

St John Vianney Catholic Primary School

Progression in Calculation

Aims

The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Magnitude of Calculations

Year 1 – $U + U$, $U + TU$ (numbers up to 20) including adding zero, $U - U$, $TU - U$ (numbers up to 20) including subtracting zero, $U \times U$, $U \div U$ **Year 2** - $TU + U$, $TU +$ multiples of 10, $TU + TU$, $U + U + U$, $TU - U$, $TU -$ tens, $TU - TU$, $TU \times U$, $U \div U$

Year 3 – add numbers with up to three-digits, $HTU +$ multiples of 10, $HTU +$ multiples of 100, subtract numbers up to three-digits, $HTU - U$, $HTU -$ multiples of 10, $HTU -$ multiples of 100, $HTU - HTU$, $TU \times U$, $TU \div U$

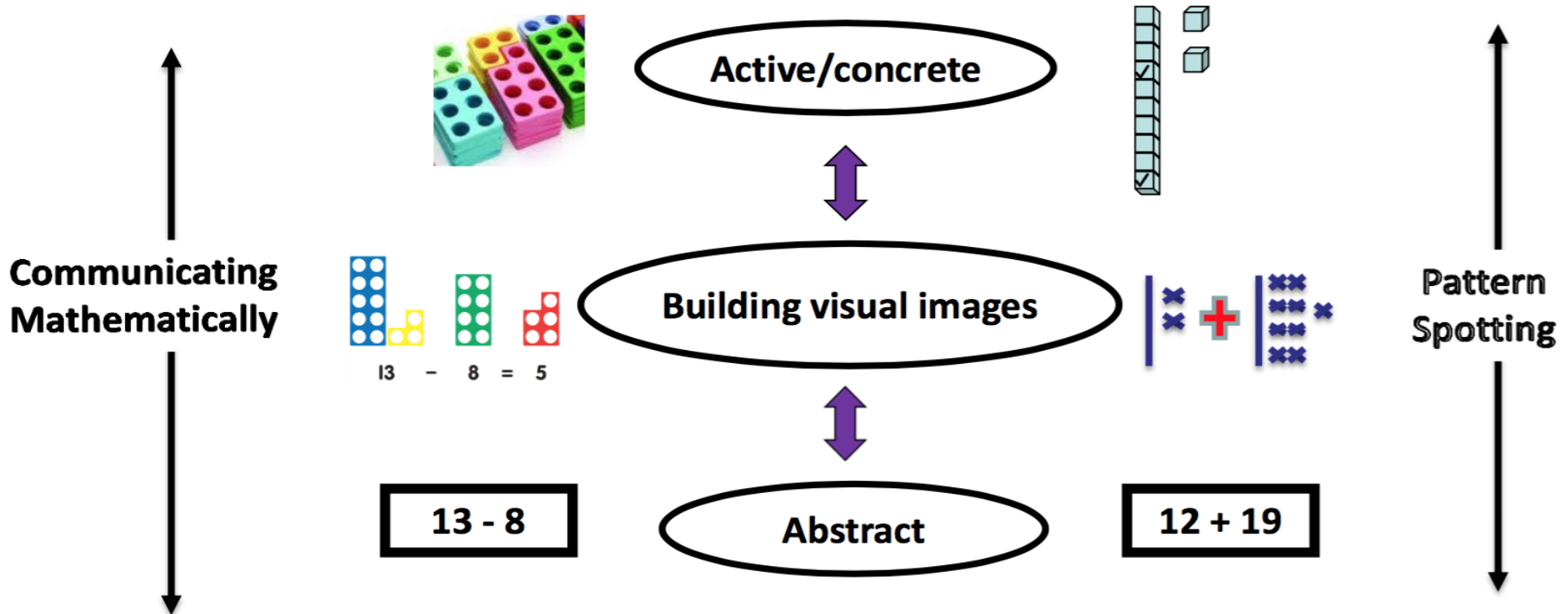
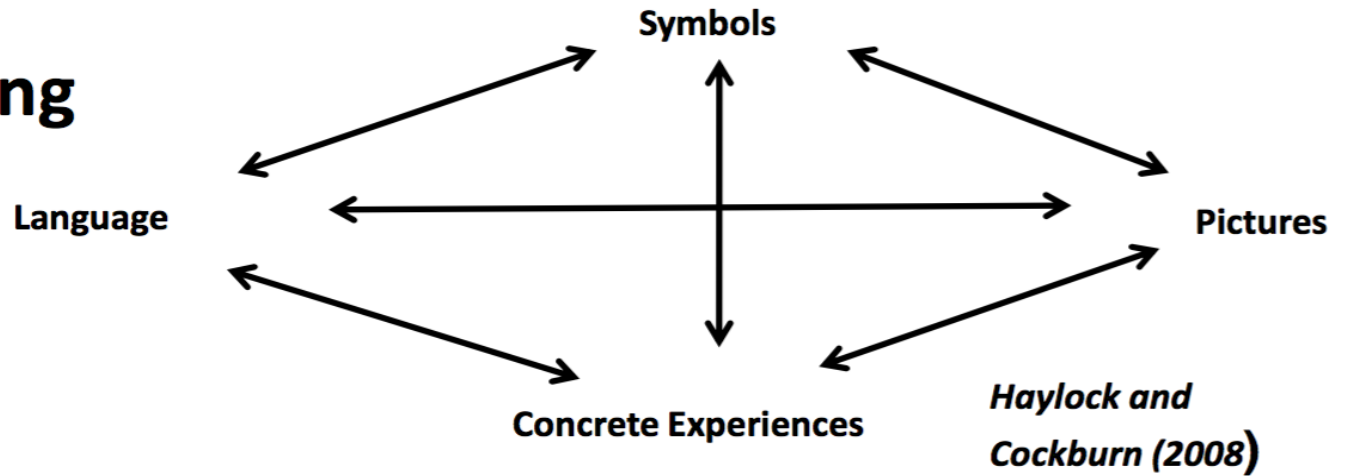
Year 4 - add and subtract numbers with up to four-digits, $ThHTU + ThHTU$, $ThHTU - ThHTU$, add and subtract decimals with up to two decimal places in the context of money, multiply three numbers together, $TU \times U$, $HTU \times U$, $TU \times U$, multiply by zero and one, $TU \div U$, $HTU \div U$

Year 5 – add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places, $ThHTU \times U$, $ThHTU \times TU$, $HTU \times TU$, multiply whole numbers and decimals with up to three-decimal places by 10, 100 and 1000, divide numbers with up to four-digits by U (including remainders as fractions and decimals and rounding according to the context)

Year 6 - add and subtract numbers with more than four-digits, add and subtract decimals with up to three decimal places, multiply numbers with up to four-digits by TU , multiply numbers with up to two-decimal places by a whole number, divide numbers up to four-digits by TU (interpreting remainder according to the context), divide decimals up to two-decimal places by U or TU

Structuring Learning

Children must have concrete experiences that enable them to create visual images. They should be encouraged to articulate their learning and to become pattern spotters.



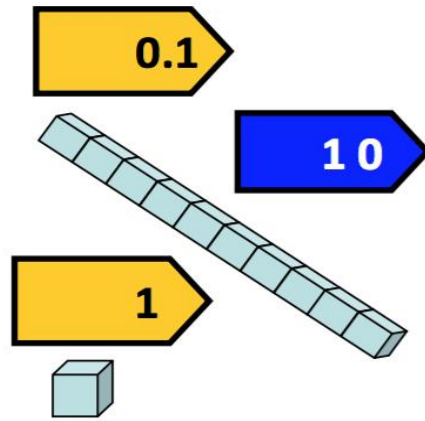
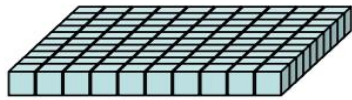
bead string



count stick

place value apparatus

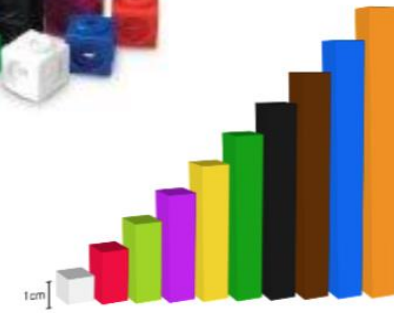
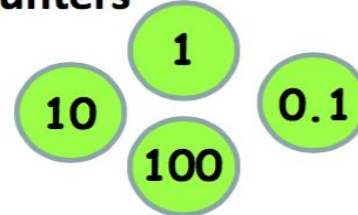
Hundreds 100s	Tens 10s	Units/Ones 1s



Multilink

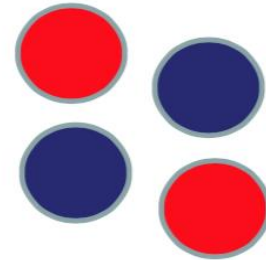
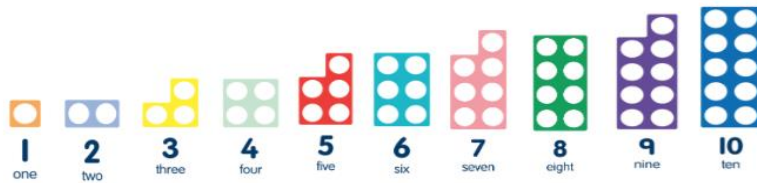


place value counters



Cuisenaire

Numicon



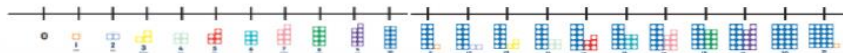
double sided counters

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100
101	102	103	104	105	106	107	108	109	110
111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128	129	130
131	132	133	134	135	136	137	138	139	140
141	142	143	144	145	146	147	148	149	150
151	152	153	154	155	156	157	158	159	160
161	162	163	164	165	166	167	168	169	170
171	172	173	174	175	176	177	178	179	180
181	182	183	184	185	186	187	188	189	190
191	192	193	194	195	196	197	198	199	200

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

number grids
100 and 200

number line



Models and images for counting on and back in ones and tens

5, 6, 7, ?

8

10, 9, 8, 7, ?

6



50, 60, 70, ?

80

100, 90, 80, 70, ?

60



5 ... 6

6 ... 5



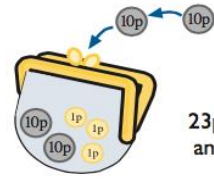
5 and 1 more is 6
1 less than 6 is 5



Imagine one more spot

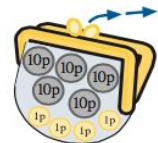


Imagine one less spot



23p
33p, 43p...

23p and 10p more is 33p
and 10p more makes 43p



54p
44p, 34p...

54p in the purse. Take 10p out,
another 10p and so on

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

36 ... 46,
56, 66

76 ... 66,
56, 46

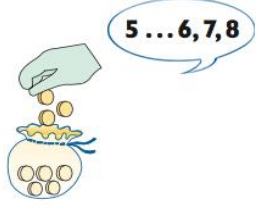
6 + 10 = 16 96 - 10 = 86
16 + 10 = 26 86 - 10 = 76
26 + 10 = 36 76 - 10 = 66
36 + 10 = 46 etc.
36 + 20 = 56 76 - 30 = 46

5 ... 6, 7

7 ... 6, 5



5 and 2 more is 7
2 less than 7 is 5



5 ... 6, 7, 8

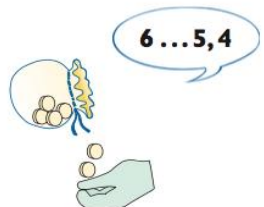
5 in the bag and 3 more

5 ... 6, 7, 8

8 ... 7, 6, 5

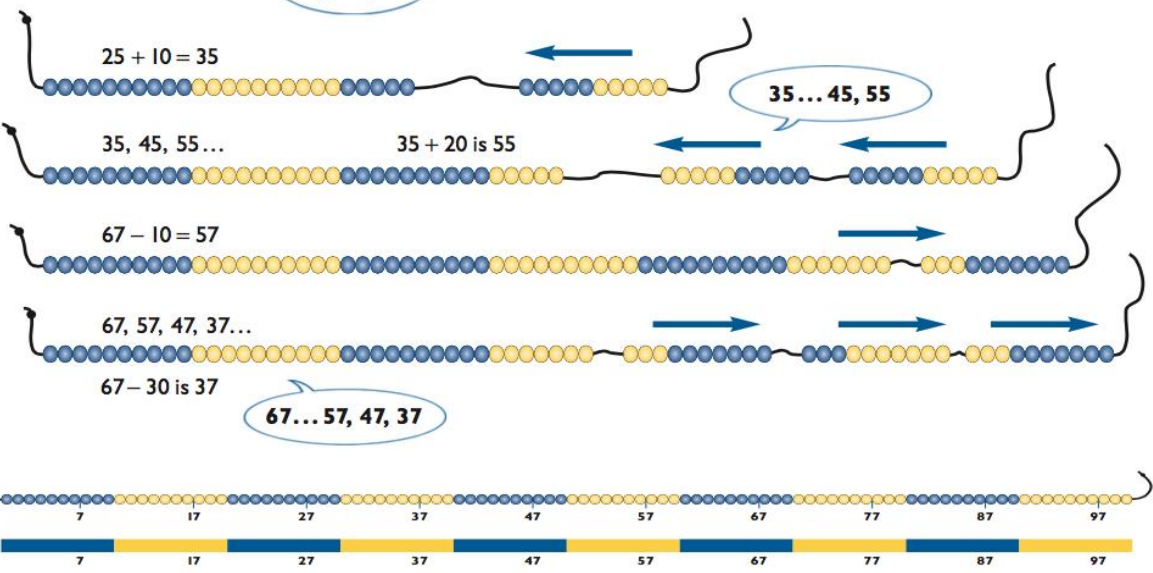


5 and 3 more is 8
3 less than 8 is 5



6 ... 5, 4

6 in the bag, take 2 out

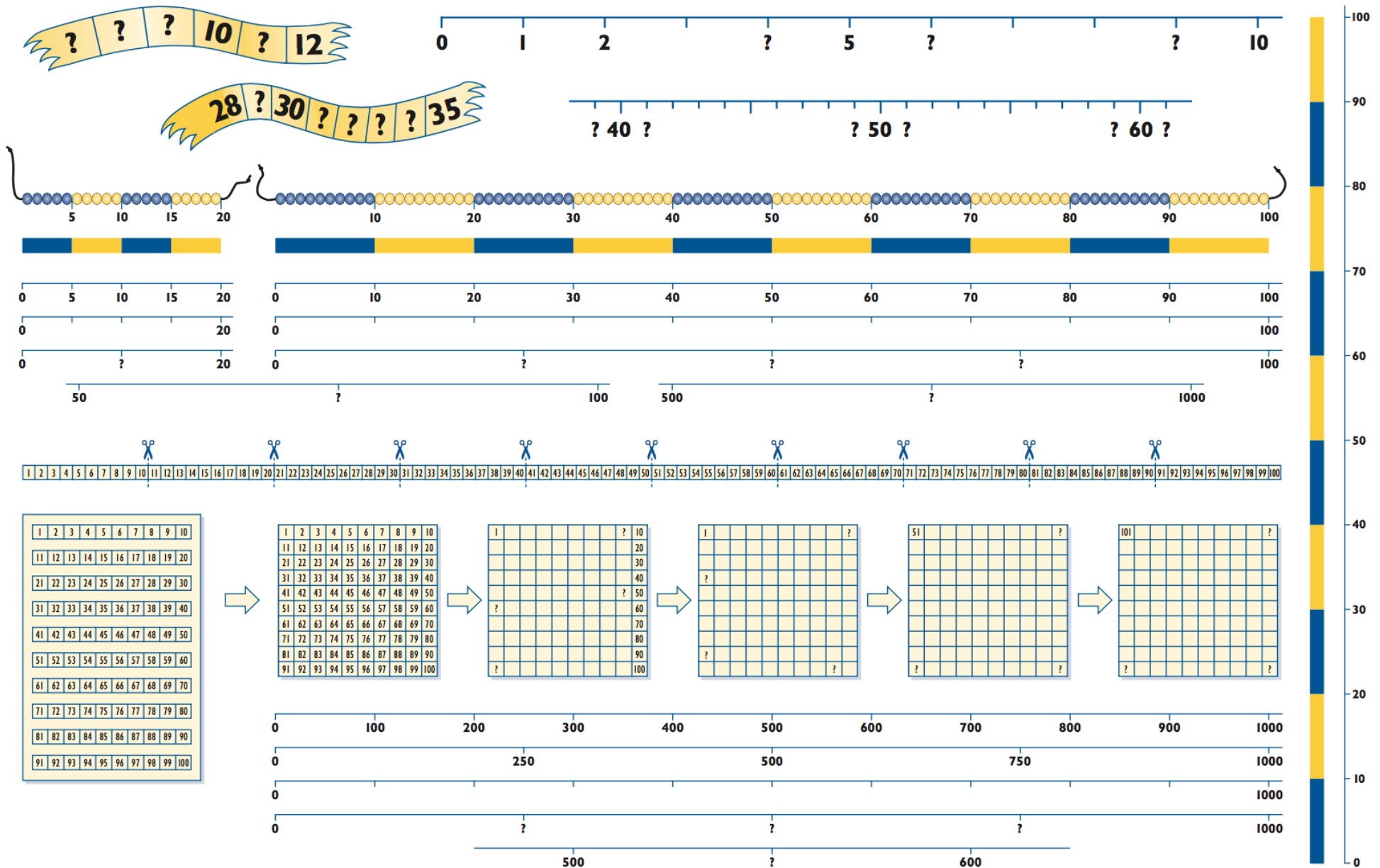


Potential Difficulties

Children may:

- say the number names in order but do not associate them with the number of objects they are counting, even when they are able to touch the objects;
- count the objects that are organised linearly or in a table, but lose track when counting objects that are randomly dispersed, or that they cannot touch, as their counting is unsystematic;
- when counting from 9 to 10, 19 to 20, 29 to 30 etc and 99 to 100, not understand that the zeros are placeholders and later are unable to record the answers to one more than numbers such as 7899;
- when counting on or back from a given number, include the given number in their counting rather than starting from the previous or next number or counting the 'jumps';
- recite accurately the counting numbers when starting from one or zero, but have difficulty counting from other starting numbers and when counting backwards;
- understand the patterns of the digits within a decade, for example 30, 31, 32, ..., 39 but struggle to recall the next multiple of 10;
- count confidently but cannot relate the 'teens' numbers to other numbers in the number system and do not recognise the relationship between 1, 11 and 21;
- count on or back in ones and tens but do not associate this with adding or subtracting ones and tens;
- be able to count on but do not understand how to apply this to addition or the combining of two or more groups of objects;
- associate counting on with addition but not as a way of finding the difference between two numbers that are close together;
- know the procedures for counting on or back when adding or subtracting but do not understand when it is more efficient to count on, for example when subtracting 19 from 22, or to count back when subtracting 3 from 22;
- confuse the vocabulary of counting 'backwards', 'forwards', 'up', 'down', 'on', 'back from' etc and count in the wrong direction.

Models and images for ordering numbers to 100



Potential Difficulties

Children may:

- be able to order numbers from smallest to largest as this reflects the left to right images of numbers they are familiar with, but are less confident ordering from largest to smallest;
- lack an understanding of the distance between numbers and do not recognise that while 79 and 82 are close there are many more numbers between 19 and 62;
- not distinguish 13 from 30 when spoken and between 17 and 71 when written;
- order sets of consecutive numbers but not sets made up of more widely dispersed numbers such as 73, 9, 38, 16 etc;
- not recognise the pattern of the decades and cannot use this pattern to order numbers 60, 61, 62, 63 ... 70, 71, 72, 73 ... 80, 81, 82, 83 ...;
- associate ordered numbers with the numbers on a number track but do not understand the structure of a hundred square or see it as a rearranged number track;
- complete sequences of missing numbers when presented with empty boxes that model a number track, but cannot complete number grids or use number lines as their mental images of the number system with tens as landmarks are limited;
- associate numbers on a number track with ordered adjacent boxes but do not understand that on a number line, numbers can always be placed between two adjacent numbers;
- not appreciate that the spaces between numbers on a number line are less important than the order of the numbers;
- recognise the image of a hundred square when all the numbers are represented but cannot imagine alternative grids, for example those made up of the even numbers or multiples of 5 or 10;
- have to count from 1 to find the number before or after a given number as they are insecure when counting from other starting numbers;
- complete sequences of numbers but do not understand the relative positions of numbers, for example that the position of 47 relative to 42 is the same as 67 relative to 62;
- not understand the importance of the most significant digits when ordering numbers to identify that 75 is bigger than 57 and later that 0.1 is bigger than 0.07;
- count in 10s but do not know what comes after 29 or before 71;
- have difficulty with the vocabulary 'more', 'most', 'less', 'in between' etc and cannot interpret meanings when solving word problems such as 'who has the least?'

Models and images for partitioning and recombining

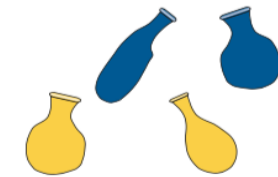
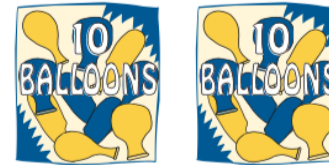
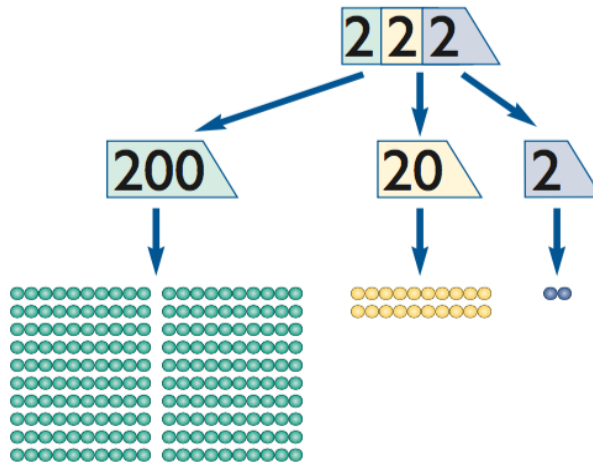
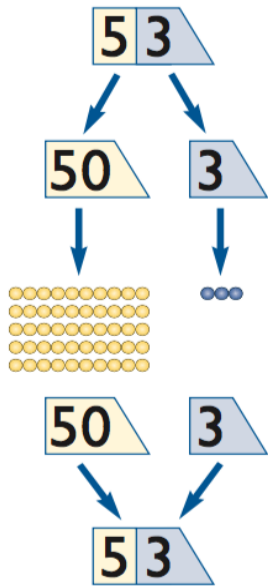
6 is 5 and 1 more
 $6 = 5 + 1$

7 is 5 and 2 more
 $7 = 5 + 2$

8 is 5 and 3 more
 $8 = 5 + 3$

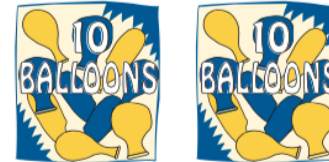
9 is 5 and 4 more
 $9 = 5 + 4$

10 is 5 and 5 more
 $10 = 5 + 5$



$10 + 10 + 4$
 $= 20 + 4$
 $= 24$ balloons

We need 35 balloons
 $35 = 30 + 5$. We need 3 packs of 10 and 5 single balloons

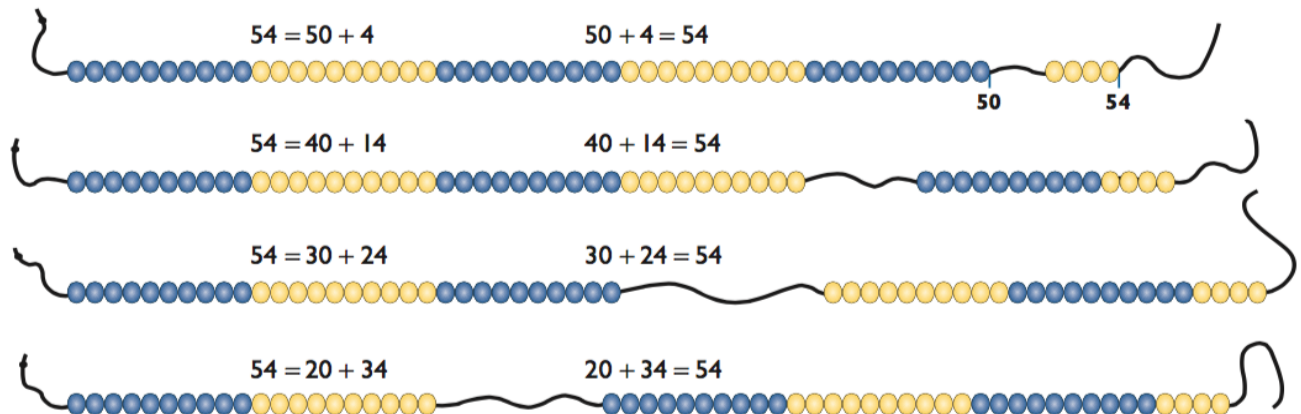


$30p + 4p = 34p$

41p could be made from $10p + 10p + 10p + 10p + 1p$

or $20p + 20p + 1p$

or lots of other ways with 20p, 10p and 1p coins.



Potential Difficulties

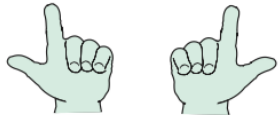
Children may:

- record incorrectly numbers they hear, for example hearing thirty-one but reversing the digits and recording 13;
- interpret the two digits in a two-digit number as separate single-digit numbers, for example associate 47 with 4 and 7 not with 40 and 7, as they do not have a secure understanding of place value;
- be able to partition 47 into 40 and 7 but not into 30 and 17 or 20 and 27 etc, for later use when subtracting numbers that involve crossing boundaries or when using 'chunking' as a method of division;
- partition numbers into tens and units, but are less confident when partitioning in other ways, for example 17 into 5s and a bit to get 15 and 2 to help when doubling, or 16 into 15 + 1 and 8 into 5 + 3 when adding 16 and 8;
- recombine tens numbers with units, for example 30 and 2 to get 32, but are less assured recombining tens numbers with two-digit numbers, for example 40 and 38 and recording 438;
- associate combining with joining together rather than with addition, for example writing 40 combined with 2 as 402 rather than $40 + 2 = 42$;
- recognise from saying a number how it can be partitioned, for example that seventy-three can be partitioned as 70 and 3, but are confused by the 'teens' as how they are said does not help with partitioning;
- rely on their interpretation of the value of the digits in a two-digit number and have no images to draw upon to help them 'see' the different ways numbers can be partitioned and recombined;
- misunderstand zero as a placeholder and say that 103 has no tens rather than 10 tens;
- be over-reliant on the language of number to help them recombine and record three-digit numbers, for example saying one hundred and six and recording 1006;
- not associate the components of a number with their magnitudes when they partition and recombine and make errors of scale, for example partition 35 and 47 into 30 and 5 and 40 and 7 when adding, but sum the wrong components $30 + 7 = 37$ and $40 + 5 = 45$ and recombine as $37 + 45 = 712$;
- be over-reliant on partitioning when other calculation strategies are more efficient, for example working through $32 - 3$ as $20 + 12 - 3$ etc rather than counting back 3.

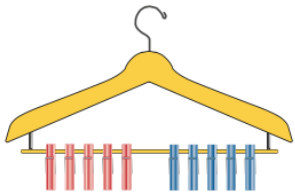
Models and images for addition and subtraction facts to 20



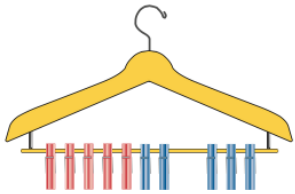
$1 + 1 = 2$ $2 - 1 = 1$
 double 1 is 2 half of 2 is 1



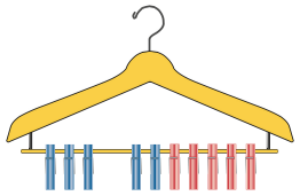
$2 + 2 = 4$ $4 - 2 = 2$
 double 2 is 4 half of 4 is 2
 ...



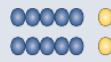
$10 = 5 + 5$



$10 = 7 + 3$



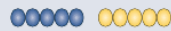
$10 = 3 + 7$



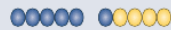
$6 + 6 = 12$ double 6 = 12
 $12 - 6 = 6$ half of 12 = 6



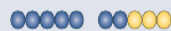
$7 + 7 = 14$ double 7 = 14
 $14 - 7 = 7$ half of 14 = 7 etc.



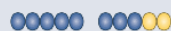
$10 = 5 + 5$
 $10 - 5 = 5$



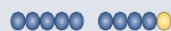
$10 = 6 + 4$ $4 + 6 = 10$
 $10 - 4 = 6$ $10 - 6 = 4$



$10 = 7 + 3$ $3 + 7 = 10$
 $10 - 3 = 7$ $10 - 7 = 3$



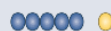
$10 = 8 + 2$ $2 + 8 = 10$
 $10 - 2 = 8$ $10 - 8 = 2$



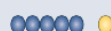
$10 = 9 + 1$ $1 + 9 = 10$
 $10 - 1 = 9$ $10 - 9 = 1$



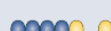
$7 + ? = 10$ $10 - ? = 7$



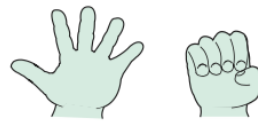
$6 = 5 + 1$ $1 + 5 = 6$
 $6 - 1 = 5$ $6 - 5 = 1$



$7 = 5 + 2$ $2 + 5 = 7$
 $7 - 2 = 5$ $7 - 5 = 2$



$7 = 4 + 3$ $3 + 4 = 7$
 $7 - 3 = 4$ $7 - 4 = 3$ etc.



$5 + ? = 10$ $10 - 5 = ?$



$6 + ? = 10$ $? + 6 = 10$
 $10 - 6 = ?$ $10 - 4 = 6$



$9 + ? = 10$ $? + 9 = 10$
 $10 - 9 = ?$ $10 - ? = 9$
 ...



$8 + ? = 10$



$7 + ? = 10$



$7 + 3 = 10$



$15 + 5 = 20$



$20 = 10 + 10$
 $20 - 10 = 10$



$20 = 15 + 5$ $5 + 15 = 20$
 $20 - 5 = 15$ $20 - 15 = 5$
 ($10 + 5 + 5 = 10 + 10 = 20$)



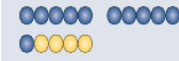
$20 = 11 + 9$ $9 + 11 = 20$
 $20 - 9 = 11$ $20 - 11 = 9$
 ($10 + 1 + 9 = 10 + 10 = 20$)



$20 = 12 + 8$ $8 + 12 = 20$
 $20 - 8 = 12$ $20 - 12 = 8$
 ($10 + 2 + 8 = 10 + 10 = 20$)



$20 = 13 + 7$ $7 + 13 = 20$
 $20 - 7 = 13$ $20 - 13 = 7$
 etc. ($10 + 3 + 7 = 10 + 10 = 20$)



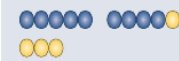
$15 = 11 + 4$ $4 + 11 = 15$
 $15 - 4 = 11$ $15 - 11 = 4$



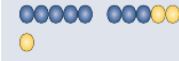
$15 = 12 + 3$ $3 + 12 = 15$
 $15 - 3 = 12$ $15 - 12 = 3$
 etc.



$17 = 13 + 4$ $4 + 13 = 17$
 $17 - 4 = 13$ $17 - 13 = 4$



$13 = 9 + 4$ $4 + 9 = 13$
 $13 - 4 = 9$ $13 - 9 = 4$





$11 = 8 + 3$ $3 + 8 = 11$
 $11 - 3 = 8$ $11 - 8 = 3$

Potential Difficulties







Children may:



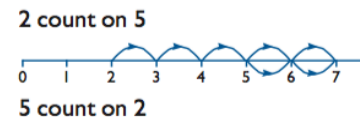
- learn the pairs of numbers that total 20, but not know the pairs that total each of the numbers up to 20;
- be able to add and subtract correctly when told the operation, but cannot decide which operation to use as they do not understand the meaning of, or the relationships between, addition and subtraction;
- be insecure with facts to 20 as they have not progressed from facts to 5, to 10 and to 20, and cannot make links between the different facts;
- think they have to learn that $3+7=10$ and $7+3=10$ as they do not understand the commutativity of addition;
- interpret the equals sign as 'makes' or 'gives an answer of' rather than 'equals' or think of it as a balance, and cannot interpret $9 = 2 + 7$ correctly or use this fact to establish $19 = 12 + 7$ etc;
- not be able to complete number sentences with the empty box in each of the three positions as they only see the empty box as the answer on the right-hand side;
- not record facts correctly as they have few strategies for remembering facts that involve visual images, rhymes or mnemonics etc;
- be unable to identify patterns which they can use to extend their knowledge of number facts, e.g. $19 - 1 = 18$, $19 - 2 = 17$, $19 - 3 = 16$, $19 - 4 = 15$;
- not use visual images enough to help them, for example when doubling 4 not 'see' how the fingers on each hand show $4 + 4$, or imagine a balance of 6 red and 6 blue beads when halving 12;
- not associate number facts such as $13+5=18$ with $18-5=13$ etc as they do not understand that addition and subtraction are inverse operations;
- think that they must always take the smaller number from the larger number, and later have to correct this misunderstanding when introduced to subtraction that involves crossing boundaries as in $62 - 37$, or when introduced to negative numbers;
- confuse the vocabulary of addition and subtraction when applying their knowledge of facts to solving problems, for example, associating 'how many?' with adding to find a total, when the question asks for 'how many more?'



Models and images for understanding addition and subtraction


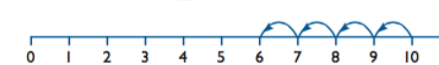
 $5 = 3 + 2$
 $3 + 2 = 5$

 $20 = 12 + 8$

  $10 = 5 + 5$
  $10 = 1 + 9$
  $10 = 2 + 8$

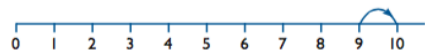
 $2 + 5 = 7$ 2 count on 5
 $5 + 2 = 7$ 5 count on 2


 How many more forks do we need?
 $3 + \square = 5$ 

 10 grapes, eat two. How many left?
 9, 8 8 left
 10 grapes, eat one, how many left? 9. And another? 8. Another, 7...

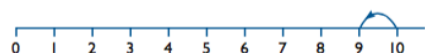



9 and 1 more is 10
 9 add 1 equals 10
 $9 + 1 = 10$


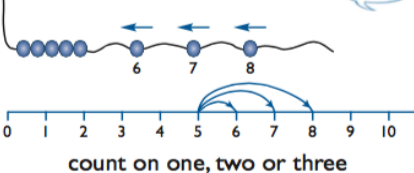



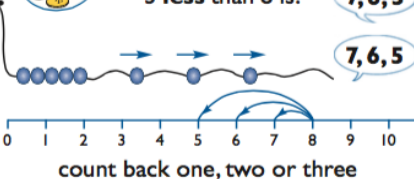



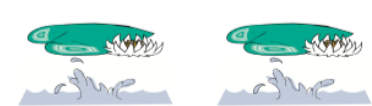
1 less than 10 is 9
 10 subtract 1 equals 9
 $10 - 1 = 9$

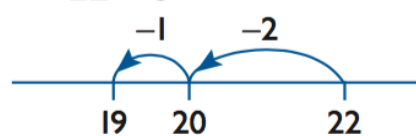


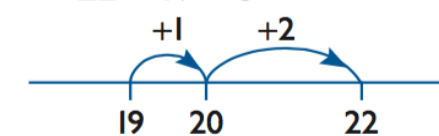
 $6 + 3 + 4$
 $10 + 3 = 13$

 5 and 1 more is? **6**
 5 and 2 more is? **6, 7**
 5 and 3 more is? **6, 7, 8**
 count on one, two or three

 1 less than 8 is? **7**
 2 less than 8 is? **7, 6**
 3 less than 8 is? **7, 6, 5**
 count back one, two or three



 $5 - \square = 3$ $\square - 2 = 3$


$22 - 3$


$22 - 19 = 3$




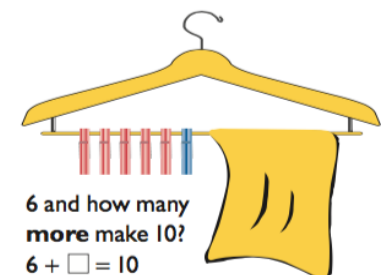
The difference is?

The difference between 11 and 14 is 3.
 $14 - 11 = 3$
 $11 + \square = 14$



Which line has **most** money?
 How much **more**?


 6 and how many more make 10?
 $6 + \square = 10$

Potential Difficulties

Children may:

- count forwards from one, as they are used to doing so when counting objects but cannot count from any starting number;
- count forwards confidently but lose their place and lack fluency when counting backwards, particularly when crossing the tens boundaries;
- know the answer to one more or one less, but when adding or subtracting larger numbers make mistakes as they include the first number when counting on/back, e.g. say '16, 15, 14' when subtracting 3 from 16;
- continue to rely on a combining model when adding two or more groups, having to count every object in the combined group rather than counting on from the larger group;
- not identify the number of items in a small group (subitise) and so have to count them out, consequently losing track when subtracting a small number mentally;
- use the take-away model when subtracting, e.g. $22 - 3$, counting back to 19, but continue to apply the model to calculations such as $24 - 17$ rather than use the difference model and count up from 17;
- count on or back without reference to the tens landmarks or knowledge of number facts and place value, e.g. counting on in ones from 15 when working out $26 - 15$ rather than bridging through 20;
- add and subtract by counting on or back but not recognise the inverse relationship between the operations and so not be able to derive the associated subtraction facts from addition facts or vice versa;
- count on or back in tens and ones but not combine these processes, for example when subtracting nine, subtract ten and then adjust by adding one;
- associate + with addition and - with subtraction and do calculations such as $4 + 3$ and $6 - 4$, but not be able to find missing numbers in statements such as $\blacksquare + 4 = 9$ and $\blacksquare - 4 = 5$;
- recognise what calculation to do when word problems include the words add or take away, but are less confident when other language is used such as fewer, sum, total or difference;
- when shown, use resources or models such as number lines to help with simple calculations, but not be able to apply these to unfamiliar contexts or to solving simple word problems.

Models and images for understanding multiplication and division



$2 + 2 + 2 + 2 + 2 = 10$
 $2 \times 5 = 10$
 2 multiplied by 5
 5 pairs
 5 hops of 2



$5 + 5 + 5 + 5 + 5 + 5 = 30$
 $5 \times 6 = 30$
 5 multiplied by 6
 6 groups of 5
 6 hops of 5

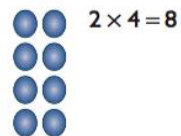


$10p + 10p + 10p + 10p + 10p = 50p$
 $10p \times 5 = 50p$
 5 hops of 10



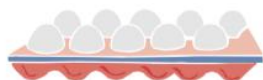
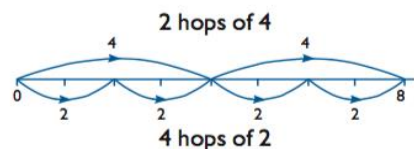
$4 \times 2 = 8$

$2 \times 4 = 8$



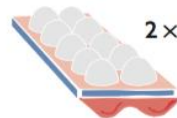
$2 \times 4 = 8$

$4 \times 2 = 8$



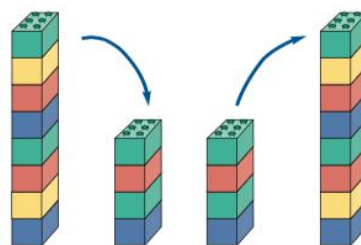
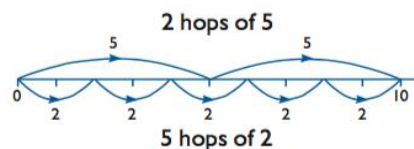
$5 \times 2 = 10$

$2 \times 5 = 10$



$2 \times 5 = 10$

$5 \times 2 = 10$



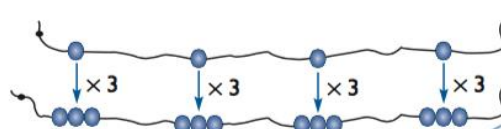
half of 8 is 4
 $8 \div 2 = 4$

double 4 is 8
 $4 \times 2 = 8$

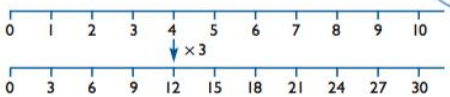


I'm 3 times as tall as you.
 I'm 3 metres tall.

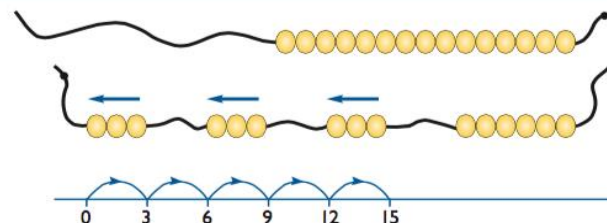
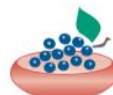
I'm only 1 metre tall.



Three times as many



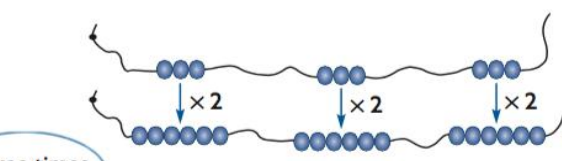
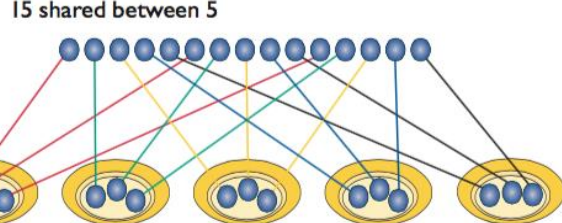
$4 \times 3 = 12$



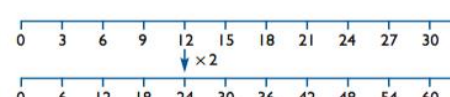
How many 3s in 15?
 $15 \div 3 = 5$



5 hops in 15. How big is each hop?
 $15 \div 5 = 3$



Twice as many



$12 \times 2 = 24$

Potential Difficulties

Children may:

- be able to double four by pairing two groups of four and counting the group of eight but do not associate this addition with the equivalent multiplication four multiplied by two;
- halve by sharing or forming pairs and counting but may not associate it with division by two or division between two;
- halve and double independently, without recognising they are inverse operations, for example, knowing that half of eight is four means that double four is eight;
- interpret $12 \div 3$ as 12 shared between 3 and use objects or pictures to share out the 12, but lose track of their recording as the numbers increase as they have no other strategy available such as counting in steps or groups;
- interpret 6×2 as six lots of 2 and apply repeated addition of two, rather than doubling the six;
- not be proficient in counting forwards and backwards in equal steps, and so make mistakes when carrying out repeated addition or repeated subtraction;
- understand multiplication as repeated addition and resort to this method without using known facts, for example writing 6×7 as $7 + 7 + 7 + 7 + 7 + 7$ and counting in 7s despite knowing that $5 \times 7 = 35$;
- understand division as repeated subtraction and use a method of counting back but lose track of the number of steps to the answer;
- carry out division by sharing or grouping but cannot cope with a remainder and do not recognise that a remainder must always be less than the divisor;
- associate \times with multiplication and \div with division and do calculations 8×2 and $16 \div 2$ but are not able to find missing numbers in statements such as $6 \times \blacksquare = 12$ and $\blacksquare \div 5 = 3$;
- recognise what calculation to do when word problems include the words times or share, but are less confident when other language is used such as product, divided by, remainder, and mistakenly associate 'how many?' and 'how much?' with addition or subtraction;
- understand multiplication as repeated addition, and division as repeated subtraction, but not as scaling up and down to prepare the way for later work in measures and on ratio.

Introduction

Written methods of calculations are based on mental strategies. Each of the four operations builds on mental skills which provide the foundation for jottings and informal written methods of recording. Skills need to be taught, practised and reviewed constantly. These skills lead on to more formal written methods of calculation.

Strategies for calculation need to be represented by models and images to support, develop and secure understanding. This, in turn, builds fluency. When teaching a new strategy it is important to start with numbers that the child can easily manipulate so that they can understand the methodology.

The transition between stages should not be hurried as not all children will be ready to move on to the next stage at the same time, therefore the progression in this document is outlined in stages. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy. We aim for the majority of children to be fluent in all four operations by the end of year four.

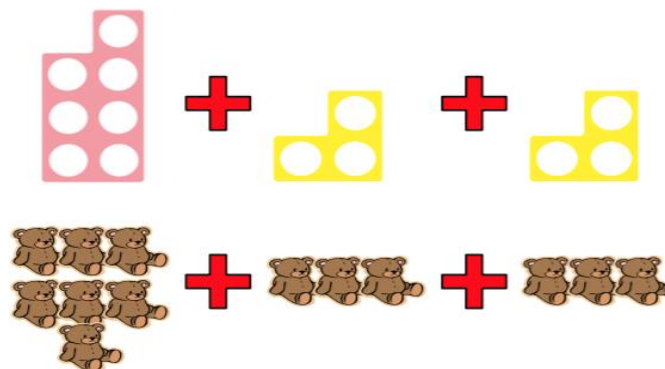
A sound understanding of the number system is essential for children to carry out calculations efficiently and accurately.

Structures of Addition (Haylock and Cockburn 2008)

Children should experience problems with all the different addition structures in a range of practical and relevant contexts e.g. money and measurement

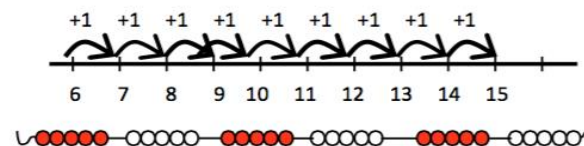
Aggregation

*Union of two sets
How many/much altogether?
The total*



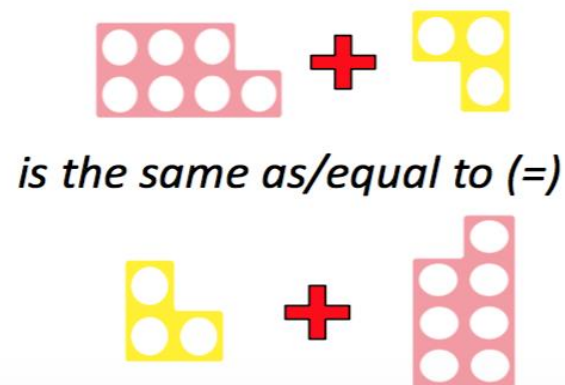
Augmentation

*Start at and count on
Increase by
Go up by*



Commutative law

*Understand addition can be done in any order
Start with bigger number when counting on
(Explain to children that subtraction does not have this property)*



Addition

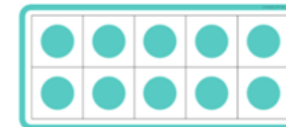
ADDITION: Vocabulary Progression:

- Using correct mathematical language is crucial for thinking, learning and communicating mathematically.
- We encourage children to explain what they are doing and why they are doing it.
- When children are first introduced to new vocabulary, it is not essential that children remember these words - but this modelling will help them become familiar with the terms, gradually beginning to use them accurately and with understanding in later years.

	EYFS	KS1		KS2			
	Reception	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Number	Zero, One, Two... Twenty Number How Many...? Count, Count (up) to, Count on (from, to), Count back (from, to) Is the Same as More / Less Few Pattern Pair	Twenty-One – One Hundred Numeral Forward / Backwards Odd / Even Equal To Equivalent To Most / Least Many Multiple Of	Two Hundred – One Thousand Tally Sequence Continue Predict Rule > Greater Than < Less Than	Factor Of Relationship Roman Numerals	Ten Thousand – Hundred Thousand – Million Next Consecutive Integer Positive Negative Above Zero Below Zero Minus Negative Numbers	Factor Pair ≥ Greater Than / Equal To ≤ Less Than / Equal To Formula Divisibility Square Number Prime Number Ascending / Descending Order	Factorise Prime Factor Digital Tool
Addition	Add / More / And Make / Sum / Total Altogether One More, Two More ...	Addition Near Double Equals Is the Same as Number Bonds / Pairs Ten More	One Hundred More Number Facts Tens Boundary	Hundreds Boundary	Inverse	Ones Boundary Tenths Boundary	

EYFS – use **quantities** and **objects** to add **two single digit** numbers and **count on** to find the answer.

We start EYFS with **LOTS** of number recognition using **CONCRETE** resources.



We start by knowing a hand is 5 (without counting).

We can then look for patterns and spot 5 using 5 frames.

Once, we can spot 5, we can use this pattern to spot 10 (know our hands).

We can also spot 10 using a 10 frame.



is 4



is 3



is 6



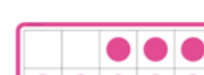
is 7



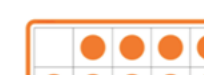
is 2



is 1



is 8



is 9

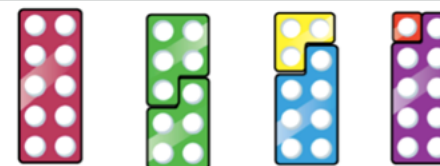
We love using ourselves to spot 5 and 10!

Once we can spot 5 and 10, we can look for patterns in numbers using 5 frames.

Then, we can spot patterns in numbers using 10 frames.



We love using Numicon to recognise number.

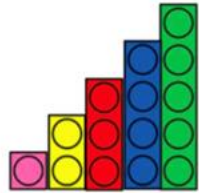


How many ways can you build 10?

We can then use Numicon to make 10.

EYFS – use **quantities** and **objects** to add **two single digit** numbers and **count on** to find the answer.

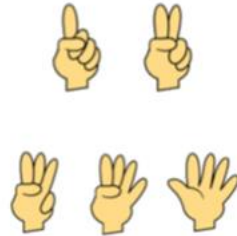
Once EYFS has developed **number recognition**, finding '**one more**' can then be explored using **CONCRETE** resources.



How many items have you used for each step?

How many more have you used each time?

Construction



Children use their counting and comparing skills

Fingers



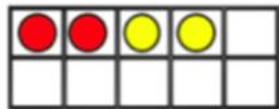
Use a 5 frame (or 10 frame) to represent numbers and then make one more.

Concrete Objects



One more than 4 makes 5

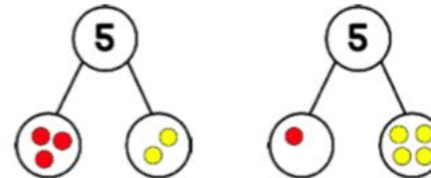
We love grouping ourselves!



First, there were 2 people on a bus.
Then, 2 people got on.
Now, there are 4 people on the bus.

Number Stories

Pictorial Representation

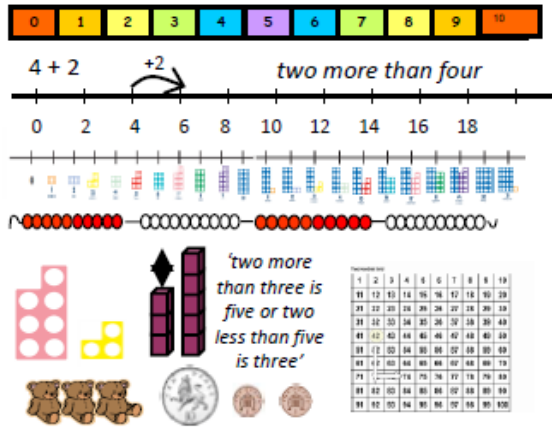

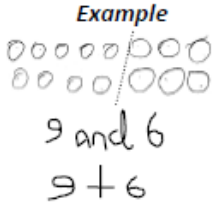
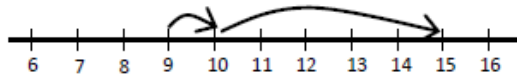
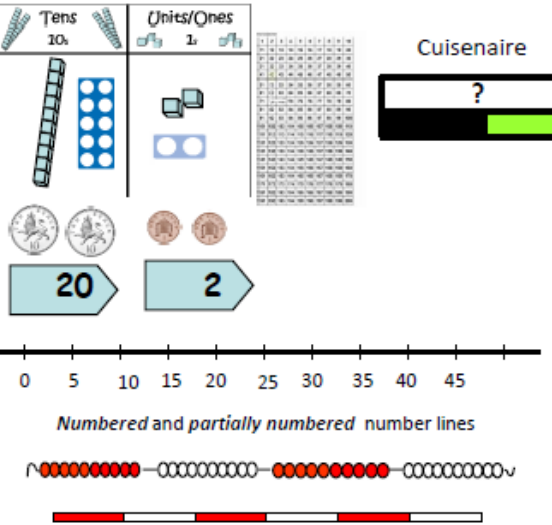
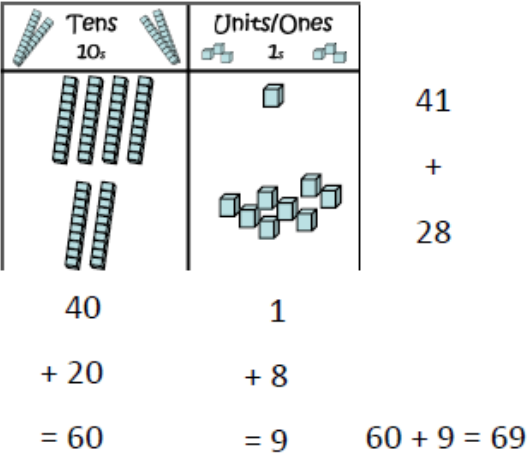


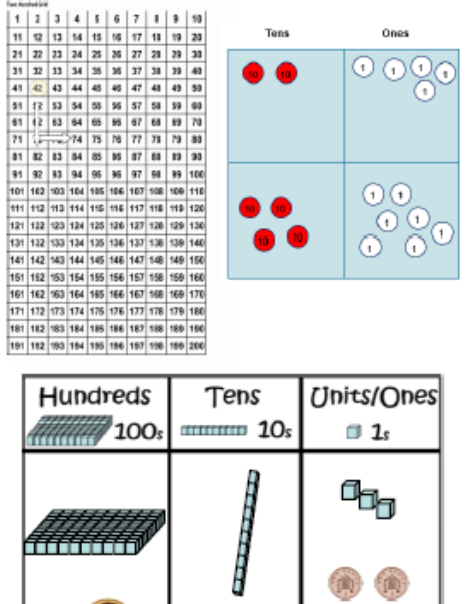
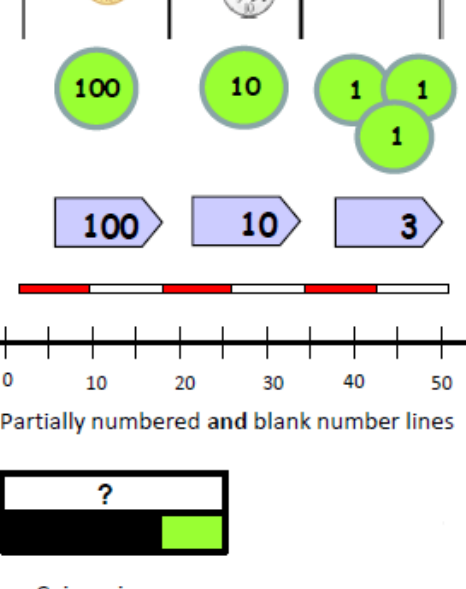
Could you show your counters on a 5 frame or part whole model?

Part Whole and 5 Frame (extending to 10 frame)

Addition

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.
Addition and subtraction should be taught together.

End of Year Expectations	Possible Concrete and Visual Representations	Children's Recording	Fluency
<p>Year 1</p> <p>Children must experience combining two, and then more than two, groups of objects using counting on and the language of addition e.g. add, plus</p> <p>Children must experience increasing numbers e.g. what is two more than seven?</p> <p>Compare quantities to say how many less and/or how many more</p>	 <p>Use practical resources such as bears, counters, cubes and number lines/hundred grids and progress to a resource such as Numicon to encourage counting in groups rather than ones</p>	<p>If using Numicon, children could use printed Numicon icons and stick these in - progressing to recording number sentences alongside</p>  <p>Children may record pictorially progressing to recording number sentences alongside</p>  	<p>Count forwards, to and across 100, beginning with 0 or 1 or from any given number</p> <p>Switch count between tens and ones e.g. 10, 20, 30, 31, 32, 33 ...</p> <p>Represent and use number bonds up to 20 (establish addition and subtraction as related operations)</p> <p>Find one more than a number</p> <p>Find ten more than a number</p> <p>Count in multiples of 2s, 5s and 10s starting on multiples to highlight pattern recognition</p>
<p>Year 2</p> <p>Children should be able to partition numbers in different ways e.g. as $2+2+2+1$ or $5+3$ or 23 as $20+3$ or $10+13$</p> <p>Children should use concrete objects, pictorial representations and add numbers in different contexts e.g. money, measures</p> <p>ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD MORE THAN TWO NUMBERS</p> <p>Children should understand the language of sum</p> <p>Ensure children understand that addition is commutative (can be done in any order)</p>	 <p>Use Numicon, number grids, place value apparatus/Dienes, place value grids, place value cards, Encourage children to partition numbers rather than counting in ones.</p>	<p>Children apply, develop and secure their understanding of place value</p> <p>Use jottings and record number sentences</p> 	<p>Show increasing fluency in deriving pairs of numbers up to 10 and then up to 20</p> <p>Use knowledge to derive and use number facts up to 100</p> <p>Add numbers mentally including TU + U, TU + tens, TU + TU, U + U + U</p>

End of Year Expectations	Possible Concrete and Visual Representations	Teacher Modelling/Children's Recording	Fluency
<p>Year 3</p> <p>Add numbers with up to three-digits (leading to formal written column method)</p> <p>ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD MORE THAN TWO NUMBERS WITH DIFFERING NUMBERS OF DIGITS</p> <p>Children should partition numbers, up to 1000, in different ways e.g. $100 + 40 + 6$ or $100 + 30 + 16$</p> <p>Solve problems in different contexts including missing number problems</p>		<p>Children apply, develop and secure their understanding of place value and begin to record in columns</p> <p><i>Manipulatives SHOULD be used alongside algorithms</i></p> <p>Column addition (no exchanging) with up to three-digits</p> $\begin{array}{r} 40 + 1 \\ + 20 + 8 \\ \hline 60 + 9 = 69 \end{array}$ <p><i>Expanded recording without exchange</i></p> $\begin{array}{r} 100 + 40 + 1 \\ + 100 + 20 + 8 \\ \hline 200 + 60 + 9 = 269 \end{array}$ <p><i>Expanded recording</i></p>	<p>Count in ones, tens and hundreds maintaining fluency through varied and frequent practice</p> <p>Count from 0 in multiples of 4, 8, 50 and 100</p> <p>Find 10 or 100 more than a number</p> <p>Mentally add HTU + ones, HTU + tens, HTU + hundreds</p> <p>Perform mental calculations with two-digit numbers, the answer could exceed 100</p>
<p>Year 4</p> <p>Add numbers with up to four-digits (formal written column method) including numbers with up to two decimal places in the context of money</p> <p>ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD MORE THAN TWO NUMBERS INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS</p> <p>Solve two-step problems in different contexts including missing number problems</p>		<p>Column addition (with exchanging)</p> $\begin{array}{r} 143 \\ + 128 \\ \hline 271 \\ 1 \end{array}$ <p><i>Compact (column) recording</i></p> <p>HTU</p> $\begin{array}{r} 789 \\ + 642 \\ \hline 1431 \\ 11 \end{array}$ <p><i>Compact (column) recording</i></p> <p>Add decimals in the context of money</p> $\begin{array}{r} \text{£ } 7.89 \\ + \text{£ } 6.42 \\ \hline \text{£ } 14.31 \\ 11 \end{array}$	<p>Count in 6s, 7s, 9s, 25s and 100s</p> <p>Find 1000 more than a number</p> <p>Perform mental calculations with increasingly large numbers to aid fluency</p>

Addition

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.
Addition and subtraction should be taught together.

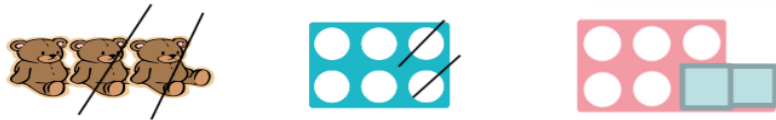
End of Year Expectations	Possible Concrete and Visual Representations	Teacher Modelling/Children's Recording	Fluency
<p>Year 5</p> <p>Add numbers with more than four-digits and decimals up to three places (formal written column method)</p> <p>N.B. ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD MORE THAN TWO NUMBERS INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS</p> <p>Solve multi-step problems selecting and justifying methods</p> <p>Perform mental calculations with increasingly large numbers</p>	<p>Cuisenaire</p>	<p><i>Manipulatives could be used alongside algorithms</i></p> $\begin{array}{r} 2141 \\ + 1128 \\ \hline 3269 \end{array}$ $\begin{array}{r} 21.41 \\ + 1.12 \\ \hline 22.88 \end{array}$ <p>Column addition (no exchanging)</p>	<p>Count forwards in powers of ten up to 100000</p> <p>Count forwards in positive and negative whole numbers through zero</p> <p>Practise mental calculations with increasingly large numbers</p> <p>Practise fluency of written methods</p>
<p>Year 6</p> <p>Add numbers with more than four-digits and decimals up to three places (formal written column method)</p> <p>N.B. ENSURE CHILDREN HAVE THE OPPORTUNITY TO ADD MORE THAN TWO NUMBERS, INCLUDING DECIMALS, WITH DIFFERING NUMBERS OF DIGITS</p> <p>Solve more complex calculations mentally</p> <p>Solve multi-step problems in contexts, deciding which operations and methods to use and why</p>	<p>Partially numbered and blank number lines</p>	$\begin{array}{r} 5189 \\ + 3128 \\ \hline 8317 \\ 11 \end{array}$ $\begin{array}{r} 51.89 \\ + 3.128 \\ \hline 55.018 \\ 11 \end{array}$ <p>Column addition (with exchanging)</p> <p><i>Addition with decimals up to three decimal places including in different contexts e.g. money and measures</i></p>	<p>Count in tens and hundreds increasing fluency of order and place value</p> <p>Perform increasingly complex mental calculations and those with increasingly large numbers to aid fluency</p>

Structures of Subtraction (Haylock and Cockburn 2008)

Children should experience problems with all the different subtraction structures in a range of practical and relevant contexts e.g. money and measurement

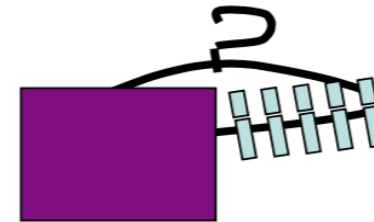
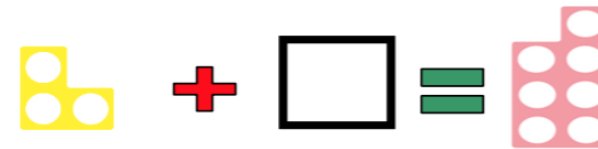
Partitioning

Take away
... how many left?
How many are not?
How many do not?



Inverse-of-addition

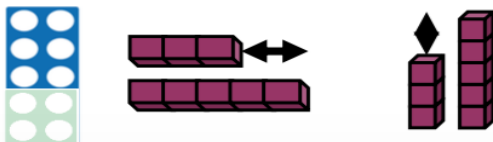
What must be added?
How many (much) more needed?



There are ten pegs on the hanger – how many are covered?

Comparison

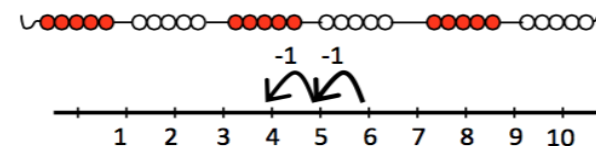
What is the difference?
How many more?
How many less (fewer)?
How much greater?
How much smaller?



'two more than three is five or two less than five is three'

Reduction

Start at and reduce by
Count back by
Go down by



Subtraction

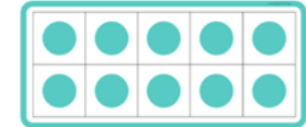
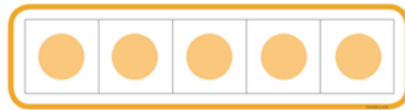
SUBTRACTION: Vocabulary Progression:

- Using correct mathematical language is crucial for thinking, learning and communicating mathematically.
- We encourage children to explain what they are doing and why they are doing it.
- When children are first introduced to new vocabulary, it is not essential that children remember these words - but this modelling will help them become familiar with the terms, gradually beginning to use them accurately and with understanding in later years.

	EYFS	KS1		KS2			
	Reception	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Number	Zero, One, Two...Twenty Number How Many...? Count, Count (up) to, Count on (from, to), Count back (from, to) Is the Same as More / Less Few Pattern	Twenty-One – One Hundred Numeral Forward / Backwards Odd / Even Equal To Equivalent To Most / Least Many Multiple Of	Two Hundred – One Thousand Tally Sequence Continue Predict Rule > Greater Than < Less Than	Factor Of Relationship Roman Numerals	Ten Thousand – Hundred Thousand – Million Next Consecutive Integer Positive / Negative Above / Below Zero Minus Negative Numbers	Factor Pair ≥ Greater Than / Equal To ≤ Less Than / Equal To Formula Divisibility Square Number Prime Number Ascending / Descending Order	Factorise Prime Factor Digital Tool
Subtraction	How many more to make...? How many more is...than...? Take away How many are left over? How many have Gone? One less, two less... How much less/fewer is...?	Half, Halve Subtract Equals Is the Same as Number Bands / Pairs Missing Number Ten More / Less	One Hundred More / Less Number Facts Tens Boundary	Hundreds Boundary	Inverse	Ones Boundary Tenths Boundary	

EYFS – use **quantities** and **objects** to subtract **two single digit** numbers and **count back** to find the answer.

As with addition, we start EYFS with **LOTS** of number recognition using **CONCRETE** resources



We start by knowing a hand is 5 (without counting).

We can then look for patterns and spot 5 using 5 frames.

Once, we can spot 5, we can use this pattern to spot 10 (know our hands).

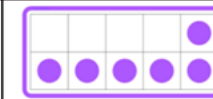
We can also spot 10 using a 10 frame.



is 4



is 3



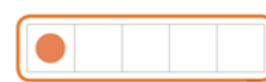
is 6



is 7



is 2



is 1



is 8



is 9

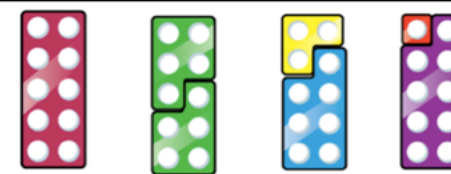
We love using ourselves to spot 5 and 10!

Once we can spot 5 and 10, we can look for patterns in numbers using 5 frames.

Then, we can spot patterns in numbers using 10 frames.



We love using Numicon to recognise number.



How many ways can you build 10?

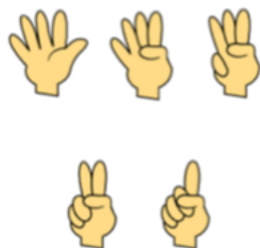
We can then use Numicon to make 10.

EYFS – subtract **two single-digit** numbers and count on or back to find the answer

Once EYFS has developed **number recognition**, finding 'one less' can then be explored using **CONCRETE** resources.



How many items have you used for each step?
Children should see the link that one less than a number is the next number that they are saying.



Children use their counting and comparing skills to find one less.



Use a 5 frame (or 10 frame) to represent numbers and then make one less.



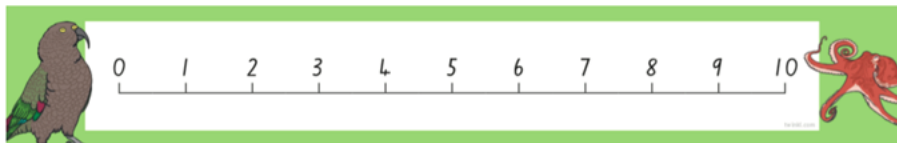
1 less than 5 makes 4

Construction

Fingers

5 or 10 Frame

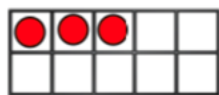
We love using objects (including ourselves) to jump back!



We use number lines to count/jump back.



First, there were 5 people on a bus.



Then, 2 people got off.



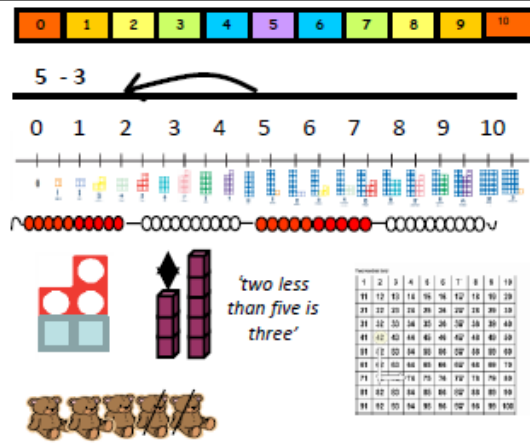
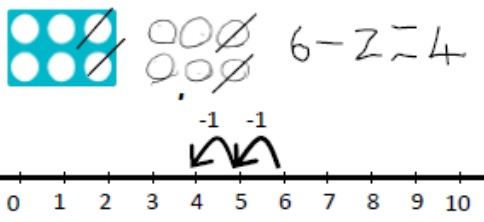
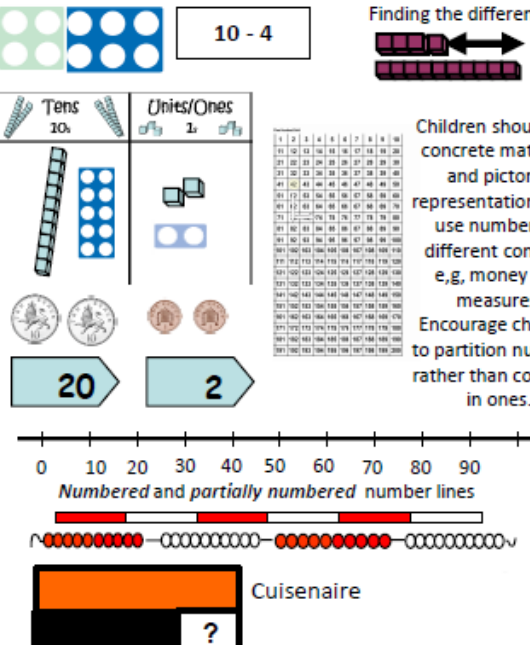
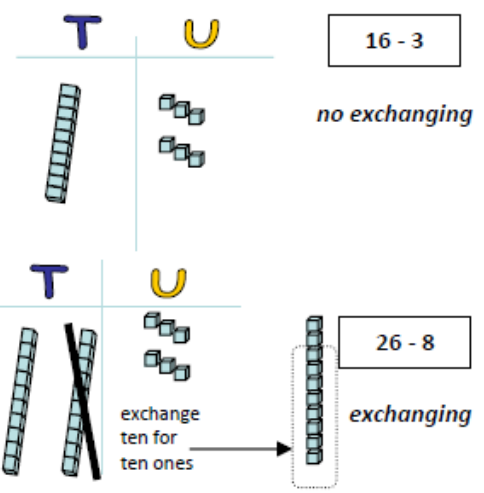
Now, these are 3 people on the bus.

Numicon

Number Stories


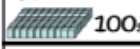








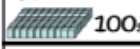








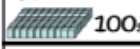








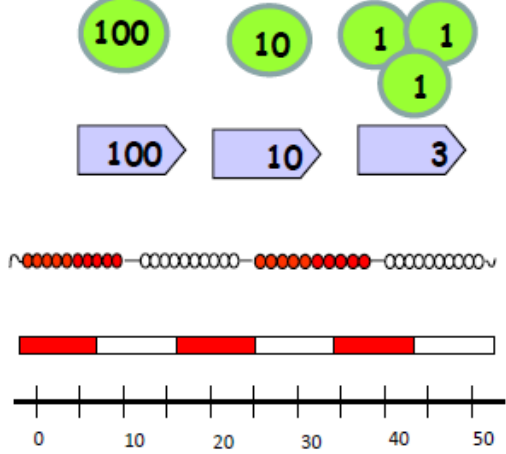
Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.
Addition and subtraction should be taught together.

End of Year Expectations	Possible Concrete and Visual Representations	Children's Recording	Fluency
<p>Year 1</p> <p>Understand subtraction as taking away What is ... less than ...?</p> <p>Compare quantities to say how many less and/or how many more</p>	 <p>5 - 3</p> <p>two less than five is three'</p> <p>Use practical resources such as bears, counters, cubes and number lines/hundred grids and progress to a resource such as Numicon to encourage counting back in groups rather than ones</p>	<p>Children may begin recording pictorially progressing to recording number sentences alongside</p>  <p>Children could use printed Numicon icons and stick these in, again progressing to recording number sentences alongside</p>	<p>Count backwards (including crossing 100) any given number</p> <p>Switch count between ones and tens e.g. 33, 32, 31, 30, 20, 10</p> <p>Represent and use subtraction facts linked to number bonds up to 20 (establish addition and subtraction as related operations)</p> <p>Find one less than a number</p> <p>Find ten less than a number</p> <p>Count back in multiples of 2s, 5s and 10s starting on multiples to highlight pattern</p>
<p>Year 2</p> <p>Understand subtraction as taking away <u>and</u> finding the difference</p> <p>Ensure children understand that subtraction is not commutative (can not be done in any order)</p> <p>Children should be able to partition numbers in different ways</p>	 <p>10 - 4</p> <p>Finding the difference</p> <p>Children should use concrete materials and pictorial representations, and use numbers in different contexts e.g. money and measures. Encourage children to partition numbers rather than counting in ones.</p> <p>20 - 2</p> <p>Cuisenaire</p>	<p>Children apply, develop and secure their understanding of place value and begin to record using jottings and number sentences</p>  <p>16 - 3</p> <p>no exchanging</p> <p>26 - 8</p> <p>exchanging</p>	<p>Practise addition and subtraction facts to 20</p> <p>Show increasing fluency in deriving subtraction facts for numbers up to 10 and then up to 20</p> <p>Use known facts to 20 to derive new facts e.g. $3 + 7 = 30 + 70$</p> <p>Use knowledge to derive and use subtraction number facts up to 100</p>

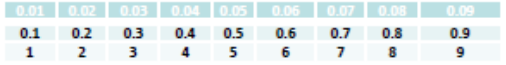



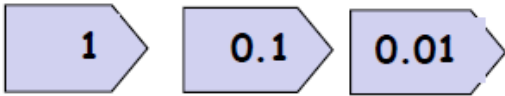

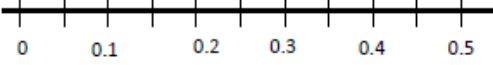








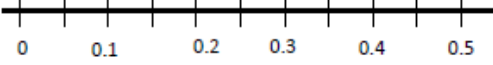

Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

End of Year Expectations	Possible Concrete and Visual Representations	Teacher Modelling/ Children's Recording	Fluency												
<p>Year 3</p> <p>Subtract numbers with up to three-digits (formal written column method)</p> <p>Children apply, develop and secure their understanding of place value and begin to record in columns</p>	<p>Cuisenaire</p>  <p>Base 10 Blocks</p> <table border="1" data-bbox="548 614 974 933"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Units/Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </tbody> </table>	Hundreds	Tens	Units/Ones										<p><i>Children SHOULD use manipulatives alongside algorithms to transition between practical and abstract</i></p> <p><i>no exchange</i></p> $\begin{array}{r} 68 - 23 \\ \hline 40 + 5 = 45 \end{array}$ <p><i>with exchange</i></p> $\begin{array}{r} 63 - 28 \\ \hline 30 + 5 = 35 \end{array}$ <p><i>Column subtraction (no exchange)</i></p> $\begin{array}{r} 148 - 121 \\ \hline 27 \end{array}$	<p>Count back in ones, tens and hundreds maintaining fluency through varied and frequent practice</p> <p>Switch count between hundreds, tens and ones e.g 500, 400, 300, 290, 280, 270, 269, 268, 267</p> <p>Mentally subtract HTU + ones, HTU + tens, HTU + hundreds</p> <p>Perform mental calculations with two-digit numbers</p> <p>Find ten and a hundred less than a number with up to three-digits</p>
Hundreds	Tens	Units/Ones													
															
															
															
<p>Year 4</p> <p>Subtract numbers with up to four-digits (formal written column method)</p> <p>Understand subtraction as the inverse of addition</p> <p>Solve two-step problems deciding upon the appropriate operations and methods and justifying choices made</p>		<p><i>Column subtraction (with exchange)</i></p> $\begin{array}{r} 723 - 317 \\ \hline 406 \end{array}$ $\begin{array}{r} 67 \overset{11}{\cancel{2}} 3 \\ - 367 \\ \hline 356 \end{array}$ $\begin{array}{r} 67 \overset{11}{\cancel{2}} 3 \\ - \pounds 3.67 \\ \hline \pounds 3.56 \end{array}$ <p><i>Ensure children can solve calculations where zero is a place holder</i></p>	<p>Count back in 6, 7, 9, 25 and 1000</p> <p>Count back through zero to include negative numbers</p> <p>Find 1000 less than a number</p> <p>Continue to practise mental calculations with increasingly large numbers to aid fluency</p>												

Subtraction

Pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly. Addition and subtraction should be taught together.

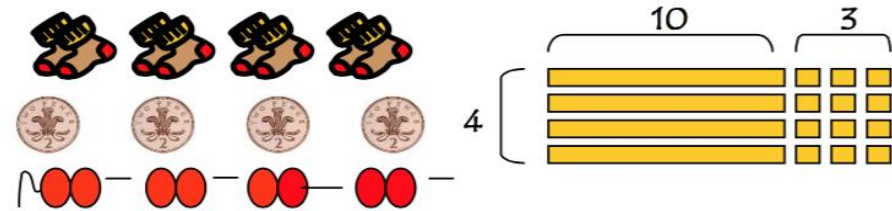
End of Year Expectations	Possible Concrete and Visual Representations	Teacher Modelling/ Children's Recording	Fluency									
<p>Year 5</p> <p>Subtract larger numbers (formal written column method)</p> <p>N.B. ENSURE CHILDREN HAVE THE OPPORTUNITY TO SUBTRACT DECIMALS WITH DIFFERING NUMBERS OF DIGITS</p> <p>Solve multi-step problems selecting and justifying methods</p> <p>Subtract numbers mentally with increasingly large numbers</p>	 <table border="1" data-bbox="689 443 1048 705"> <tr> <td>U</td> <td>1/10</td> <td>1/100</td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td>1</td> <td>0.1</td> <td>0.01</td> </tr> </table> <p></p> <p>Cuisenaire</p>   	U	1/10	1/100				1	0.1	0.01	<p><i>Children might use manipulatives alongside algorithms</i></p> <p>Column subtraction (no exchanging)</p> $\begin{array}{r} 13548 \\ - 12128 \\ \hline 1420 \end{array}$ <p>Column subtraction (with exchanging)</p> $\begin{array}{r} ^2 ^1 ^1 ^1 \\ 13423 \\ - 12678 \\ \hline 745 \end{array}$ <p><i>Ensure children can solve calculations where zero is a place holder</i></p>	<p>Count backwards in powers of ten up to one million</p> <p>Count backwards in positive and negative whole numbers through zero</p> <p>Practise mental calculations with increasingly large numbers</p>
U	1/10	1/100										
												
1	0.1	0.01										
<p>Year 6</p> <p>Subtract multi-digit numbers including numbers with up to three decimal places (formal written column method)</p> <p>ENSURE CHILDREN HAVE THE OPPORTUNITY TO SUBTRACT DECIMALS, WITH DIFFERING NUMBERS OF DIGITS</p> <p>Solve multi-step problems in contexts, deciding which operations and methods to use and why</p> <p>Solve more complex calculations mentally</p>	  	<p>Column subtraction (no exchanging)</p> $\begin{array}{r} 1.48 \\ - 1.21 \\ \hline 0.27 \end{array}$ <p>Column subtraction (with exchanging)</p> $\begin{array}{r} 6 ^1 ^1 \\ 7.23 \\ - 3.67 \\ \hline 3.56 \end{array}$ <p><i>Subtraction with decimals up to three decimal places including in different contexts e.g. money and measures</i></p>	<p>Undertake mental calculations with increasingly large numbers and more complex calculations</p>									

Structures of Multiplication (Haylock and Cockburn 2008)

Children should experience problems with all the different multiplication structures in a range of practical and relevant contexts e.g. money and measurement

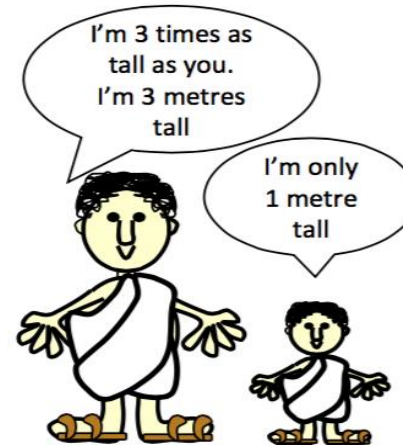
Repeated addition

So many lots (sets) of so many
How many (how much) altogether
Per, each



Scaling

Scaling, scale factor
Doubling, trebling
So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)



Commutative law

Scaling, scale factor
Doubling, trebling
So many times bigger than (longer than, heavier than, and so on)
So many times as much as (or as many as)

a x b and b x a are equal



4 x 2 is the same as/equal to 2 x 4

Multiplication

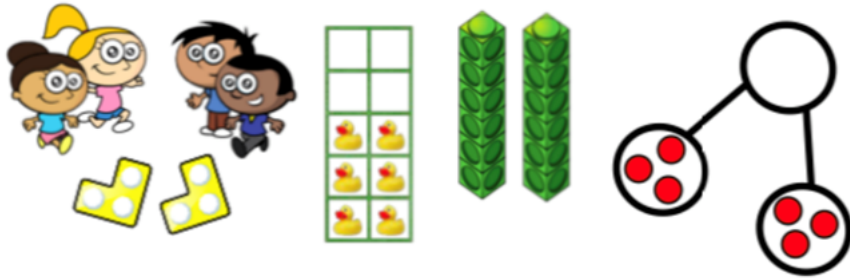
MULTIPLICATION: Vocabulary Progression:

- Using correct mathematical language is crucial for thinking, learning and communicating mathematically.
- We encourage children to explain what they are doing and why they are doing it.
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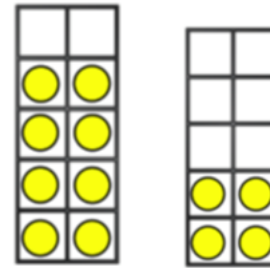
	EYFS	KS1		KS2			
	Reception	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Number	Ones / Tens / Digit The Same Number As As Many As More / Larger / Bigger / Greater Fewer / Smaller / Less Most / Biggest / Largest / Greatest Compare / Order First, Second, Third... Last / Last But One Before / After / Next / Between	Equal To Half-Way Between Above / Below One More / Ten More One Less / Ten Less	Place Value Stands for Represents Exchange	One Hundred More One Hundred Less	One Thousand More One Thousand Less		
Multiplication	Doubling Number Patterns	Multiplication Multiply Multiplied By Multiple Grouping Array	Groups of Times Once, Twice, Three Times... Ten Times Repeated Addition Group in Pairs / Three / Tens Equal Groups of Row / Column Multiplication Table Multiplication / Division Fact	Factor Product	Inverse Square / Squared Cube / Cubed		

EYFS – solve problems by **doubling**.

As with division, we start EYFS with **LOTS** of **concrete** resources.

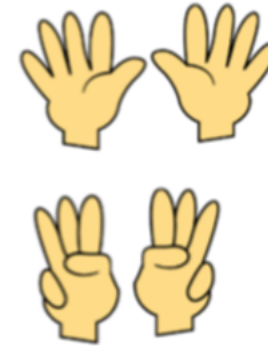


Build Doubles using Concrete Resources

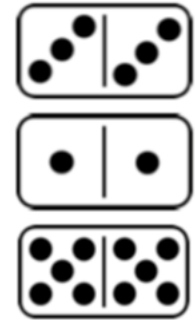


Building numbers in pairs

10 Frame



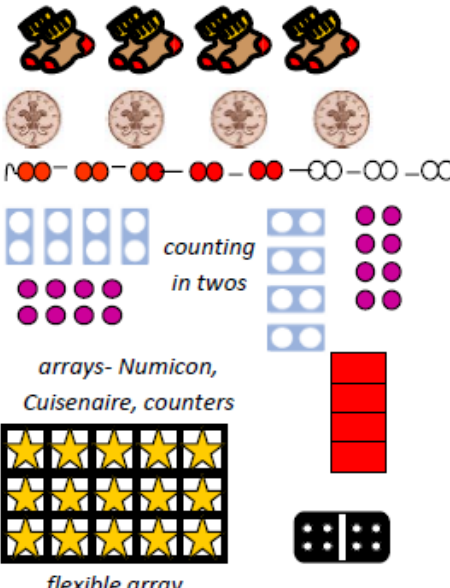
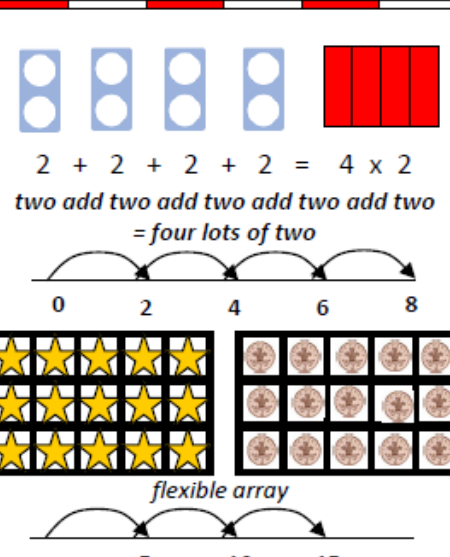
Fingers



Matching Pairs

Multiplication

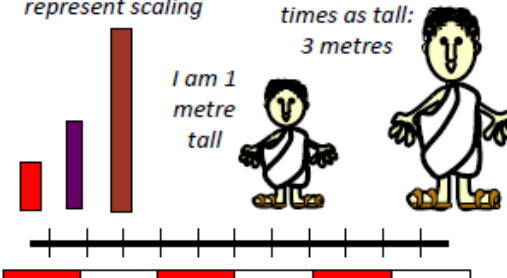
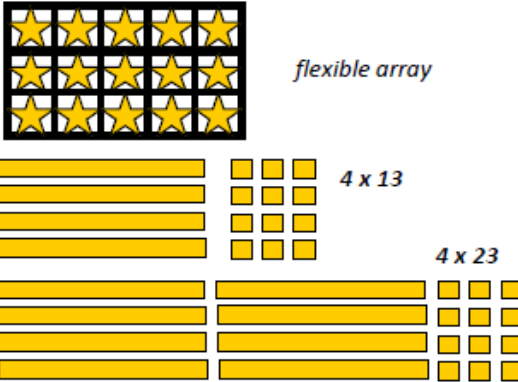
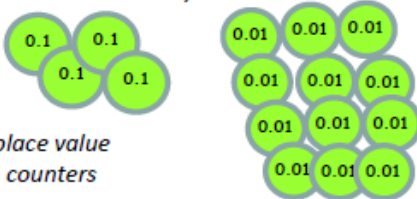
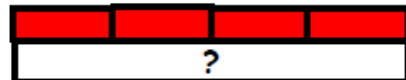
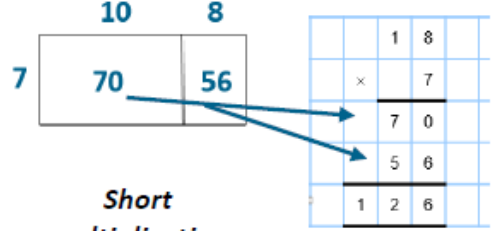
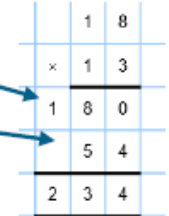

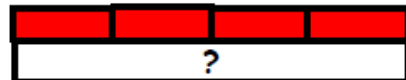
Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly.
 Multiplication and division should be taught together.

End of Year Expectations	Possible concrete and visual representation	Children's Recording	Fluency
<p>Year 1</p> <p>Solve single step practical problems involving multiplication</p> <p>Use concrete objects, pictorial representations to explore grouping</p> <p>Make connections between arrays, number patterns and counting in twos, fives and tens</p> <p>Double numbers and quantities</p>	 <p>counting in twos</p> <p>arrays- Numicon, Cuisenaire, counters</p> <p>flexible array</p>	<p>Practical only e.g. link to small world</p> <p>Using concrete objects, pictorial representations and arrays with the support of an adult – take photographs/draw pictures – if using Numicon small icons could be stuck in</p> <p>four lots of two is eight</p> <p>two lots of four is eight</p> <p>track with cuisenaire</p>	<p>Count in twos, fives and tens from different multiples</p> <p>e.g. 6, 8, 10, 12 etc</p> <p>Emphasise number patterns</p> <p>Double number and quantities</p>
<p>Year 2</p> <p>Understand multiplication as repeated addition</p> <p>Calculate mathematical statements for multiplication within the tables and write them using symbols</p> <p>Understand and solve problems involving arrays</p> <p>Ensure children understand that multiplication is commutative (can be done in any order)</p> <p>Understand that multiplication and division are inverse operations</p>	 <p>$2 + 2 + 2 + 2 = 4 \times 2$</p> <p>two add two add two add two add two = four lots of two</p> <p>flexible array</p>	<p>Record practical work as number sentences</p> <p>$4 \times 2 = 8$</p> <p>$2 \times 4 = 8$</p> <p>track with cuisenaire</p>	<p>Count in twos, threes, fives from zero and tens from any number</p> <p>e.g. 6, 8, 10, 12 etc</p> <p>Emphasise number patterns</p> <p>Introduction to multiplication tables. Practise to become fluent in multiplication facts for 2, 5 and 10</p> <p>Solve multiplication problems mentally</p>

Multiplication – multiplication and division should be taught together– refer to structures of multiplication

End of Year Expectations	Possible concrete and visual representation	Teacher Modelling/Children's Recording	Fluency
<p>Year 3</p> <p>Develop reliable written methods</p> <p>Understand and solve scaling problems</p> <p>Solve problems involving multiplication including correspondence</p>	<p><i>Cuisenaire to represent scaling</i></p> <p><i>Statue is 3 times as tall: 3 metres</i></p> <p><i>flexible array</i></p> <p><i>arrays</i></p> <p><i>place value counters</i></p>	<p><i>Children must use manipulatives alongside algorithms</i></p> <p>4×13 'four <u>lots of</u> thirteen'</p> <p><i>Expanded methods – grid and area</i></p> $\begin{array}{r rr} & 10 & 3 \\ \hline 4 & 40 & 12 \\ \hline \end{array}$ <p>$40 + 12 = 52$</p> <p><i>Progressing to developing fluency in short multiplication</i></p> $\begin{array}{r} 1\ 3 \\ \times 4 \\ \hline 5\ 2 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1\ 3\ 3 \\ \times 4 \\ \hline 5\ 3\ 2 \\ \hline 1\ 1 \end{array}$ <p><i>Start with digits that are below five so children can practise method without encountering difficulty with multiplication tables</i></p>	<p>Count from 0 in multiples of 4, 8, 50 and 100</p> <p>Use multiples of 2, 3, 4, 5, 8, 10, 50 and 100</p> <p>Practise mental recall of multiplication tables – 3, 4 and 8x times tables</p> <p>Connect the 2, 4 and 8 times tables using doubling</p> <p>Develop efficient mental methods using commutativity and multiplication facts to derive related facts e.g. $4 \times 4 \times 12 = 12 \times 4 \times 5 = 12 \times 20$</p>
<p>Year 4</p> <p>Multiplying three numbers</p> <p>Solve two-step problems</p> <p>Multiplying by 0 and by 1</p> <p>Develop fluency in short multiplication using formal written layout</p> <p>Solve problems involving multiplication including using the distributive law, integer scaling problems and harder correspondence problems</p>	<p>4×13</p> <p><i>place value counters</i></p>	<p><i>Progressing to developing fluency in short multiplication</i></p> $\begin{array}{r} 1\ 3 \\ \times 4 \\ \hline 5\ 2 \\ \hline 1 \end{array} \qquad \begin{array}{r} 1\ 3\ 3 \\ \times 4 \\ \hline 5\ 3\ 2 \\ \hline 1\ 1 \end{array}$ <p><i>Start with digits that are below five so children can practise method without encountering difficulty with multiplication tables</i></p>	<p>Count in multiples of 6, 7, 9, 25 and 1000</p> <p>Recall and use multiplication facts up to 12×12 with increasing fluency</p> <p>Derive multiplication facts with up to three-digits</p> <p>Recognise and use factor pairs and commutativity in mental calculations</p> <p>Use the distributive law</p> <p>Combine knowledge of number facts and rules of arithmetic to solve mental and written calculations e.g. $2 \times 6 \times 5 = 10 \times 6$</p>

Multiplication - multiplication and division should be taught together– refer to structures of multiplication

End of Year Expectations	Possible concrete and visual representation	Teacher Modelling/Children's Recording	Fluency
<p>Year 5</p> <p>Multiply decimals with up to three decimal places</p> <p>Identify multiples and factors including finding all factor pairs of a number, and common factors of two numbers</p> <p>Solve problems involving all four operations where larger numbers are used by decomposing them into their factors</p> <p>Multiply whole numbers and those involving decimals by 10, 100 & 1000</p> <p>Understand and use multiplication and division as inverses including in problems involving missing numbers and balancing equations</p> <p>Solve problems involving multiplication and division including scaling by simple fractions</p> <p>Know and use the vocabulary of prime numbers, prime factors and composite (non-prime)</p> <p>Recognise and square and cube numbers and associated notation</p>	<p><i>Cuisenaire to represent scaling</i></p> <p>Statue is 3 times as tall: 3 metres</p> <p>I am 1 metre tall</p>  <p><i>flexible array</i></p>  <p><i>arrays</i></p>  <p><i>place value counters</i></p> 	<p><i>Children might use manipulatives alongside algorithms</i></p>  <p><i>Short multiplication</i></p> $\begin{array}{r} 10 \quad 8 \\ 7 \quad 70 \quad 56 \end{array}$ $\begin{array}{r} 1 \quad 8 \\ \times 7 \\ \hline 7 \quad 0 \\ 5 \quad 6 \\ \hline 1 \quad 2 \quad 6 \end{array}$ $\begin{array}{r} 1 \quad 3 \quad 2 \quad 4 \\ \times 6 \\ \hline 7 \quad 9 \quad 4 \quad 4 \end{array}$ $\begin{array}{r} 3 \quad . \quad 2 \quad 4 \\ \times 6 \\ \hline 1 \quad 9 \quad . \quad 4 \quad 4 \end{array}$ $\begin{array}{r} 1 \quad 1 \quad 2 \\ \times 10 \quad 8 \\ \hline 10 \quad 100 \quad 80 \\ 3 \quad 30 \quad 24 \end{array}$  <p><i>Long multiplication</i></p> $\begin{array}{r} 1 \quad 3 \quad 2 \quad 4 \\ \times 2 \quad 6 \\ \hline 7 \quad 9 \quad 4 \quad 4 \end{array}$ $\begin{array}{r} 3 \quad . \quad 2 \quad 4 \\ \times 2 \quad 6 \\ \hline 1 \quad 9 \quad . \quad 4 \quad 4 \end{array}$ $\begin{array}{r} 2 \quad 6 \quad 4 \quad 8 \quad 0 \\ \times 1 \quad 1 \quad 2 \\ \hline 3 \quad 4 \quad 4 \quad 2 \quad 4 \end{array}$ $\begin{array}{r} 6 \quad 4 \quad . \quad 8 \quad 0 \\ \times 1 \quad 2 \\ \hline 8 \quad 4 \quad . \quad 2 \quad 4 \end{array}$	<p>Count forwards in steps of powers of 10 from any given number up to 1 000 000</p> <p>Practise and extend use of formal written method of short multiplication</p> <p>Apply all multiplication tables frequently. Commit them to memory and use them confidently to make larger calculations</p> <p>Multiply numbers mentally drawing upon known facts</p>
<p>Year 6</p> <p>Multiply numbers up to 4-digit x TU</p> <p>Multiply numbers with up to two decimal places x whole number</p> <p>Multiply multi-digit numbers up to four-digits by a two-digit whole number</p> <p>Multiply single-digit numbers with up to two-decimal places by whole numbers</p> <p>Solve problems involving all four operations</p>	<p><i>arrays</i></p>  <p><i>place value counters</i></p> 	<p><i>Long multiplication</i></p> $\begin{array}{r} 1 \quad 3 \quad 2 \quad 4 \\ \times 2 \quad 6 \\ \hline 7 \quad 9 \quad 4 \quad 4 \end{array}$ $\begin{array}{r} 3 \quad . \quad 2 \quad 4 \\ \times 2 \quad 6 \\ \hline 1 \quad 9 \quad . \quad 4 \quad 4 \end{array}$ $\begin{array}{r} 2 \quad 6 \quad 4 \quad 8 \quad 0 \\ \times 1 \quad 1 \quad 2 \\ \hline 3 \quad 4 \quad 4 \quad 2 \quad 4 \end{array}$ $\begin{array}{r} 6 \quad 4 \quad . \quad 8 \quad 0 \\ \times 1 \quad 2 \\ \hline 8 \quad 4 \quad . \quad 2 \quad 4 \end{array}$	<p>Undertake mental calculations with increasingly large numbers</p> <p>Continue to use all multiplication tables to calculate mathematical statements in order to maintain fluency</p>

Structures for Division (Haylock and Cockburn 2008)

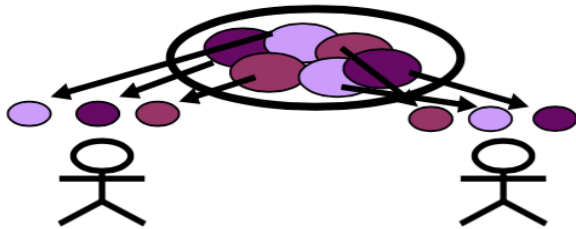
Children should experience problems with the different division structures in a range of practical and relevant contexts e.g. money and measurement

Equal-sharing

Sharing equally between
How many (much) each?

6 shared equally by 2

$$6 \div 2$$

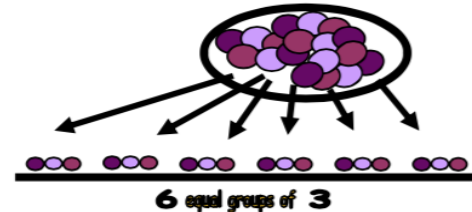


Inverse of multiplication (Grouping)

So many lots (sets/groups) of so many
Share equally in to groups of ...

$$18 \div 3$$

18 divided into
equal groups of 3s



Divide twelve into equal
groups of four



Make 12



Overlay
groups of
four

= 3

Ratio structure

comparison

inverse of scaling structure of multiplication
scale factor (decrease)

Barney earns three times more than Fred. If
Barney earns £900 how much does Fred earn?

Jo's journey to school is three times as
long as Ella's. If Jo walks to school in
30 minutes how long does it take Ella?

Division

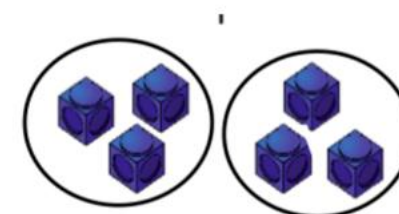
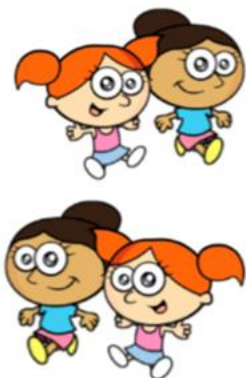
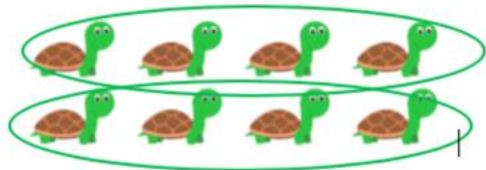
DIVISION: Vocabulary Progression:

- Using correct mathematical language is crucial for thinking, learning and communicating mathematically.
- We encourage children to explain what they are doing and why they are doing it.
- When children are first introduced to new vocabulary, it is not essential that children remember these words - but this modelling will help them become familiar with the terms, gradually beginning to use them accurately and with understanding in later years.

	EYFS	KS1		KS2			
	Reception	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Number	Ones / Tens / Digit The Same Number As As Many As More / Larger / Bigger / Greater Fewer / Smaller / Less Most / Biggest / Largest / Greatest Compare / Order First, Second, Third... Last / Last But One Before / After / Next / Between	Equal To Half-Way Between Above / Below One More / Ten More One Less / Ten Less	Place Value Stands for Represents Exchange	One Hundred More One Hundred Less	One Thousand More One Thousand Less		
Division	Sharing Halving Number Patterns	Division Dividing Grouping Array	Groups of Times Repeated Addition Divide / Divide By Share Equally Left / Left Over One / Two / Three / Ten Each Group in Pairs / Three / Tens Equal Groups of Row / Column	Factor Product Remainder	Inverse Square / Squared Cube / Cubed		

EYFS – solve problems by **halving** and **sharing**.

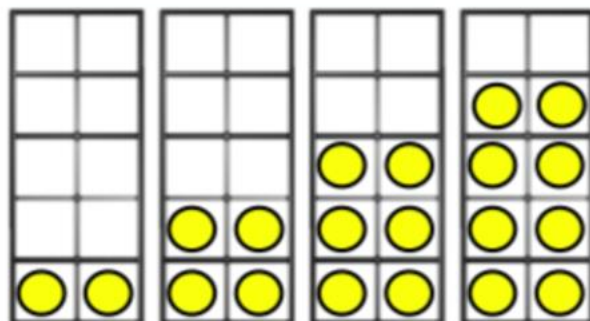
As with multiplication, we start EYFS with **LOTS** of **concrete** resources.



Making Equal Groups

We love putting ourselves in groups

6 in 2 groups (halves)



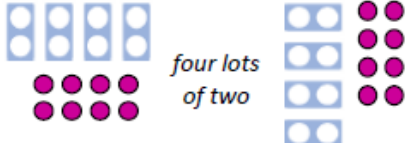



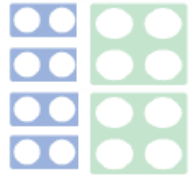
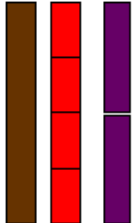

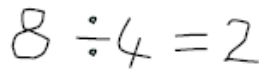




Base 10 – Doubles and Halves

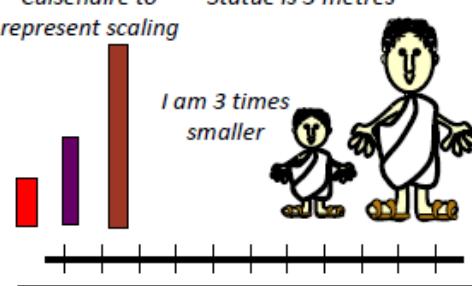
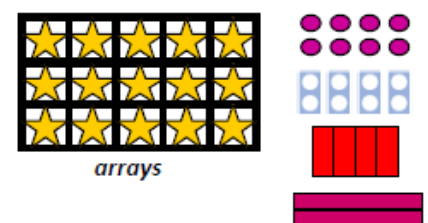
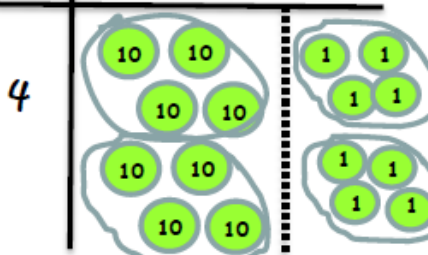
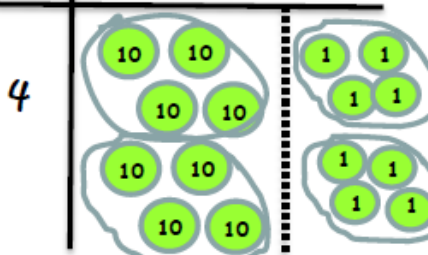

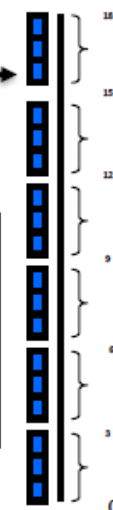
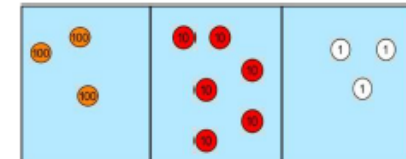
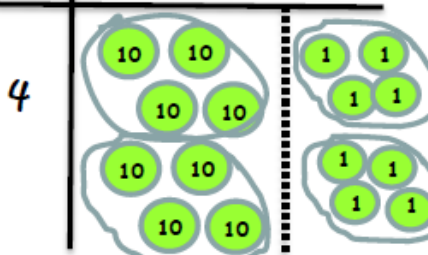
We love chopping things in halves!

Division

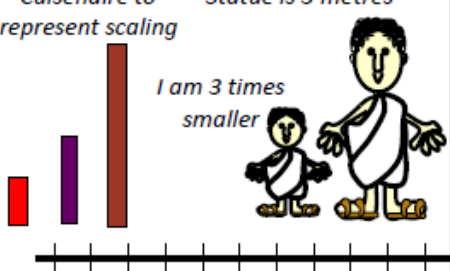

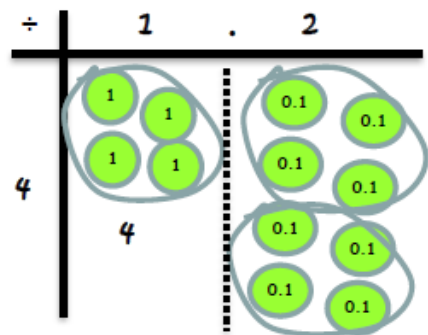
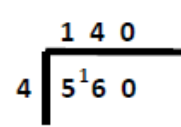
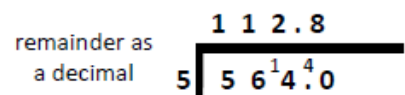
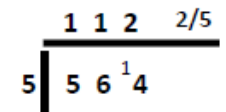
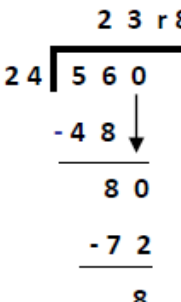
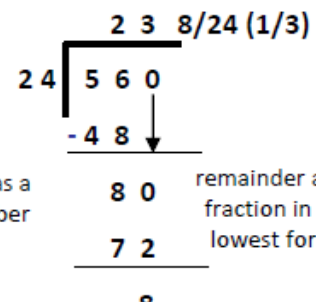
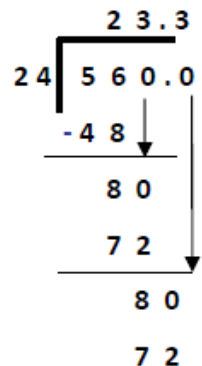
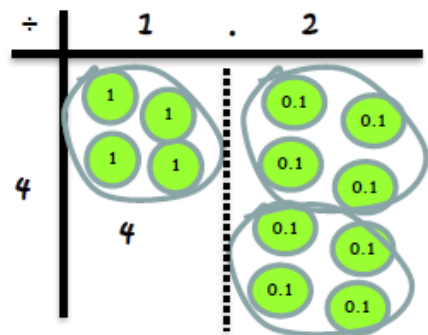
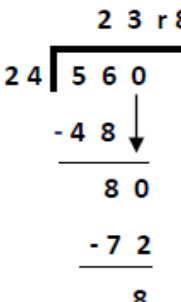
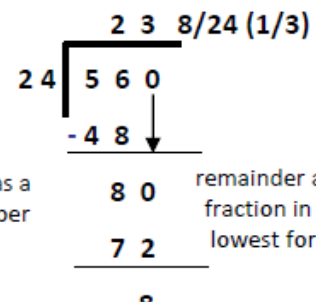
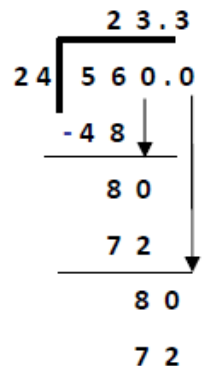
Pupils develop the concept of multiplication and division and are enabled to use these operations flexibly.
 Multiplication and division should be taught together.

End of Year Expectations	Possible concrete and visual representation	Teacher Modelling/Children's Recording	Fluency
<p>Year 1</p> <p>Solve single step practical problems involving division</p> <p>Use concrete objects, pictorial representations</p> <p>Understand division as grouping and sharing</p> <p>Use the language of 'sharing equally between'</p>	<p><i>counting in groups of twos</i></p>  <p><i>straw bundles</i></p>   <p><i>Numicon and counter arrays</i></p>  <p><i>Cuisenaire</i></p> <p><i>doubling</i></p>  <p><i>flexible array</i></p> 	<p>Practical only e.g. link to small world</p> <p>Using concrete objects, pictorial representations and arrays with the support of an adult – take photographs/draw pictures – if using Numicon small icons could be stuck in</p>  <p><i>Eight can be divided into four equal groups of two or two equal groups of four</i></p> 	<p>Count in twos, fives and tens from different multiples</p> <p>e.g. 6, 8, 10, 12 etc</p> <p>Emphasise patterns</p> <p>Find simple fractions eg half and quarter, of objects, numbers and quantities</p>
<p>Year 2</p> <p>Solve single step practical problems involving division</p> <p>Use concrete objects, pictorial representations</p> <p>Understand division as grouping</p> <p>Find halves and then quarters</p> <p>Work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete quantities and to arrays</p>	<p><i>four lots of two</i></p> <p><i>two lots of four</i></p> <p><i>doubling</i></p> <p><i>flexible array</i></p> 	<p>Record as number sentences using \div and $=$</p> <p>$8 \div 4$</p> <p>Eight divided into four equal groups = two in each group</p>    <p><i>Eight can be divided into four equal groups of two or two equal groups of four</i></p>	<p>Count back in twos, threes, fives from zero and tens from any number</p> <p>e.g. 12, 10, 8, 6 etc</p> <p>Emphasise patterns</p> <p>Connect ten times table to place value and five times table to divisions on a clock face</p> <p>Introduction to multiplication tables. Practise to become fluent in division facts for 2, 5 and 10</p> <p>Solve division problems involving grouping and sharing</p>

Division - multiplication and division should be taught together– refer to structures of division

End of Year Expectations	Possible concrete and visual representation	Teacher Modelling/Children's Recording	Fluency																																
<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;">Year 3</div> <ul style="list-style-type: none"> Develop a reliable written method for division Solve problems involving missing numbers Solve problems including those that involve scaling Recognise, find and name $\frac{1}{2}$ and $\frac{1}{4}$ of an object, shape or quantity Understand the link between unit fractions and division Connect $\frac{1}{10}$ to division by 10 Count in tenths 	<p><i>Cuisenaire to represent scaling</i></p> <p><i>Statue is 3 metres</i></p> <p><i>I am 3 times smaller</i></p>  <p><i>arrays</i></p>  <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $88 \div 4$ </div> <div style="display: flex; align-items: center; justify-content: center;"> <div style="font-size: 2em; margin-right: 10px;">÷</div> <table style="border-collapse: collapse;"> <tr> <td style="border-right: 1px solid black; padding: 5px 10px;">2</td> <td style="padding: 5px 10px;">2</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 5px 10px;">4</td> <td style="padding: 5px 10px;">  </td> </tr> </table> </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> <table style="border-collapse: collapse;"> <tr> <td style="border: 1px solid black; width: 20px; height: 20px; text-align: center;">?</td> <td style="border: 1px solid black; width: 20px; height: 20px; text-align: center;">?</td> <td style="border: 1px solid black; width: 20px; height: 20px; text-align: center;">?</td> <td style="border: 1px solid black; width: 20px; height: 20px; text-align: center;">?</td> </tr> </table> </div>	2	2	4		?	?	?	?	<p><i>Children should use manipulatives alongside algorithms</i></p>  <p>Repeated subtraction - chunking</p> <p>Ensure children see/understand the link between grouping on a number line and vertical recording for chunking</p> <div style="display: flex; align-items: center; justify-content: center; margin: 10px 0;"> $95 \div 5 = 19$ <div style="border: 1px solid black; padding: 5px; margin-left: 20px;"> <p style="text-align: center; margin: 0;">Fact Box</p> <p style="margin: 0;">$2 \times 5 = 10$</p> <p style="margin: 0;">$5 \times 5 = 25$</p> <p style="margin: 0;">$10 \times 5 = 50$</p> </div> </div> <div style="display: flex; align-items: center; justify-content: center; margin: 10px 0;"> <table style="border-collapse: collapse;"> <tr> <td style="padding: 5px 10px;">95</td> <td style="padding: 5px 10px;">-</td> <td style="padding: 5px 10px;">50</td> <td style="padding: 5px 10px;">(10 × 5)</td> </tr> <tr> <td style="padding: 5px 10px;">45</td> <td></td> <td></td> <td></td> </tr> <tr> <td style="padding: 5px 10px;">- 25</td> <td></td> <td></td> <td>(5 × 5)</td> </tr> <tr> <td style="padding: 5px 10px;">20</td> <td></td> <td></td> <td></td> </tr> <tr> <td style="padding: 5px 10px;">- 20</td> <td></td> <td></td> <td>(4 × 5)</td> </tr> <tr> <td style="padding: 5px 10px;">0</td> <td></td> <td></td> <td></td> </tr> </table>  </div> <div style="display: flex; align-items: center; justify-content: center; margin: 10px 0;">  <div style="margin-left: 20px;"> $\div \begin{array}{r} 121 \\ 3 \overline{) 363} \end{array}$ </div> </div> <div style="display: flex; align-items: center; justify-content: center; margin: 10px 0;"> $321 \div 3$ </div> <p>Short division- no remainders</p> <div style="display: flex; align-items: center; justify-content: center; margin: 10px 0;"> $560 \div 4$ <div style="margin-left: 20px;"> $\div \begin{array}{r} 140 \\ 4 \overline{) 560} \end{array}$ </div> </div>	95	-	50	(10 × 5)	45				- 25			(5 × 5)	20				- 20			(4 × 5)	0				<p>Recall and use related division facts for the 3, 4 and 8x tables (Continue to practise other tables)</p> <p>Write and calculate mathematical statements for division using what is known</p> <p>Use division facts to derive related division facts e.g. using $6 \div 3 = 2$ to work out $60 \div 3 = 20$</p> <hr/> <p>Continue to practise recalling division facts for multiplication tables up to 12×12</p> <p>Practise mental methods and extend this to three-digit numbers for example $200 \times 3 = 600$ into $600 \div 3 = 200$</p> <p>Use place value, known and derived facts to divide mentally, including dividing by 1</p> <p>Recognise and use factor pairs and commutativity in mental calculations</p>
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4																																			
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<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;">Year 4</div> <ul style="list-style-type: none"> Become fluent in the formal written method of short division with exact answers when dividing by a one-digit number Divide one- or two-digit numbers by 10 or 100, identifying value of digits as tenths or hundredths Solve two-step problems in different contexts, choosing the appropriate operation, working with increasingly harder numbers including correspondence questions e.g. three cakes shared equally between 10 children 																																			

Division - multiplication and division should be taught together– refer to structures of division

End of Year Expectations	Possible concrete and visual representation	Teacher Modelling/Children's Recording	Fluency				
<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;">Year 5</div> <p>Identify factors, including finding all factor pairs of a number, and common factors of two numbers</p> <p>Practise and extend the formal written method of short division: numbers up to four-digits by a one-digit number</p> <p>Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding as appropriate for the context</p> <p>Use multiplication and division as inverses</p> <p>Solve problems involving division including scaling down</p> <p>Divide whole numbers and those involving decimals by 10, 100 & 1000</p>	<p style="text-align: center;"><i>Cuisenaire to represent scaling</i></p> <p style="text-align: center;"><i>Statue is 3 metres</i></p> <p style="text-align: center;"><i>I am 3 times smaller</i></p>  <p style="text-align: center;"><i>flexible arrays</i></p>  <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> $4.8 \div 4$ </div>  <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <table style="border-collapse: collapse; text-align: center;"> <tr> <td style="border: 1px solid black; width: 20px; height: 20px;">?</td> <td style="border: 1px solid black; width: 20px; height: 20px;">?</td> <td style="border: 1px solid black; width: 20px; height: 20px;">?</td> <td style="border: 1px solid black; width: 20px; height: 20px;">?</td> </tr> </table> </div>	?	?	?	?	<p style="text-align: center;"><i>Children might use manipulatives alongside algorithms</i></p> <p>without remainder</p> $560 \div 4$  <p>remainder as a decimal</p> $564 \div 5$  <p>remainder as a fraction</p> $564 \div 5$  <p>long division</p> $560 \div 24$   	<p>Count backwards in steps of powers of 10 for any given number up to 1 000 000</p> <p>Count backwards with positive/negative whole numbers through zero</p> <p>Practise mental calculation with increasingly large numbers</p> <p>Apply all multiplication tables and related division facts frequently, commit them to memory and use them to confidently to make larger calculations</p>
?	?	?	?				
<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;">Year 6</div> <p>Divide numbers up to 4-digits by a 2-digit whole number using formal written methods of long division, interpret remainders as whole numbers, fractions or by rounding, as appropriate for the context</p> <p>Divide numbers with up to 2 decimal places by 1-digit and 2-digit whole numbers, initially in practical contexts involving money and measures</p> <p>Understand the relationship between unit fractions and division</p> <p>Recognise division calculations as the inverse of multiplication</p> <p>Solve problems involving division</p>	 <div style="border: 1px solid black; padding: 5px; margin: 10px auto; width: fit-content;"> <table style="border-collapse: collapse; text-align: center;"> <tr> <td style="border: 1px solid black; width: 20px; height: 20px;">?</td> <td style="border: 1px solid black; width: 20px; height: 20px;">?</td> <td style="border: 1px solid black; width: 20px; height: 20px;">?</td> <td style="border: 1px solid black; width: 20px; height: 20px;">?</td> </tr> </table> </div>	?	?	?	?	<p>remainder as a whole number</p> $560 \div 24$  <p>remainder as a fraction in its lowest form</p> $560 \div 24$  <p>remainder as a decimal</p> $560.0 \div 24$ 	<p>Practise division for larger numbers, using the formal written methods of short and long division</p> <p>Continue to use all multiplication tables and division facts to maintain fluency</p> <p>Perform mental calculations, including with mixed operations and larger numbers</p>
?	?	?	?				