Maths reasoning

KS2

National Curriculum

Fluency

Pupils become fluent in the fundamentals of maths, including through varied and frequent practise with increasing complex so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.

Reasoning

Pupils reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language

Problem solving

Pupils can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Maths in Key Stage 2

- pupils become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value.
- pupils should develop their ability to solve a range of problems, including with simple fractions and decimal place value.
- By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table.
- pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic.
- pupils are introduced to the language of algebra as a means for solving a variety of problems.
- By the end of year 6, pupils should be fluent in written methods for all four operations.

Fluency

Refers to knowing key mathematical facts and methods and recalling these efficiently.

It is widely acknowledged that practice, drill and memorisation are essential if students are to become mathematically fluent.

Reasoning

Reasoning is the critical skill that enables a student to make use of all other **mathematical** skills.

Students recognise that **mathematics** makes sense and can be understood.

Example of reasoning activities

True or false?

- 324 is a multiple of 9?
- When I count in 10's, I will say the number 10100
- When I count backwards in 50's from 10 I will say the number -200

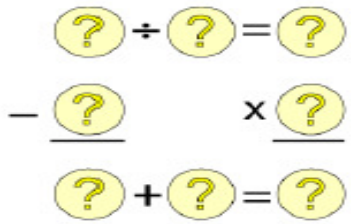
Example of reasoning activities

Odd one out

- Which is the odd one out in each of these collections of 4 fractions 6/10 3/5 18/20 9/15

  30/100 3/10 6/20 3/9 Why?
- **What do you notice?**
- Find 4/6 of 24
- Find 2/3 of 24 What do you notice?
- Can you write any other similar statements?

Year 3

	National Curriculum Statement	All students		
		Fluency	**Reasoning**	**Problem Solving**
Multiplication and Division	Solve problems including missing number problems involving multiplication and division, positive integer scaling problems and correspondence problems in which *n* objects are connected to *m* objectives.	• Fill in the boxes: 5 x ☐ = 15 ☐ x 4 = 32 48 ☐ 6 = 8 • Jemima has a toy car measuring 8cm. Aisha has a toy train that is 8 times as long as the car. How long is the train? • Kainat is making buns. For every 40g of flour she needs 1 egg. If she uses 5 eggs, how many grams of flour does she use? If she uses 400g of flour, how many eggs doe she need? • Tami is planting bulbs. She has 76 bulbs altogether. How many complete rows of eight can she plant?	• 12 buns are shared between 3 boys. 16 buns are shared between 4 girls. Who gets more buns, boys or girls? Explain your answer. • For every 3 boys in class there are 2 girls. Which of the combinations of boys and girls could be correct? 18 boys and 12 girls 15 boys and 10 girls 21 boys and 9 girls 12 boys and 8 girls Show your thinking using a picture. • Each tary can fit 8 gingerbread men on. I make two trays. Two people are correct, who do you agree with? Explain why.  Simone: 8 + 2 Frazer: 8 x 2 Aliyha: 8 + 8 Jem: 8 x 8	• Use the numbers 1 - 8 to fill the circles below:  • Lottie is counting the number of legs in her house. People and cats live in Lottie's house. People have 2 legs; cats have 4 legs. If there are 26 legs altogether, how many cats and people might there be? • William has 3 t-shirts and 4 pairs of trousers, how many different outfits can he make? • How many different combinations of numbers can you find that would fit in the empty boxes? 5 x ☐ = 10 x ☐

Year 4

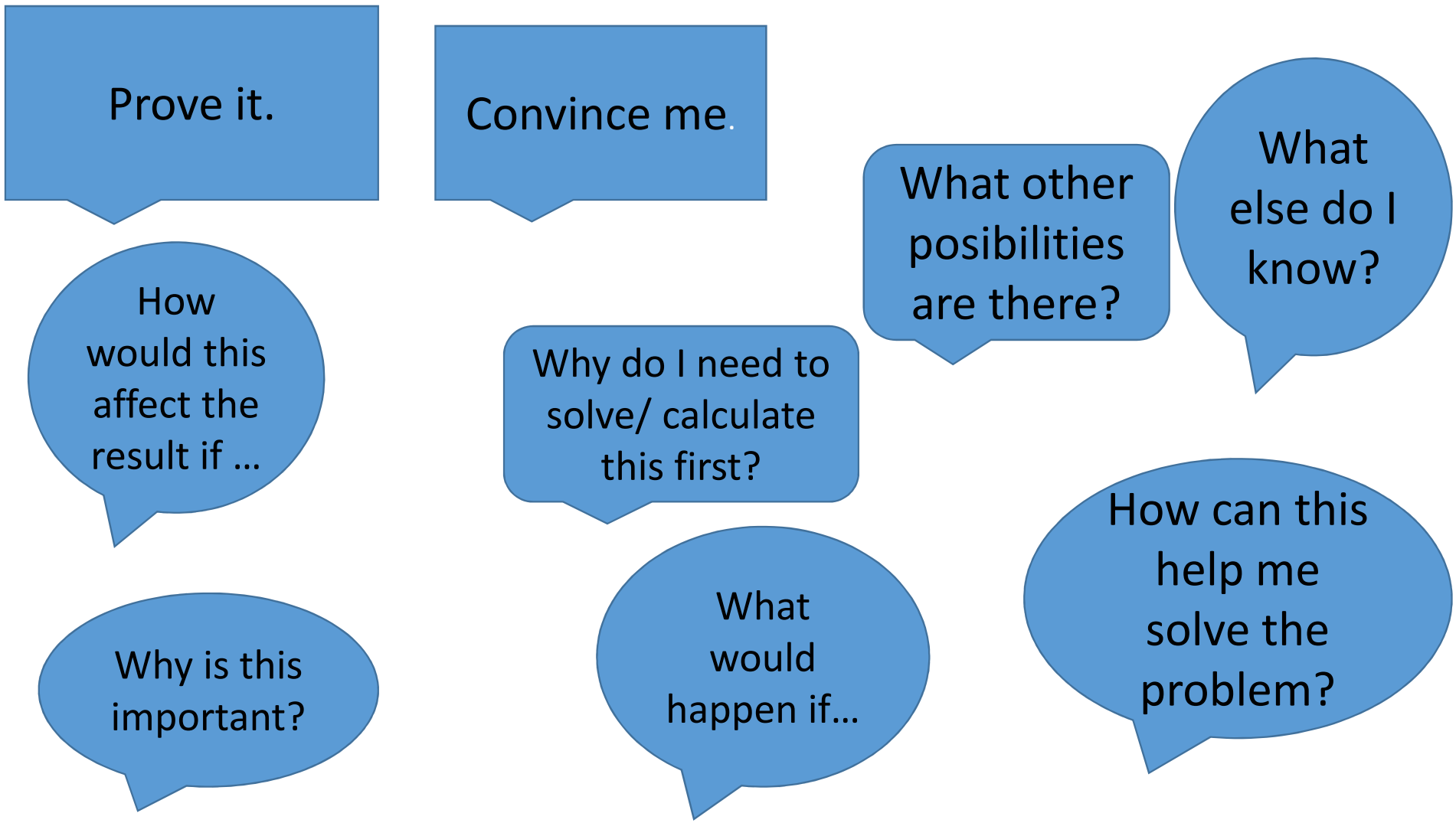
	National Curriculum Statement	All students		
		Fluency	Reasoning	Problem Solving
Multiplication and Division	Recognise and use factor pairs and commutatively in mental calculations.	• Use 16 cubes.  How many different arrays can you make? Think about making towers of cubes that are equal in height. Can you write a multiplication sentence to describe the towers? The numbers in your multiplication sentences are the factors of 16! • 7 x 5 = ■ = 5 x ■ • Find the missing numbers 12 x 6 = 6 x ___ 2 x 3 x 5 = __ x 5 2 x 7 x 5 = __ x 5 • 13 x 12 can be solved by using factor pairs, eg 13 x 3 x 4 or 13 x 2 x 6. What factor pair could you use to solve 17 x 8?	• Fill in the missing numbers 25 x 3 = ☐ x ☐ x ☐ • Use factor pairs to solve 15 x 8. Is there more than one way you can do it? • Multiply a number by itself and then make one factor one more and the other one less. What do you notice? Does this always happen? Eg 4 x 4 = 16 6 x 6= 36 5 x 3 = 15 7 x 5= 35 Try out more examples to prove your thinking.	• Place <, >, or = in these number sentences to make them correct: 50 x 4 ■ 4 x 50 4 x 50 ■ 40 x 5 200 x 5 ■ 3 x 300 • The school has a singing group of more than 12 singers but less than 32. They sing together in different ways. Sometimes they sing in pairs and sometimes in groups of 3, 4 or 6. Whatever size groups they are in, no one is left out and everyone is singing. How many singers are there in the school choir? 

Year 5

	National Curriculum Statement	All students		
		Fluency	Reasoning	Problem Solving
Multiplication and Division	Multiply numbers up to 4 digits by a one or two digit number using a formal written method, including long multiplication for 2 digit numbers.	• Solve the calculation: 3 4 6 × 2 7 • Calculate: 5612 x 4 654 x 34 • Mo Farah runs 135 miles a week. How far does he run each year? • There are 18 rows of 16 stickers on a sheet. How many stickers on 1 sheet? How many stickers on 10 sheets? 	• Spot the mistake and make a correction. 527 x 42 10540 2018 12648 • Laura thinks that a 4 should be placed in the empty box. Do you agree? 4 7 ☐ × 2 3 1 0 9 0 2 • What goes in the missing box? 12☐ 2 ÷ 6 = 212 14☐4 ÷ 7 = 212 Prove your answer.	• Using the digits 1, 2, 3 and 4 in any order in the bottom row of the number pyramid, how many different totals can you make? What is the smallest/ largest total?  • Find the missing digits: 5 2 ☐ × ☐ 7 1 5 ☐ 3 0 3 6 4 7 1 ☐ 2 7 7 • Start with 0; choose a path through the maze. Which path has the highest/ lowest total? You can go up, down, left or right. S +6 x5 x2 -4 +7 X8 +9 x7 x6 x5 +3 x4 +9 E

Year 6

	National Curriculum Statement	All students		
		Fluency	Reasoning	Problem Solving
Four Operations	Use their knowledge of the order of operations to carry out calculations involving the four operations.	• 4(72 ÷ 9) × (1923 -382) • Add brackets to make this calculation correct; 25 + 10 – 3 × 20 – 15 = 20 • Sarah had 7 bags with 5 sweets in each. She added one more to each bag. Circle the calculation below that shows the correct working out. 7 (5 + 1)= 42 7 × 5 + 1 = 36 7 × 5 + 1 = 42 • Work out 3 + 4 x 7 6 + (25 x 9) – 1 10 – 32 20 + 15 ÷ 5 100 – 17 x 4	• Choose operations to go in the boxes to make the number sentences true: 5 □ 3 □ 8 = 23 5 □ 3 □ 8 = 29 • Daniel completed the following calculation and got the answer 168 2(30 ÷ 5) + 14 = 168 Can you explain what he did and where he made the mistake? • Amy says You can do multiplication and division in any order. This is the same for addition and subtraction. Is she correct? Can you include some calculations to support your answer?	• **Countdown** Ask children to choose 1 or 2 numbers from the 'top' (25/50/75/100) and 4 or 5 numbers from the 'bottom' 1-10. Children make a target number. • Write different number sentences using the digits 3, 4, 5 and 8 before the equals sign that use: -one operation - two operations, no brackets - two operations, brackets • The mass of a box of chocolates is 290g. The box contains 7 identical chocolates.  Manish eats 3 chocolates. The mass of the box is now 194g Find the weight of them empty box.


Prove it.
Convince me.
What other posibilities are there?
What else do I know?
How would this affect the result if …
Why do I need to solve/ calculate this first?
Why is this important?
What would happen if…
How can this help me solve the problem?

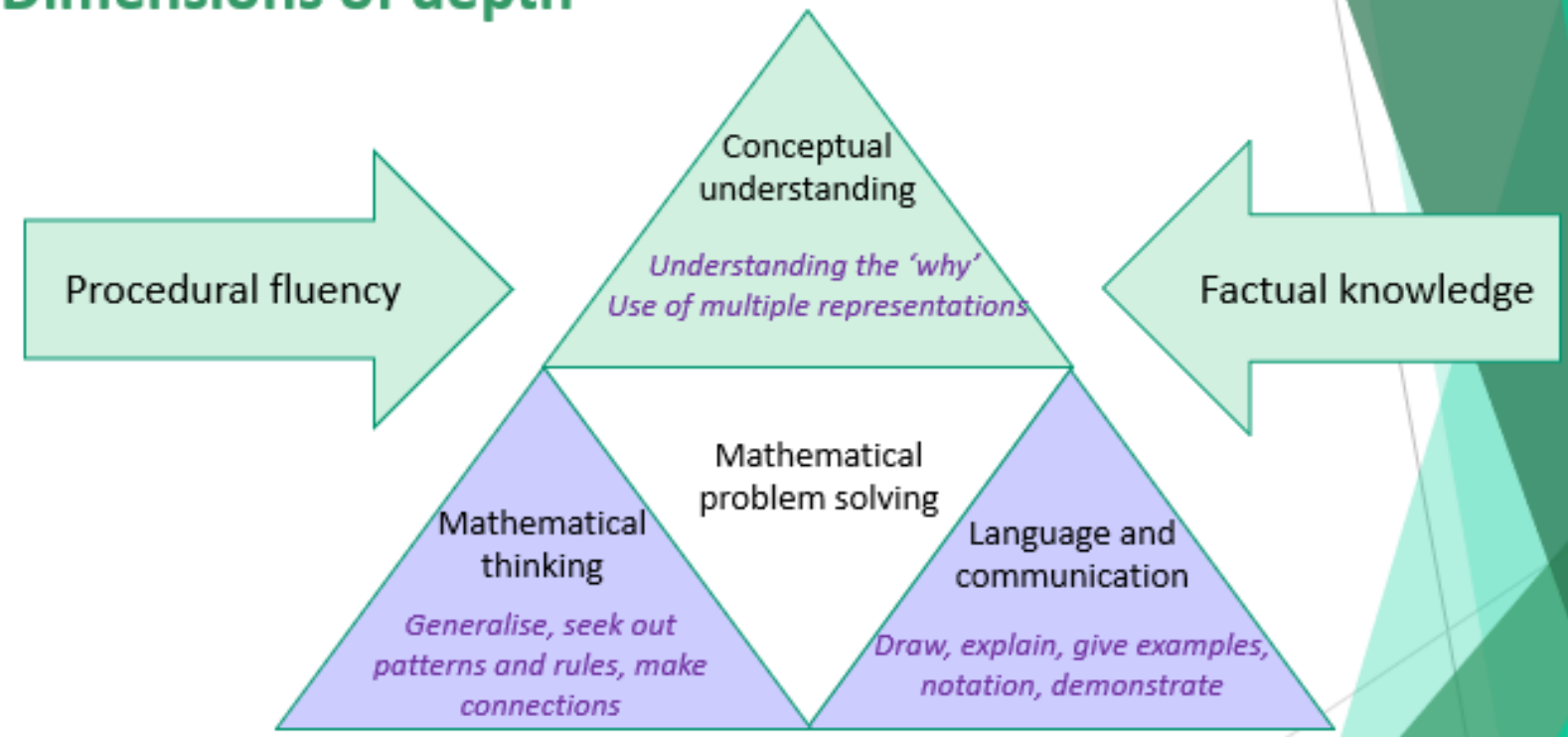
Maths Talk

Key Questions

Math Talk

Sentence Starters

Dimensions of depth

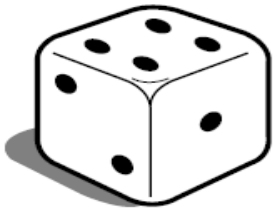


Mastering Mathematics, Helen Drury, 2014

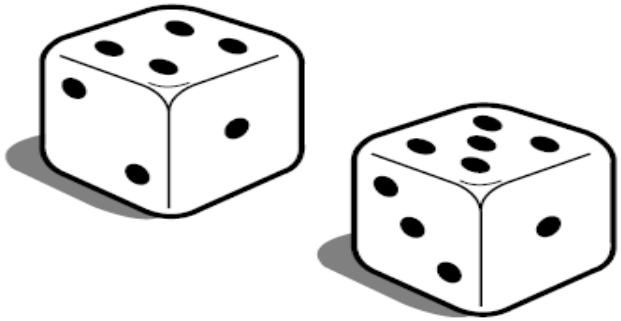
Roly poly

The dots on opposite faces of a dice add up to 7.

1. Imagine rolling one dice.
   The score is the total number of dots you can see.
   You score 17.
   Which number is face down?
   How did you work out your answer?



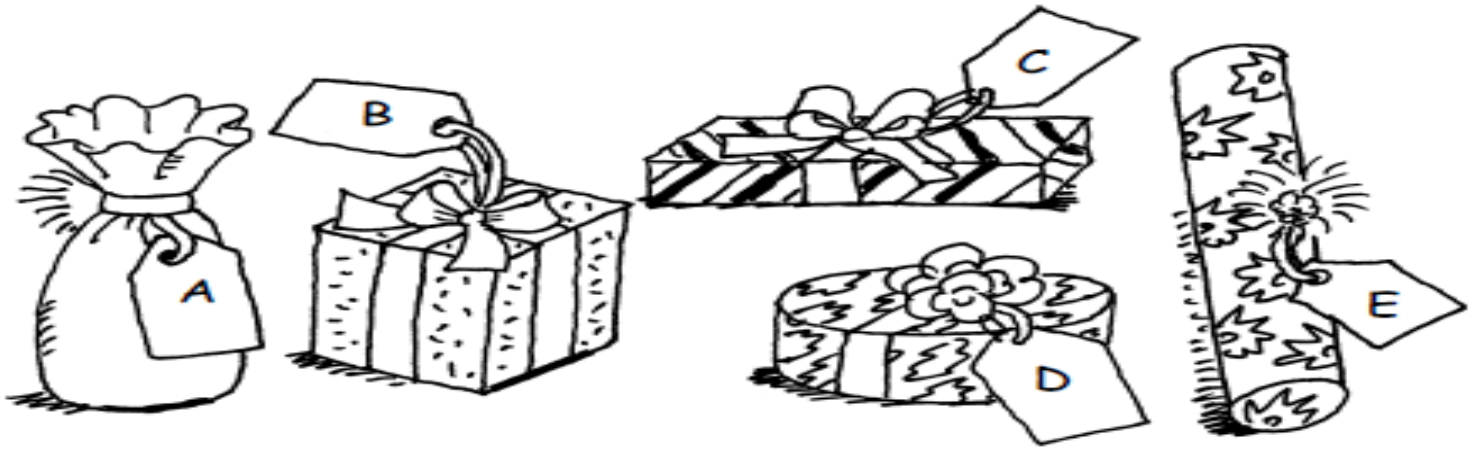
2. Imagine rolling two dice.
   The dice do not touch each other.



   The score is the total number of dots you can see.
   Which numbers are face down to score 30?

Presents

Gurmit paid £21 for five presents.



For A and B he paid a total of £6.
For B and C he paid a total of £10.
For C and D he paid a total of £7.
For D and E he paid a total of £9.

How much did Gurmit pay for each present?

Guess the domino.

https://nrich.maths.org/6995

What are you aiming to do?

The team has to find the rule on the ruler's card, using the minimum number of tests. A test is asking whether a particular domino obeys the rule.

As a team:

- Finding out what others think
- Giving reasons for ideas
- Being concise
- Reflecting on what has been said
- Allowing everyone to contribute.

Getting started

Make teams of 4 or 5. Choose one person to be the “Ruler".

You will need:

- The set of domino rule cards
- A piece of coloured paper (the box)
- A set of dominoes

Tackling the problem

- The team has the dominoes
- The ruler selects a rule card from the pack and keeps it hidden from the rest of the team
- Before deciding on a domino to present to the ruler for testing - the team must discuss and agree as a team why they think it would make a good test case
- If the domino obeys the rule then the ruler puts it in the box, if it does not obey the rule it is placed outside the box.
- If someone thinks they know the rule, they must share their reasoning with the rest of the team, who discuss and check their reasoning before deciding to guess the rule.
- The ruler then reveals the rule.
- If the team is right the task can start again with a different ruler. If they are wrong, they discuss where their error in reasoning may have been before moving on.

Key questions

- Was there a choice of domino to test that was particularly useful? Why?
- Were there any ideas that proved really helpful?
- How well did you listen to others in your group?
- How did you ensure that everyone had a chance to contribute?