

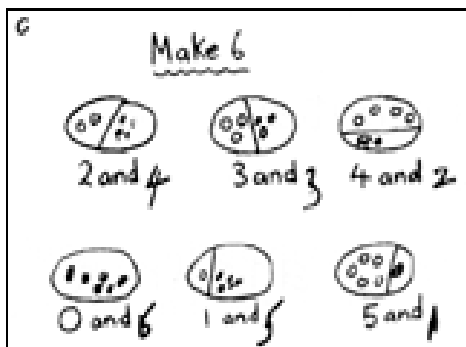


St John's C of E Primary School

Progression in written calculations document 2014

Written with guidance from the Lancashire mathematics team

Stage 1



Children are taught that addition is the combining of two or more amounts. They begin by counting all of the items in the groups, then move on to counting on from the largest amount. Children are encouraged to develop a mental image of the size of numbers. They learn to think about addition as combining amounts in practical, real life situations. They begin to record addition number sentences such as $2 + 4 = 6$ and $8 = 3 + 5$ and $3 + 2 + 4 = 9$

Stage 2

Children move on to using Base 10 equipment to support their developing understanding of addition.

$$11 + 5 = 16$$

11 cubes are lined up (1 ten and 1 unit/one).

5 cubes are added to the line of 11 giving a total of 16.



If possible, use two different colours of base 10 equipment so that the initial amounts can still be seen.

Stage 3

Children continue to use the Base 10 equipment to support their calculations, including exchanging 10 units/ones for 1 ten when the total of the units/ones is 10 or more. They will record their own drawings of the Base 10 equipment, using lines for 10 rods and dots for the unit blocks.

$$34 + 23 = ?$$

The units/ones are added first $4 + 3 = 7$

The tens are added next

$$30 + 20 = 50$$

Both answers are put together $50 + 7 = 57$

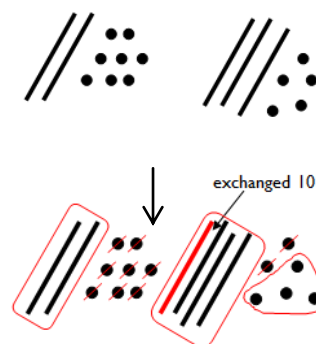


$$28 + 36 = ?$$

The units/ones are added first

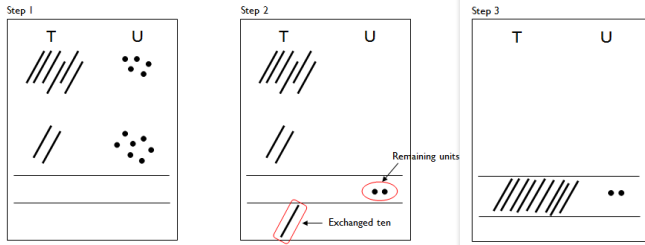
$8 + 6 = 14$ with ten units/ones exchanged for 1 ten.

A ring is put around the units/ones not exchanged – this is the units part of the answer. The tens are then added, including the exchanged ten, to complete the sum.



Stage 4

$65 + 27$



Step 1

Step 2

Step 3

$$\begin{array}{r} \text{T} \quad \text{U} \\ 6 \quad 5 \\ + 2 \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} \text{T} \quad \text{U} \\ 6 \quad 5 \\ + 2 \quad 7 \\ \hline \quad 2 \\ | \end{array}$$

$$\begin{array}{r} \text{T} \quad \text{U} \\ 6 \quad 5 \\ + 2 \quad 7 \\ \hline 9 \quad 2 \\ | \end{array}$$

Stage 5

$$\begin{array}{r} \text{HTU} \\ 625 \\ + 48 \\ \hline 673 \\ 1 \end{array} \quad \begin{array}{r} 367 \\ + 85 \\ \hline 452 \\ 11 \end{array} \quad \begin{array}{r} 321 \\ + 7 \\ + 48 \\ \hline 376 \\ 1 \end{array}$$

$$\begin{array}{r} \pounds 3.48 \\ + \pounds 0.78 \\ \hline \pounds 4.26 \\ 1 \quad 1 \end{array}$$

This is the final stage of the method, and should be continued to be used for all written addition calculations.

The example top left would be 'said' as follows:

$5 + 8 = 13$, put 3 down and carry the 10

$20 + 40 + 10$ that was carried over = 70 (*7 written in the tens column*)

$600 + 0 = 600$ (*6 written in the hundreds column*)

Children will be expected to use this method for adding numbers with more than 3 digits, numbers involving decimals and adding any number of amounts together.

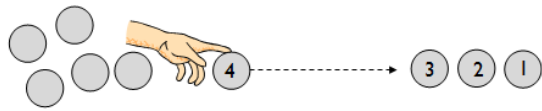
Children should not be made to go onto the next stage if:

- 1) they are not ready.
- 2) they are not confident.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

Stage 1

Children will subtract two numbers by taking one away from the other and counting how many are left.



Children are encouraged to develop a mental image of the size of numbers. They learn to think about subtraction as 'take away' in practical, real life situations. They begin to record subtraction number sentences such as $8 - 5 = 3$



Stage 4A

$$\begin{array}{r} 89 \\ - 57 \\ \hline 30 \end{array} \rightarrow \begin{array}{r} 80 \\ - 50 \\ \hline 30 \end{array} \rightarrow \begin{array}{r} 9 \\ - 7 \\ \hline 2 \end{array} = 32$$

The calculation should be read as subtract 7 from 9 or 9 subtract 7.

Children move from using the Base 10 method to expanded vertical method, using base 10 notation and arrow cards. Children learn to subtract the least significant digits first (start with the numbers on the right and work from right to left). The answer to each individual subtraction is written underneath before these answers are recombined.

Stage 2

Children move on to using Base 10 equipment alongside a number track to support their developing understanding of subtraction.

$13 - 4 = ?$

13 cubes are lined up. 4 cubes are removed from the end of the line leaving 9 left. It is important that children keep track of how many have been removed.



Touch count and remove the number to be taken away.



Touch count to find the number that remains.



Stage 4B

This stage involves exchange.

It is clear that there are not enough units to subtract 6 from 1, so one of the tens from the 70 is exchanged for 10 ones.

The initial number 71 is rearranged as 60 and 11 to make the calculation easier.

This would be recorded by the children as:

$$\begin{array}{r} 70 \\ - 40 \\ \hline 30 \end{array} \rightarrow \begin{array}{r} 70 \\ - 40 \\ \hline 30 \end{array} \rightarrow \begin{array}{r} 1 \\ - 6 \\ \hline 5 \end{array} = 25$$

$$\begin{array}{r} 60 \\ - 40 \\ \hline 20 \end{array} \rightarrow \begin{array}{r} 11 \\ - 6 \\ \hline 5 \end{array} = 25$$

$$\begin{array}{r} 60 \\ - 40 \\ \hline 20 \end{array} \rightarrow \begin{array}{r} 11 \\ - 6 \\ \hline 5 \end{array} = 25$$

Stage 3

Children continue to use the Base 10 equipment to support their calculations. They will record their own drawings of the Base 10 equipment, using lines for 10 rods and dots for the unit blocks.

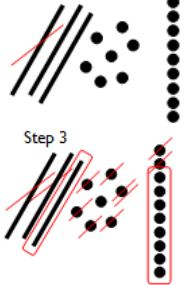
$39 - 17 = ?$

39 is drawn. 17 is crossed out. A ring is drawn around what is left to give the answer giving 22



$37 - 19 = ?$

37 is drawn. 9 units cannot be crossed out, so a ten is crossed out and exchanged for 10 ones which are in a line. 19 is crossed out. A ring is drawn around what is left to give the answer 18



Stage 5

This final stage is the compact method of decomposition. The example shows how the same calculation would be carried out using the previous method and the final method.

$$\begin{array}{r} 754 \\ - 86 \\ \hline \end{array}$$

Stage 4B

$$\begin{array}{r} 600 \\ - 700 \\ \hline 600 \end{array} \rightarrow \begin{array}{r} 140 \\ - 50 \\ \hline 80 \end{array} \rightarrow \begin{array}{r} 14 \\ - 6 \\ \hline 8 \end{array} = 668$$

becomes

Stage 5

$$\begin{array}{r} 6141 \\ - 764 \\ \hline 668 \end{array}$$

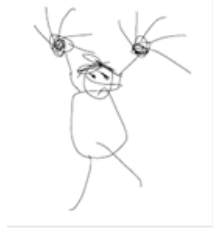
This is the final stage of the process and will continue to be used with larger numbers and numbers involving decimals.

Progression in Written Multiplication

Stage 1

Children are encouraged to develop a mental image of the size of numbers. They learn to think about equal groups or sets of objects in practical, real life situations.

They begin to record these situations using pictures.



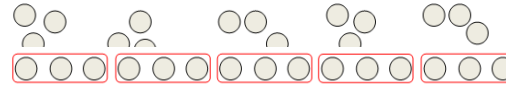
A child's jotting showing fingers on each hand as a double.



A child's jotting showing double three as three cookies on each plate.

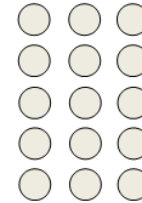
Stage 2

Children understand that multiplication is repeated addition and that can be done by counting in equal steps/groups.

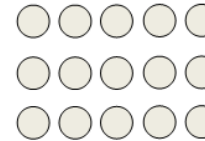


or

Children can then be introduced to the image of a rectangular array, initially through real items such as egg boxes, baking trays, ice cube trays, wrapping paper etc. and using these to show that counting up in equal groups can be a quicker way of finding a total.



$$3 + 3 + 3 + 3 + 3 = 15$$

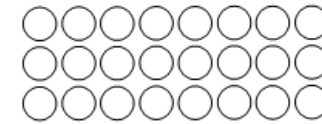


$$5 + 5 + 5 = 15$$

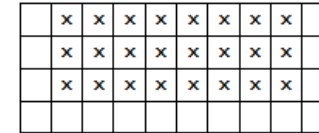
Children also understand that 3×5 is the same as 5×3

Stage 3

Children continue to use arrays and create their own to represent multiplication calculations



$$3 \times 8 = 8 + 8 + 8 = 24$$



$$3 \times 8 = 8 + 8 + 8 = 24$$

Stage 4

Children will continue to use arrays to lead into the grid method of multiplication.

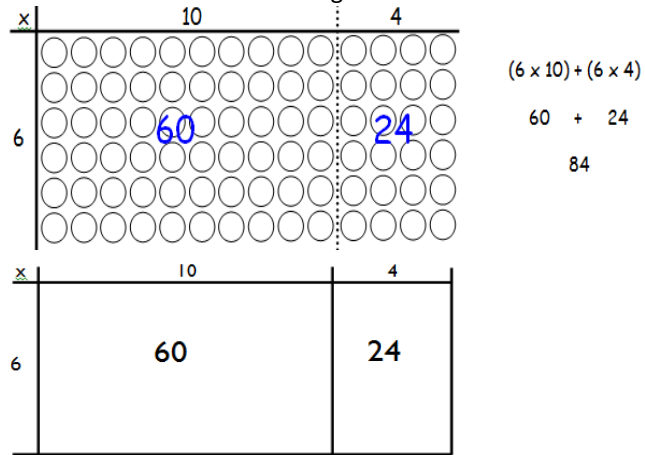
$$14 \times 6$$

The 14 is partitioned (split) into 10 and 4.

The answer to 6×10 is found = 60

The answer to 6×4 is found = 24

The two answers are added together $60 + 24 = 84$

**Stage 5**

This is the final stage, the array is removed and children use the grid method.

$$23 \times 8$$

$$\begin{array}{r} \times \quad 20 \quad 3 \\ 8 \quad \boxed{160} \quad \boxed{24} \\ \hline 160 \\ + \quad 24 \\ \hline 184 \end{array}$$

$$346 \times 9$$

$$\begin{array}{r} \times \quad 300 \quad 40 \quad 6 \\ 9 \quad \boxed{2700} \quad \boxed{360} \quad \boxed{54} \\ \hline 2700 \\ + \quad 360 \\ + \quad 54 \\ \hline 3114 \\ \small 11 \end{array}$$

The grid method can be used for multiplying any numbers, including long multiplication and multiplication involving decimals.

$$4.92 \times 3$$

$$\begin{array}{r} \times \quad 4 \quad 0.9 \quad 0.02 \\ 3 \quad \boxed{12} \quad \boxed{2.7} \quad \boxed{0.06} \\ \hline 12 \\ + \quad 2.7 \\ + \quad 0.06 \\ \hline 14.76 \end{array}$$

$$72 \times 38$$

$$\begin{array}{r} \times \quad 70 \quad 2 \\ 30 \quad \boxed{2100} \quad \boxed{60} \\ 8 \quad \boxed{560} \quad \boxed{16} \\ \hline 2100 \\ + \quad 560 \\ + \quad 60 \\ + \quad 16 \\ \hline 2736 \\ \small 1 \end{array}$$

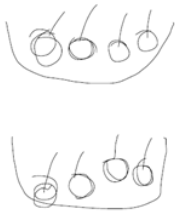
Stage 1

Children are encouraged to develop a mental image of the number system in their heads to use for calculation. They should experience practical calculation opportunities involving **equal groups** and **equal sharing**.



They may develop ways of recording calculations using pictures.

A child's jotting showing halving six spots between two sides of a ladybird.



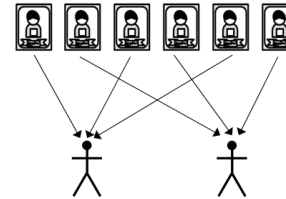
A child's jotting showing how they shared the apples at snack time between two groups.

Stage 2

Children explore practical contexts where they share equally and group equally. $6 \div 2 = ?$

Equal sharing (6 shared equally between 2)

6 football stickers are shared equally between 2 people, how many do they each get? Children may solve this by using a 'one for you, one for me' strategy until all of the stickers have been given out.



Equal grouping (How many groups of 2 are there in 6?)

There are 6 football stickers, how many people can have 2 stickers each?



Stage 3

Children continue to use practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation.

$12 \div 3 = ?$ Children begin to read this calculation as, 'How many groups of 3 are there in 12?'



At this stage, children will also be introduced to division calculations that result in remainders.

$13 \div 4 = 3$ remainder 1



Stage 4

$43 \div 8$



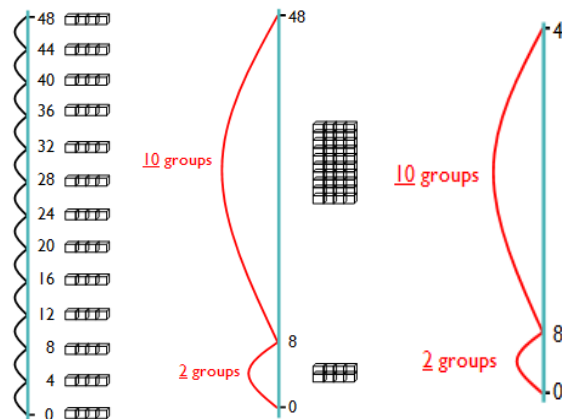
$43 \div 8 = 5$ remainder 3

At this stage, children also learn if the remainder should be rounded up or down e.g. $62 \div 8 = 7$ remainder 6

I have 62p. Sweets are 8p each. How many can I buy?
 Answer: 7 (the remaining 6p is not enough for another sweet)
 Apples are packed into boxes of 8. There are 62 apples. How many boxes do I need?
 Answer: 8 (the remaining 6 apples still need to be placed into a box)

Stage 5

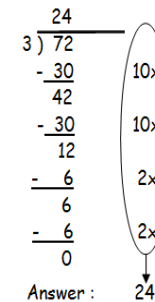
The previous method of repeated subtraction on a number line is continued, but using a vertical number line alongside practical equipment. The repeated subtraction is made more efficient by subtracting 'chunks' of the divisor.



Stage 6

This is the final stage, in which children use the 'chunking' method.

$72 \div 3$



1x	3
2x	6
5x	15
10x	30

Children should write key facts in a menu box. This will help them in identifying the largest group they can subtract in one chunk.

Stage 6 continued

$196 \div 6$

$$\begin{array}{r} 32 \text{ r } 4 \\ 6 \overline{) 196} \\ \underline{- 180} \\ 16 \\ \underline{- 12} \\ 4 \end{array}$$

1x	6
2x	12
4x	24
5x	30
10x	60
20x	120

The key facts in the menu box should be extended to include 4x and 20x.

Answer : 32 remainder 4 or 32 r 4

Stage 6 continued

$972 \div 36$

$$\begin{array}{r} 27 \\ 36 \overline{) 972} \\ \underline{- 720} \\ 252 \\ \underline{- 252} \\ 0 \end{array}$$

Answer : 27

Stage 6 continued

$362 \div 17$

$$\begin{array}{r} 21.29 \\ 17 \overline{) 362} \\ \underline{- 340} \\ 22 \\ \underline{- 17} \\ 5.0 \\ \underline{- 3.4} \\ 1.60 \\ \underline{- 1.53} \\ 0.07 \end{array}$$

To enable children to continue the calculation, they need to understand that 5 is the same as 5.0

When recalling and deriving multiplication and division facts, children should also identify decimal equivalents of times tables,
 e.g. if $2 \times 17 = 34$, I know that $0.2 \times 17 = 3.4$
 if $9 \times 17 = 153$, $0.9 \times 17 = 15.3$
 so $0.09 \times 17 = 1.53$

Children should not be made to go onto the next stage if:

- 1) they are not ready.
- 2) they are not confident.

Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.

