Progression Towards a Written Method for Addition

Y3

End of Year Objective:
Add numbers with up to three digits, using formal written method of columnar addition.*

**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4*

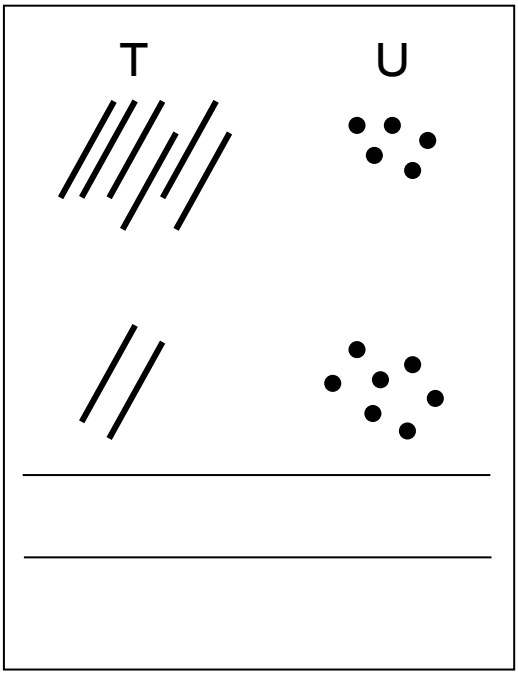
It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Children will build on their knowledge of using Base 10 equipment from Y2 and continue to use the idea of exchange.

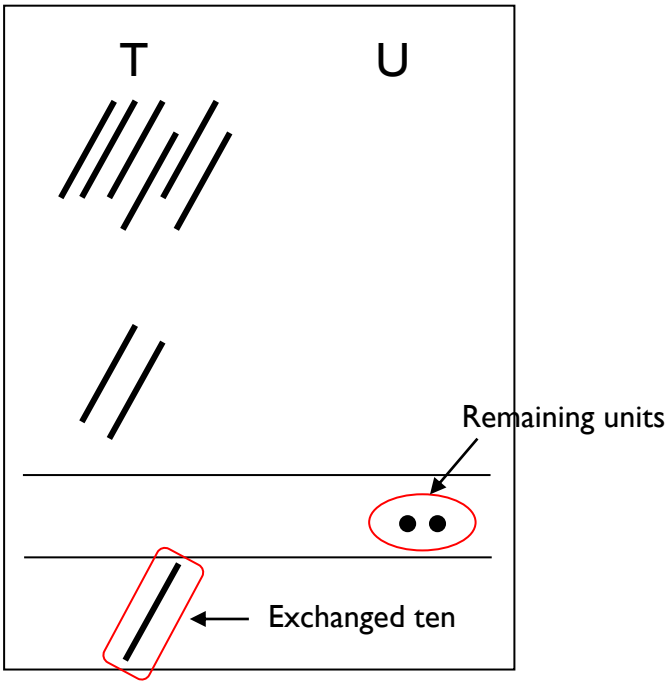
Children should add the **least significant digits** first (i.e. start with the units/ones), and in an identical method to that from year 2, should identify whether there are greater than ten units which can be exchanged for one ten.

They can use a place value grid to begin to set the calculation out vertically and to support their knowledge of exchange between columns (as in Step 1 in the diagram below).e.g. 65 + 27

Step 1



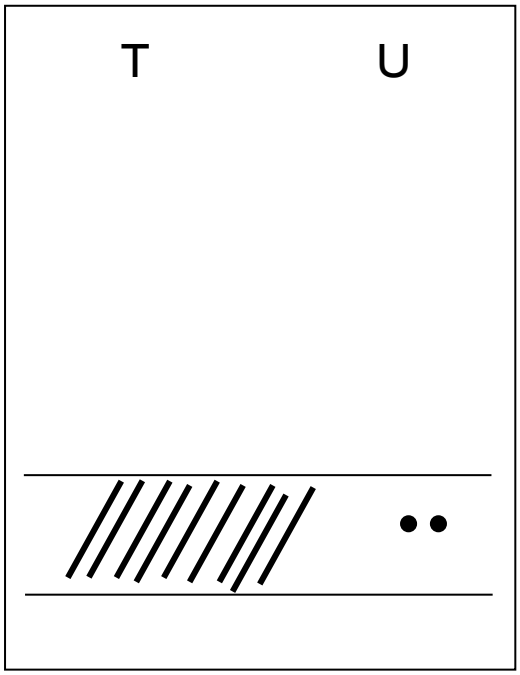
Step 2



Children would exchange ten units/ones for a ten, placing the exchanged ten below the equals sign. Any remaining units/ones that cannot be exchanged for a ten move into the equals sign as they are the units part of the answer (as in the diagram in Step 2 above).

If there are any tens that can be exchanged for a hundred, this can be done next. If not, the tens move into the equals sign as they are the tens part of the answer (as in the diagram in Step 3 below).

Step 3



Written method

Step 1

	T	U
	6	5
+	2	7

Step 2

	T	U
	6	5
+	2	7
		2
	1	

Step 3

	T	U
	6	5
+	2	7
	9	2
	1	

Children should utilise this practical method to link their understanding of exchange to how the column method is set out. Teachers should model the written method alongside this practical method initially.

This should progress to children utilising the written and practical methods alongside each other and finally, and when they are ready, to children utilising just the written method.

By the end of year 3, children should also extend this method for three digit numbers.

Progression Towards a Written Method for Subtraction

Y3

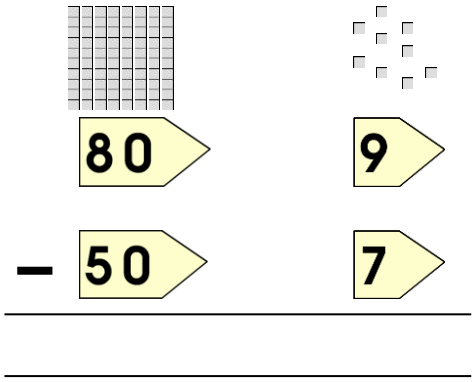
End of Year Objective:
Subtract numbers with up to three digits, using formal written method of columnar subtraction.*

**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4*

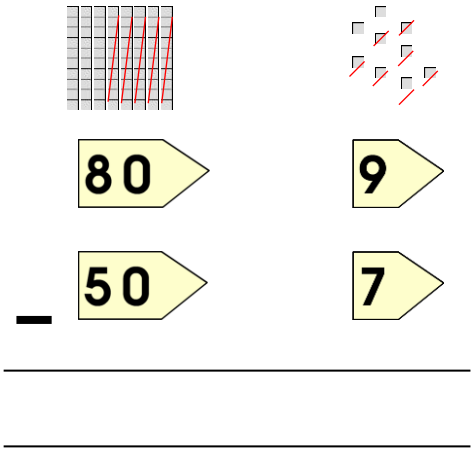
It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Children will build on their knowledge of using Base 10 equipment from year 2 and continue to use the idea of exchange. This process should be demonstrated using arrow cards to show the partitioning and Base 10 materials to represent the first number, removing the units and tens as appropriate (as with the more informal method in year 2).

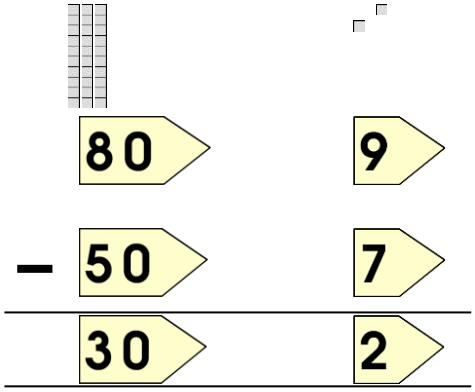
Step 1



Step 2



Step 3



Emphasise that the second (bottom) number is being subtracted from the first (top) number rather than the lesser number from the greater.

This will be recorded by the children as:

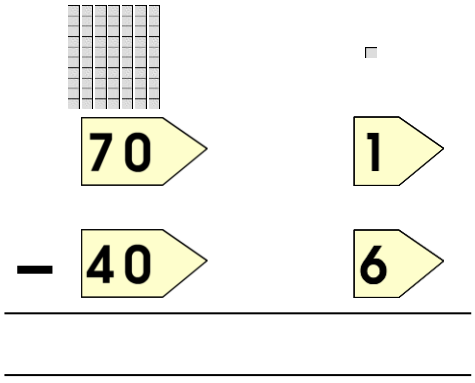
	80	→	9	
-	50	→	7	
	30	→	2	= 32

Children can also use jottings of the Base 10 materials (as in year 2) to support with their calculation, as in the example below.

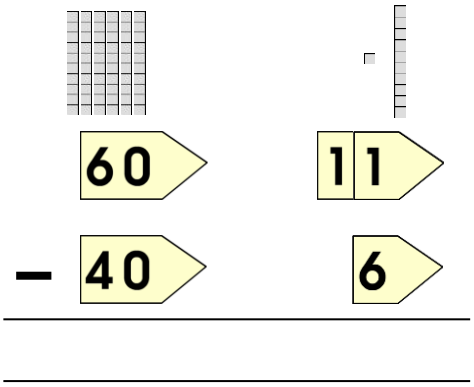
80 → 9
- 50 → 7
30 → 2 = 32

From this the children will begin to solve problems which involve exchange. Children need to consider whether there are enough units/ones to remove 6. In this case there are not (Step 1) so they need to exchange a ten into ten ones to make sure that there are enough, as they have been doing in the method for year 2 (Step 2). They should be able to see that the number is just partitioned in a different way, but the amount remains the same (71 = 70 + 1 = 60 + 11).

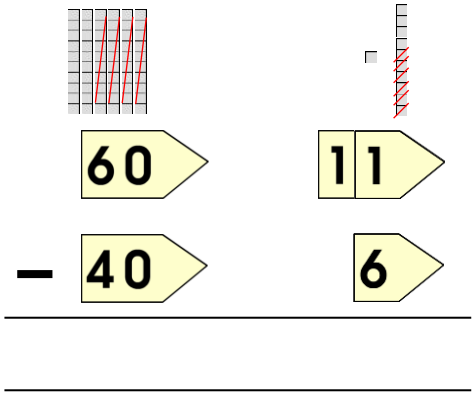
Step 1



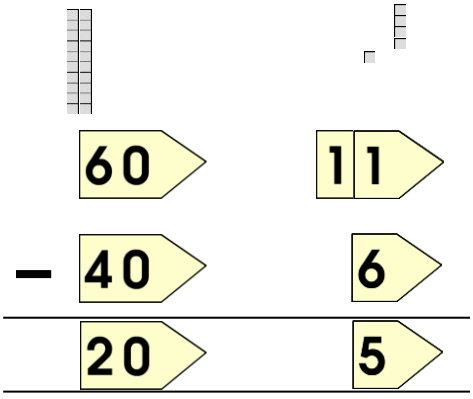
Step 2



Step 3



Step 4



This will be recorded by the children as:

60
70 → 11
- 40 → 6
20 → 5 = 25

By the end of year 3, children should also extend this method for three digit numbers.

Progression Towards a Written Method for Multiplication

Y3

End of Year Objective:
Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods.*

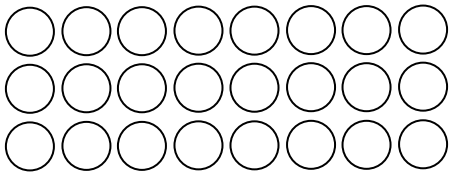
**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4*

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Initially, children will continue to use arrays where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

3 x 8

They may show this using practical equipment:



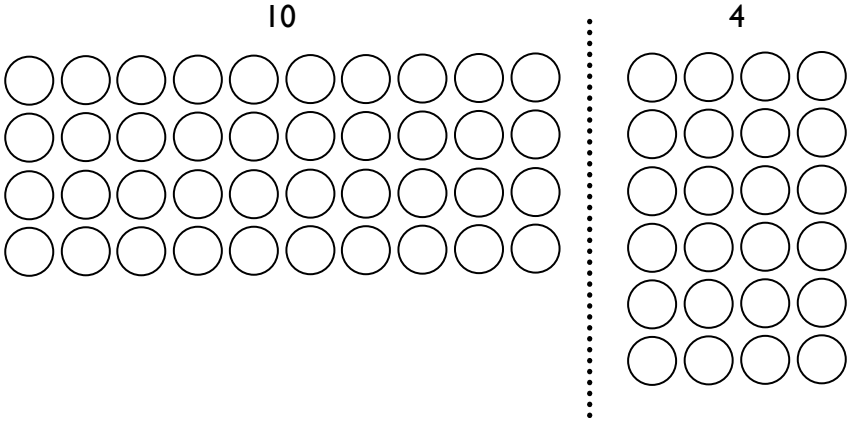
3 x 8 = 8 + 8 + 8 = 24

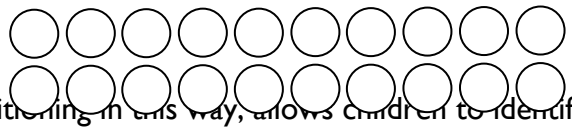
or by jottings using squared paper:

	x	x	x	x	x	x	x	x	
	x	x	x	x	x	x	x	x	
	x	x	x	x	x	x	x	x	

3 x 8 = 8 + 8 + 8 = 24

As they progress to multiplying a two-digit number by a single digit number, children should use their knowledge of partitioning two digit numbers into tens and units/ones to help them. For example, when calculating 14 x 6, children should set out the array, then partition the array so that one array has ten columns and the other four.





Partitioning in this way, allows children to identify that the first array shows 10 x 6 and the second array shows 4 x 6. These can then be added to calculate the answer:

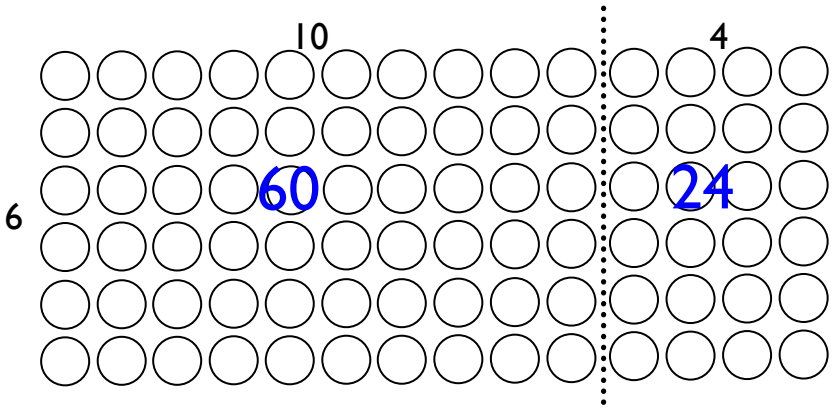
(6 x 10) + (6 x 4)

= 60 + 24

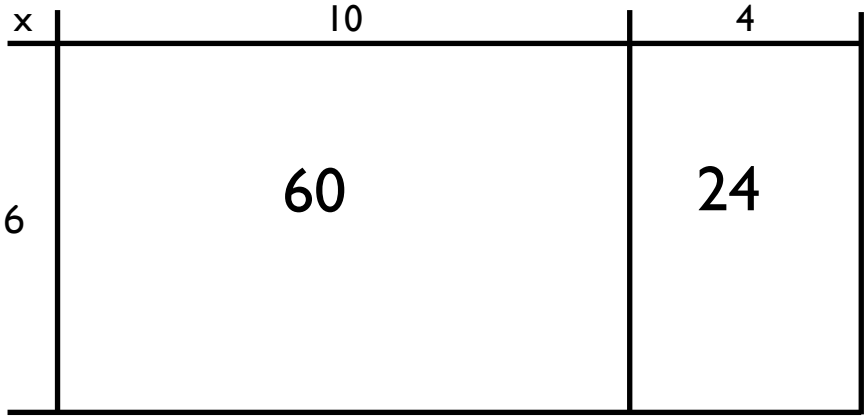
= 84

NB There is no requirement for children to record in this way, but it could be used as a jotting to support development if needed.

This method is the precursor step to the grid method. Using a two-digit by single digit array, they can partition as above, identifying the number of rows and the number of columns each side of the partition line.



By placing a box around the array, as in the example below, and by removing the array, the grid method can be seen.



It is really important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording.

From this, children can use the grid method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20. Children should be encouraged to set out their addition in a column at the side to ensure the place value is maintained. When children are working with numbers where they can confidently and correctly calculate the addition mentally, they may do so.

13 x 8

x	10	3
8	80	24

$$\begin{array}{r} 80 \\ +\ 24 \\ \hline 104 \end{array}$$

When children are ready, they can then progress to using this method with other two-digit numbers.

37 x 6

x	30	7
6	180	42

$$\begin{array}{r} 180 \\ +\ 42 \\ \hline 222 \end{array}$$

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Progression Towards a Written Method for Division

Y3

End of Year Objective:
Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, progressing to formal written methods.*

**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4*

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Initially, children will continue to use division by grouping (including those with remainders), where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

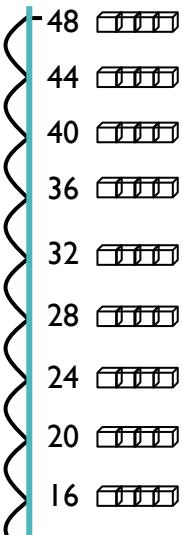
43 ÷ 8 =

43 ÷ 8 = 5 remainder 3

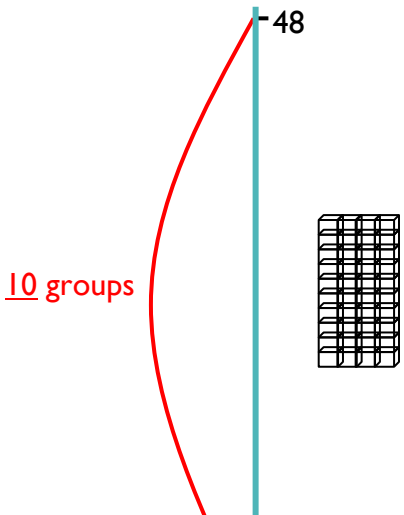
In preparation for developing the 'chunking' method of division, children should first use the repeated subtraction on a vertical number line alongside the continued use of practical equipment. There are two stages to this:

Stage 1 – repeatedly subtracting individual groups of the divisor
Stage 2 – subtracting multiples of the divisor (initially 10 groups and individual groups, then 10 groups and other multiples in line with tables knowledge)
After each group has been subtracted, children should consider how many are left to enable them to identify the amount remaining on the number line.

Stage 1
48 ÷ 4 = 12 (groups of 4)



Stage 2
48 ÷ 4 = 10 (groups of 4) + 2 (groups of 4)
= 12 (groups of 4)



Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

Progression Toward Mental Calculation Strategies (Addition and Subtraction)

Y3

End of Year Objective:
Add and subtract numbers mentally, including: a three-digit number and ones; a three-digit number and tens; a three-digit number and hundreds.

Rapid Recall

Children should be able to:

- recall and use addition and subtraction facts for 100 (multiples of 5 and 10)
- derive and use addition and subtraction facts for 100
- derive and use addition and subtraction facts for multiples of 100 that total 1000

Mental Strategies

Partition and combine multiples of hundreds, tens and ones

Partitioning numbers is a core strategy for adding and subtracting pairs of numbers. Children can either partition both of the numbers in the calculation, or keep the first number the same and just partition the second. *(See Y2 for more information).*

Examples of calculations:

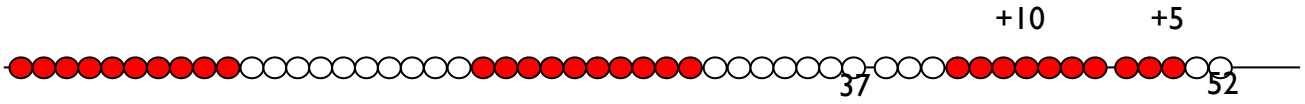
526 + 200	counting on in hundreds
137 + 40	counting on in tens
272 + 8	counting on in ones
428 - 200	counting back in hundreds
323 - 70	counting back in tens
693 - 8	counting back in ones
37 + 15	37 add 10 and 5 = 37 add 10 add 5 (crossing tens boundaries)
42 – 25	42 take away 20 and 5 = 42 take away 20 take away 5 (crossing tens boundaries)

Prerequisite skills:

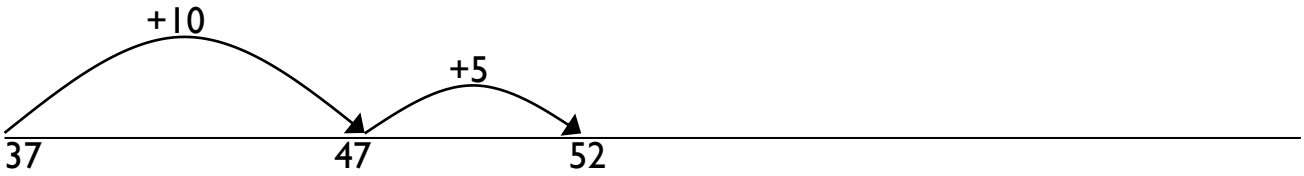
- Count forwards and backwards in ones, tens and hundreds from any one-, two- or three-digit number
- Understand place value and understand which digit changes if one, ten or hundred is added or subtracted
- Partition numbers into hundreds, tens and ones

Addition

37 + 15 = 52 (shown using a beadstring)



37 + 15 = 52 (shown using a numberline)

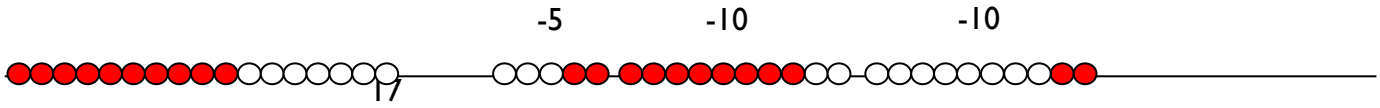


37 + 15 = (shown using number sentences)

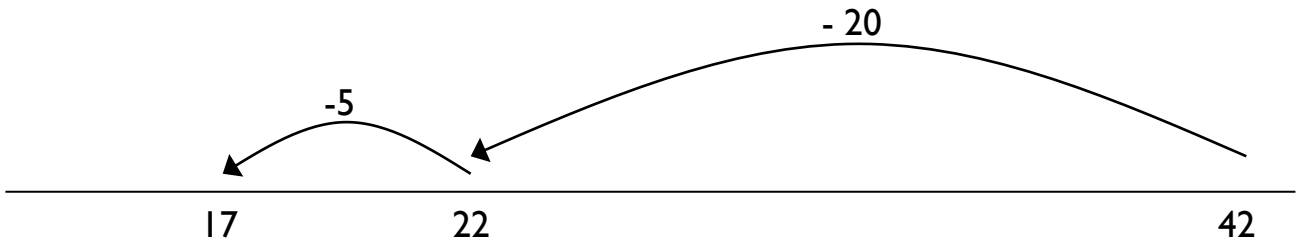
37 + 10 = 47
47 + 5 = 52

Subtraction

42 - 25 = 17 (shown using a beadstring)



42 – 25 = 17 (shown using a numberline)



42 – 25 = 17 (shown using number sentences)

42 – 20 = 22
22 – 5 = 17

Reorder numbers in a calculation

In Y3, children need to build on their knowledge gained in Y2 and continue to reorder calculations to make them more efficient. *(See Y2 for more information).*

Examples of calculations:

23 + 54	54 + 23
12 + 19 + 12	12 + 12 + 19 (using knowledge of doubles)
6 + 8 + 4	6 + 4 + 8 (using knowledge of number bonds to 10)
70 + 50 + 30	70 + 30 + 50 (using knowledge of number bonds to 100)

Prerequisite skills:

- Understand the place value of numbers to identify which number is the greater
- Understand that reordering works for addition but not subtraction* *(because children are not at the level when they are solving calculations such as 16 – 3 – 6, when reordering would be appropriate).*

Identify and use knowledge of number bonds within a calculation

Number bonds to 10 and 100 can be used to make calculations more efficient when combined with other strategies such as reordering and partitioning.

Examples of calculations:

42 + 38	42 + 30 + 8 (recognising that 2 and 8 is a number bond to 10, so the answer will be a multiple of 10)
60 – 28	60 – 20 – 8 (utilising knowledge that 10 – 8 = 2, so 40 – 8 = 32)
120 – 50	120 – 20 – 30 (utilising knowledge of number bonds to 100, leaving an answer of 70)

Prerequisite skills:

- Know, or quickly derive, number bonds to 10 and 100
- Identify number bonds within other numbers, e.g. identifying 7 + 3 within the calculation 57 + 33
- Identify that when adding two two-digit numbers, that 57 + 43 = 100 but 57 + 53 does not and why

Find differences by counting up through the next multiple of 10 or 100

In Y3, children need to build on their knowledge and understanding gained in Y2 to find larger differences that cross 10 and 100 boundaries. Some of these calculations are preparing children for time and money calculations throughout KS2.

Examples of calculations:

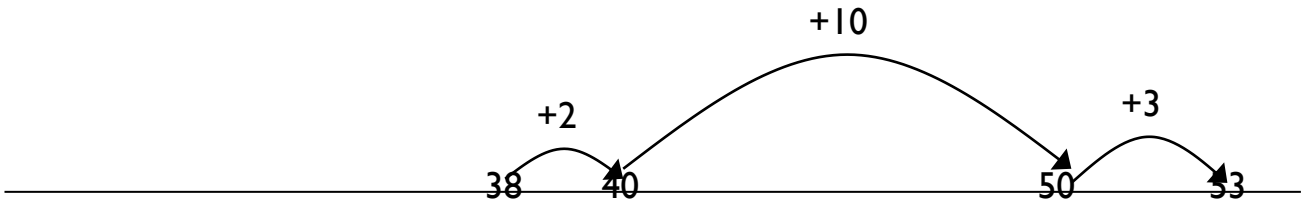
60 – 43	useful for time calculations, e.g. a journey time from 2:43 until 3:00
53 – 38	efficient because the numbers are close to each other
104 – 95	efficient because the numbers are close to each other
200 – 86	useful for money calculations, e.g. change from £2 when spending 86p

Prerequisite skills:

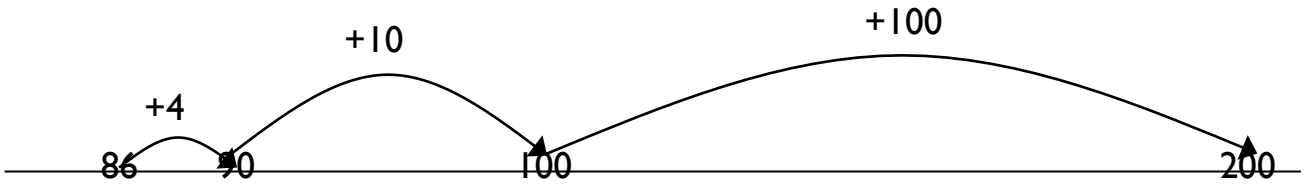
- Understand the place value of numbers to identify which number is the greater or lesser
- Establish whether numbers are close together or near to multiples of 10 or 100
- Place numbers appropriately on an unmarked numberline
- Count forwards and backwards in ones and tens

Children could use empty numberlines to record the calculation.

53 – 38 = 15



200 – 86 = 114



Bridge through 10 when adding or subtracting a single digit number (partitioning, e.g. 58 + 5 = 58 + 2 + 3 or 76 – 8 = 76 – 6 - 2)

In Y3, children need to consolidate their knowledge and understanding gained in Y2. *(See Y2 for more information).*

Examples of calculations

35 + 7	as 35 + 5 + 2
97 + 6	as 97 + 3 + 3
178 + 5	as 178 + 2 + 3
42 – 7	as 42 – 2 – 5
204 – 6	as 204 – 4 – 2
371 – 5	as 371 – 1 – 4

Prerequisite skills:

- Partition numbers in different ways, e.g. 5 as 2 + 3 to enable 58 + 5 as 58 + 2 + 3
- Know, or quickly derive, number bonds to 10

Children could use empty numberlines to record the calculation.

198 + 6 = 204

153 – 7 = 146

The bridging strategy can then be linked with the partitioning strategy for efficient addition and subtraction of two two-digit numbers.

Add or subtract 9, 19, 29 etc by rounding and compensating

In Y3, children need to build on their knowledge and understanding gained in Y2 *(See Y2 for more information)* to add and subtract one less than a multiple of 10 up to 89 to two and three-digit numbers.

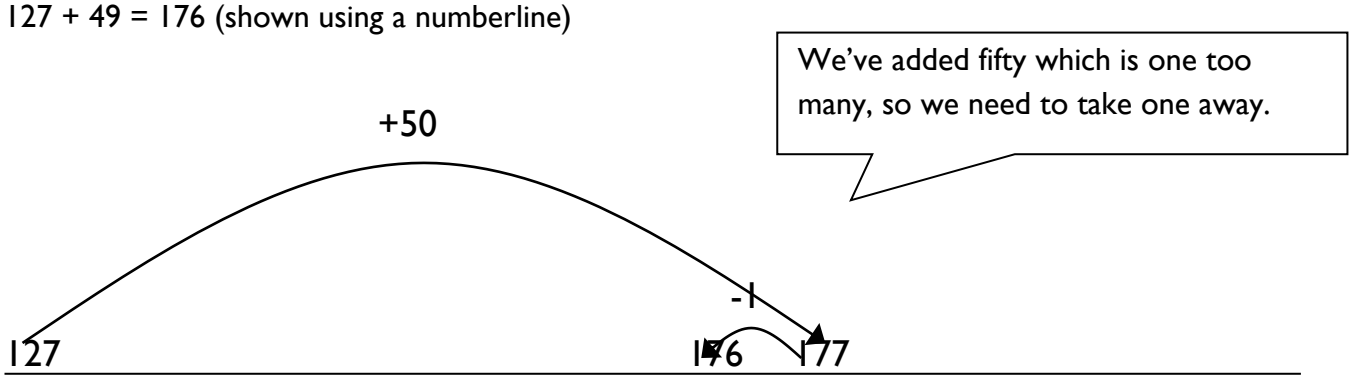
Examples of calculations

34 + 29	as 34 + 30 - 1
127 + 49	as 127 + 50 - 1
96 - 39	as 96 - 40 + 1
273 - 59	as 273 - 60 + 1

Prerequisite skills:

- Identify the difference between the number being added and subtracted and the multiple of 10
- Understand that the adjustment needs to be the opposite of the operation carried out

127 + 49 = 176 (shown using a numberline)

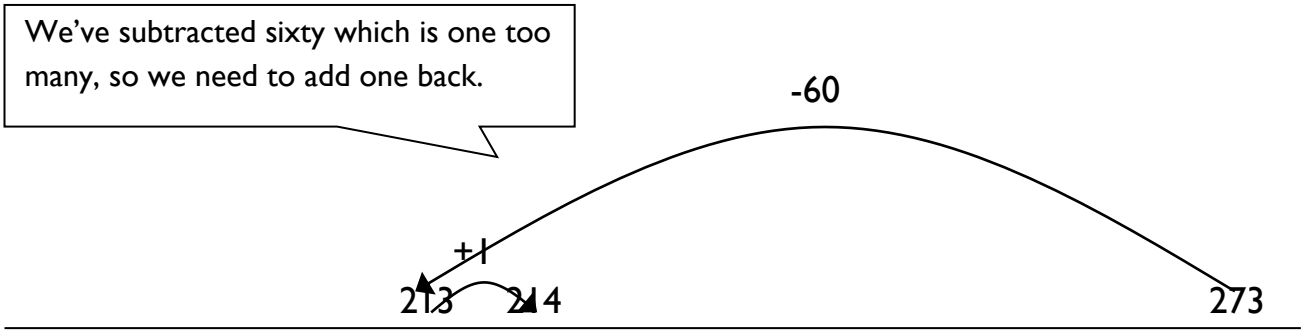


127 + 49 = 176 (shown using number sentences)

127 + 50 = 177

177 – 1 = 176

273 - 59 = 214 (shown using a numberline)



273 - 59 = 214 (shown using number sentences)

273 – 60 = 213
213 + 1 = 214