recognise when fractions are, and are not, in their simplest form.

Children use their understanding of common factors to simplify fractions. They learn that when the numerator and denominator have no common factors greater than 1, the fraction is in its simplest form.

The step begins with fractions with one common factor (greater than 1) and moves on to fractions with several common factors. Children are encouraged to look for the greatest possible number to divide by, but also understand that simplification can be performed in more than one step.

Pictorial representations and fraction walls can be used to support understanding.

### Things to look out for

- Children may partially simplify a fraction instead of finding the simplest form, for example  $\frac{6}{24} = \frac{3}{12}$
- When simplifying mixed numbers, children may divide the whole number as well as the numerator and denominator.

## Key questions

- What are the common factors of \_\_\_\_\_ and \_\_\_\_\_?
- Why is it better to identify the greatest possible number that both the numerator and denominator can be divided by?
- Does the simplified fraction have the same value?
- Do the numerator and denominator have any more common factors?
- How can you tell if a fraction is in its simplest form?
- When simplifying a mixed number, why does the integer not change?

## Possible sentence stems

- Both the numerator and the denominator can be divided by \_\_\_\_\_
- To simplify the fraction, I will divide the numerator and denominator by \_\_\_\_\_
- \_\_\_\_\_ in its simplest form is \_\_\_\_\_

## National Curriculum links

- Use common factors to simplify fractions; use common multiples to express fractions in the same denomination



# Equivalent fractions and simplifying

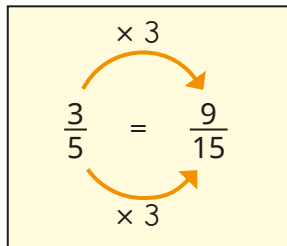
## Key learning

- Here are some fractions.

$\frac{4}{5}$	$\frac{30}{60}$	$\frac{7}{8}$	$\frac{42}{48}$	$\frac{2}{6}$	$\frac{1}{2}$	$\frac{8}{10}$	$\frac{16}{48}$
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Find the pairs of equivalent fractions.

- Jack uses multiplication to find equivalent fractions.



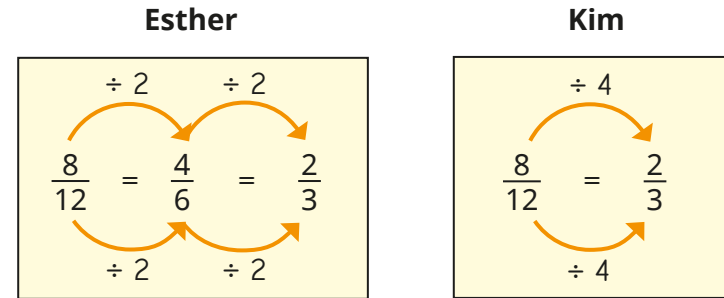
Use Jack's method to complete the equivalent fractions.

$\frac{4}{5} = \frac{\square}{20}$     
  $\frac{4}{5} = \frac{20}{\square}$     
  $\frac{\square}{7} = \frac{9}{21}$     
  $\frac{4}{7} = \frac{\square}{21}$

- Use division to write the fractions in their simplest form.

$\frac{12}{15} = \frac{4}{\square}$     
  $\frac{12}{20} = \frac{\square}{5}$     
  $\frac{16}{24} = \frac{2}{\square}$   
 $\frac{10}{12} = \frac{\square}{\square}$     
  $\frac{6}{30} = \frac{\square}{\square}$     
  $\frac{24}{40} = \frac{\square}{\square}$

- Esther and Kim are simplifying fractions.



What is the same? What is different?

Use one of their methods to simplify the fractions.

$\frac{2}{12}$	$\frac{4}{12}$	$\frac{6}{12}$	$\frac{6}{24}$	$\frac{8}{24}$	$\frac{16}{24}$
----------------	----------------	----------------	----------------	----------------	-----------------

- Mo is simplifying  $2\frac{4}{10}$



To simplify  $2\frac{4}{10}$ , keep the whole number the same and simplify the fraction.

$$2\frac{4}{10} = 2\frac{2}{5}$$

Use Mo's method to simplify the mixed numbers.

$3\frac{4}{10}$	$4\frac{12}{20}$	$6\frac{16}{30}$	$2\frac{16}{40}$
-----------------	------------------	------------------	------------------

# Equivalent fractions and simplifying

## Reasoning and problem solving

Tom and Aisha are simplifying an improper fraction.

**Tom**

**Aisha**

$$\frac{36}{8} = 4\frac{4}{8} = 4\frac{1}{2}$$

$$\frac{36}{8} = \frac{9}{2} = 4\frac{1}{2}$$

various answers

Whose method do you prefer?

Explain your answer.



Tiny is simplifying  $4\frac{12}{16}$

$$4\frac{12}{16} = 1\frac{3}{4}$$

Explain Tiny's mistake.



Tiny has divided the whole number by 4 instead of just simplifying the fraction.

Here are some fractions.



$$\frac{5}{15}$$

$$\frac{2}{4}$$

$$\frac{4}{16}$$

$$\frac{8}{16}$$

$$\frac{5}{10}$$

$$\frac{3}{9}$$

$$\frac{6}{12}$$

$$\frac{2}{8}$$

Which of the fractions:

- simplify to  $\frac{1}{2}$
- simplify to  $\frac{1}{3}$
- simplify to  $\frac{1}{4}$ ?

What patterns can you see?

What is the relationship between the numerator and the denominator?

Identify three more fractions that could go in each list.



simplifies to  $\frac{1}{2}$ :

$$\frac{2}{4}, \frac{8}{16}, \frac{5}{10}, \frac{6}{12}$$

simplifies to  $\frac{1}{3}$ :

$$\frac{5}{15}, \frac{3}{9}$$

simplifies to  $\frac{1}{4}$ :

$$\frac{4}{16}, \frac{2}{8}$$

multiple possible answers

# Equivalent fractions on a number line

## Notes and guidance

In this small step, children use number lines to count forwards and backwards in fractions and to find equivalent fractions.

Children start by revising counting fractions above 1 on a number line to ensure they are able to count in fractions accurately. Using a number line clearly shows that finding equivalent fractions does not change the value of the fraction. Encourage children to draw extra intervals on number lines to support them in placing the fractions. Number lines can also be used to support children in finding the difference between fractions. This will be revised later in the block when adding and subtracting fractions.

Encourage children to spot patterns on number lines when simplifying, rather than thinking about fractions individually.

## Things to look out for

- Children may find it difficult to place a fraction on the number line when the denominator is greater than the value of the divisions on the number line.
- When crossing 1, children may not be confident in converting mixed numbers/improper fractions.

## Key questions

- How many intervals are there on the number line? What is each interval worth?
- What equivalent fractions have you found?
- Is this fraction in its simplest form? How do you know?
- Can you divide the number line into more intervals to place the fractions more accurately?
- How will you place one sixteenth on a number line that is counting in eighths?
- Which fraction was the easiest/hardest to label? Why?

## Possible sentence stems

- From my number line, I can see that \_\_\_\_\_ is equivalent to \_\_\_\_\_
- When I count in eighths, I can change \_\_\_\_\_ into \_\_\_\_\_ because they are equivalent.

## National Curriculum links

- Use common factors to simplify fractions; use common multiples to express fractions in the same denomination

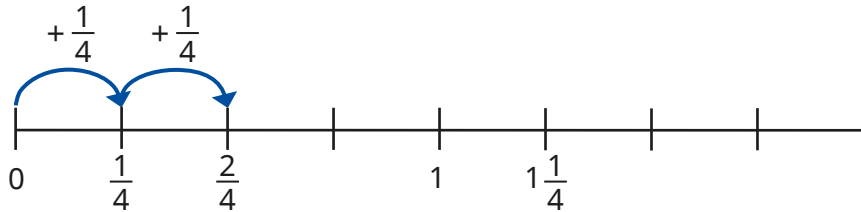
# Equivalent fractions on a number line

## Key learning

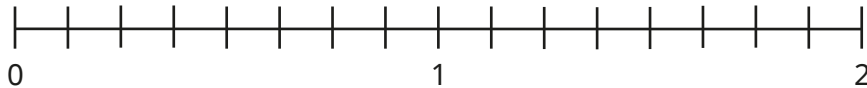
- Jack is counting in quarters.

He writes each number on a number line.

Complete the number line.

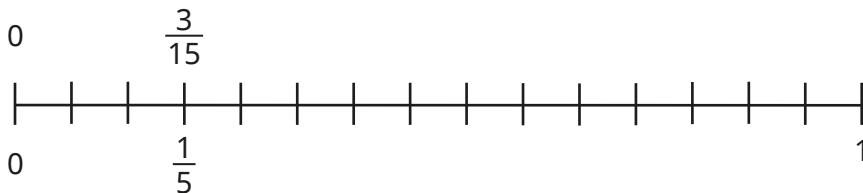


- Use the number line to count forward in eighths.



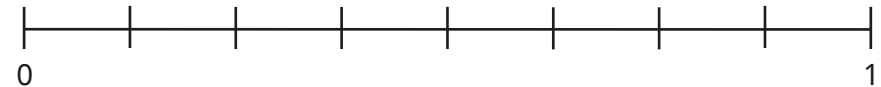
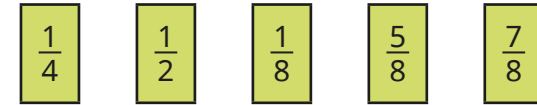
Which of the fractions can be simplified?

- Count in fifteenths on this number line and then write the fractions in their simplest form.



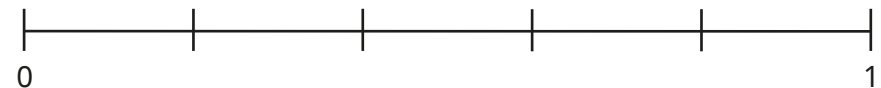
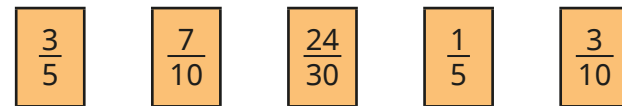
What patterns can you see?

- Label the fractions on the number line.

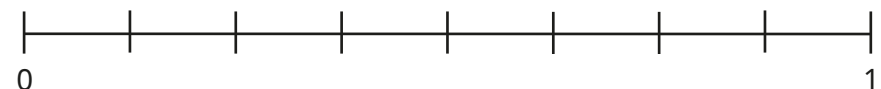
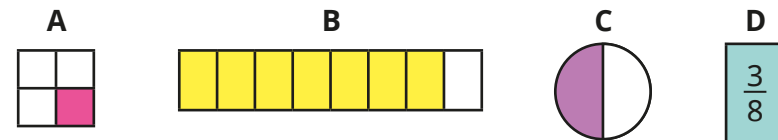


What is the difference between the greatest and smallest fraction?

- Label the fractions on the number line.



- Label A, B, C and D on the number line.



# Equivalent fractions on a number line

## Reasoning and problem solving

Rosie is counting back in tenths.

She starts at  $2\frac{1}{10}$  and counts back 7 tenths.

What number does Rosie end on?

Show this on a number line.

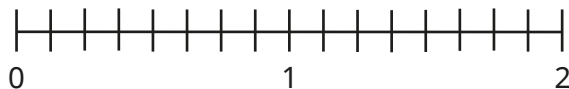
Simplify the fraction.



$$1\frac{4}{10} = \frac{14}{10}$$

$$1\frac{2}{5} = \frac{7}{5}$$

How many ways can you show a difference of one quarter on the number line?



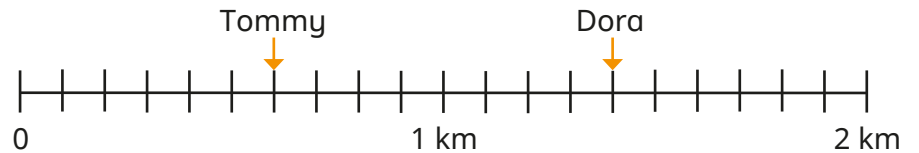
multiple possible answers, e.g.

$$\frac{7}{8} \text{ to } 1\frac{1}{8}$$

$$1\frac{1}{4} \text{ to } 1\frac{1}{2}$$

Dora and Tommy are completing a 2 km race.

The number line shows how far they have run so far.



How much further has Dora run than Tommy?

How much further do they each need to run?

Write your answers in their simplest form.

Huan has completed  $\frac{1350}{2000}$  of the race.

Label the number line to show how far Huan has run so far.



$$\frac{4}{5} \text{ km} \quad / \quad \text{Dora: } \frac{3}{5} \text{ km} \quad \text{Tommy: } 1\frac{2}{5} \text{ km}$$

$$\text{halfway between } 1\frac{3}{10} \text{ and } 1\frac{4}{10}$$

## Compare and order (denominator)

### Notes and guidance

In this small step, children compare and order fractions with the same denominator. Building on the skills covered in the previous steps, they first need to use their knowledge of equivalent fractions to find a common denominator in order to compare.

Children begin by using bar models to help compare fractions. They first work with pairs of fractions where one denominator is a multiple of the other, building on learning from Year 5.

They then look at pairs of fractions where the denominators are not multiples of each other, using their knowledge of multiples and common multiples. Encourage children to find the first common multiple, but allow them to explore different methods. Once children are confident expressing fractions with a common denominator, they use this to order fractions.

### Things to look out for

- Some children may compare the numerators without looking at the denominators and finding equivalent fractions.
- Children may not always find the most efficient common multiple when multiplying the denominators, for example expressing  $\frac{1}{6}$  and  $\frac{2}{9}$  as  $\frac{9}{54}$  and  $\frac{12}{54}$  rather than  $\frac{3}{18}$  and  $\frac{4}{18}$

### Key questions

- How could you use a number line or a bar model to help you compare the fractions?
- If the denominators are the same, how do you compare the fractions?
- Is one denominator a multiple of the other?
- If one denominator is not a multiple of the other, what do you need to do to be able to compare the fractions?
- How is comparing mixed numbers different from comparing proper fractions? How is it similar?

### Possible sentence stems

- I am comparing \_\_\_\_\_ and \_\_\_\_\_. I can use \_\_\_\_\_ as the common denominator.
- If one denominator is not a multiple of the other, I need to find a \_\_\_\_\_

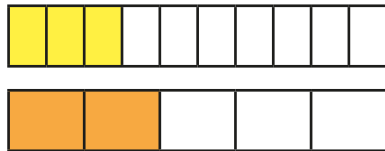
### National Curriculum links

- Compare and order fractions, including fractions  $> 1$
- Use common factors to simplify fractions; use common multiples to express fractions in the same denomination

# Compare and order (denominator)

## Key learning

- The bar models show  $\frac{3}{10}$  and  $\frac{2}{5}$



Which fraction is greater? How do you know?

- Alex is comparing  $\frac{1}{5}$  and  $\frac{4}{15}$

She uses equivalent fractions to help.

$$\frac{1}{5} = \frac{3}{15} \quad \frac{3}{15} < \frac{4}{15} \text{ so } \frac{1}{5} < \frac{4}{15}$$

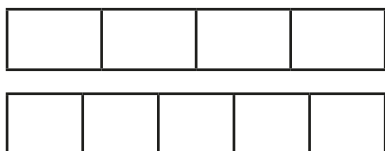
Use Alex's method to compare the fractions.

$$\frac{3}{20} \text{ and } \frac{1}{10}$$

$$\frac{3}{4} \text{ and } \frac{20}{36}$$

$$1\frac{2}{5} \text{ and } 1\frac{10}{30}$$

- Use the bar models to compare  $\frac{3}{4}$  and  $\frac{2}{5}$



$$\frac{3}{4} \bigcirc \frac{2}{5}$$

- Aisha is comparing  $\frac{5}{6}$  and  $\frac{3}{4}$  by finding the first common multiple of the denominators.

$$\frac{5}{6} = \frac{10}{12} \quad \frac{3}{4} = \frac{9}{12}$$

$$\frac{10}{12} > \frac{9}{12} \text{ so } \frac{5}{6} > \frac{3}{4}$$

Use Aisha's method to compare the fractions.

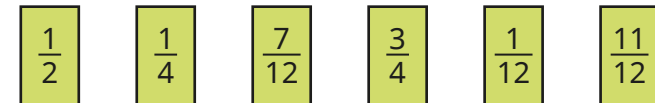
$$\frac{4}{5} \bigcirc \frac{3}{4}$$

$$\frac{3}{5} \bigcirc \frac{4}{7}$$

$$\frac{3}{4} \bigcirc \frac{7}{10}$$

$$2\frac{2}{5} \bigcirc 2\frac{3}{8}$$

- Write the fractions in descending order.



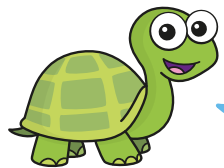
- Write the fractions in ascending order.



# Compare and order (denominator)

## Reasoning and problem solving

Tiny compares  $\frac{3}{5}$  and  $\frac{7}{15}$



$\frac{7}{15}$  is greater  
because 7 is greater  
than 3

No

Is Tiny correct?

Explain your answer.



Use the digit cards to complete  
the statements.

5

3

$$\frac{\square}{4} > \frac{\square}{6}$$

$$\frac{\square}{4} < \frac{6}{\square}$$

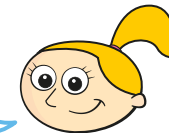
$$\frac{5}{4} > \frac{3}{6}$$

$$\frac{3}{4} < \frac{6}{5} \text{ or } \frac{5}{4} < \frac{6}{3}$$

Eva, Teddy and Amir are reading  
the same book.



I read  $7\frac{3}{4}$  pages  
in 10 minutes.



Eva



I read  $\frac{15}{2}$  pages  
in 10 minutes.

Amir

Eva

I read  $3\frac{1}{2}$  pages  
in 5 minutes.



Teddy

Who is reading the fastest?

How do you know?





## Compare and order (numerator)

### Notes and guidance

In the previous small step, children compared and ordered fractions using a common denominator. They now compare and order fractions with the same numerator.

Bar models are a useful representation to explore fractions with the same numerator, starting with unit fractions and then moving on to non-unit fractions. This will lead to the understanding that if the numerators are the same, then the greater the denominator, the smaller the fraction.

Children could visualise or place fractions on a number line and think about whether it is greater than or less than  $\frac{1}{2}$  or if it is close to 0 or 1. Understanding can then be built on to compare fractions greater than 1

Children should consider whether it is more efficient to find a common numerator or a common denominator.

### Things to look out for

- $\frac{1}{4}$  may be seen as smaller than  $\frac{1}{5}$  because 4 is less than 5
- Children may need to be encouraged to use their knowledge of 0, 1 and  $\frac{1}{2}$  to help compare fractions, for example  $\frac{6}{10} > \frac{2}{7}$  because  $\frac{6}{10} > \frac{1}{2}$  and  $\frac{2}{7} < \frac{1}{2}$

### Key questions

- How can you compare the fractions shown in the bar model?
- Do you need to change one or both numerators? Why?
- Is this fraction closer to 0 or 1?
- Is this fraction greater or less than  $\frac{1}{2}$ ?
- Is it more efficient to find a common numerator or a common denominator?

### Possible sentence stems

- When the numerators are the same, the \_\_\_\_\_ the denominator, the \_\_\_\_\_ the fraction.
- I know \_\_\_\_\_ is greater than  $\frac{1}{2}$  because ...
- I know \_\_\_\_\_ is closer to 1 than \_\_\_\_\_ because ...

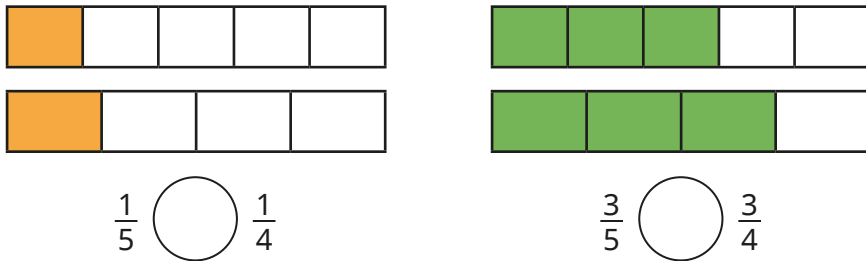
### National Curriculum links

- Compare and order fractions, including fractions  $> 1$

# Compare and order (numerator)

## Key learning

- Write  $<$ ,  $>$  or  $=$  to compare the fractions.



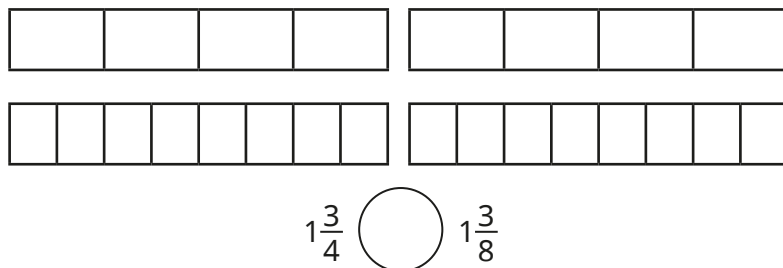
Complete the sentence.

When the numerators are the same, the \_\_\_\_\_ the denominator, the \_\_\_\_\_ the fraction.

- Write  $<$ ,  $>$  or  $=$  to compare the fractions.



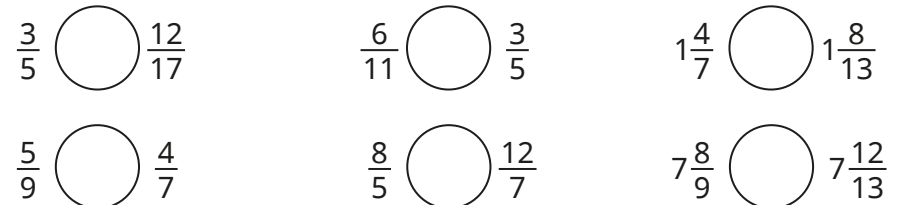
- Complete the bar models and write  $<$ ,  $>$  or  $=$  to compare the fractions.



- Whitney is comparing  $\frac{2}{5}$  and  $\frac{6}{13}$  using a common numerator.

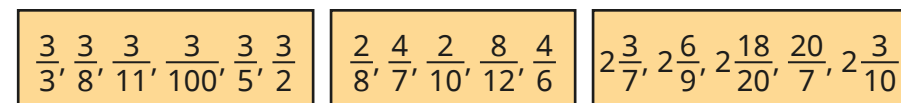
$$\frac{2}{5} = \frac{6}{15} \quad \frac{6}{15} < \frac{6}{13} \quad \text{so} \quad \frac{2}{5} < \frac{6}{13}$$

Use Whitney's method to compare the numbers.



- Dani and Tom have completed a quiz.  
 Dani answered 7 out of 12 of her questions correctly.  
 Tom answered 21 out of 30 of his questions correctly.  
 Who got a greater fraction of their questions correct?

- Write each set of fractions in ascending order.



# Compare and order (numerator)

## Reasoning and problem solving

Brett is comparing  $\frac{3}{7}$  and  $\frac{6}{11}$

How many different ways can he work this out?

Find a pair of fractions where it would be more efficient to find:

- a common numerator
- a common denominator.

Compare answers with a partner.



multiple possible answers, e.g. common numerator:

$$\frac{3}{7} = \frac{6}{14}, \frac{6}{14} < \frac{6}{11}$$

$$\text{so } \frac{3}{7} < \frac{6}{11}$$

What could the missing number be, to make the statement true?

$$\frac{1}{5} > \frac{1}{\square} > \frac{1}{12}$$

Is there more than one answer?

How do you know?



6, 7, 8, 9, 10 or 11

Two different pieces of wood have had a fraction of their length chopped off.



Here are the pieces now, showing the fraction that is left.



B  
90 cm

Which piece of wood was longer to begin with?

Explain your answer.



The second piece of wood was 1 m long before it was cut.

How long was the first piece of wood?

# Add and subtract simple fractions

## Notes and guidance

Before beginning, it may be appropriate to revise adding and subtracting fractions with the same denominator to remind children that where the denominators are the same, they need to add/ subtract the numerators and leave the denominator unchanged. In this small step, children build on previous learning in this block and Year 5 to use equivalent fractions to add and subtract fractions where one denominator is a multiple of the other.

Children may be familiar with some common additions and subtractions such as  $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$  and this is a good example on which to build. They start by using bar models before moving on to finding the first common multiple of the denominators.

As the focus is on addition and subtraction of simple fractions, children are not yet required to work with improper fractions and mixed numbers as this will be looked at later in the block.

### Things to look out for

- Children may not realise the need to make the denominators equal before adding.
- Children may add both the numerators and the denominators, for example  $\frac{1}{2} + \frac{1}{4} = \frac{2}{6}$
- Children may not always simplify their answers.

## Key questions

- Do the fractions have the same denominator?
- When are two fractions equivalent?
- How can you find a common denominator?
- How many of the fractions do you need to convert?
- Now the denominators are the same, how do you add/ subtract the fractions?

## Possible sentence stems

- Fractions must have the same \_\_\_\_\_ before they can be added or subtracted.
- The denominator has been multiplied by \_\_\_\_\_, so to make the equivalent fraction, multiply the numerator by \_\_\_\_\_
- When fractions have the same \_\_\_\_\_, to add or subtract them I just \_\_\_\_\_ the \_\_\_\_\_

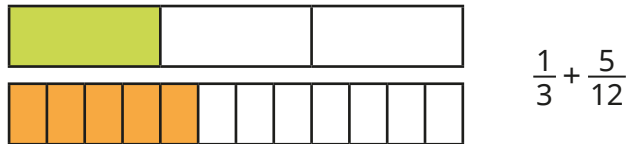
## National Curriculum links

- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions

# Add and subtract simple fractions

## Key learning

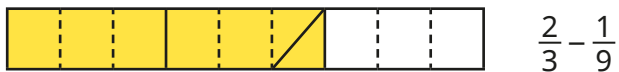
- Use the bar model to help add the fractions.



Work out the additions.

▶  $\frac{1}{3} + \frac{1}{12}$       ▶  $\frac{1}{3} + \frac{7}{12}$       ▶  $\frac{2}{3} + \frac{1}{12}$

- Use the bar model to work out the subtraction.



Work out the subtractions.

▶  $\frac{2}{3} - \frac{2}{9}$       ▶  $\frac{1}{3} - \frac{2}{9}$       ▶  $\frac{2}{3} - \frac{5}{9}$

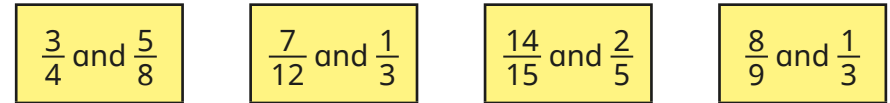
- Here is a method for working out  $\frac{7}{10} + \frac{7}{30}$

$$\frac{7}{10} = \frac{21}{30} \quad \frac{21}{30} + \frac{7}{30} = \frac{28}{30} = \frac{14}{15}$$

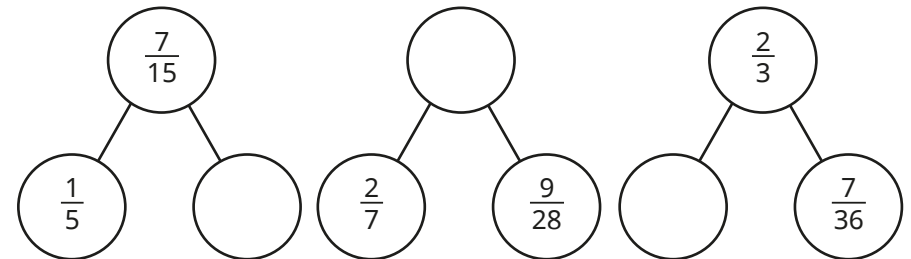
Use this method to work out the additions.

▶  $\frac{2}{9} + \frac{7}{27}$       ▶  $\frac{8}{15} + \frac{1}{5}$       ▶  $\frac{3}{16} + \frac{3}{8} + \frac{1}{4}$

- Find the difference between each pair of fractions.



- Complete the part-whole models.



- Ms Lee has a full tin of paint.
  - She uses  $\frac{1}{5}$  of the paint on Monday.
  - She uses  $\frac{1}{20}$  on Tuesday.
  - She uses  $\frac{3}{10}$  on Wednesday.



How much paint does she have left?

# Add and subtract simple fractions

## Reasoning and problem solving



Tiny is adding fractions.

Here are Tiny's workings.

$$\frac{3}{5} + \frac{1}{15} = \frac{4}{20} = \frac{1}{5}$$

$$\frac{10}{15} = \frac{2}{3}$$

Explain Tiny's mistake.

Find the correct answer.



Use the same digit in both boxes to complete the calculation.

$$\frac{\square}{20} + \frac{1}{\square} = \frac{9}{20}$$

$$\frac{4}{20} + \frac{1}{4} = \frac{9}{20}$$

$$\frac{5}{20} + \frac{1}{5} = \frac{9}{20}$$

Find all the possible answers.



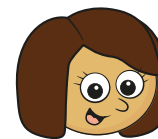
Find the missing number.



$$\frac{3}{5} + \frac{1}{20} = \frac{3}{4} - \frac{\square}{10}$$

1

Kim subtracts  $\frac{3}{5}$  from a fraction.



The answer is  $\frac{8}{45}$

$\frac{7}{9}$

What fraction has Kim subtracted  $\frac{3}{5}$  from?

Give your answer in its simplest form.

# Add and subtract any two fractions

## Notes and guidance

Following on from the previous small step, children add and subtract fractions where the denominators are not multiples of each other.

Children may need to revisit how to find a common denominator before completing the calculations. They use bar models and then move on to finding the first common multiple of the denominators. Once this is secure, they add up to three fractions or subtract fractions with different denominators.

Children add fractions with answers greater than one, but do not add and subtract mixed numbers until the next step.

Encourage children to simplify answers and convert improper fractions to mixed numbers as appropriate.

### Things to look out for

- Children may add both the numerators and the denominators, for example  $\frac{1}{3} + \frac{1}{4} = \frac{2}{7}$
- Children may not always simplify their answers.
- Children may leave answers as improper fractions, for example  $\frac{7}{5}$

## Key questions

- Do the fractions have the same denominator?
- What is the first common multiple of \_\_\_\_\_ and \_\_\_\_\_?
- How many of the fractions do you need to convert?
- How do you know if your answer is in its simplest form?
- Do you need to convert your answer to a mixed number? Why or why not?

## Possible sentence stems

- The lowest common multiple of \_\_\_\_\_ and \_\_\_\_\_ is \_\_\_\_\_
- To add/subtract the fractions, I could convert them both to \_\_\_\_\_
- When fractions have the same \_\_\_\_\_, to add or subtract them you just \_\_\_\_\_ the \_\_\_\_\_

## National Curriculum links

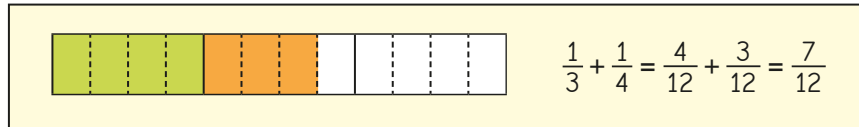
- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- Identify common factors, common multiples and prime numbers

# Add and subtract any two fractions

## Key learning

- Esther is working out  $\frac{1}{3} + \frac{1}{4}$

She finds a common denominator to work out the answer.



Use Esther's method to work out the additions.

$$\frac{1}{4} + \frac{2}{3}$$

$$\frac{1}{4} + \frac{1}{5}$$

$$\frac{1}{4} + \frac{3}{5}$$

- What common denominator would you use to add each pair of fractions?

$$\frac{2}{4} \text{ and } \frac{1}{5}$$

$$\frac{1}{6} \text{ and } \frac{2}{5}$$

$$\frac{1}{3} \text{ and } \frac{5}{7}$$

$$\frac{3}{8} \text{ and } \frac{4}{7}$$

Find the sum of each pair.

- On Friday, Scott walked  $\frac{5}{6}$  km to school, then  $\frac{3}{4}$  km to the shop and then  $\frac{4}{5}$  km home.

How far did he walk altogether?

- Annie is calculating  $\frac{7}{9} - \frac{1}{2}$

She finds the first common multiple of 9 and 2

$$\text{first common multiple of 9 and 2 is 18} \quad \frac{7}{9} - \frac{1}{2} = \frac{14}{18} - \frac{9}{18} = \frac{5}{18}$$

Use this method to find the differences.

$$\frac{2}{3} - \frac{1}{5}$$

$$\frac{4}{9} - \frac{1}{6}$$

$$\frac{5}{7} - \frac{1}{3}$$

$$\frac{11}{12} - \frac{3}{8}$$

- Kim has  $\frac{3}{4}$  kg of carrots and  $\frac{2}{5}$  kg of potatoes.

She is calculating the total mass of the carrots and potatoes.

$$\frac{3}{4} + \frac{2}{5} = \frac{15}{20} + \frac{8}{20} = \frac{23}{20} = 1\frac{3}{20} \text{ kg}$$

Use Kim's method to find the sums.

Give your answers as mixed numbers.

$$\frac{3}{4} + \frac{3}{5}$$

$$\frac{7}{8} + \frac{1}{3}$$

$$\frac{5}{6} + \frac{5}{7}$$

$$\frac{13}{20} + \frac{2}{3}$$

- Write  $<$ ,  $>$  or  $=$  to complete the statements.

$$\frac{1}{3} + \frac{1}{5} \bigcirc \frac{4}{5} - \frac{1}{3}$$

$$\frac{1}{3} - \frac{1}{5} \bigcirc \frac{4}{5} - \frac{1}{3}$$



# Add and subtract any two fractions

## Reasoning and problem solving

Huan and Dora are working out  $\frac{1}{4} + \frac{5}{6}$   
Here are their methods.

**Huan**

$$\frac{1}{4} + \frac{5}{6} = \frac{6}{24} + \frac{20}{24} = \frac{26}{24} = 1\frac{2}{24}$$

**Dora**

$$\frac{1}{4} + \frac{5}{6} = \frac{3}{12} + \frac{10}{12} = \frac{13}{12} = 1\frac{1}{12}$$

Who is correct?

Explain your answer.



Both are correct.

Fill in the boxes to make the calculation correct.

$$1\frac{\square}{10} = \frac{4}{\square} + \frac{\square}{10}$$



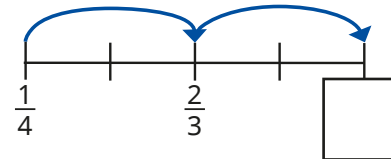
multiple possible answers, e.g.

$$1\frac{1}{10} = \frac{4}{5} + \frac{3}{10}$$

$$1\frac{7}{10} = \frac{4}{5} + \frac{9}{10}$$

The jumps on the number line are equal.

What is the missing value on the number line?



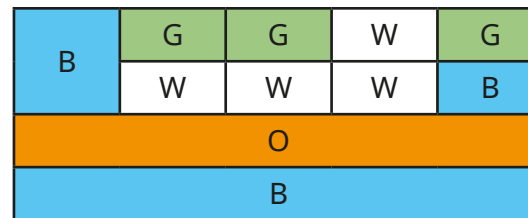
$1\frac{1}{12}$

A wall has been painted in different colours.

$\frac{1}{4}$  of the wall is orange (O).

What fraction of the wall is blue (B)?

What fraction of the wall is white (W)?



blue =  $\frac{2}{5}$

white =  $\frac{1}{5}$

# Add mixed numbers

## Notes and guidance

Children encountered mixed numbers in the answers to additions in the previous small step. They now add two mixed numbers, building on their experience of this in Year 5

Children explore adding the wholes and fractional parts separately. This is usually the most efficient method, but converting to improper fractions and then adding is an alternative. Some children may need to revisit converting between improper fractions and mixed numbers.

Questions begin with fractions with the same denominator and then move on to fractions with different denominators. Children can still draw models to represent adding fractions, particularly if these are useful for pairs of fractions with unequal denominators.

### Things to look out for

- Children may make errors in the partitioning or recombining of the integer and fractional parts.
- Children may make arithmetical errors when converting to improper fractions with larger numbers.
- The cognitive load is significant when finding solutions to these multi-step problems, so providing scaffolding/ partially started solutions may be useful.

## Key questions

- How can you partition the mixed numbers?
- How can the addition be rewritten to make it easier?
- In this question, is it easier to deal with wholes and fractions or to use improper fractions? Why?
- How do you convert a mixed number into an improper fraction?
- Are there any improper fractions in the answer? What can you do about this?

## Possible sentence stems

- Mixed numbers can be partitioned into a \_\_\_\_\_ part and a \_\_\_\_\_ part.
- A fraction is improper when the \_\_\_\_\_ is greater than the \_\_\_\_\_
- \_\_\_\_\_ is made up of \_\_\_\_\_ wholes and \_\_\_\_\_

## National Curriculum links

- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- Identify common factors, common multiples and prime numbers

# Add mixed numbers

## Key learning

- What method would you use to work out the additions?

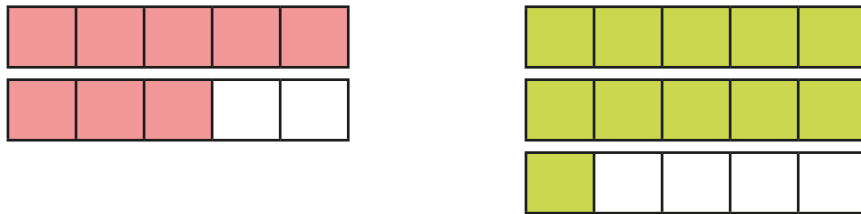
$$3\frac{2}{7} + 4$$

$$3\frac{2}{7} + \frac{4}{7}$$

$$3\frac{2}{7} + 4\frac{4}{7}$$

How are they similar? How are they different?

- Aisha uses a bar model to help work out  $1\frac{3}{5} + 2\frac{1}{5} = 3\frac{4}{5}$



Use bar models to help work out the additions.

$$1\frac{2}{7} + 3\frac{2}{7}$$

$$3\frac{2}{7} + 1\frac{4}{7}$$

$$2\frac{1}{7} + 3\frac{5}{7}$$

$$2\frac{1}{7} + 3\frac{6}{7}$$

- Work out the total of each pair of fractions.

$$\frac{3}{11} + \frac{2}{11}$$

$$1\frac{3}{11} + \frac{2}{11}$$

$$1\frac{3}{11} + 1\frac{2}{11}$$

$$2\frac{3}{11} + 1\frac{2}{11}$$

How did you work them out?

Compare methods with a partner.

- Rosie and Amir are working out  $1\frac{1}{2} + 2\frac{1}{6}$

**Rosie**

$$1 + 2 = 3$$

$$\frac{1}{2} + \frac{1}{6} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6}$$

$$3 + \frac{4}{6} = 3\frac{4}{6} = 3\frac{2}{3}$$

**Amir**

$$1\frac{1}{2} + 2\frac{1}{6} = \frac{3}{2} + \frac{13}{6}$$

$$= \frac{9}{6} + \frac{13}{6}$$

$$= \frac{22}{6} = 3\frac{4}{6} = 3\frac{2}{3}$$

Whose method do you prefer?

Explain your answer.

Use your preferred method to add the mixed numbers.

$$3\frac{1}{2} + 2\frac{3}{8}$$

$$2\frac{1}{9} + 2\frac{2}{5}$$

$$34\frac{1}{9} + 5\frac{5}{8}$$

$$2\frac{3}{4} + 1\frac{4}{5}$$

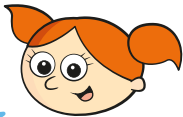
- A jug contains  $2\frac{3}{4}$  litres of juice.  
Another jug contains  $3\frac{3}{5}$  litres of juice.  
How much juice is there altogether?





# Add mixed numbers

## Reasoning and problem solving

Alex, Whitney and Teddy are trying to run 10 km between them.

I ran  $2\frac{1}{2}$  km.  Alex

 Whitney I ran  $3\frac{4}{5}$  km.

I ran  $3\frac{6}{10}$  km.  Teddy

How far have they run?  
How much further do they need to run?

$9\frac{9}{10}$  km

---

$\frac{1}{10}$  km

On Saturday and Sunday, Nijah ran a total of  $4\frac{1}{2}$  km. Suggest how far Nijah ran on each day. Find more than one answer.

multiple possible answers, e.g.  
 $2\frac{1}{3}$  km and  $2\frac{1}{6}$  km

The numbers in the row and column add up to make the totals shown.

$2\frac{1}{4}$	$\square\frac{\square}{8}$	$\frac{1}{2}$	$= 3\frac{7}{8}$
$\frac{1}{\square}$			
$3\frac{1}{12}$			
$= 5\frac{1}{2}$			

Find the missing values.

$1\frac{1}{8}$   
 $\frac{1}{6}$

# Subtract mixed numbers

## Notes and guidance

In this small step, children subtract two mixed numbers, building on the learning from Year 5. Children make links between what is the same and what is different when subtracting mixed numbers compared to adding them.

To introduce this step, children subtract mixed numbers that have the same denominator and do not break the whole. They then subtract fractions with different denominators and complete questions that break the whole. When breaking the whole, children can exchange one whole or convert mixed numbers to improper fractions. Bar models are useful tools to illustrate both methods, and number lines can be used to help find the difference.

### Things to look out for

- When breaking the whole, children may be unsure how to exchange.
- Children may make errors when partitioning mixed numbers, for example they may not correctly convert  $3\frac{3}{4}$  to  $2\frac{7}{4}$
- Children should think about which method is most appropriate for the question, rather than relying on just one method.

## Key questions

- How can you partition the mixed number?
- How can the subtraction be rewritten to make it easier?
- In this question, is it easier to deal with wholes and fractions or to use improper fractions? Why?
- How do you convert a mixed number into an improper fraction?

## Possible sentence stems

- This calculation will/will not cross the whole because ...
- A fraction is equal to one whole when the \_\_\_\_\_ is equal to the \_\_\_\_\_
- The mixed number can be partitioned into \_\_\_\_\_ and \_\_\_\_\_
- \_\_\_\_\_ can be written as \_\_\_\_\_ wholes and \_\_\_\_\_

## National Curriculum links

- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- Identify common factors, common multiples and prime numbers

# Subtract mixed numbers

## Key learning

- What method would you use to work out the subtractions?

$$3\frac{7}{8} - 1$$

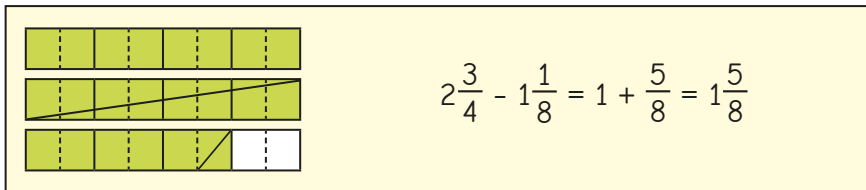
$$3\frac{7}{8} - \frac{3}{8}$$

$$3\frac{7}{8} - 1\frac{3}{8}$$

Compare methods with a partner.

How is this similar to addition? How is it different?

- Tom uses bar models to help work out  $2\frac{3}{4} - 1\frac{3}{8}$



Use bar models to help work out the subtractions.

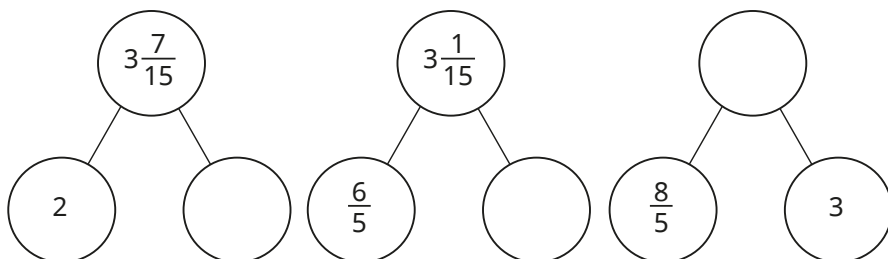
$$2\frac{3}{4} - 1\frac{5}{8}$$

$$3\frac{3}{4} - 2\frac{3}{8}$$

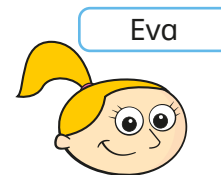
$$2\frac{1}{2} - 1\frac{3}{10}$$

$$4\frac{1}{3} - 2\frac{1}{3}$$

Complete the part-whole models.



- Eva and Tommy are working out  $3\frac{3}{5} - 1\frac{7}{10}$



Eva

$$3\frac{3}{5} = 2\frac{8}{5}$$

$$3\frac{3}{5} - 1\frac{7}{10} = 2\frac{8}{5} - 1\frac{7}{10} = 2\frac{16}{10} - 1\frac{7}{10} = 1\frac{9}{10}$$

I cannot subtract the wholes and fractions separately because  $\frac{3}{5}$  is less than  $\frac{7}{10}$  I will exchange 1 whole for 5 fifths.

I will convert both mixed numbers to improper fractions and then subtract them.



Tommy

$$3\frac{3}{5} - 1\frac{7}{10} = \frac{18}{5} - \frac{17}{10} = \frac{36}{10} - \frac{17}{10} = \frac{19}{10} = 1\frac{9}{10}$$

Choose a method to work out the subtractions.

$$4\frac{4}{5} - 1\frac{9}{10}$$

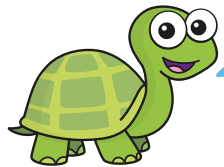
$$2\frac{1}{7} - 1\frac{1}{3}$$

$$3\frac{5}{12} - 1\frac{7}{9}$$

$$3\frac{5}{11} - 1\frac{4}{5}$$

# Subtract mixed numbers

## Reasoning and problem solving



I cannot work out  $3\frac{1}{3} - 1\frac{1}{2}$  because  $\frac{1}{2}$  is greater than  $\frac{1}{3}$

No

Is Tiny correct?

Explain your answer.



Jack is calculating  $4\frac{2}{7} - 2\frac{6}{7}$   
He adds  $\frac{1}{7}$  to both numbers.

$$4\frac{2}{7} - 2\frac{6}{7} = 4\frac{3}{7} - 3,$$

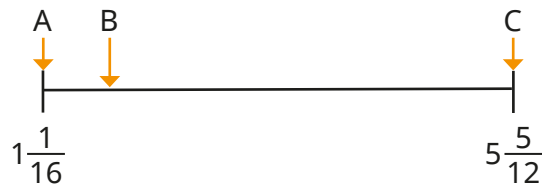
so the answer is  $1\frac{3}{7}$



He increased both numbers by  $\frac{1}{7}$  so the difference remained constant.

Explain why Jack is correct.

On the number line, C is  $3\frac{2}{3}$  more than B.



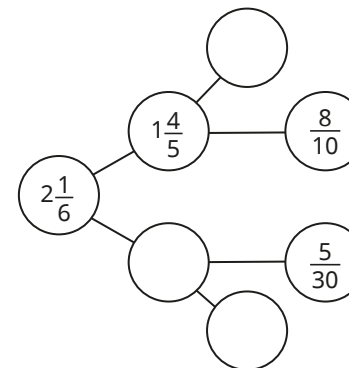
$1\frac{3}{4}$

$1\frac{11}{16}$

What is the value of B?

What is the difference between A and B?

Complete the part-whole model.



1  
 $\frac{11}{30}$   
 $\frac{1}{5}$

# Multi-step problems

## Notes and guidance

In this small step, children apply the skills they have learnt in previous steps to solving problems in real-life contexts.

The problems may involve more than one calculation and children need to choose the operations and consider what order to perform them in; this will need careful modelling. Encourage children to think about the most appropriate method to perform any of the calculations. Sharing methods could help children gain a flexible approach to solving the problems.

Children also need to ensure that they write fractions in their simplest form and convert between improper fractions and mixed numbers where appropriate.

### Things to look out for

- For longer word problems, the questions may need to be broken down into separate sections to scaffold learning.
- If their understanding is not secure, children may need to revise earlier learning before completing the problems.
- Children may need support to set out solutions with several parts clearly.
- Some children may struggle with the maths because they are overwhelmed by the context of a question.

## Key questions

- What can you work out first?
- What do you need to know to work out the answer?
- Can you draw a diagram to represent the problem?
- Can you work out the answer to this part of the problem mentally or do you need another method?
- What can you do next?

## Possible sentence stems

- First, I need to work out ...
- The calculation I need to do is ...
- Next, I need to work out ...

## National Curriculum links

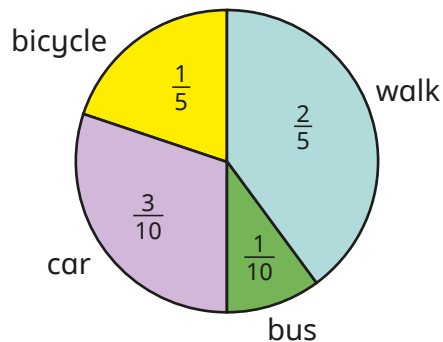
- Use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Solve problems involving addition, subtraction, multiplication and division



# Multi-step problems

## Key learning

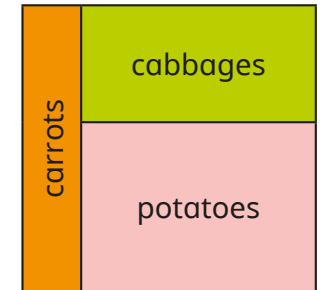
- Children in Class 6 were asked how they travel to school. The results of the survey are shown in the pie chart.



What fraction of children do not get the bus to school?

- Dr Fisher has  $\frac{7}{8}$  of a tank of petrol in his car. He drives to see his friend and uses  $\frac{1}{5}$  of a tank of petrol. What fraction of a tank of petrol is left in the tank?
- A family buys two equal-sized boxes of cereal. In one week, they eat  $\frac{2}{3}$  of box A and  $\frac{3}{5}$  of box B. How many boxes of cereal do they eat that week? How many boxes of cereal will they need for three weeks?

- Here is a vegetable patch.  $\frac{1}{5}$  of the patch is for carrots and  $\frac{3}{8}$  of the patch is for cabbages.

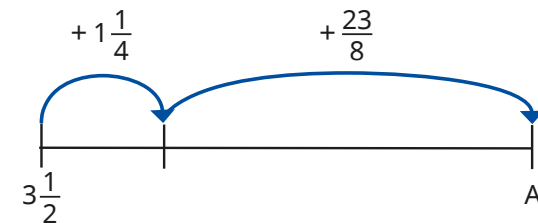


What fraction of the patch is for potatoes?

How much more of the patch is for the potatoes than for the cabbages?

Give all your answers in their simplest form.

- What is the value of A?



- Whitney has 5 bags of raisins. On Monday, she eats  $\frac{2}{3}$  of a bag and gives  $\frac{4}{5}$  of a bag away. On Tuesday, she eats  $1\frac{1}{3}$  bags and gives  $\frac{2}{3}$  of a bag away. How many bags of raisins does Whitney have left?

# Multi-step problems

## Reasoning and problem solving

Annie and Mo are going on a trip.



Annie

My suitcase has a mass of  $29\frac{1}{2}$  kg.

My suitcase is  $2\frac{1}{5}$  kg lighter than Annie's.



Mo

What is the total mass of the suitcases?

There is a weight allowance of 32 kg per suitcase.

How much below the weight allowance are Annie and Mo's suitcases?

$$56\frac{4}{5} \text{ kg}$$

$$\text{Annie: } 2\frac{1}{2} \text{ kg}$$

$$\text{Mo: } 4\frac{7}{10} \text{ kg}$$

Find the value of



$$\heartsuit + 3\frac{4}{9} = 6\frac{1}{3}$$

$$8\frac{1}{10} - \heartsuit = \text{Sun}$$

$$\text{Sun} = 5\frac{19}{90}$$

Complete the calculation.

$$2\frac{9}{12} + 3\frac{15}{20} - 2\frac{3}{4} - 2\frac{75}{100} = \boxed{\phantom{00}}$$

1

How can you make this calculation simpler?

Autumn Block 4

# Fractions B

## Small steps

Step 1

Multiply fractions by integers

Step 2

Multiply fractions by fractions

Step 3

Divide a fraction by an integer

Step 4

Divide any fraction by an integer

Step 5

Mixed questions with fractions

Step 6

Fraction of an amount

Step 7

Fraction of an amount – find the whole

# Multiply fractions by integers

## Notes and guidance

Building on their learning in Year 5, this small step provides practice in multiplying fractions and mixed numbers by integers.

A variety of representations can show that multiplying fractions by integers is the same as repeated addition of a fraction. As when adding and subtracting fractions, the denominator does not change. Children recognise that they need to multiply the numerator by the integer. When multiplying mixed numbers, children can either partition them into wholes and parts, multiplying each of them by the integer, or convert the mixed number to an improper fraction and then multiply the numerator by the integer.

### Things to look out for

- Children may multiply both the denominator and numerator by the integer, or only multiply the numerator of the part in a mixed number and not the whole.
- Children may make mistakes when converting between mixed numbers and improper fractions.
- Children should be encouraged to give their answers in their simplest form and convert any improper fractions to mixed numbers.

## Key questions

- How is multiplying fractions by integers similar to addition of fractions? How is it different?
- What happens to the denominator when you multiply a fraction by an integer?
- Do you find it easier to partition the mixed number first or to convert it to an improper fraction?
- Is  $\frac{2}{3} \times 7$  equal to  $7 \times \frac{2}{3}$ ? Why?

## Possible sentence stems

- To multiply a fraction by an integer, I need to multiply the numerator by \_\_\_\_\_
- To multiply a mixed number by an integer, I can partition it into \_\_\_\_\_ and \_\_\_\_\_ and then multiply them both by the integer.
- To multiply a mixed number by an integer, I can convert the mixed number to an \_\_\_\_\_ and then ...

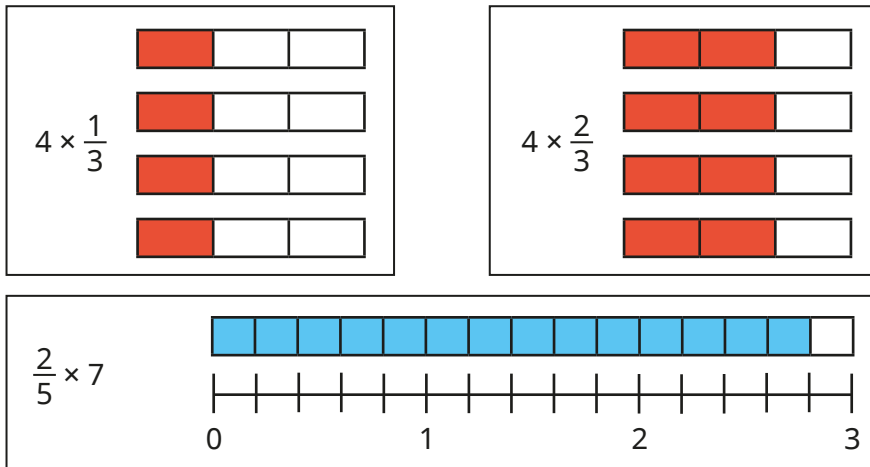
## National Curriculum links

- Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams (Y5)

# Multiply fractions by integers

## Key learning

- Use the diagrams to work out the multiplications.



- Complete the calculations.

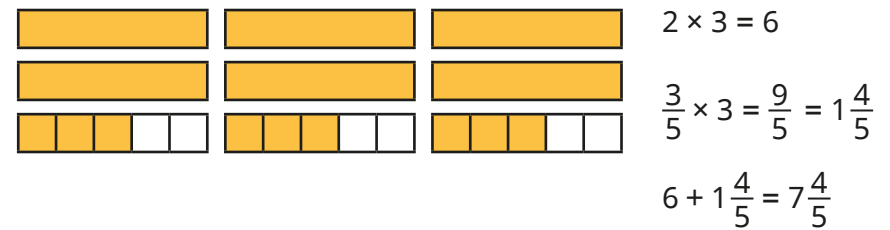
$\frac{3}{5} \times \underline{\quad} = \frac{9}{5} = \underline{\quad}$        $\frac{2}{7} \times \underline{\quad} = \frac{\square}{7} = 1\frac{1}{7}$

- Huan works out  $4 \times \frac{7}{8}$

$$4 \times \frac{7}{8} = \frac{28}{8} = 3\frac{4}{8}$$

How can you improve Huan's answer?

- Eva partitions  $2\frac{3}{5}$  to help her work out  $2\frac{3}{5} \times 3$



Use Eva's method to work out the multiplications.

$2\frac{5}{6} \times 3$	$1\frac{3}{7} \times 5$	$3 \times 2\frac{2}{3}$	$1\frac{1}{6} \times 4$
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- Tommy works out  $2\frac{3}{5} \times 3$  by converting the mixed number to an improper fraction first.

$$2\frac{3}{5} = \frac{13}{5} \quad \frac{13}{5} \times 3 = \frac{39}{5} \quad \frac{39}{5} = 7\frac{4}{5}$$

Use Tommy's method to work out the multiplications.

$2\frac{2}{5} \times 3$	$1\frac{5}{7} \times 3$	$2 \times 1\frac{3}{4}$	$1\frac{1}{6} \times 2$
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# Multiply fractions by integers

## Reasoning and problem solving

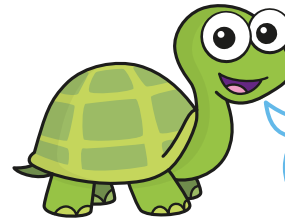
There are 12 children in a class.  
The teacher has 4 litres of orange juice.



Each child gets  $\frac{1}{5}$  litre of orange juice.  
How much orange juice will be left over?

$1\frac{3}{5}$  litres

Tiny is working out  $4 \times 3\frac{2}{5}$



The answer  
is  $12\frac{8}{20}$

Is Tiny correct?  
Explain your reasoning.

No

A classroom desk is  $1\frac{1}{3}$  m long.  
The classroom is 6 m wide.

Will 5 desks fit side by side in the classroom?

Explain your answer.

No  
 $5 \times 1\frac{1}{3} = 6\frac{2}{3}$

$$A \times 3\frac{1}{5} = B$$

B is an integer.  
Work out possible values of A and B.

multiple possible  
answers, e.g. A = 5  
and B = 16

# Multiply fractions by fractions

## Notes and guidance

Building on the previous step, children multiply a fraction by another fraction.

Children use concrete and pictorial representations to support them, including folding paper, diagrams and bar models.

By exploring the pictorial representations, children identify the fact that fractions can be multiplied by multiplying both the numerators and denominators. They may need to be reminded that answers should be given in their simplest form.

As the fractions children multiply in this step are all proper, they could be stretched to explain why their answer is always smaller than the fractions given in the question.

## Things to look out for

- Children may believe that “multiplication always makes numbers bigger”, but should realise that this is not the case when multiplying by a number less than 1
- The processes for different operations could get mixed up and children may unnecessarily convert to a common denominator as if they are adding or subtracting fractions.

## Key questions

- How can you show the calculation as a diagram?
- What is the same and what is different about “half of” a number and “ $\frac{1}{2} \times$ ” a number?
- When you multiply two fractions, is the product greater than or smaller than each of the fractions? Why?
- Why are all of your answers less than 1?

## Possible sentence stems

- To show \_\_\_\_\_, I have split my diagram into \_\_\_\_\_ equal sections.
- To find the product, I need to ...
- When multiplying a pair of fractions, I need to multiply the \_\_\_\_\_ and multiply the \_\_\_\_\_

## National Curriculum links

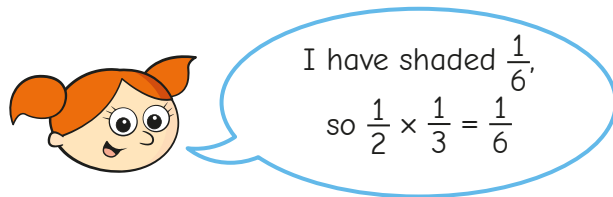
- Multiply simple pairs of proper fractions, writing the answer in its simplest form



# Multiply fractions by fractions

## Key learning

- Alex is using a piece of paper to work out  $\frac{1}{2} \times \frac{1}{3}$   
First, she folds the piece of paper in half.  
Then she folds the half into thirds.  
Alex shades the fraction that she has created.



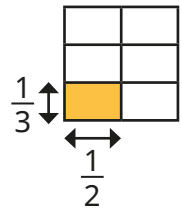
Use Alex's method to work out the multiplications.

$$\frac{1}{4} \times \frac{1}{2}$$

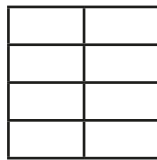
$$\frac{1}{4} \times \frac{1}{3}$$

$$\frac{1}{4} \times \frac{1}{4}$$

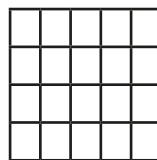
- Whitney is using diagrams to represent multiplying fractions.  
Shade the diagrams to work out the multiplications.



$$\frac{1}{3} \times \frac{1}{2} = \underline{\hspace{2cm}}$$



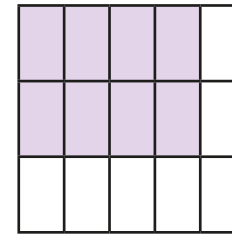
$$\frac{1}{4} \times \frac{1}{2} = \underline{\hspace{2cm}}$$



$$\frac{1}{5} \times \frac{1}{4} = \underline{\hspace{2cm}}$$

Can any of your answers be simplified?

- Dani is using a diagram to work out  $\frac{2}{3} \times \frac{4}{5}$



Explain why the diagram shows  $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

Use similar diagrams to work out  $\frac{2}{3} \times \frac{2}{5}$  and  $\frac{2}{3} \times \frac{3}{5}$

- Dexter has spotted a connection between the numerators and the denominators in the question and answer.

$$\frac{3}{4} \times \frac{1}{5} = \frac{3}{20} \quad \frac{4}{5} \times \frac{3}{7} = \frac{12}{35} \quad \frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$$

What connection has Dexter spotted?

Use the connection to work out the multiplications.

$$\frac{2}{5} \times \frac{1}{3}$$

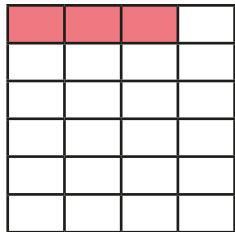
$$\frac{3}{4} \times \frac{3}{5}$$

$$\frac{2}{7} \times \frac{4}{5}$$

# Multiply fractions by fractions

## Reasoning and problem solving

Aisha uses this diagram to work out the product of two fractions.



What fractions has Aisha multiplied?

What is the answer?

$$\frac{3}{4} \times \frac{1}{6} \text{ or } \frac{1}{6} \times \frac{3}{4}$$

$$\frac{3}{24} = \frac{1}{8}$$

Find the missing numbers.

$$\text{red splat} \times \frac{3}{6} = \frac{6}{12} = \frac{2}{2}$$

Is there more than one answer?

multiple possible answers, e.g.

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$$

$$\frac{2}{6} \times \frac{3}{2} = \frac{6}{12} = \frac{1}{2}$$

Work out the missing numbers.

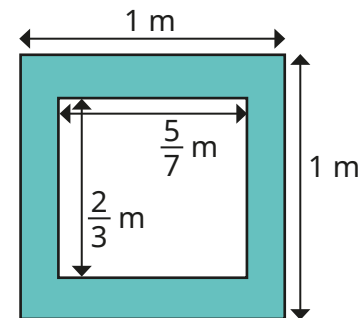
$$\frac{1}{2} \times \frac{1}{\square} = \frac{1}{16}$$

$$\frac{\square}{6} \times \frac{3}{5} = \frac{21}{30}$$

$$\frac{3}{\square} \times \frac{4}{5} = \frac{3}{5}$$

8, 7, 4

What is the area of the shaded region?



$\frac{11}{21} \text{ m}^2$

# Divide a fraction by an integer

## Notes and guidance

In this small step, children are introduced to dividing fractions by integers for the first time. They focus on dividing fractions where the numerator is a multiple of the integer they are dividing by, for example  $\frac{3}{5}$  divided by 3, or  $\frac{6}{7}$  divided by 2

Bar models are used initially to represent fractions and to explore how to divide a fraction by an integer. Children complete the number sentence alongside the representation to encourage them to notice that the denominator stays the same and the numerator is divided by the integer. The idea of unitising could be used to support children with dividing fractions by integers. For example, if they know that 6 ones shared between 2 is equal to 3 ones, and 6 eggs shared between 2 is equal to 3 eggs, then 6 sevenths shared between 2 is equal to 3 sevenths. Links can be made to previous representations when multiplying fractions, for example by looking at the equivalence of  $\frac{4}{7} \div 2$  and  $\frac{4}{7} \times \frac{1}{2}$

### Things to look out for

- Children may divide both the numerator and denominator by the integer.
- Children may be tempted to use an abstract procedure, rather than think carefully about what the question is asking.

## Key questions

- How could you represent the fraction?
- How could you split the fraction into \_\_\_\_\_ equal parts?
- What do you notice about the numerators in the question and the answer?
- What do you notice about the denominators in the question and the answer?
- What changes and what stays the same?
- How can you show the division as a bar model?

## Possible sentence stems

- If you divide \_\_\_\_\_ into equal groups, then each group is \_\_\_\_\_ because \_\_\_\_\_  $\div$  \_\_\_\_\_ = \_\_\_\_\_
- \_\_\_\_\_ ones divided by \_\_\_\_\_ is equal to \_\_\_\_\_ ones, so \_\_\_\_\_ eighths divided by \_\_\_\_\_ is equal to \_\_\_\_\_ eighths.

## National Curriculum links

- Divide proper fractions by whole numbers

# Divide a fraction by an integer

## Key learning

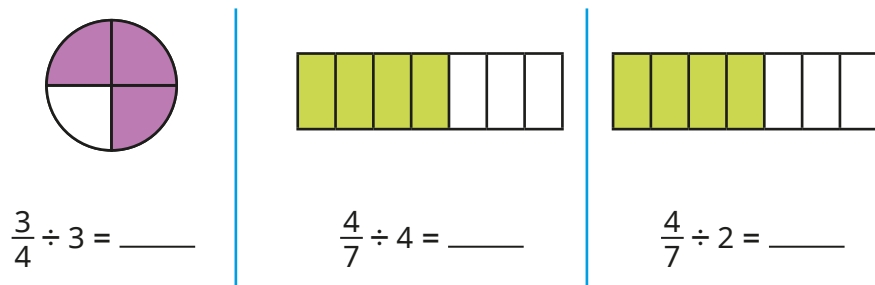
- Filip has  $\frac{2}{5}$  of a chocolate bar.

He shares it with his friend.

What fraction of the chocolate bar do they each get?



- Use the diagrams to help you work out the divisions.



- Use the division fact  $12 \div 4 = 3$  to work out the divisions.

▶  $12,000 \div 4$     ▶  $12 \text{ m} \div 4$     ▶  $12\text{p} \div 4$     ▶  $\frac{12}{19} \div 4$

- Complete the divisions.

▶  $\frac{6}{11} \div 3$     ▶  $\frac{15}{17} \div 5$     ▶  $\frac{49}{50} \div 7$     ▶  $\frac{96}{101} \div 12$

- A cake has a mass of  $\frac{8}{9}$  kg.

- ▶ What is the mass of each piece if the cake is cut into 8 equal pieces?
- ▶ What is the mass of each piece if the cake is cut into 4 equal pieces?
- ▶ What is the mass of each piece if the cake is cut into 2 equal pieces?

- Find the missing integers.

▶  $\frac{15}{16} \div \text{_____} = \frac{5}{16}$

▶  $\frac{20}{23} \div \text{_____} = \frac{4}{23}$

▶  $\frac{15}{16} \div \text{_____} = \frac{3}{16}$

▶  $\frac{20}{23} \div \text{_____} = \frac{5}{23}$

- Mo works out  $1\frac{3}{5} \div 2$  using improper fractions.

$$1\frac{3}{5} \div 2 = \frac{8}{5} \div 2 = \frac{4}{5}$$

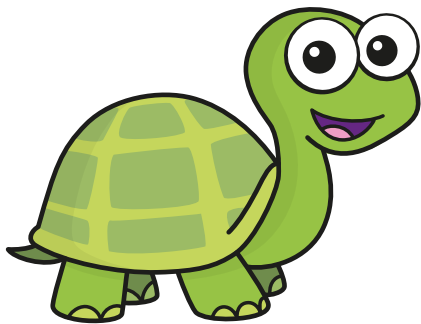
Use Mo's method to work out the divisions.

▶  $1\frac{1}{3} \div 2$     ▶  $1\frac{7}{9} \div 4$     ▶  $2\frac{5}{8} \div 3$     ▶  $3\frac{3}{4} \div 5$

# Divide a fraction by an integer

## Reasoning and problem solving

Dividing by 2 is the same as finding half of a number, so  $\frac{4}{11} \div 2$  is the same as  $\frac{1}{2} \times \frac{4}{11}$



Yes

Do you agree with Tiny?

Explain your answer.

Amir and Alex each have a piece of ribbon that is  $\frac{99}{100}$  m long.

- Amir cuts his ribbon into 9 equal pieces.
- Alex cuts her ribbon into 3 equal pieces.

Whose pieces of ribbon are longer?

By how much?

Give your answer in centimetres.

Compare methods with a partner.

Alex  
22 cm

What could the missing numbers be?

$$\frac{\square}{21} \div 4 = \frac{\square}{21}$$

Can any of your answers be simplified?

multiple possible answers, e.g.

$$\frac{4}{21} \div 4 = \frac{1}{21}$$

$$\frac{12}{21} \div 4 = \frac{3}{21} = \frac{1}{7}$$

# Divide any fraction by an integer

## Notes and guidance

In this small step, children build on their learning from the previous step to divide fractions where the numerator is not a multiple of the integer they are dividing by.

Children continue to use models and draw diagrams to divide fractions into equal parts. There are two methods that children could use throughout this step. They could use their prior knowledge of equivalent fractions combined with learning from the previous step to find an equivalent fraction where the numerator is a multiple of the integer they are dividing by. Alternatively, through the use of diagrams, children could explore the link between multiplying by a unit fraction and dividing by an integer. When using this method, children should be encouraged to spot the pattern that the numerator stays the same and the denominator is multiplied by the integer.

Encourage children to compare methods and decide which is more efficient, and why.

### Things to look out for

- Following on from the previous step, children may try to divide the numerator by the integer anyway even when it is not a multiple, for example  $\frac{3}{5} \div 2 = \frac{1.5}{5}$
- Children may become over-reliant on quick tricks.

## Key questions

- How can you split a fraction into equal parts? What is each part of the fraction worth?
- How can you show the division as a bar model?
- How is  $\frac{1}{3} \div 2$  similar to  $\frac{1}{3} \times \frac{1}{2}$ ?
- What fractions are equivalent to \_\_\_\_\_?
- Why does finding an equivalent fraction help you to divide a fraction by an integer?
- What multiplication can you use to work out \_\_\_\_\_  $\div$  \_\_\_\_\_?

## Possible sentence stems

- The bar is split into \_\_\_\_\_ equal parts.
- I am dividing each \_\_\_\_\_ by \_\_\_\_\_, so I must split each part into \_\_\_\_\_ equal parts.
- \_\_\_\_\_ is equivalent to \_\_\_\_\_, so \_\_\_\_\_  $\div$  \_\_\_\_\_ is equal to \_\_\_\_\_  $\div$  \_\_\_\_\_


## National Curriculum links

- Divide proper fractions by whole numbers

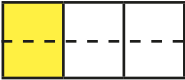
# Divide any fraction by an integer

## Key learning

- Teddy divides one third into 2 equal parts.



Each part is worth  $\frac{1}{6}$   
so  $\frac{1}{3} \div 2 = \frac{1}{6}$



Draw diagrams to work out the divisions.


$\frac{1}{3} \div 3$

$\frac{2}{3} \div 3$

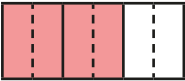
$\frac{1}{5} \div 3$

$\frac{2}{5} \div 3$

- Annie is dividing  $\frac{2}{3}$  by 4



I will find an equivalent fraction to help me divide the numerator equally.



$$\frac{2}{3} = \frac{4}{6}$$

$$\frac{4}{6} \div 4 = \frac{1}{6}$$

Use equivalent fractions to work out the divisions.

$\frac{3}{5} \div 2$

$\frac{1}{3} \div 3$

$\frac{2}{3} \div 3$


$\frac{4}{5} \div 8$

- Jack is dividing fractions by integers.

$$\frac{2}{5} \div 3 = \frac{2}{15}$$

$$\frac{3}{4} \div 5 = \frac{3}{20}$$

$$\frac{5}{7} \div 6 = \frac{5}{42}$$



I've noticed something!

What has Jack noticed?

- Work out the divisions.

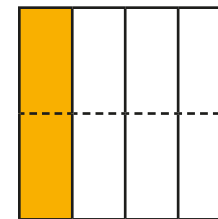
$\frac{1}{8} \div 3$

$\frac{5}{9} \div 2$

$\frac{2}{7} \div 3$

$\frac{11}{12} \div 15$

- Use the diagram to explain why  $\frac{1}{4} \div 2 = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$



- Work out the missing numbers.

$$\frac{1}{3} \div 2 = \frac{3}{4} \times \frac{\square}{\square} = \frac{\square}{\square}$$

$$\frac{3}{5} \div 2 = \frac{3}{5} \times \frac{\square}{\square} = \frac{\square}{\square}$$

# Divide any fraction by an integer

## Reasoning and problem solving

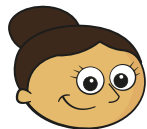
The children are working

out  $\frac{6}{7} \div 2$



$$6 \div 2 = 3, \text{ so } \frac{6}{7} \div 2 = \frac{3}{7}$$

Ron



$$\frac{1}{7} \div 2 = \frac{1}{14}, \text{ so } \frac{6}{7} \div 2 = \frac{6}{14}$$

Dora



$$\frac{6}{7} \div 2 = \frac{6}{7} \times \frac{1}{2} = \frac{6}{14}$$

Whitney

Explain why each child is correct.

Discuss each of their methods.

$\frac{3}{7}$  and  $\frac{6}{14}$   
are equivalent.

Is the statement true or false?

$$\frac{3}{5} \div 4 = \frac{3}{4} \div 5$$

True

Explain your answer.

Find the missing fractions  
and integers.

$$\underline{\hspace{2cm}} \div 4 = \frac{7}{36}$$

$$\frac{3}{20} \div \underline{\hspace{2cm}} = \frac{3}{80}$$

$$\underline{\hspace{2cm}} \div \underline{\hspace{2cm}} = \frac{2}{5}$$

$\frac{7}{9}$  or equivalent  
\_\_\_\_\_

4  
\_\_\_\_\_

multiple possible  
answers, e.g.

$\frac{4}{5}$  and 2

Is there more than one possible  
answer for each calculation?



# Mixed questions with fractions

## Notes and guidance

Children have now used all four operations with fractions in isolation. In this small step, children identify the appropriate operation(s) to use in a given situation.

Bar models are used to explore word problems and to support children in selecting the correct operation(s). Children start by choosing the correct single operation to solve a problem and move on to explore multi-step problems using all four operations. This step provides a good opportunity to revisit learning from earlier in the year. They can consolidate their knowledge of the order of operations, and also topics such as measure from earlier years.

### Things to look out for

- Children may find it difficult to identify the different steps within a problem.
- Children may perform the operations in the wrong order.
- If there are a lot of steps, children may get confused about where they are in the solution to the problem.
- The presence of a fraction in the question may make it feel harder for children, and they could be prompted by considering a similar question with integer values.

## Key questions

- Do you need to find the whole or a part? Where can you show this on the bar model?
- What type of calculation do you need to do? How can you tell?
- Does it matter in which order you perform the calculations? Why/why not?
- Which operation should you perform first/second?
- What happens when you insert brackets into the calculation?

## Possible sentence stems

- In this calculation, first I need to do \_\_\_\_\_ and then ...
- To solve the problem, I need to find the \_\_\_\_\_ of the two fractions.

### National Curriculum links

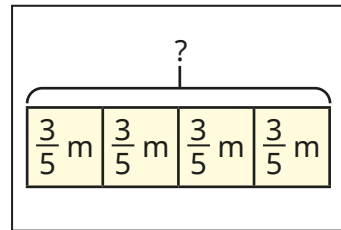
- Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- Multiply simple pairs of proper fractions, writing the answer in its simplest form
- Divide proper fractions by whole numbers
- Solve problems involving addition, subtraction, multiplication and division

# Mixed questions with fractions

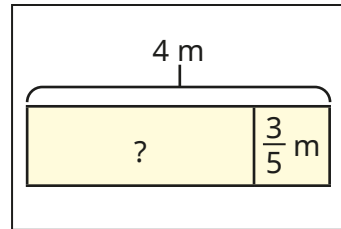
## Key learning

- Match the bar models to the correct problems.

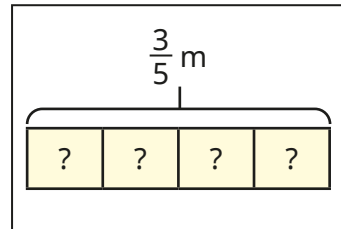
A piece of ribbon is 4 m long. Tom cuts  $\frac{3}{5}$  off. How much ribbon is left?



Nijah has 4 pieces of ribbon. Each piece is  $\frac{3}{5}$  m long. How much ribbon does Nijah have altogether?

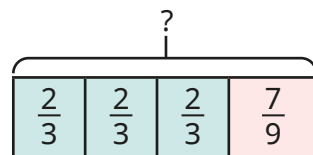


A piece of ribbon is  $\frac{3}{5}$  m long. Brett cuts it into 4 equal parts. How long is each part?



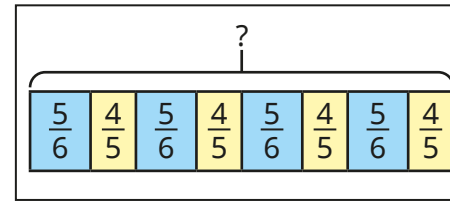
Work out the answer to each problem.

- Find the total length of the bar. Is there more than one way to find the answer?

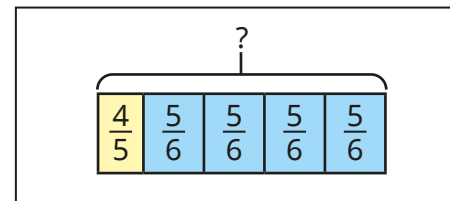


- Find the difference between  $\frac{3}{4} \times 3$  and  $\frac{3}{4} + 3$

- Match the bar models to the calculations.



$$\left(\frac{4}{5} + \frac{5}{6}\right) \times 4$$



$$\frac{4}{5} + \left(\frac{5}{6} \times 4\right)$$

- Work out the calculations.

$$\triangleright 3\frac{1}{3} + \frac{3}{4} \times 2$$

$$\triangleright 3\frac{1}{3} + \frac{3}{4} \div 2$$

$$\triangleright 3\frac{1}{3} + \frac{3}{4} \div 2$$

- Scott has one-quarter of a bag of sweets.

Kim has two-thirds of a bag of sweets.

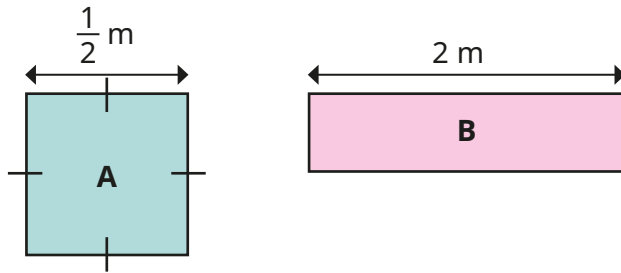
They combine their sweets and share them between themselves and Esther.

What fraction of a bag of sweets does each child get?

## Mixed questions with fractions

### Reasoning and problem solving

Square A and rectangle B have the same area.  
Find the difference between their perimeters.

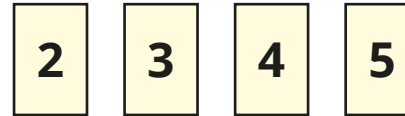


$$2\frac{1}{4} \text{ m}$$

Add two sets of brackets to make the calculation correct.

$$\frac{1}{2} + \frac{1}{4} \times 8 + \frac{1}{6} \div 2 + 1 = 6\frac{1}{18}$$

$$\left(\frac{1}{2} + \frac{1}{4}\right) \times 8 + \frac{1}{6} \div (2 + 1)$$



Using each digit once only, find as many solutions to the calculation that are between 1 and 2 as you can.

$$\frac{\boxed{1}}{\boxed{\phantom{00}}} + \boxed{\phantom{00}} \times \frac{\boxed{\phantom{00}}}{\boxed{\phantom{00}}}$$

Compare answers with a partner.

multiple possible answers, e.g.

$$\frac{1}{3} + 2 \times \frac{4}{5}$$

$$\frac{1}{4} + 2 \times \frac{3}{5}$$

$$\frac{1}{5} + 2 \times \frac{3}{4}$$

$$\frac{1}{3} + 4 \times \frac{2}{5}$$

$$\frac{1}{4} + 3 \times \frac{2}{5}$$

$$\frac{1}{5} + 3 \times \frac{2}{4}$$

# Fraction of an amount

## Notes and guidance

In Year 5, children used bar models to pictorially represent unit and non-unit fractions of an amount. The main focus of this small step is on understanding that the denominator is the number of parts the whole is divided into, and the numerator represents the number of those parts that are selected.

Bar models are a useful way for children to realise the connection between parts and wholes of an amount. By the end of this step, children should be able to find fractions of an amount in different contexts. Encourage them to divide by the denominator and multiply by the numerator, understanding why they are doing this and what they are finding in each step.

### Things to look out for

- Children may divide by the numerator instead of the denominator.
- Support may be needed for children who are not fluent with times-tables facts.
- Children may only find the value of the unit fraction and not multiply by the numerator to find the value of the whole fraction.

## Key questions

- How do multiplication and division help us when finding fractions of an amount?
- What does dividing the whole amount by the denominator work out?
- How are the parts and wholes represented in a fraction?
- What bar model could you draw to represent the calculation?
- What is the difference between a unit fraction and a non-unit fraction?

## Possible sentence stems

- The whole is divided into \_\_\_\_\_ equal parts. Each part is worth \_\_\_\_\_
- The numerator is \_\_\_\_\_, so the fraction is worth \_\_\_\_\_
- If one fifth is equal to \_\_\_\_\_, then \_\_\_\_\_ fifths are equal to \_\_\_\_\_

## National Curriculum links

- Associate a fraction with division and calculate decimal fraction equivalents

# Fraction of an amount

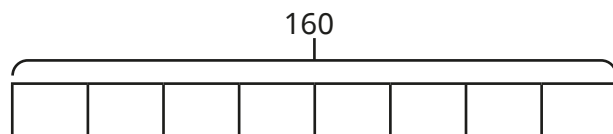
## Key learning

- Complete the sentences.
  - ▶ To find one-half of an amount, divide the amount by \_\_\_\_\_
  - ▶ To find one-third of an amount, divide the amount by \_\_\_\_\_
  - ▶ To find one-quarter of an amount, divide the amount by \_\_\_\_\_
  - ▶ To find one-tenth of an amount, divide the amount by \_\_\_\_\_
  - ▶ To find one-eighteenth of an amount, divide the amount by \_\_\_\_\_

- Work out the fractions of the amounts.

$\frac{1}{5}$ of 20	$\frac{1}{4}$ of 40	$\frac{1}{5}$ of 30
$\frac{1}{10}$ of £20	$\frac{1}{8}$ of 40 m	$\frac{1}{10}$ of 90 g

- Use the bar model to find the missing numbers.

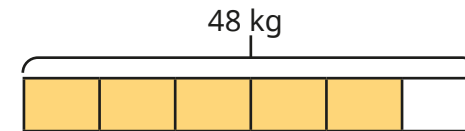


- ▶  $\frac{1}{8}$  of 160 = \_\_\_\_\_
- ▶  $\frac{5}{8}$  of 160 = \_\_\_\_\_
- ▶ \_\_\_\_\_ of 160 = 60

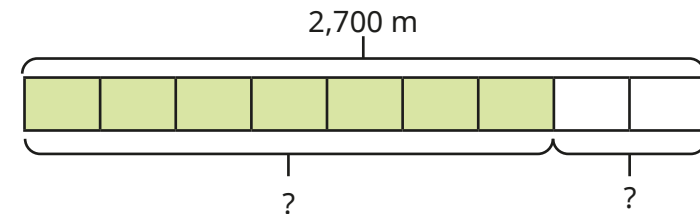
- A cook has 48 kg of potatoes.

He uses  $\frac{5}{6}$  of the potatoes.

How many kilograms of the potatoes does he have left?



- Use the bar model to complete the calculations.



- ▶  $\frac{7}{9}$  of 2,700 m = \_\_\_\_\_ m
- ▶  $\frac{2}{9}$  of 2,700 m = \_\_\_\_\_ m

- Work out the fractions of the amounts.

$\frac{3}{8}$ of 40	$\frac{5}{6}$ of 18	$\frac{3}{4}$ of 160	$\frac{4}{7}$ of 35
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# Fraction of an amount

## Reasoning and problem solving

Fill in the missing numbers.

$$\frac{\square}{6} \text{ of } \pounds 300 = \pounds 250$$

5

$$420 \text{ g} = \frac{\square}{12} \text{ of } 720 \text{ g}$$

7

Amir is saving to buy a new game that costs  $\pounds 120$

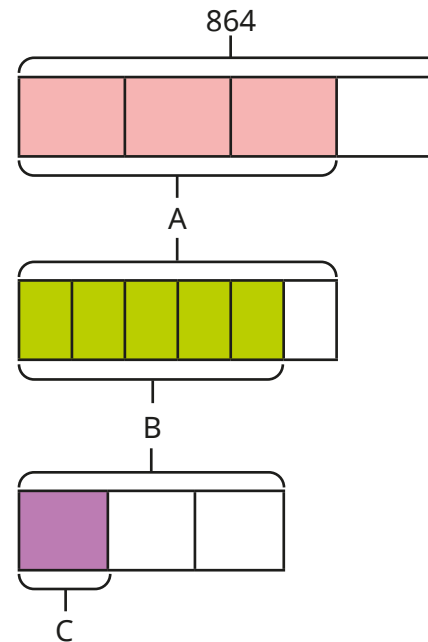
He has saved  $\frac{2}{3}$  of the money so far.

How much more money does Amir need to save?

$\pounds 40$

Compare methods with a partner.

Find the values of A, B and C.



$$A = 648$$

$$B = 540$$

$$C = 180$$

Compare methods with a partner.

# Fraction of an amount – find the whole

## Notes and guidance

In the previous step, children found a fraction of an amount. In this small step, they find the whole amount given a fraction of it.

Using a bar model to represent the parts and the whole is a useful support to children when working through this step. When finding the whole from a unit fraction, a pictorial representation helps children to understand why they simply need to multiply the given amount by the denominator. They then find a unit fraction from a given non-unit fraction and use this to find the whole.

Draw attention to the fact that, when calculating the whole, their answer will be greater than the number in the question. This will help children to sense check their answer.

Fluency with times-tables facts is very helpful here; some children may need a times-table square as support.

## Things to look out for

- Children may misinterpret  $\frac{3}{4}$  of \_\_\_\_\_ = 24 as “Find  $\frac{3}{4}$  of 24”
- Without pictorial support, children may find it difficult to work out whether to divide or multiply by the numerator/denominator.

## Key questions

- How many equal parts are there altogether?
- How many equal parts do you know the value of?
- What is the value of each equal part?
- How can you find the whole?
- Should the whole be greater than or less than the value you are given? Why?

## Possible sentence stems

- If one-sixth is equal to \_\_\_\_\_, then the whole is equal to \_\_\_\_\_
- If five-sixths is equal to \_\_\_\_\_, then one-sixth is equal to \_\_\_\_\_ and the whole is equal to \_\_\_\_\_
- The whole is split into \_\_\_\_\_ equal parts.
- To find one part, I need to divide by \_\_\_\_\_  
To find the whole, I need to multiply by \_\_\_\_\_

## National Curriculum links

- Associate a fraction with division and calculate decimal fraction equivalents

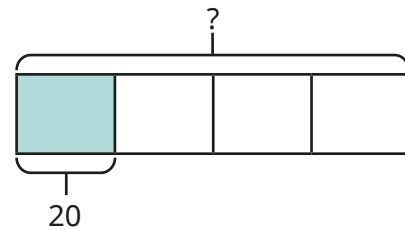
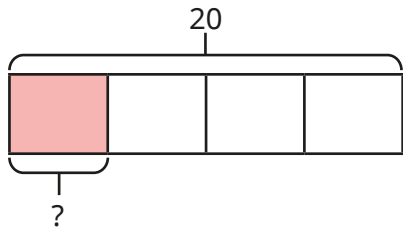
# Fraction of an amount – find the whole

## Key learning

- Complete the calculations.

$$\frac{1}{4} \text{ of } 20 = \underline{\quad}$$

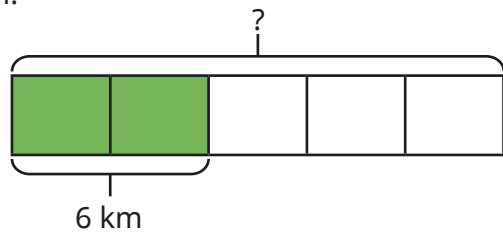
$$\frac{1}{4} \text{ of } \underline{\quad} = 20$$



What is the same about the calculations?

What is different?

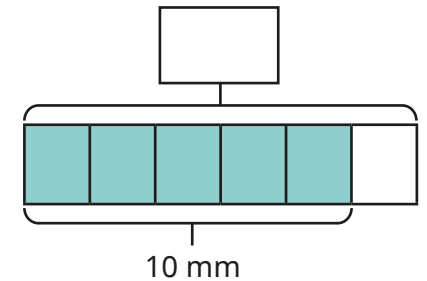
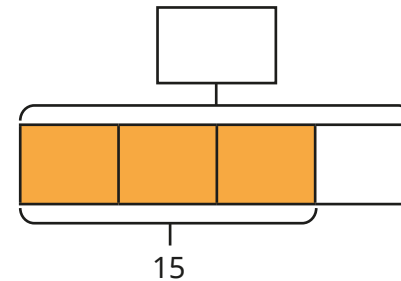
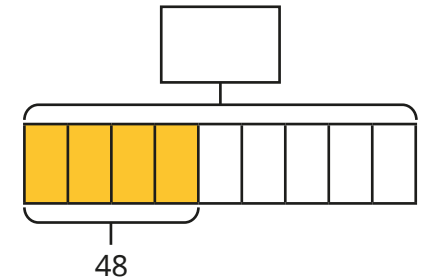
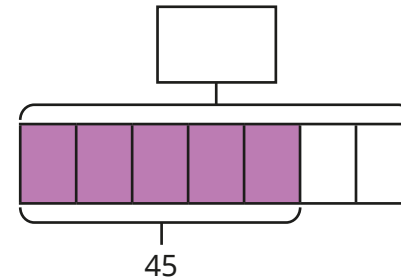
- Tommy runs  $\frac{2}{5}$  of a race for his running club.  
He runs 6 km.



How far is  $\frac{1}{5}$  of the race?

How far is the race altogether?

- Work out the missing wholes.



- Find the missing numbers.

▶  $\frac{3}{5}$  of  $\underline{\quad}$  = 21

▶ £180 =  $\frac{3}{7}$  of  $\underline{\quad}$

▶  $\frac{\square}{3}$  of 60 = 40

▶  $\frac{2}{\square}$  of 80 = 32



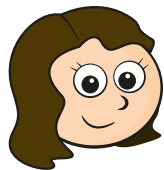
# Fraction of an amount – find the whole

## Reasoning and problem solving

Miss Rose lights a candle before she has a bath.

After her bath,  $\frac{2}{5}$  of the candle is left.

This part of the candle measures 13 cm.



Before my bath, the candle measured 65 cm.

No

Is Miss Rose correct?

Explain your reasoning.

Class 6 voted for their favourite ice cream flavour.

The table shows the fraction of the class that voted for each flavour.

Strawberry	$\frac{1}{4}$
Raspberry	$\frac{1}{6}$
Vanilla	$\frac{1}{12}$
Chocolate	$\frac{3}{8}$
Bubblegum	$\frac{1}{8}$

6 children in the class voted for strawberry.

How many children are there in Class 6?

How many children voted for chocolate?

24

9

Autumn Block 5

# Converting units

## Small steps

Step 1

Metric measures

Step 2

Convert metric measures

Step 3

Calculate with metric measures

Step 4

Miles and kilometres

Step 5

Imperial measures

# Metric measures

## Notes and guidance

Building on their experiences from earlier years, children recognise, read and write all metric measures for length, mass and capacity. This is the first time they will be introduced to tonnes as a measure for mass.

Highlight the difference between capacity (the amount an object can contain) and volume (the amount actually in an object). Children consider the most appropriate unit of measure and develop their estimation skills in context. Although metric units of measurement are used throughout, children may mention imperial units of measurement. The relationship between metric and imperial units will be explored later in the block.

Refer to the mass of an object, rather than its weight. The mass remains constant, whereas the weight of an object depends on the effect of gravity.

## Things to look out for

- Children may use the terms “weight” and “mass” interchangeably.
- Based on real-world experience, children may be more familiar with imperial measures, for example “miles” rather than “kilometres”.

## Key questions

- Which units could you use to measure length/mass/capacity?
- Which is the most appropriate unit to measure the \_\_\_\_\_ of a \_\_\_\_\_? Why?
- Why do you think \_\_\_\_\_ is not an appropriate estimate?
- Why would you not use kilometres to measure the length of the classroom? What would you use?
- What is the difference between capacity and volume?

## Possible sentence stems

- The best unit to measure the \_\_\_\_\_ of a \_\_\_\_\_ would be \_\_\_\_\_ because ...

## National Curriculum links

- Solve problems involving the calculation and conversion of units of measure, using decimal notation up to 3 decimal places where appropriate
- Use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to 3 decimal places

# Metric measures

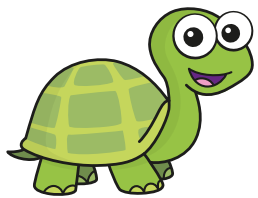
## Key learning

- Sort the units of measurement into the table.



Length	Mass	Capacity

- Tiny is thinking about volume and capacity.



The volume of the glass is the same as its capacity because they measure the same thing.



Do you agree with Tiny?  
Talk about it with a partner.

- Estimate the length of your classroom using appropriate units.  
Compare answers with a partner.

- Choose the most appropriate unit for each measurement.

- ▶ the length of a table



- ▶ the mass of a car



- ▶ the capacity of a water bottle



- Choose the most accurate estimate for each measurement.

- ▶ the mass of an apple



- ▶ the height of a door




- ▶ the capacity of a glass



# Metric measures


## Reasoning and problem solving


 It is impossible to measure the mass of a car in grams!

Do you agree with Amir?  
Explain your thinking.

No


Ron's dog is about  $\frac{1}{4}$  of the height of the door.  
 Ron is three times the height of his dog.

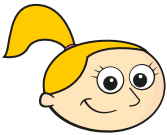


Estimate the height of Ron and his dog.

A door is approximately 2 m tall, so Ron's height is approximately 150 cm and the dog's height is 50 cm.

Whitney and Eva are measuring the length of a football pitch.

I am going to measure in metres.
 
 Whitney


 I am using kilometres to measure the pitch.
   
Eva

metres

Which unit of measurement is more appropriate?  
 Explain your reasoning.

# Convert metric measures

## Notes and guidance

In previous years, children learnt how to multiply and divide numbers by 10, 100 and 1,000. In Year 5, children learnt how to convert between metric measurements of length and mass. This small step recaps this learning and also introduces conversions between metric measurements for capacity.

Children convert between units both ways, for example from metres to centimetres and centimetres to metres. When making these conversions, children may need to be reminded about decimal place value.

When comparing measurements with different units, children need to convert them to the same unit. During this small step, highlight the inverse relationship between multiplication and division. It is important that children understand the role of zero as a place value holder when performing some calculations.

## Things to look out for

- Children may think that you multiply by 1,000 when converting measurements from metres to kilometres because they know that kilometres are a greater unit of measurement than metres. This may also happen when converting between units of mass and capacity.

## Key questions

- What is the same and what is different about kilometres and kilograms?
- What is the same and what is different about 1.5 km and 1.500 km?
- What do you notice about the conversions from metres to kilometres and grams to kilograms?
- Do you need to multiply or divide by 10/100/1,000? How do you know?

## Possible sentence stems

- There are \_\_\_\_\_ grams in one kilogram, so there are \_\_\_\_\_ grams in \_\_\_\_\_ kilograms.

## National Curriculum links

- Solve problems involving the calculation and conversion of units of measure, using decimal notation up to 3 decimal places where appropriate
- Use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to 3 decimal places

# Convert metric measures

## Key learning

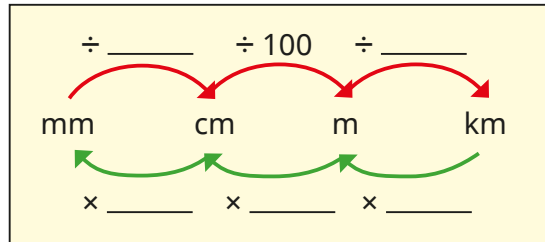
- There are 1,000 g in 1 kg and 1,000 kg in 1 tonne.

Use this fact to complete the tables.

g	kg
3,000	
	4
2,500	

kg	tonnes
7,000	
	8
9,500	

- Complete the diagram to show the conversions.



Use the diagram to complete the table.

mm	cm	m	km
1,500,000			
	250,000		
			3.4

- Complete the bar model.

1 litre	1 litre	1 litre	1 litre	$\frac{1}{2}$ litre
1,000 ml				

Complete the sentences.

- ▶  $4\frac{1}{2}$  litres = \_\_\_\_\_ ml
- ▶ \_\_\_\_\_ litres = 2,000 ml
- ▶ 3 litres = \_\_\_\_\_ ml
- ▶ 2,500 ml = \_\_\_\_\_ litres

- Write <, > or = to compare the measurements.

100 ml  0.1 l

15 cm  1.5 m

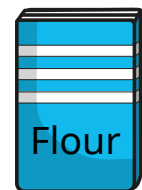
25 l  2,500 ml

1,500 mm   $1\frac{1}{2}$  m

4,020 ml  4.2 l

1.5 km  150 m

- A bag of flour has a mass of 200 g. Scott uses 3 bags of flour when baking. How much flour does he use? Write your answer in kilograms.





# Convert metric measures

## Reasoning and problem solving

Put the capacities in order, starting with the smallest.

3 litres	3,500 ml
0.4 litres	0.035 litres
450 ml	330 ml

Compare answers with a partner.

0.035 litres  
330 ml  
0.4 litres  
450 ml  
3 litres  
3,500 ml

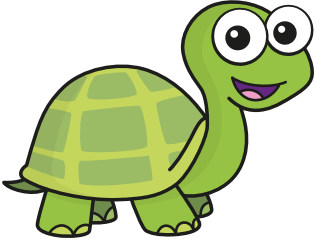
Dani thinks that 12,000 g is greater than 20 kg because  $12,000 > 20$

Do you agree?

Explain your answer.

No

These measurements are all the same length.



34,000 mm	3,400 cm
34 m	0.034 km

Do you agree with Tiny?

Explain your answer.

Yes

# Calculate with metric measures

## Notes and guidance

Building on the previous step, children use and apply their conversion skills to solve measurement problems in context.

The use of pictorial representations, such as bar models and number lines, to represent the problem helps children to choose the correct operation(s) to solve the problem. Children need to be secure with the four operations to find the correct numerical answers. Some of the problems involve finding a fraction of an amount (covered earlier this term) and adding and subtracting decimals, which will be revisited in the Spring Term.

### Things to look out for

- When finding a fraction of a unit of measurement, such as  $\frac{1}{2}$  of 1 kilogram, children may not notice the relationship between kilograms and grams and therefore will not be able to confidently write this as 500 g, which is easier to work with.
- When adding or subtracting amounts with different numbers of decimal places, children may not line up the place value columns accurately.
- Children may not convert all values to the same unit of measure before calculating.

## Key questions

- What operation are you going to use? Why?
- How could you use a bar model to help you understand the question?
- How many grams are there in one kilogram?
- Does it matter if the items in the question are measured in different units? Why?
- How can you convert between metres and centimetres?

## Possible sentence stems

- There are \_\_\_\_\_ in a \_\_\_\_\_
- To convert from \_\_\_\_\_ to \_\_\_\_\_, multiply/divide by \_\_\_\_\_

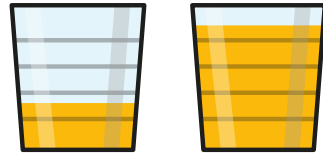
## National Curriculum links

- Solve problems involving the calculation and conversion of units of measure, using decimal notation up to 3 decimal places where appropriate
- Use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to 3 decimal places

# Calculate with metric measures

## Key learning

- Esther drinks 250 ml of juice.  
Kim drinks 3 times as much.



- ▶ How much does Kim drink?  
Give your answer in litres.

- ▶ How much do Esther and Kim drink in total?

- Brett has a piece of ribbon measuring 1.75 m.  
He is given a second piece of ribbon.  
Now he has 296 cm of ribbon in total.



How long is the second piece of ribbon in centimetres?

- A parcel has a mass of 440 grams.

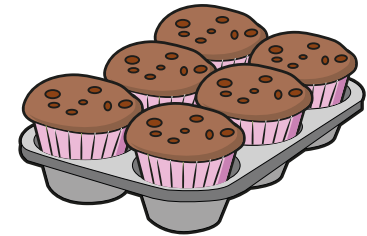


- ▶ What is the mass of 27 of these parcels?  
Give your answer in kilograms.

- ▶ A postal worker can carry a maximum of 12.5 kg.  
How many of these parcels can she carry?

- One gram of silver costs £0.55  
How much does half a kilogram of silver cost?
- Aisha uses these ingredients to make muffins.

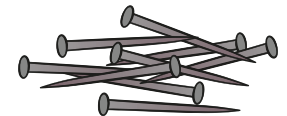
600 g caster sugar  
0.6 kg butter  
18 eggs  
 $\frac{3}{4}$  kg flour  
10 g baking powder



The mass of each egg is 50 g.

What is the total mass of the ingredients in kilograms?

- There are 28 nails in a packet.  
Each nail has a mass of 2 g.



- ▶ What is the total mass of nails in 60 packets?  
Give your answer in kilograms.
- ▶ The mass of nails in a large box is 0.5 kg.  
How many nails does it hold?

## Calculate with metric measures

### Reasoning and problem solving

Ron makes a stack of his comic books.

Each comic book is 2.5 mm thick.



The total height of the stack is 11.5 cm.  
How many comic books does he have?

46

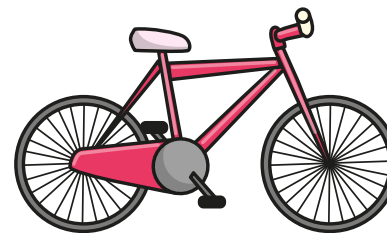
The total mass of a box and a crate is 3.4 kg.

The crate is 900 g heavier than the box.

What is the mass of the crate?

2.15 kg or 2,150 g

Teddy, Annie and Jack cycle as far as they can in one hour.



- Teddy cycles  $\frac{5}{6}$  of the distance that Jack cycles.
- Annie cycles 1,350 m less than Teddy.
- Jack cycles 5.4 km.

How far does Teddy cycle?

How far does Annie cycle?

How far do the three children cycle in total?

Teddy: 4,500 m  
or 4.5 km

Annie: 3,150 m  
or 3.15 km

Total: 13,050 m  
or 13.05 km

# Miles and kilometres

## Notes and guidance

In Year 5, children explored the relationship between some imperial and metric units of measurement. This small step focuses on the relationship between miles and kilometres.

Children need to know that one mile is a greater distance than one kilometre. They learn that 5 miles is approximately equal to 8 km. Using this fact, they solve conversions from miles to kilometres and from kilometres to miles. Children need to know that the symbol “ $\approx$ ” means “is approximately equal to”.

To provide context, distances measured in miles in the UK could be compared to distances measured in kilometres in Europe.

## Things to look out for

- Children may think that a kilometre is longer than a mile, since the same distance measured in kilometres is given by a greater number than if it was measured in miles. For example, 15 miles is approximately 24 km.
- Children may try to use additive reasoning rather than multiplicative reasoning when converting between miles and kilometres. 10 miles  $\approx$  16 km, so children may add 5 to both when finding out how many kilometres are equal to 15 miles.

## Key questions

- Which is further, one mile or one kilometre?
- What does the word “approximately” mean?
- What does the symbol “ $\approx$ ” mean?
- How can you use the key fact of 5 miles  $\approx$  8 km to calculate how many kilometres are approximately equal to 20 miles?
- When might you need to convert between miles and kilometres?

## Possible sentence stems

- \_\_\_\_\_ miles are approximately equal to 8 km.
- 10 miles are approximately equal to \_\_\_\_\_ km.

## National Curriculum links

- Solve problems involving the calculation and conversion of units of measure, using decimal notation up to 3 decimal places where appropriate
- Use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to 3 decimal places

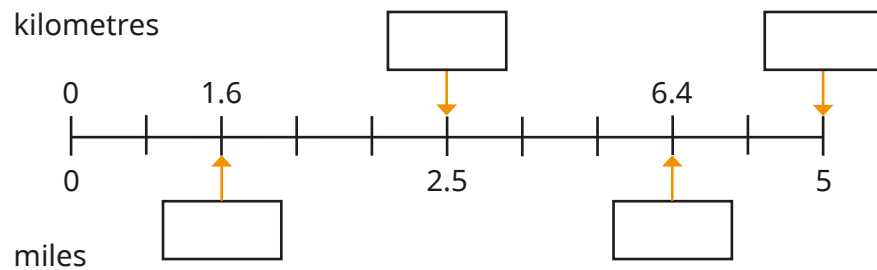
# Miles and kilometres

## Key learning

- Use the fact 5 miles  $\approx$  8 km to complete the conversions.

- ▶ 10 miles  $\approx$  \_\_\_\_\_ km
- ▶ 15 miles  $\approx$  \_\_\_\_\_ km
- ▶ 25 miles  $\approx$  \_\_\_\_\_ km
- ▶ 32 km  $\approx$  \_\_\_\_\_ miles
- ▶ 40 km  $\approx$  \_\_\_\_\_ miles
- ▶ 64 km  $\approx$  \_\_\_\_\_ miles

- Fill in the missing numbers on the number line.



- Complete the conversions.

- ▶ 7.5 miles  $\approx$  \_\_\_\_\_ km
- ▶ 160 km  $\approx$  \_\_\_\_\_ miles
- ▶ 96 miles  $\approx$  \_\_\_\_\_ km
- ▶ \_\_\_\_\_ km  $\approx$  55 miles
- ▶ \_\_\_\_\_ miles  $\approx$  320 km
- ▶ \_\_\_\_\_ km  $\approx$  250 miles

- Use a map of your local area. Find something that is approximately:

- 1 mile away from your school
- 1 km away from your school
- 2 miles away from your school
- 2 km away from your school

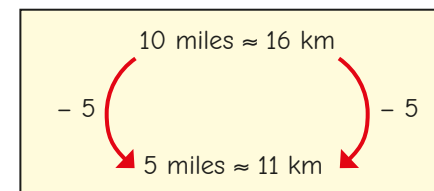
Compare answers with a partner.

- Write  $<$ ,  $>$  or  $=$  to compare the distances.

100 km  100 miles      48 km  28 miles

1.6 km  1 mile      0.5 miles  1 km

- Here are Tiny's workings to convert 5 miles to kilometres.

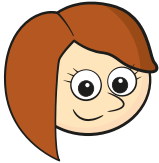


Explain Tiny's mistake.

# Miles and kilometres


## Reasoning and problem solving

Rosie and Tommy are running a 5-mile race.



I have run 6.4 km so far.

Rosie



I have run 3.8 miles so far.

Tommy

Who has the furthest left to run?  
Compare methods with a partner.

Tommy

Mo cycles 45 miles over the course of 3 days.

On day 1, he cycles 16 km.

On day 2, he cycles 10 miles further than he did on day 1

How far does he cycle on day 3?

Give your answer in miles and in kilometres.

15 miles  
24 km

The distance from Acton to Bigton is 120 miles.

Mr Smith is driving from Acton to Bigton. He stops at a service station 84 km into his journey.

How far does Mr Smith have left to travel?

Give your answer in miles and kilometres.

67.5 miles  
108 km

# Imperial measures

## Notes and guidance

In this small step, children continue to explore imperial measures and the relationships between imperial and metric measures. Children need to know and use the following facts:

- 1 inch  $\approx$  2.5 cm
- 1 stone = 14 pounds
- 1 foot = 12 inches
- 1 gallon = 8 pints
- 1 pound = 16 ounces

They use these facts to perform related conversions, both within imperial measures and between imperial and metric measures.

Attention should be drawn to the fact that the conversion between inches and cm is approximate while the others are exact.

## Things to look out for

- Children may have less prior experience of some of the imperial measures, so they may be dealing with a lot of new vocabulary.
- Some of the relationships will be new, for example children may recognise feet and inches as measuring length but not know the relationship between them.

## Key questions

- When do you use imperial measures instead of metric measures?
- Why is it easier to convert between metric measures than between imperial measures?
- Which is greater, one foot or one metre?
- Which is shorter, one centimetre or one inch?
- Which is heavier, one pound or one stone?

## Possible sentence stems

- As 1 inch is approximately equal to \_\_\_\_\_ cm, \_\_\_\_\_ inches are approximately equal to \_\_\_\_\_ cm.
- There are \_\_\_\_\_ inches in 1 foot, so there are \_\_\_\_\_ inches in \_\_\_\_\_ feet.

## National Curriculum links

- Solve problems involving the calculation and conversion of units of measure, using decimal notation up to 3 decimal places where appropriate
- Use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation up to 3 decimal places



# Imperial measures

## Key learning

- Sort the units of measurement into the table.

millilitre	centimetre	mile	gram	litre
stone	inch	metre	millimetre	tonne
gallon	ounce	pound	foot	kilometre

	Length	Mass	Capacity
Metric			
Imperial			

- 1 inch  $\approx$  2.5 cm

1 foot = 12 inches

Use these key facts to complete the conversions.

- ▶ 2 inches  $\approx$  \_\_\_\_\_ cm
- ▶ \_\_\_\_\_ inches  $\approx$  7.5 cm
- ▶ \_\_\_\_\_ inches  $\approx$  25 cm
- ▶ 12 inches  $\approx$  \_\_\_\_\_ cm
- ▶ 2 feet = \_\_\_\_\_ inches
- ▶ 5 feet = \_\_\_\_\_ inches
- ▶ 20 feet = \_\_\_\_\_ inches
- ▶ 100 feet = \_\_\_\_\_ inches

- 1 gallon = 8 pints

Use this key fact to complete the conversions.

- ▶ 2 gallons = \_\_\_\_\_ pints
- ▶ 10 gallons = \_\_\_\_\_ pints
- ▶ \_\_\_\_\_ gallons = 40 pints
- ▶ \_\_\_\_\_ gallons = 104 pints

- 1 pound (lb) = 16 ounces
- 1 stone = 14 pounds (lb)

Use these key facts to complete the conversions.

- ▶ 2 pounds = \_\_\_\_\_ ounces
- ▶ 5 pounds = \_\_\_\_\_ ounces
- ▶ \_\_\_\_\_ pounds = 240 ounces
- ▶ 2 stones = \_\_\_\_\_ lb
- ▶ 5 stones = \_\_\_\_\_ lb
- ▶ \_\_\_\_\_ stones = 154 lb

- Scott's bike has a mass of 24 pounds. Nijah's bike has a mass of  $1\frac{1}{2}$  stones. What is the difference between the mass of the two bikes?
- At sports day, Huan jumps 2 feet and 3 inches. Dora jumps 15 cm further than Huan. How far does Dora jump?

# Imperial measures

## Reasoning and problem solving

At sports day, the children drink a total of 60 gallons of water.

Each child drinks 3 pints.

How many children are at the sports day?

Compare methods with a partner.



160 children



Mr Hall is 6 foot 2 inches tall.

Ms Lee is 162 cm tall.

Who is taller?

How much taller are they?

Compare methods with a partner.



Mr Hall: 185 cm

He is 23 cm taller than Ms Lee.



Amir wants to make a cake.

Here are some of the ingredients he needs:

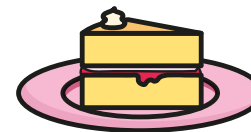
- 8 ounces caster sugar
- 6 ounces flour
- 6 ounces butter

This is what he has in his cupboards:

- 0.5 lb caster sugar
- 0.25 lb flour
- $\frac{3}{8}$  lb butter

Does Amir have enough ingredients to bake the cake?

If not, how much more does he need to buy?



Amir has the exact amount of caster sugar and butter.

He does not have enough flour. He needs another 2 ounces.