PROBLEM SOLVING

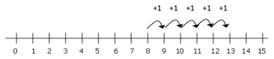
FLUENCY REASONING

Progression in Addition

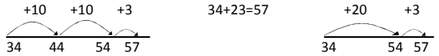
Step 1: *Number tracks Concrete objects Pictorial representations*
Use concrete objects and pictorial representations to solve simple addition problems, introducing **counting all** (aggregation: combining groups/total) and **counting-on** (augmentation: adding one more) using a number track putting the largest number first

1	2	3	4	5	6	7	8	9	10

Step 2: *Numbered lines Informal jottings Practical resources*
Continue to use practical resources to support calculation. Introduce and model the use of the number line. Children begin to use numbered lines to support their own calculations.
8 + 5 = 13

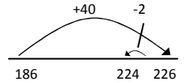


Step 3: *Empty number lines Informal jottings Practical resources*
Children will begin to use 'empty number lines' themselves starting with the larger number and counting on irrespective of the order of calculation.
First counting on in tens and ones then helping children to become more efficient by adding the 'ones' in one jump (e.g. by using the known fact 4 + 3 = 7).



34+23=57

Children will continue to use empty number lines with increasingly large numbers, using the line to record their steps



38 + 186 = 224
186 + 40 - 2 = 224

Going back 2 like this, is called compensating

Step 4: *Partitioning Horizontal recording of mental calculations*
Children will continue to use informal pencil and paper methods (jottings) to support, record and explain mental methods building on existing strategies.
Record mental methods horizontally, using partitioning (*partition into hundreds, tens and ones*)
47 +76
= 76 + 40 +7 = 116 + 7 = 123

Or 70 + 40 = 110
6 + 7 = 13
110 + 13 = 123

Leading to 70 6
+ 40 7
110 13
= 123 in the next step

Step 5: *Expanded column addition Place value resources*
Secure informal methods lead to the development of more formal written methods. Partition into hundreds, tens and ones and so on, then add the least significant digits first in preparation for carrying. Use place value resources to reinforce the concept of place value; some children will need to include place value columns to support understanding.

T ones	*H T ones*
6 7	2 6 7
+ 2 4	+ 8 5
1 1	1 2
+ 8 0	1 4 0
9 1	+ 2 0 0
	3 5 2

This step is important because it makes the link between the horizontal method and recording addition of the ones and then the tens vertically

Step 6: *Column addition Place value resources*
From this, children will begin to carry below the line.

3 6 7	2.84
+ 8 5	+ 1.36
4 5 2	4.20
1 1	1 1

Children should extend the carrying method to numbers with at least four digits, including decimals in a range of contexts including money and measurement (with units up to 2 decimal places e.g. 1.75 metres)

It is important to know that the decimal points should line up under each other, particularly when adding numbers with different numbers of decimal places

Step 7: *Column addition Addition of numbers with any numbers of digits*
Children should extend the carrying method to numbers with any number of digits, including decimals in a range of context including measures to 3 decimal places (e.g. 3.752 litres)

Mental methods of addition should be used if they are more efficient
Examples:
adding near doubles e.g. 21 + 20 = 40 + 1 = 41
adding near multiples of ten
e.g. 45 + 19 = 45 + 20 - 1 = 64

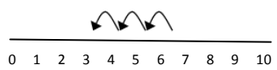
Progression in Subtraction

Step 1: *Concrete objects Pictorial representations Number tracks*
Use concrete, practical objects/resources (bead-strings, cube towers or number tracks) and pictorial representations to solve simple subtraction problems, introducing these models:
- taking away (physical removal or reduction) Jo has 4 cars and gives 3 to Jack. How many cars does she have left? 4 - 3 = 1
- finding the difference (comparison) Jo has 4 cars, Jack has 3 cars. How many more does Jo have than Jack? 4 – 3 = 1

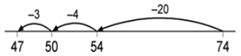
- part/whole situations (complements - finding how many in a set fit a criteria) There are 4 cars, 3 are red and the rest are blue. How many are blue? 4 – 3 = 1

Step 2: *Numbered lines Apparatus Informal jotting*
Continue to use practical resources to support calculation. Introduce and model the use of the numbered line. Children begin to use numbered lines to support their own calculations.

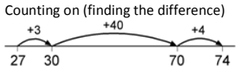
Taking away I have £6 and spend £3. How much left? 6 – 3 = 3



Step 3: *Empty number lines*
Counting back (taking away)
74 – 27 = 47



Counting on (finding the difference)



Empty number lines are not a method in themselves-they record the steps taken mentally; children should be confident in counting back and forward in ones, then larger steps

Children will continue to use empty number lines with increasingly large numbers and will begin to use informal pencil and paper methods (jottings) to support, record and explain mental methods building on existing strategies.

Step 4: *Expanded column method for subtraction Partitioning*
Initially, the children will be taught to record in columns, partitioning numbers into tens and ones, with examples that do not need the regrouping of tens and ones
8 9 – 57 = 32

8 0	9
- 5 0	7
3 0	2

Some children may continue confidently working in subtraction on a number line.

Step 5: *Expanded column method for subtraction Partitioning Regrouping*
From Step 4, the children will begin to regroup the tens into ones
71 – 46 = 25

60		1
~~70~~		1
- 40		6
20		5

Use *regrouping* rather than borrowing, we're *regrouping* one ten into ten ones…

Children should demonstrate their place value knowledge lining ones up under ones, tens under tens and so on.

754 – 86 = 668

600	140	
~~7~~00	~~5~~0	14
-	80	6
600	60	8

Estimate answers by rounding numbers. Check answers using addition (the inverse) 668+86 =

Step 6: *Column method for subtraction*
Children should apply the regrouping method to numbers with at least 4 digits, including decimals in a range of contexts including money and measurement (to 2 decimal places e.g. 1.25 metres). Number lines and mental methods should also be used, if more efficient than column method.

6 1	6 14 1	5 14 1
~~7~~ 1	~~7~~ ~~5~~ 4	~~6~~ ~~5~~. 3
- 4 6	- 8 6	- 8. 7
2 5	6 8	5 6. 6

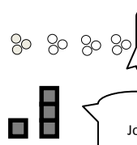
Children should continue to use an empty number line to record mental strategies where the numbers are close together or near to multiples of 10, 100 to **count on.** 1007 – 998 = 9 (998 counting on 9 to 1007)

Step 7: Children should extend the regrouping method to numbers with any number of digits, including decimals in a range of context including measures to 3 decimal places (e.g. 3.752 litres)

Progression in Multiplication

Step 1: *Concrete objects Apparatus Pictorial representations Diagrams*
Simple problems introducing the models of **arrays**, **repeated addition** and **scaling**

Arrays: 3 x 4
3 four times
4 lots of 3

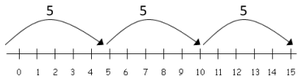


Repeated addition
4 lots of 3 3 + 3 + 3 + 3
3 x 4 3 four times

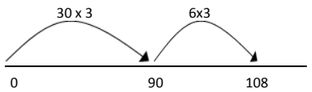
Scaling 1 x 3 = 3
John's tower is three times as tall as Grace's tower

Step 2: *Repeated addition Number lines*
5 x 3 = 15
5 three times or *5 + 5 + 5 = 15* or *3 lots of 5* or *5 x 3*

Repeated addition can be shown easily on a number line:
5 x 3 = 5 + 5 + 5



Step 3: *Empty Number lines Partitioning (grouping in tens and ones)*
36 x 3
30 x 3 = 90
6 x 3 = 18 90 + 18= 108



Step 4: *Grid method (area/array method)*

Children will continue to use arrays where appropriate leading into the grid method of multiplication for 2 digit by 1 digit, then 2 digit by 2 digit calculations.

x	10	4
6	60	24

(6 x 10) + (6 x 4)
60 + 24
= 84

This model/image supports the development of the grid method, but should not be included as a written method

Children will use this layout for recording
14 x 6 = 60 + 24 = 84

x	10	4
6	60	24

Estimate by rounding and check by repetition

47 x 34 = 1598

x	40	7
30	1200	210
4	160	28

1200
210
160
+ 28
1598

Add mentally where possible

Step 5: *Grid method*
Extended to 3 digit by 2 digit numbers and for decimals to 3 decimal places

Step 6: *Expanded short multiplication*

38
x 7
56
+ 210
266

It is important that children have a solid understanding of place value before being taught compact methods. Use place value apparatus to model this vertical recording method of multiplication

Step 7: *Short multiplication*

3 8
X 7
2 6 6
5

The steps are
7 x 8 = 56 (carry the 5 or fifty)
7 x 30 = 210

Step 8: *Long multiplication*

4 7
x 3 4
1 8 8
2
+1 4 1 0
2
1 5 9 8

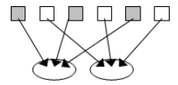
Multiply by 4, then by 30

Progression in Division

Step 1: *Concrete objects Apparatus Pictorial representations Diagrams*
Children will use concrete objects, pictorial representations and arrays in context, with the support of the teacher, to work on practical problem solving activities involving **sharing equally** (equal sets of) and **grouping** (repeated subtraction)

Children will begin to develop their recording using pictures and written descriptions and continue to use a wide range of representations including:

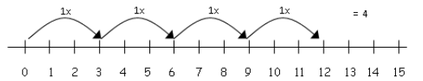
Sharing equally



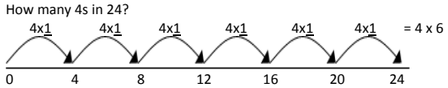
6 sweets shared between 2 people, how many do they each get?
6 ÷ 2 = 3

Grouping or repeated subtraction
There are 6 sweets. How many people can have 2 sweets each?
How many 2s in 6?
6 ÷ 2 = 3 OO OO OO

Step 2: *Counting on or back using a number line or bead bar*
12 ÷ 3 = 4
How many 3s in 12?

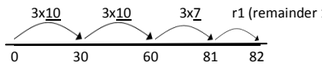


Step 3: *Empty number lines Repeated subtraction*
Children use number lines to record equal groups of the divisor, beginning with steps of one group
24 ÷ 4 = 6
How many 4s in 24?



Children move onto to more efficient multiples of the divisor, subtracting larger groups, using a number line to record their steps.

Step 4: *Empty number lines Repeated subtraction of equal groups*
Children move on to calculations involving remainders.
82 ÷ 3 = 27 r1
How many 3s in 82?
27 lots of 3, with one out of a group of three left over or 1/3



Children should complete word problems involving division, rounding remainders up or down in a real life context
(e.g. 6 eggs to a box. 19 eggs. How many whole boxes? How many 6s in 19?
19 ÷ 6 = 3r1 so 3 whole boxes made).

Step 5: *Expanded column division or chunking*
Children should begin to record repeated subtraction in columns, using increasingly large, efficient multiples of the divisor, recording their answers at the top of the calculation in line with place value columns (including remainders).

96 ÷ 6 = 16

1 6	
6) 9 6	
- 6 0	(6 x 10)
3 6	
- 3 6	(6 x 6)
0	

82 ÷ 3 = 27r1

2 7r1	
3) 8 2	
- 6 0	(3 x 20)
2 2	
- 2 1	(3 x 7)
1	

This step is important in making the link between number line recording of grouping (repeated subtraction) and the recording of short/long division.

Step 6: *Short division*

98 ÷ 7 becomes

1 4	
7) 9 8	
2	

Answer: 14

432 ÷ 5 becomes

8 6 r 2	
5) 4 3 2	
3	

Answer: 86 remainder 2

Step 7: *Long Division*

432 ÷ 15 becomes

2 8 r 12	
1 5) 4 3 2	
- 3 0 0	
1 3 2	
- 1 2 0	
1 2	

Answer: 28 remainder 12

Children will use their knowledge and understanding of division to use a formal written method to divide numbers up to 4 digits by a 2 digit number. Any remainders should be shown as fractions or decimal fractions.

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FLUENCY REASONING