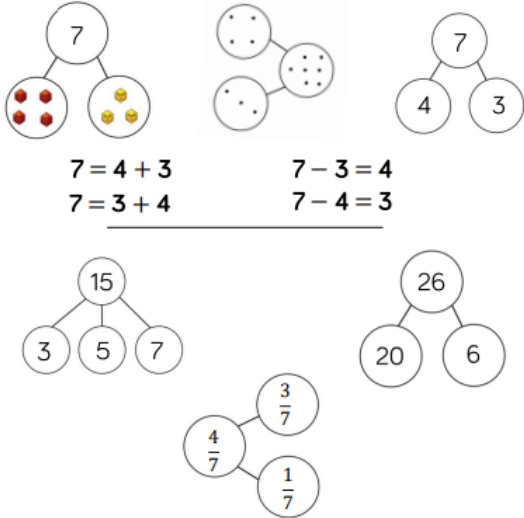
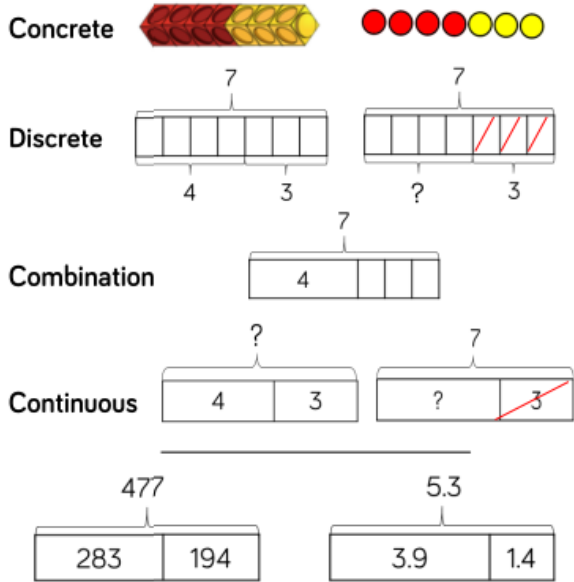


Maths Calculations Supporting Document – Addition and Subtraction

Method and benefits of use	Examples
<p>Part-Whole Model</p> <p>This part-whole model supports children in their understanding of aggregation and portioning. Due to its shape, it can be referred to as a cherry part-whole model.</p> <p>When the parts are complete and the whole is empty, children use aggregation to add parts together to find the total.</p> <p>When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.</p> <p>Part-whole models can be used to partition a number into two or more parts, or to help children partition a number into tens and ones or other place value columns</p> <p>In KS2, children can apply understanding of the part-whole model to add and subtract fractions, decimals and percentages.</p>	
<p>Bar model (single)</p> <p>The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.</p> <p>Cubes and counters can be used in a line as a concrete representation of the bar model.</p> <p>Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.</p> <p>The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.</p> <p>Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.</p> <p>In KS2, children can use bar models to represent larger numbers, decimals and fractions.</p>	

Bar model (multiple)

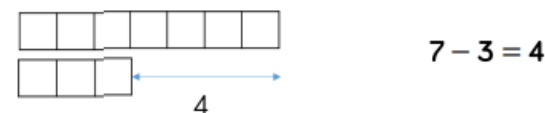
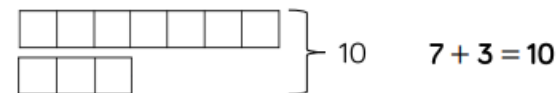
The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

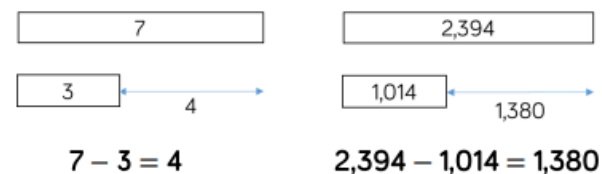
Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Discrete



Continuous



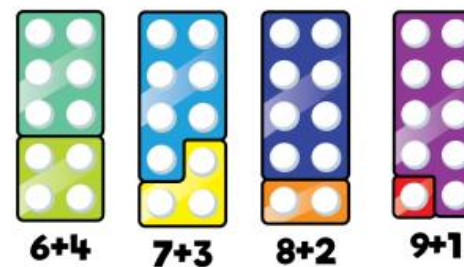
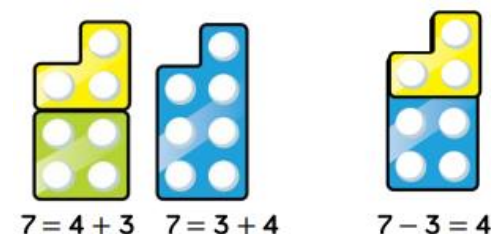
Number Shapes (Numicon)

Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use numbers shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.



Cubes

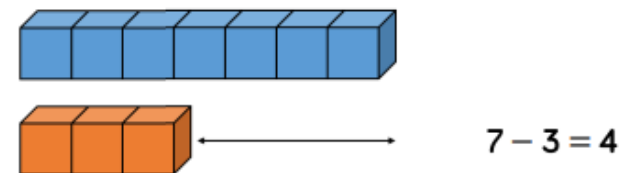
Cubes can be useful to support the children with the addition and subtraction of one-digit numbers.

When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but less efficient with larger numbers as they are difficult to subitise and children may miscount them.

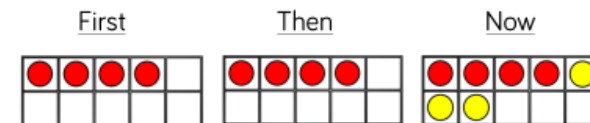
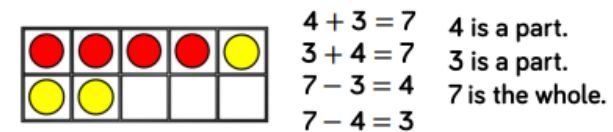


Ten Frames (within 10)

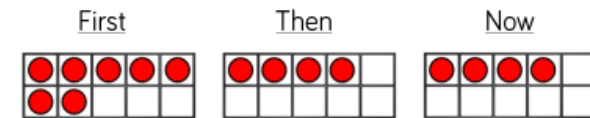
When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning. Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structure, the ten frame can enable children to find all the number bonds for a number.

Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.



$4 + 3 = 7$



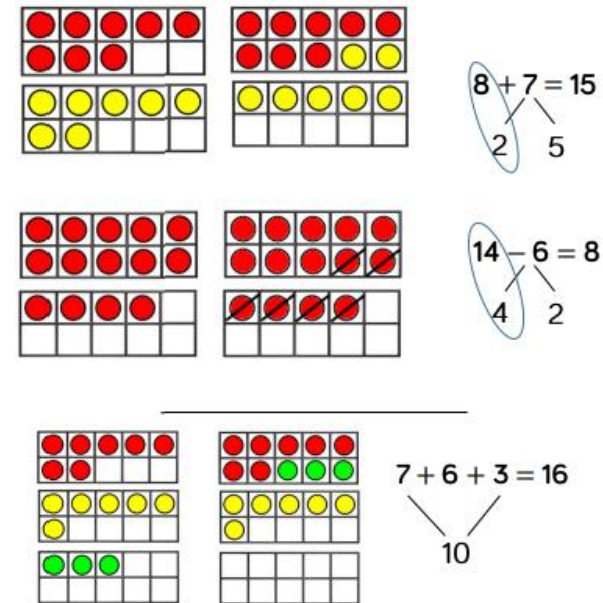
$7 - 3 = 4$

Ten Frames (within 20)

When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 tens frames. Remove the smaller number, thinking carefully about how you have portioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number separate on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond within 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.



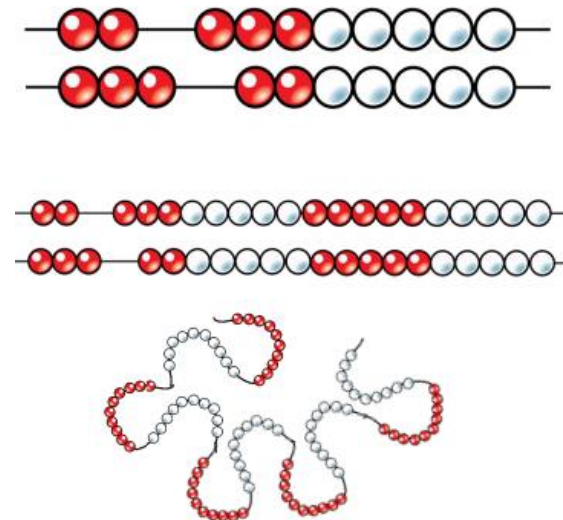
Bead Strings

Different sizes of bead strings can support children at different stages of addition and subtraction.

Bead strings to 10 are very effective at helping children to investigate number bonds up to 10. They can help children systematically find all the number bonds to 10 by moving one bead at a time to see the difference numbers they have partitioned the 10 beads into.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.



Number Tracks

Number tracks are useful to support children in their understanding of augmentation and reduction.

When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count to find the total.

When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

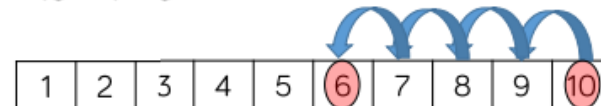
Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Place board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

$$5 + 3 = 8$$



$$10 - 4 = 6$$



$$8 + 7 = 15$$



Number Line (labelled)

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

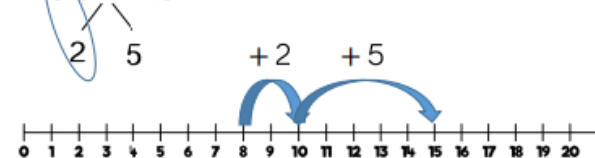
Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by tens frames so children can see how they partition the smaller number into the two separate jumps.

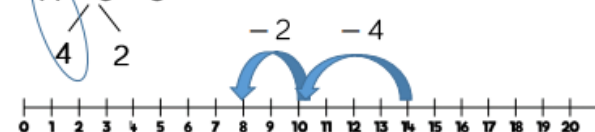
$$5 + 3 = 8$$



$$8 + 7 = 15$$



$$14 - 6 = 8$$



Number Lines (blank)

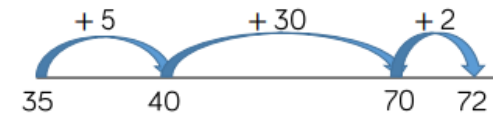
Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

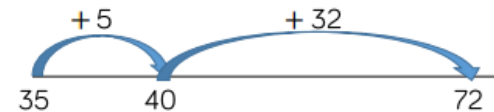
Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

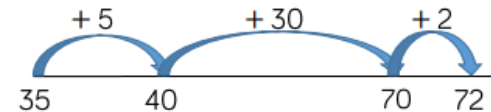
$$35 + 37 = 72$$



$$35 + 37 = 72$$



$$72 - 35 = 37$$



Straws

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

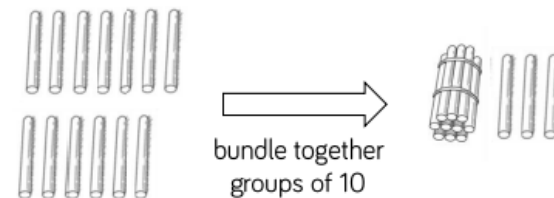
Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or others ties to make bundles of 10 straws.

When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

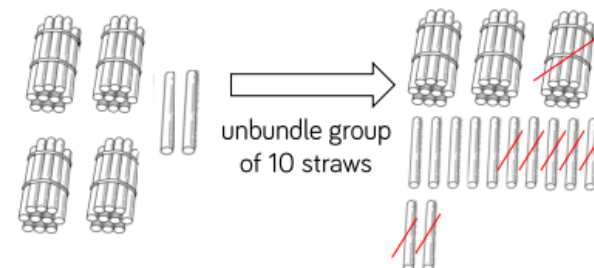
When subtracting numbers, children unbundle a group of 10 straws to represent the exchanged from 1 ten to 10 ones.

Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.

$$7 + 6 = 13$$



$$42 - 17 = 25$$



Base 10/Dienes (addition)

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first add without exchange before moving on to addition with exchange. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children.

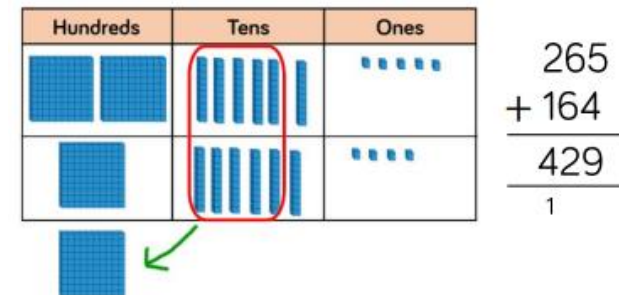
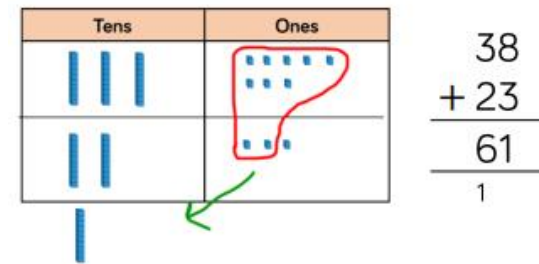
How many ones are there altogether?

Can we make an exchange (Yes or No)

How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)

How many ones do we have left? (Write ones column)

Repeat for each column

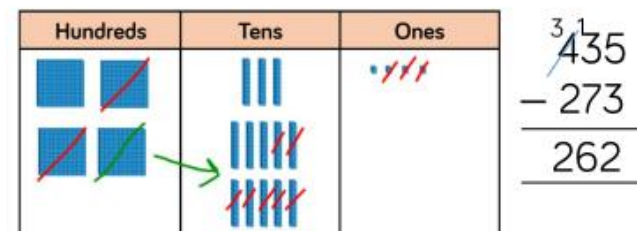
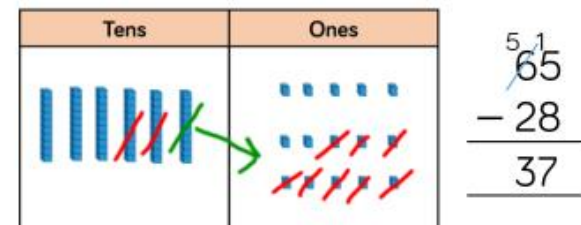


Base 10/Dienes (subtraction)

Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children move to the column on the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

This model is efficient with up to 4-digit numbers. Place value counters are more effective with larger numbers and decimals.

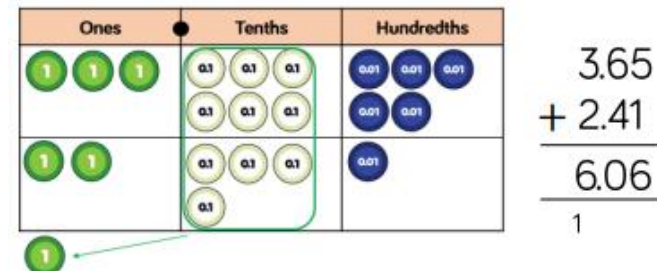
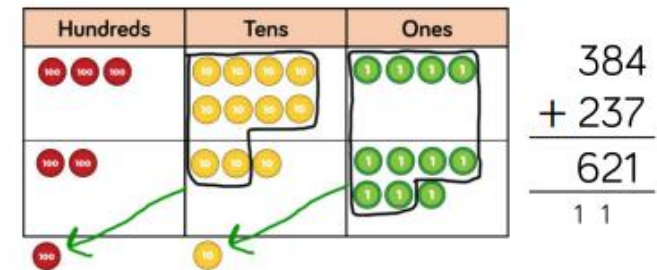


Place Value Counters (addition)

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have enough place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

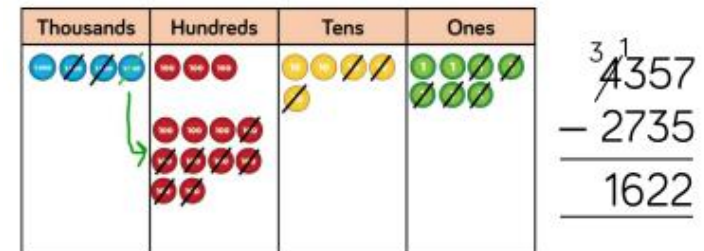
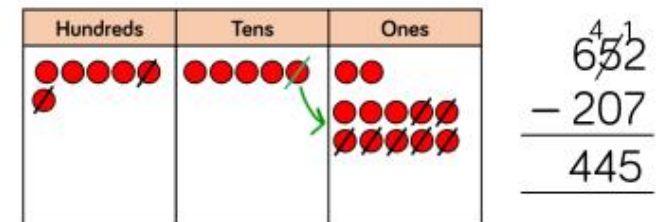


Place Value Counters (Subtraction)

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have enough place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/ten/hundreds to subtract in a column, children move to the column on the left to exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently



	Part-whole model	Bar model	Number shapes	Ten frames	Bead strings	Number tracks	Number lines	Straws	Column addition	Hundred squares	Base 10	Place Value Counters
Year 1				Within 10 and 20	10 and 20		Labelled					
Year 2				Within 20			Labelled & Blank					
Year 3												
Year 4												
Year 5												
Year 6												

Glossary

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative - numbers can be added in any order.

Complement - in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference - the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange - Change a number or expression for another of an equal value.

Minuend - A quantity or number from which another is subtracted.

Partitioning - Splitting a number into its component parts.

Reduction - Subtraction as take away.

Subitise - Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total - The aggregate or the sum found by addition.