Following the findings of the 'Coordinating mathematical success: the mathematics subject report' (July 2023), we felt we wanted to address the recommendation:

- Primary schools need to help younger pupils to learn their addition facts by heart and regularly check their recall of this knowledge.

We researched and discovered the Nottingham's Number Fluency Programme that we have adopted, to use when needed, in KS1 and for some new arrivals children in KS2.

Maths Intervention
Nottingham's Number Fluency Programme
("Knowing number facts without counting!")

Programme aims:

▪ To improve pupils' fluency with the additive facts that support all calculation involving addition or subtraction, in order to develop their mathematical literacy
▪ To use a structured and systematic teaching framework of the strategies for deriving additive root facts quickly and efficiently based on research evidence

What is number fluency?

Number fluency is getting an answer pretty quickly and with limited demands on 'working memory'. It is also about:
▪ knowing facts and how they relate to each other
(e.g. 4 + 5= 9 so 24 + 5 = 29 or 40 + 50 = 90)
▪ understanding the meaning of operations and how they relate to each other (e.g. 9 = 4 + 5 'The whole is 9 and one part is 4 so the other part must be 5.' 4 + 5 = 9 so 9 – 5 = 4 or 4 = 9 – 5 because one of the parts is equal to the whole subtract the other part.)

Number fluency is not a fixed ability: it can grow and develop through frequent opportunities to practise effective strategies.

Why should we focus on number fluency?

▪ Although according to the National Curriculum pupils in Year 1 should 'memorise and reason with number bonds to 10 and 20 in several forms (for example, 9 + 7 = 16; 16 – 7 = 9; 7 = 16 – 9)', many children do not become fluent in these facts, particularly facts bridging ten, which can hinder their progress in Key Stage 2 and beyond.
Research findings:
▪ Higher attainers in mathematics tend to use known facts or derived fact strategies whereas lower attainers use counting on approaches.
▪ The teaching of strategies is more effective in securing number fluency than taking a rote memorisation approach.
▪ Children will have more success if the strategies are taught through a few key resources.
▪ If children can commit key facts to long term memory, then working memory is freed to deeper and more complex learning.

The set of additive root facts

0 + 0	0 + 1	0 + 2	0 + 3	0 + 4	0 + 5	0 + 6	0 + 7	0 + 8	0 + 9	0 + 10
1 + 0	1 + 1	1 + 2	1 + 3	1 + 4	1 + 5	1 + 6	1 + 7	1 + 8	1 + 9	1 + 10
2 + 0	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	2 + 7	2 + 8	2 + 9	2 + 10
3 + 0	3 + 1	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6	3 + 7	3 + 8	3 + 9	3 + 10
4 + 0	4 + 1	4 + 2	4 + 3	4 + 4	4 + 5	4 + 6	4 + 7	4 + 8	4 + 9	4 + 10
5 + 0	5 + 1	5 + 2	5 + 3	5 + 4	5 + 5	5 + 6	5 + 7	5 + 8	5 + 9	5 + 10
6 + 0	6 + 1	6 + 2	6 + 3	6 + 4	6 + 5	6 + 6	6 + 7	6 + 8	6 + 9	6 + 10
7 + 0	7 + 1	7 + 2	7 + 3	7 + 4	7 + 5	7 + 6	7 + 7	7 + 8	7 + 9	7 + 10
8 + 0	8 + 1	8 + 2	8 + 3	8 + 4	8 + 5	8 + 6	8 + 7	8 + 8	8 + 9	8 + 10

A suggested progression of strategies:

Strategies for facts within 10

1. Adding one (e.g. 7 + 1 & 1 + 7)
2. Adding two to even numbers (e.g. 4 + 2 & 2 + 4)
3. Adding two to odd numbers (e.g. 5 + 2 & 2 + 5)
4. Adding zero (e.g. 3 + 0 & 0 + 3)
5. Number bonds to ten (e.g. 8 + 2 & 2 + 8)
6. Near number bonds to ten (e.g. 6 + 3, 3 + 6)
7. Doubles of numbers to five (e.g. 3 + 3, 4 + 4 & 5 + 5)
8. Near doubles of numbers to five (e.g. 3 + 4, 4 + 3, 4 + 5 & 5 + 4)
9. Number in the middle (e.g. 3 + 5, 5 + 3)

Strategies for bridging 10

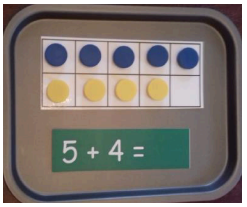
10. Adding ten to a single-digit number (e.g. 5 + 10 and 10 + 5)
11. Doubles of numbers to ten (e.g. 6 + 6, 7 + 7, 8 + 8, 9 + 9 & 10 + 10)
12. Near doubles (e.g. 5 + 6 or 6 + 5 ‘Double 5 and add one.’)
13. Bridging (e.g. 8 + 5 or 5 + 8 ‘Regroup to make a ten.’)
14. Number in the middle (e.g. 6 + 8 or 8 + 6 ‘Double the number in the middle: double 7’)
15. Compensating (e.g. 7 + 9 ‘Add ten to 7 and subtract one.’)

Use fluency sessions to work on the strategies:

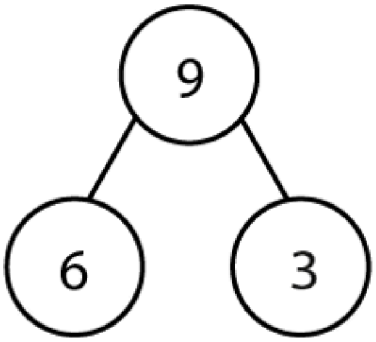
▪ as addition using commutativity
(7+1 = 1+7, 4+5 = 5+4)

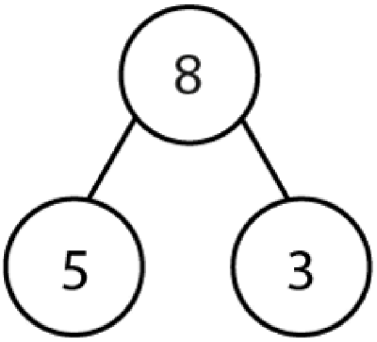
▪ as subtraction facts using inverse
(4+5=9, 9-4=5, 9-5=4, 5=9-4, 4=9-5)

▪ as related facts
(14+5=19, 24+5=29, 29-4=25, 19-5=4)



Teach a strategy at a time and encourage children to build the calculation using the two colours, recording their answers as part-whole models and equations.





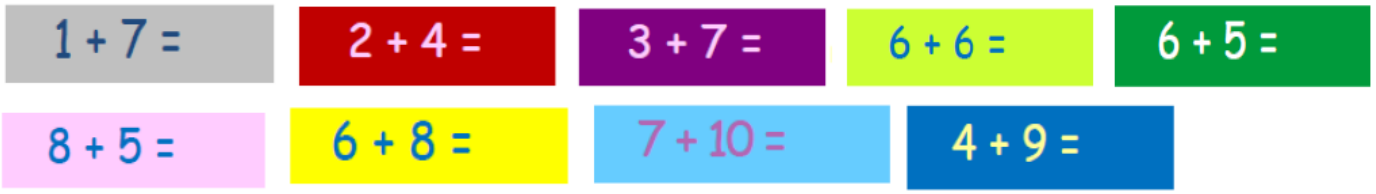
9 = 6 + 3 6 + 3 = 9 9 = 3 + 6 3 + 6 = 9 8 = 3 + 5 3 + 5 = 8 8 = 5 + 3 5 + 3 = 8
9 – 3 = 6 6 = 9 – 3 9 – 6 = 3 3 = 9 – 6 8 – 3 = 5 5 = 8 – 3 8 – 5 = 3 3 = 8 – 5

Ensure children hear and use the ‘part-part-whole’ vocabulary and the range of addition &
subtraction vocabulary, for example:

▪ 8 is the whole; 3 is a part and 5 is a part.
▪ 8 is equal to 5 and 3.
▪ 5 add 3 equals 8.
▪ 3 plus 5 is equal to 8.
▪ The sum of 5 and 3 is 8.
▪ 3 added to 5 is 8.

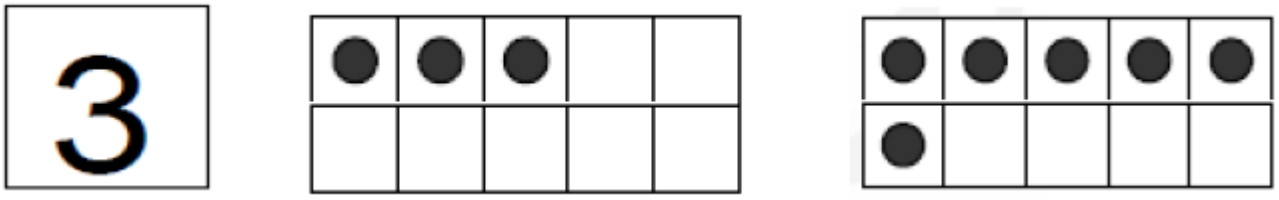
- 8 is 3 more than 5.
- 5 and 3 equals 8.
- The total of 3 and 5 is 8.
- 8 subtract 3 equals 5.
- 8 minus 5 is 3.
- 5 is 8 minus 3.
- One of the parts added to the other part is equal to the whole.
- The whole take away one of the parts is equal to the other part.
- One of the parts is equal to the whole minus the other part.

The colour-coded addition cards can be used for rehearsal of the different strategies.



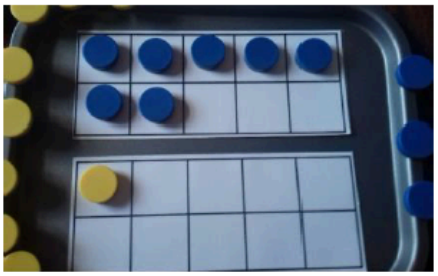
Using the ten-frame trays to help children visualise the additive root facts:

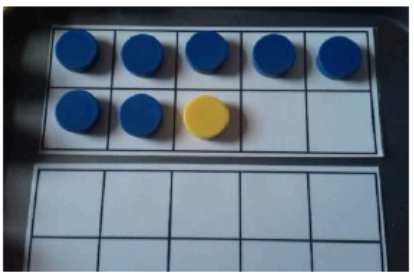
Step 1.
Although in other maths sessions children may look at random arrangements of numbers on a ten-frame in order to practise subitising and using their part-part-whole skills, ensure children are able to create and recognise without counting the numbers 1-10 on the ten-frame using this 'rule' when they are learning strategies for the additive root facts:
Always fill the top row first starting from the left (the same way you read). When the top row is full start filling the bottom row from the left.

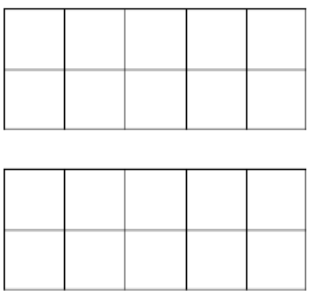


Allow children opportunities to practise building and recognising the numbers 1-10 by calling out the number and using numeral cards and ten-frame flash cards.

Step 2.
When the children can confidently recognise the numbers 1-10 **without counting** start teaching the strategies in the given order.
E.g. 'Adding one': For 1 + 7 or 7 + 1 children build both amounts (addends) and move the smallest amount of counters to create the total.







Step 3.
When children are gaining confidence with a particular strategy encourage them to visualise moving the counters (of the smallest addend) rather than actually moving them. Eventually the children should be able to picture the numbers on empty ten-frames, visualising the entire addition fact in order to picture the total.

Learning the subtraction facts

When children learn the additive root facts they will also learn the related subtraction facts.
All subtraction facts can be completed using a 'think addition' strategy. For example, if children have mastered the adding two facts, they should be introduced to the related subtraction facts in order to apply their knowledge:
When presented with 7 – 2 = they should think, "2 plus what equals 7? 2 plus 5 is 7 so 7 – 2 equals 5."
Teachers should use 'think-aloud' to model this strategy:

7-2=	7-5=
2 plus what equals 7?	*5 plus what is 7?*

Assessing progress

The 'Baseline assessment' can be used before and after children have had experience of the first ten strategies: all the strategies for the facts within ten and the adding ten to a single digit number strategy. To ascertain whether or not children use known facts or strategies rather than counting on their fingers, this assessment should be completed as individual pupil
conferencing using the 'assessment within ten' strips and recording sheet:

7 + 1	2 + 4	7 + 3	5 + 0	4 + 4	3 + 2	6 + 3	10 + 2

The 'Bridging ten assessment' can be used in a similar way to track children's progress with all
the bridging ten facts and to note the strategies used.

+	1	2	3	4	5	6	7	8	9
1	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9
2	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9
3	3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9

Knowledge/strategies for additive fluency:
Grey - Count on
Green - Double

Red - Near double
Yellow - Bonds to ten
Pink - Compensate to add 9
Purple - Bridge to make a ten
Blue - Number in the middle
Brown - Skip count by 2
Commutativity

Addition facts within ten:

<table>
<tr>
<td>

Adding one facts:

2 + 1 1 + 2
3 + 1 1 + 3
4 + 1 1+ 4
5 + 1 1 + 5
6 + 1 1 + 6
7 + 1 1 + 7
8 + 1 1 + 8
9 + 1 1 + 9

</td>
<td>

Adding two to even numbers facts:

4 + 2 2 + 4
6 + 2 2 + 6
8 + 2 2 + 8

Adding two to odd numbers facts:

3 + 2 2 + 3
5 + 2 2 + 5
7 + 2 2 + 7

</td>
<td colspan="2">

Adding zero facts:

1 + 0 0 + 1
2 + 0 0 + 2
3 + 0 0 + 3
4 + 0 0 + 4
5 + 0 0 + 5
6 + 0 0 + 6
7 + 0 0 + 7
8 + 0 0 + 8
9 + 0 0 + 9
10 + 0 0 + 10

</td>
</tr>
<tr>
<td>

Number bonds to ten facts:

10 + 0 0 + 10
9 + 1 1 + 9
8 + 2 2 + 8
7 + 3 3 + 7
6 + 4 4 + 6
5 + 5

</td>
<td>

Near number bonds to ten facts:

6 + 3 3 + 6

Number in the middle facts:

2 + 4 4 + 2
3 + 5 5 + 3
4 + 6 6 + 4

</td>
<td>

Doubles facts:

1 + 1
2 + 2
3 + 3
4 + 4
5 + 5

</td>
<td>

Near doubles facts:

2 + 3 3 + 2
3 + 4 4 + 3
4 + 5 5 + 4

</td>
</tr>
<tr>
<td colspan="4">

Adding ten facts:

1 + 10 10 + 1 2 + 10 10 + 2 3 + 10 10 + 3 4 + 10 10 + 4 5 + 10 10 + 5
6 + 10 10 + 6 7 + 10 10 + 7 8 + 10 10 + 8 9 + 10 10 + 9

</td>
</tr>
</table>

Addition facts bridging ten: