St Peter's C. of E. Junior School Calculation Policy

Date Reviewed: November 2022

Chair of Governors: Mary-Jane Tinsley

Date for Review: September 2024

The policy, having been presented to, and agreed upon by the whole staff and Governors, will be distributed to:

- Teaching staff
- School Governors

A copy of the policy will also be available:

- On the staff drive
- On the school website

This will ensure the policy is readily available to visiting teachers, support staff, volunteers and parents.

Mathematics Mastery

At St Peter's C. of E. School we believe that every child has the potential to succeed, having access to the same curriculum content as their peers, and acquiring a deep, long term, secure and adaptable understanding of the subject.

All teachers use Complete Maths as a framework for the planning and teaching of mathematics. Each new concept taught follows the following approach.

Concrete – Children should have the opportunity to use concrete objects and manipulatives to help them understand what they are doing.

Pictorial – children should build on this concrete approach by using pictorial representations. These representations can then be used to reason and problem solve.

Abstract – with the foundations firmly laid, children should be able to move on to an abstract approach using numbers and key concepts with confidence.

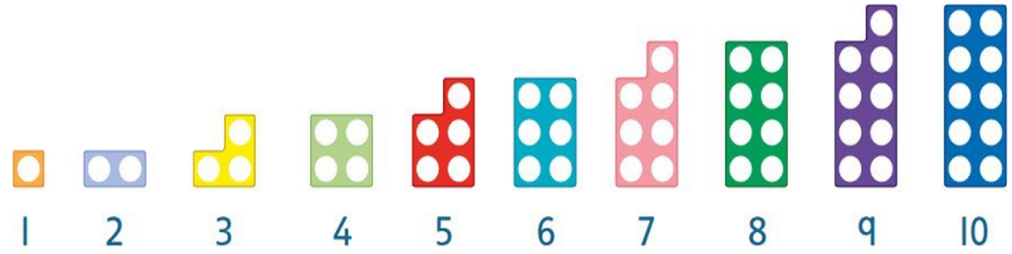
Language – teaching children to know and understand the vocabulary needed to be successful

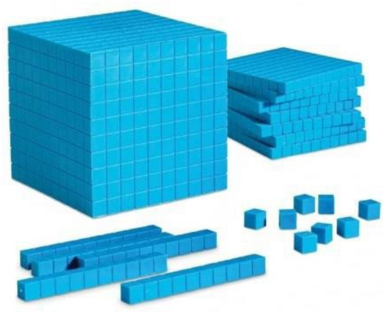
Reasoning and problem solving is encompassed in the above approaches to deepen and master all aspects of mathematics.

This policy outlines the different calculation strategies that should be taught and used from Year 3 to Year 6 in line with the requirements of the 2014 Primary National Curriculum, giving examples of concrete, pictorial and abstract calculations.

Resources for Teaching Mathematics

Teachers can use any teaching resources that they wish to use and the policy does not recommend one set of resources over another, rather that, a variety of resources are used. For each of the four rules of number, different strategies are laid out, together with examples of what concrete materials can be used and how, along with suggested pictorial representations. The principle of the concrete-pictorial-abstract (CPA) approach [Make it, Draw it, Write it] is for children to have a true understanding of a mathematical concept they need to master all three phases. At St Peter's School, the following concrete objects are available for all children to use:

Numicon	
Bead Strings	

Diennes (Base10)	
Cuisenaire	
Counters	

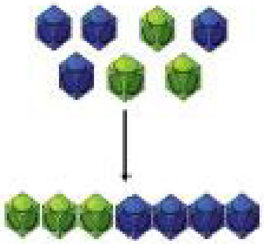
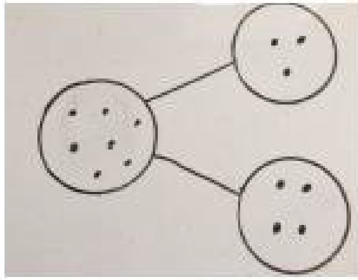
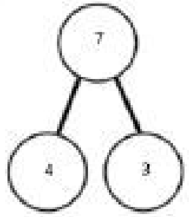
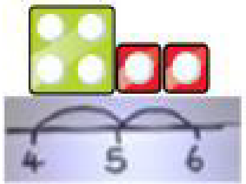
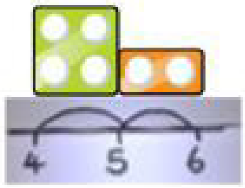
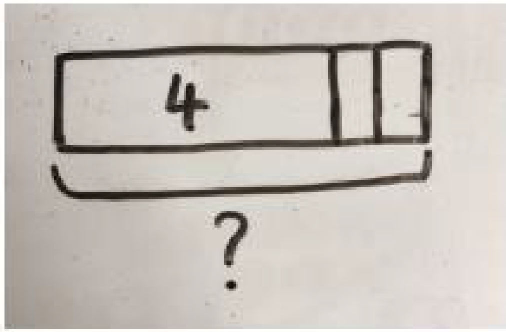
Mathematical Language

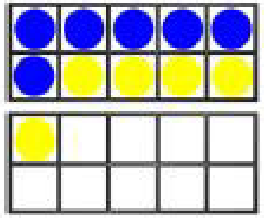
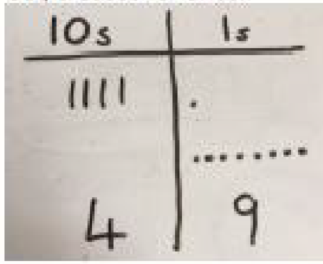
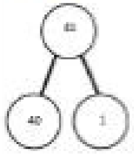
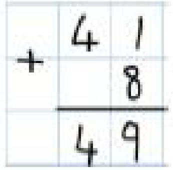
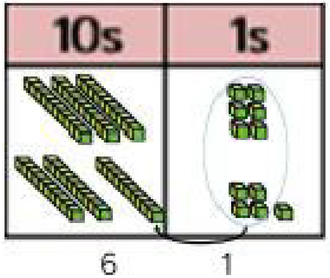
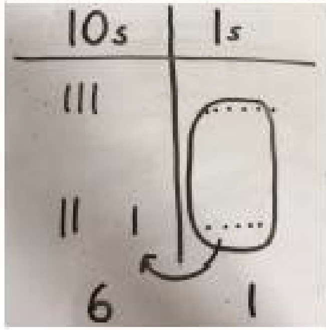
The 2014 National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning (reasoning). Indeed, in certain year groups, the non-statutory guidance highlights the requirement for children to extend their language around certain concepts. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate and precise mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers only accepting what is correct. (See Mathematics Glossary)

Correct Terminology	Old Terminology
Ones	Units
Equal to (the same as)	Equals
Zero	Oh (the letter o)
Exchange Exchanging Regrouping	Stealing Borrowing
Calculation Equation	Generic term of ‘sum’ or ‘number sentence’

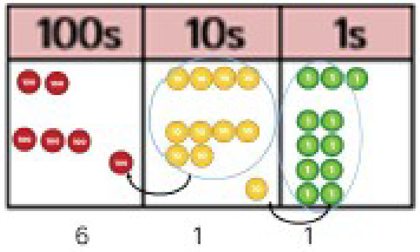
Addition

Key Vocabulary – sum, total parts and wholes, plus, add, together, more, 'is equal to', 'is the same as'

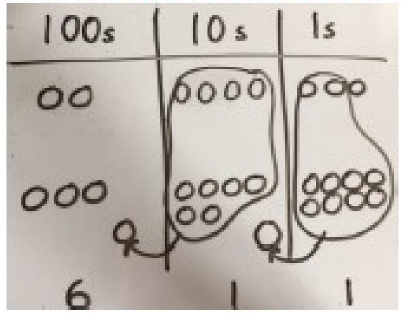
Concrete	Pictorial	Abstract
Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars). 	Children to represent the cubes using dots or crosses. They could put each part on a part whole model too. 	4 + 3 = 7 Four is a part, 3 is a part and the whole is seven. 
Counting on using number lines using cubes or Numicon.   	A bar model which encourages the children to count on, rather than count all. 	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 

Regrouping to make 10; using ten frames and counters/cubes or using Numicon. 6 + 5 	Children to draw the ten frame and counters/cubes. 	Children to develop an understanding of equality e.g. $6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$
TO + O using base 10. Continue to develop understanding of partitioning and place value. 41 + 8 	Children to represent the base 10 e.g. lines for tens and dot/crosses for ones. 	41 + 8  1 + 8 = 9 40 + 9 = 49 
TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36 + 25 	Chidlren to represent the base 10 in a place value chart. 	Looking for ways to make 10. 36 + 25= 1 5 30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61 Formal method: $\begin{array}{r} 36 \\ +25 \\ \hline 61 \\ \hline {\scriptstyle 1} \end{array}$

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.

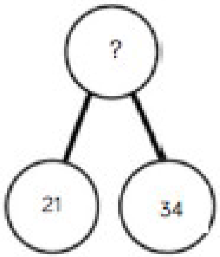


Chidren to represent the counters in a place value chart, circling when they make an exchange.



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline {\scriptstyle 1\ 1\ } \end{array}$$

Conceptual variation; different ways to ask children to solve 21 + 34



?	
21	34

Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

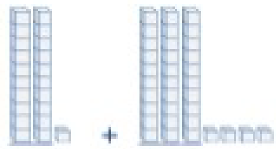
21 + 34 = 55. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \\ \hline \end{array}$$

21 + 34 =

$\boxed{}$ = 21 + 34

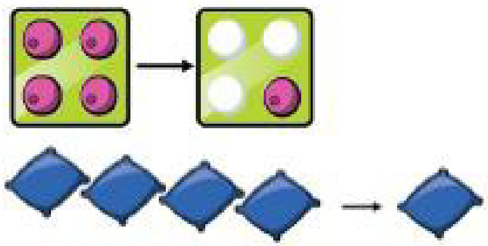
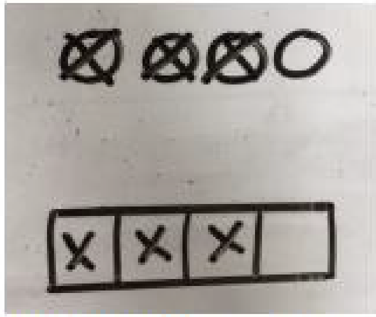
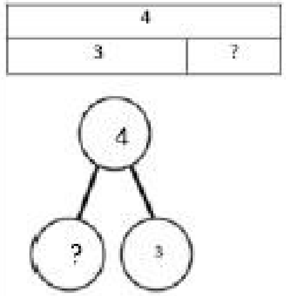
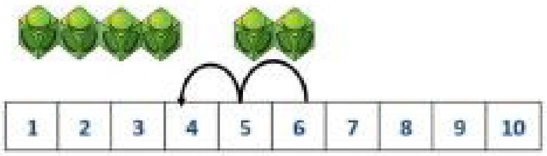
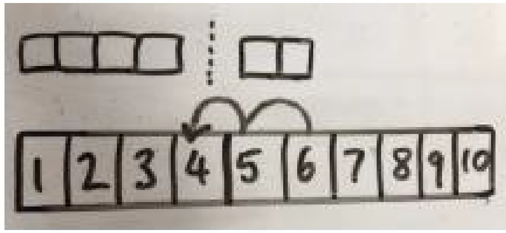
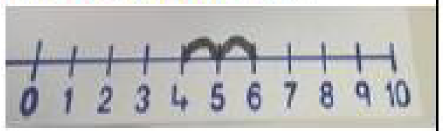
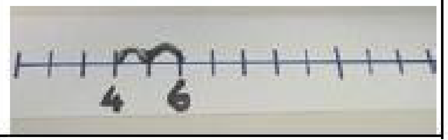
Calculate the sum of twenty-one and thirty-four.



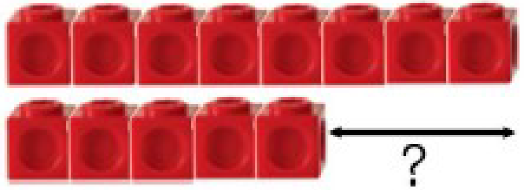
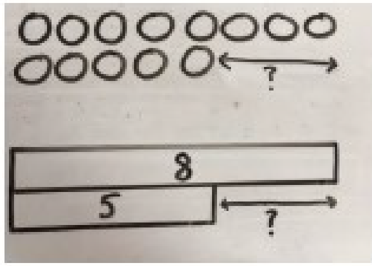
Missing digit problems:

Subtraction

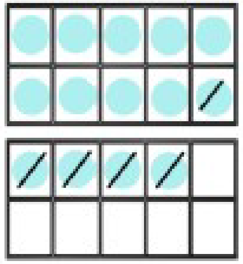
Key Vocabulary – take away, less than, the difference, subtract, minus, fewer, decrease.

Concrete	Pictorial	Abstract
Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used). 4 - 3 = 1 	Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used. 	4- 3 = □ = 4 - 3 
Counting back (using number lines or number tracks) children start with 6 and count back 2. 6 - 2 = 4 	Children to represent what they see pictorially e.g. 	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line  

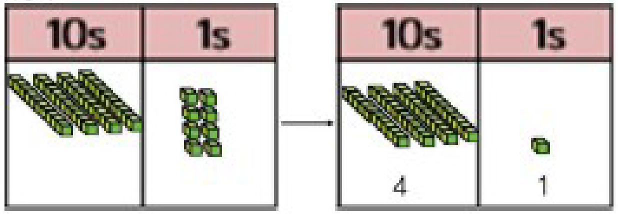
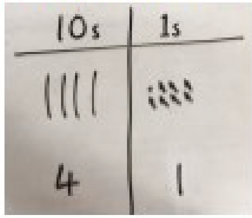
<table>
<tr>
<td>Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).

Calculate the difference between 8 and 5.
</td>
<td>Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.
</td>
<td>Find the difference between 8 and 5.

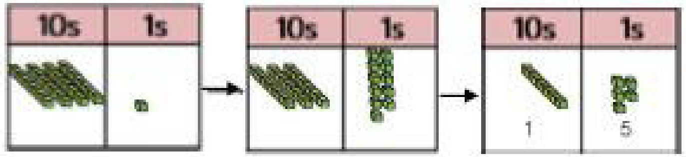
8 – 5, the difference is $\boxed{}$

Children to explore why
9 - 6 = 8 – 5 = 7 – 4 have the same difference.</td>
</tr>
<tr>
<td>Making 10 using ten frames.
14 – 5
</td>
<td>Children to present the ten frame pictorially and discuss what they did to make 10.
</td>
<td>Children to show how they can make 10 by partitioning the subtrahend.

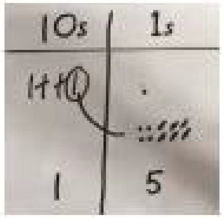
14 - 5 = 9
4 1

14 – 4 = 10
10 – 1 = 9</td>
</tr>
<tr>
<td>Column method using base 10.
48-7
</td>
<td>Children to represent the base 10 pictorially.
</td>
<td>Column method or children could count back 7.
  4 8
–   7
  4 1</td>
</tr>
</table>

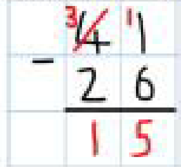
Column method using base 10 and having to exchange.
41 – 26



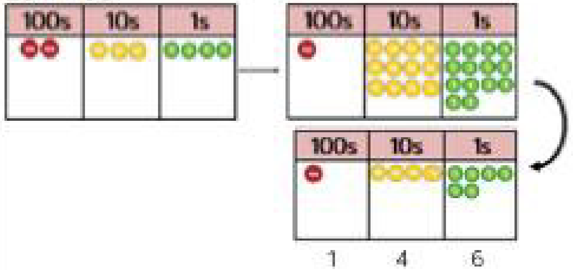
Represent the base 10 pictorially, remembering to show the exchange.



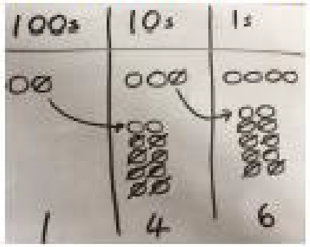
Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because 41 = 30 + 11.



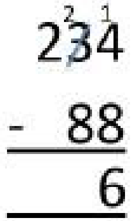
Column method using place value counters.
234 - 88



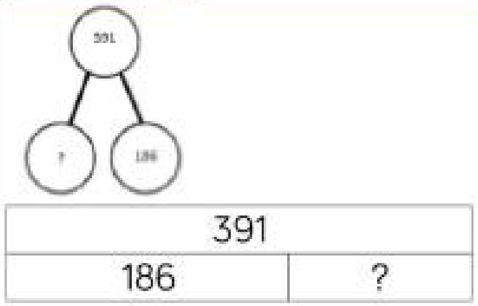
Represent the place value counters pictorially; remembering to show what has been exchanged.



Formal colum method. Children must understand what has happened when they have crossed out digits.



Conceptual variation; different ways to ask children to solve 391 - 186



Raj spent £391, Timmy spent £186.
How much more did Raj spend?

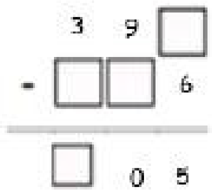
Calculate the difference between 391 and 186.

☐ = 391 – 186

391
-186

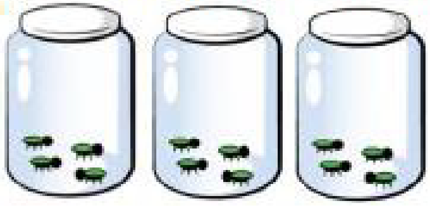
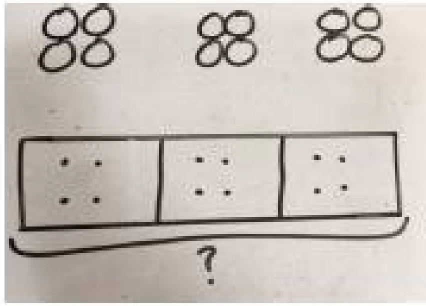
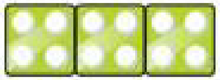
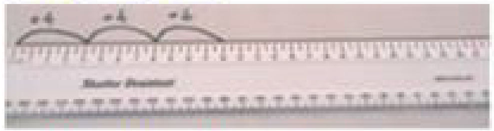
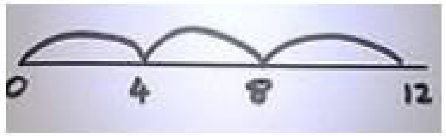
What is 186 less than 391?

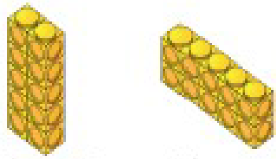
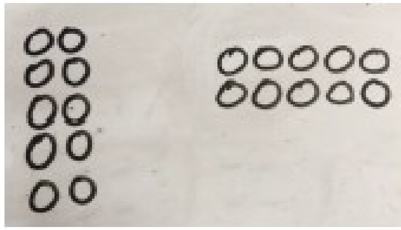
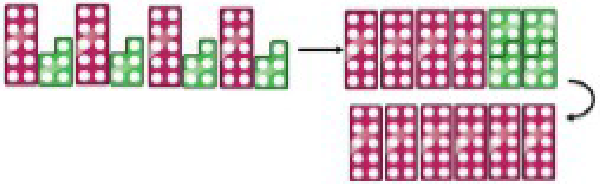
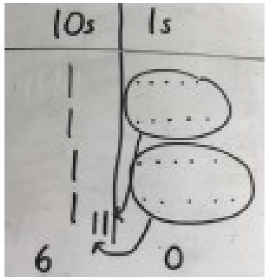
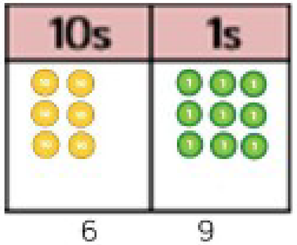
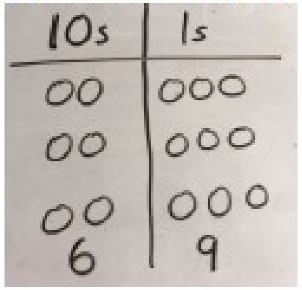
Missing digit calculations



Multiplication

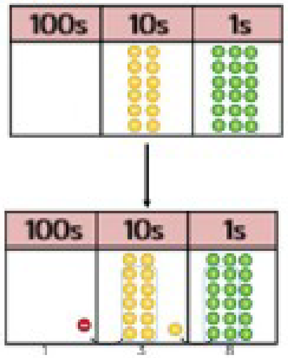
Key Vocabulary – double, times, multiplied by, the product of, groups of, lots of, equal groups

Concrete	Pictorial	Abstract
Repeated grouping/repeated addition 3 × 4 4 + 4 + 4 There are 3 equal groups, with 4 in each group.  	Children to represent the practical resources in a picture and use a bar model.  	3 × 4 = 12 4 + 4 + 4 = 12
Number lines to show repeated groups- 3 × 4   Cuisenaire rods can be used too.	Represent this pictorially alongside a number line e.g.:  	Abstract number line showing three jumps of four. 3 × 4 = 12 

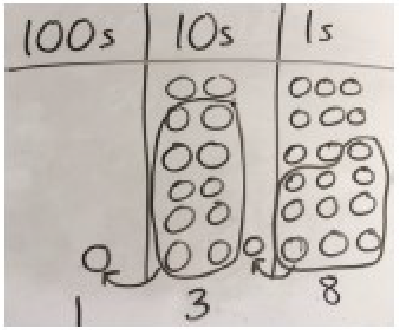
Use arrays to illustrate commutativity counters and other objects can also be used. $2 \times 5 = 5 \times 2$  2 lots of 5 5 lots of 2	Children to represent the arrays pictorially. 	Children to be able to use an array to write a range of calculations e.g. $10 = 2 \times 5$ $5 \times 2 = 10$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$
Partition to multiply using Numicon, base 10 or Cuisenaire rods. 4×15 	Children to represent the concrete manipulatives pictorially. 	Children to be encouraged to show the steps they have taken. 4×15 10 5 $10 \times 4 = 40$ $5 \times 4 = 20$ $40 + 20 = 60$ A number line can also be used 
Formal column method with place value counters (base 10 can also be used.) 3×23 	Children to represent the counters pictorially. 	Children to record what it is they are doing to show understanding. 3×23 $3 \times 20 = 60$ 20 3 $3 \times 3 = 9$ $60 + 9 = 69$ $\begin{array}{r} 23 \\ \times\ 3 \\ \hline 69 \\ \hline \end{array}$

Images taken from White Rose Hub

Formal column method with place value counters.
6 x 23



Children to represent the counters/base 10, pictorially e.g. the image below.



Formal written method

6 x 23 =

$$\begin{array}{r} 23 \\ \times\ \ 6 \\ \hline 138 \\ \hline {\scriptstyle 1\ 1} \end{array}$$

When children start to multiply 3d × 3d and 4d × 2d etc., they should be confident with the abstract:

To get 744 children have solved 6 × 124.
To get 2480 they have solved 20 × 124.

$$\begin{array}{r} 1\ 2\ 4 \\ \times\ \ \ 2\ 6 \\ \hline 7\ 4\ 4 \\ 2\ 4\ 8\ 0 \\ \hline 3\ 2\ 2\ 4 \\ \hline {\scriptstyle 1\ 1\ \ \ \ } \end{array}$$

Answer: 3224

Conceptual variation; different ways to ask children to solve 6 × 23

23	23	23	23	23	23

?

Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that 6 x 23 = 138

Find the product of 6 and 23

6 × 23 =

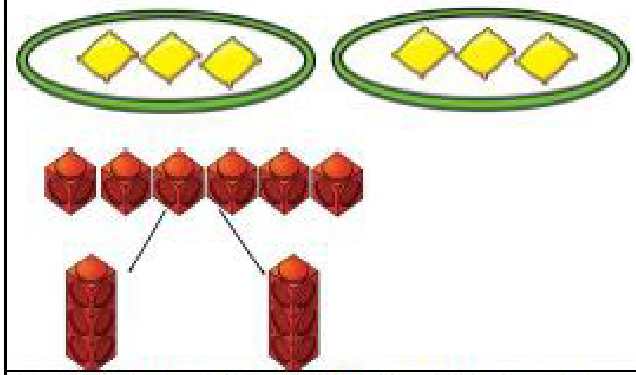
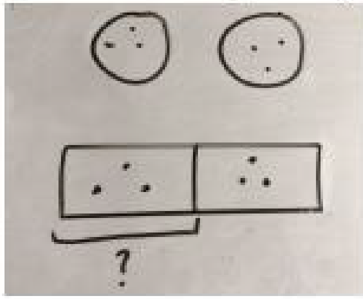
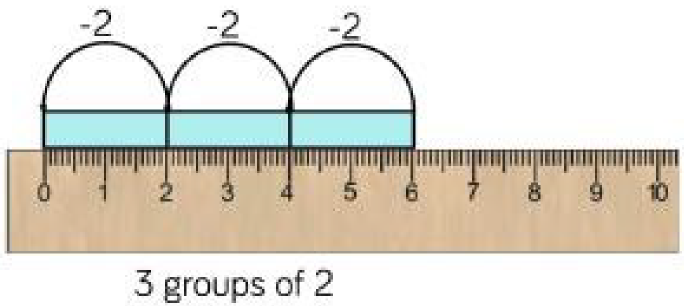
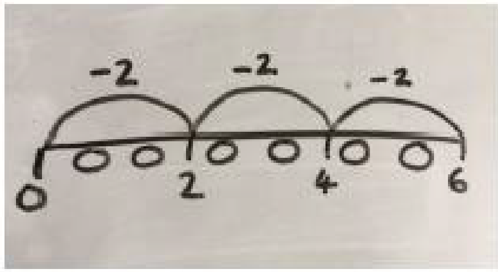
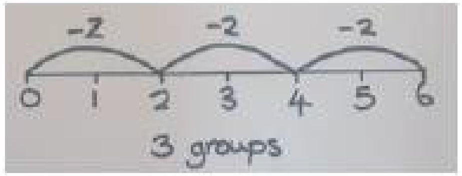
$\boxed{}$ = 6 × 23

$$\begin{array}{r} 6 \\ \times\ 23 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 23 \\ \times\ \ 6 \\ \hline \\ \hline \end{array}$$

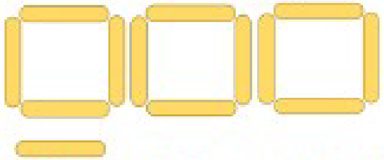
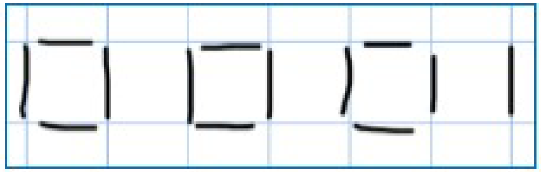
What is the calculation?
What is the product?

Division

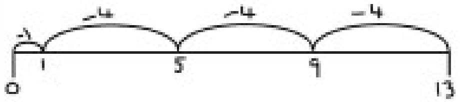
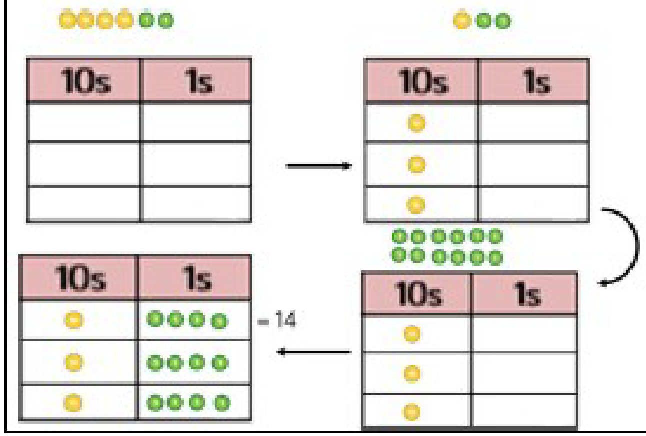
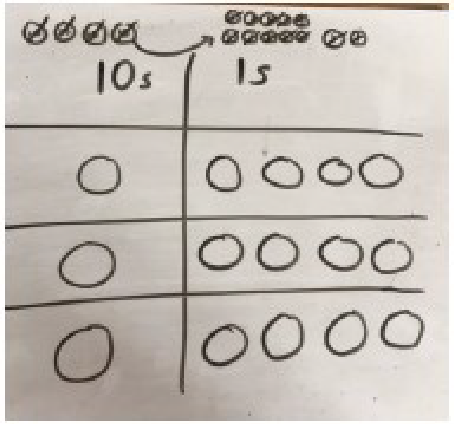
Key vocabulary – share, group, divide, divided by, half

Concrete	Pictorial	Abstract
Sharing using a range of objects. 6 ÷ 2 	Represent the sharing pictorially. 	6 ÷ 2 = 3 | 3 | 3 | Children should also be encouraged to use their 2 times tables facts.
Repeated subtraction using Cuisenaire rods above a ruler. 6 ÷ 2 	Children to represent repeated subtraction pictorially. 	Abstract number line to represent the equal groups that have been subtracted. 

<table>
<tr>
<td>2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used.
13 ÷ 4

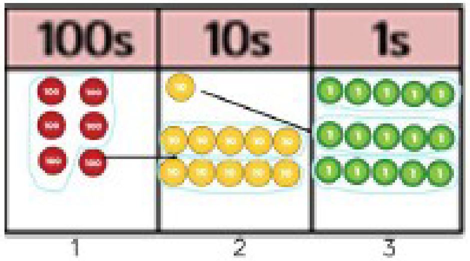
Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.

There are 3 whole squares, with 1 left over.</td>
<td>Children to represent the lollipop sticks pictorially.

There are 3 whole squares, with 1 left over.</td>
<td>13 ÷ 4 – 3 remainder 1

Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'

</td>
</tr>
<tr>
<td>Sharing using place value counters.
42 ÷ 3 = 14

</td>
<td>Children to represent the place value counters pictorially.

</td>
<td>Children to be able to make sense of the place value counters and write calculations to show the process.

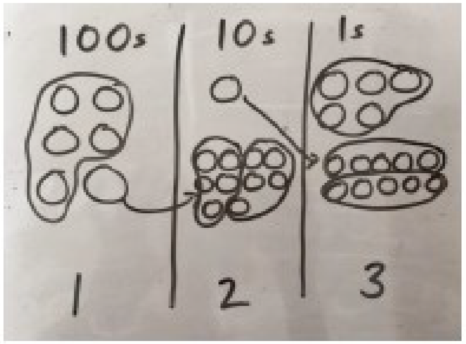
42 ÷ 3
42 = 30 + 12
30 ÷ 3 = 10
12 ÷ 3 = 4
10 + 4 = 14</td>
</tr>
</table>

Short division using place value counters to group.
615 ÷ 5



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.

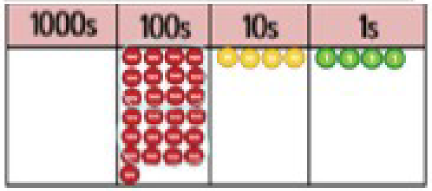


Children to the calculation using the short division scaffold.

$$\begin{array}{r} 123 \\ 5\overline{)6\,{}^{1}1\,{}^{1}5} \end{array}$$

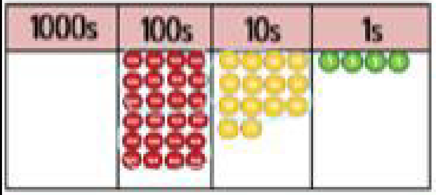
Long division using place value counters
2544 ÷ 12

We can't group 2 thousands into groups of 12 so will exchange them.



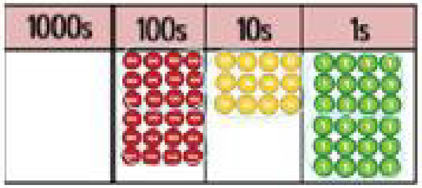
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12\overline{)2544} \\ \underline{24} \\ 1 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

```
   0 2 1
12 |2544
    24
     14
     12
      2
```

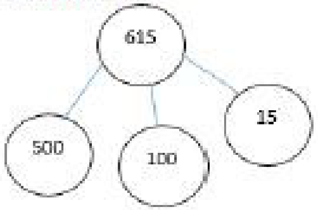


After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

```
   0 2 1 2
12 |2544
    24
     14
     12
      24
      24
       0
```

Conceptual variation; different ways to ask children to solve 615 ÷ 5

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

```
5 |615
```

$615 \div 5 =$

$\square = 615 \div 5$

What is the calculation?
What is the answer?

Images taken from White Rose Hub