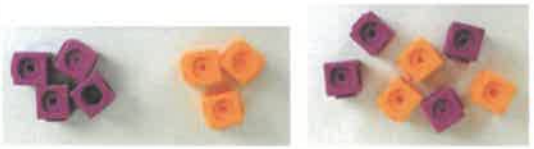
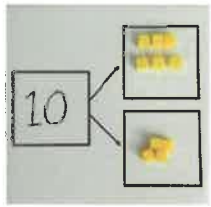
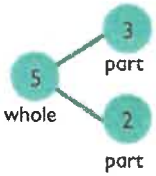
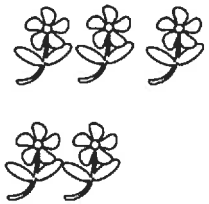
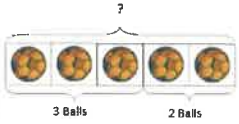
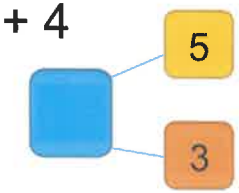
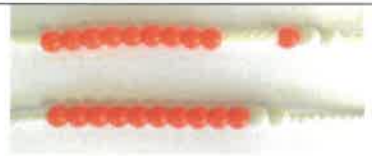
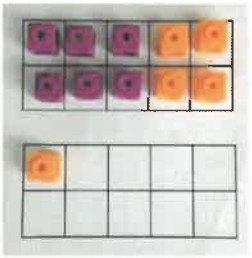
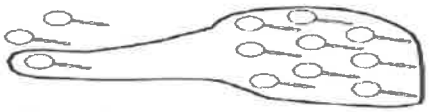
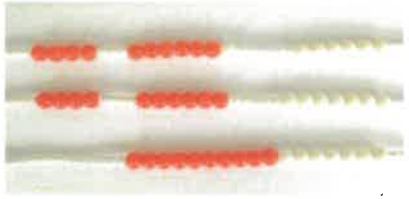
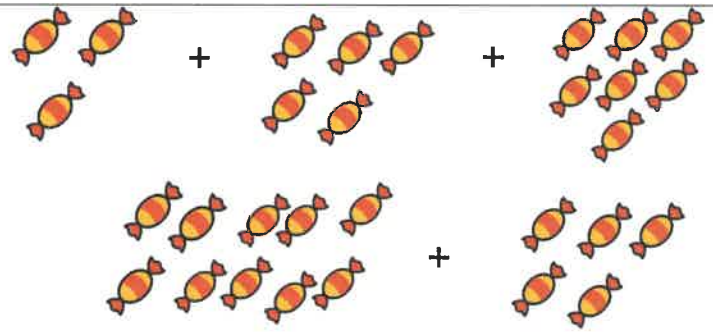
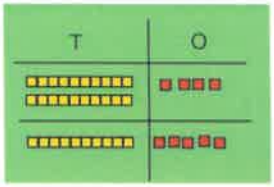
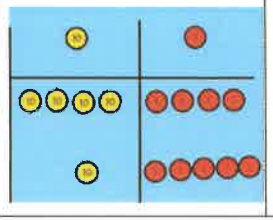
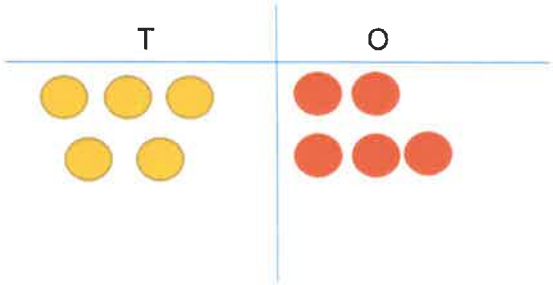
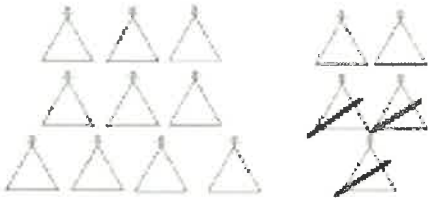
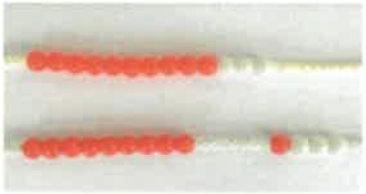
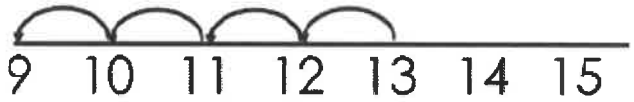
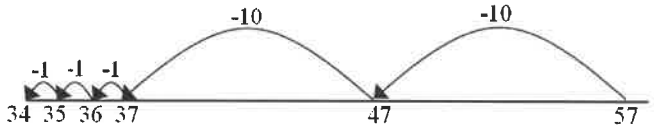
Progression in Calculations

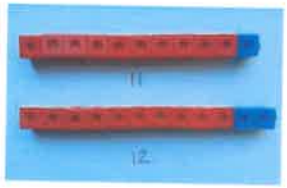
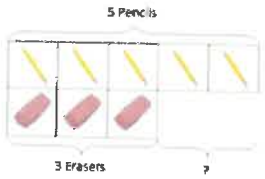
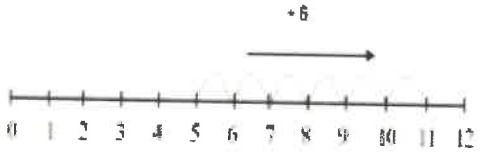
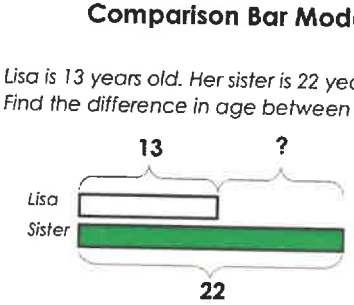
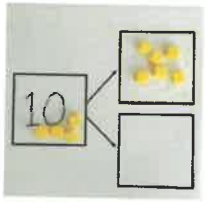
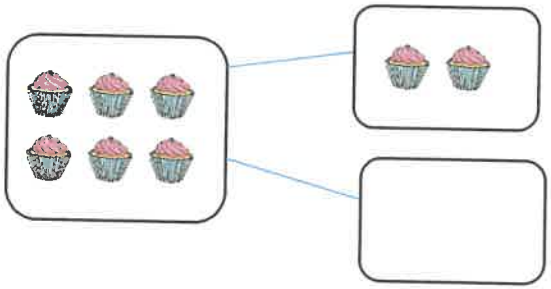
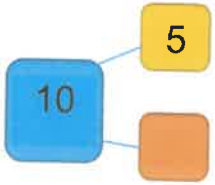
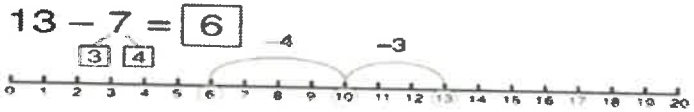
Addition

Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	  Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.

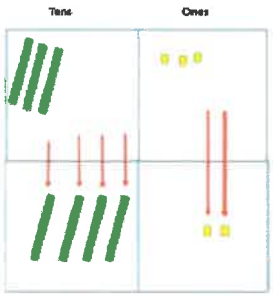
Regrouping to make 10.	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10.	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number to make 10. 9 + 5 = 14 1 4 +1 +4 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Add together three groups of objects. Draw a picture to recombine the groups to make 10.	4 + 7 + 6 = 10 + 7 10 = 17 Combine the two numbers that make 10 and then add on the remainder.
Column method- no regrouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 	Calculations 21 + 42 = 21 + 42

Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away.  6 – 2 = 4  	Cross out drawn objects to show what has been taken away.  15 – 3 = 12	18 -3= 15 8 – 2 = 6
Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.  13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  9 10 11 12 13 14 15 Start at the bigger number and count back the smaller number showing the jumps on the number line.  -10 -10 -1 -1 -1 34 35 36 37 47 57 This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

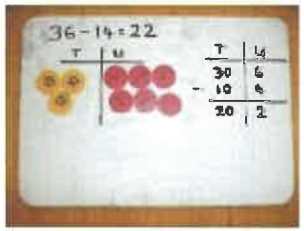
<table>
<tr>
<td>Find the difference</td>
<td>Compare amounts and objects to find the difference.
 
Use cubes to build towers or make bars to find the difference
 
Use basic bar models with items to find the difference</td>
<td> 
Count on to find the difference.
Draw bars to find the difference between 2 numbers.
  </td>
<td>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</td>
</tr>
<tr>
<td>Part Part Whole Model</td>
<td> 
Link to addition- use the part whole model to help explain the inverse between addition and subtraction.
If 10 is the whole and 6 is one of the parts. What is the other part?
10 - 6 =</td>
<td>Use a pictorial representation of objects to show the part part whole model.
  </td>
<td> 
Move to using numbers within the part whole model.</td>
</tr>
<tr>
<td>Make 10</td>
<td>14 – 9 =
 
Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.</td>
<td> 
Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</td>
<td>16 – 8=
How many do we take off to reach the next 10?
How many do we have left to take off?</td>
</tr>
</table>

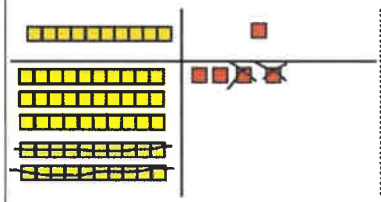
Column method without regrouping



Use Base 10 to make the bigger number then take the smaller number away.

Show how you partition numbers to subtract. Again make the larger number first.

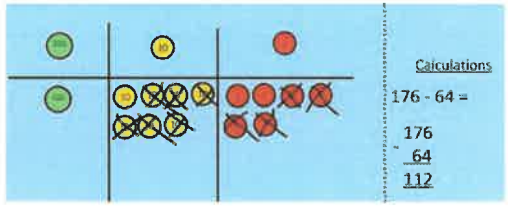




Calculations

54
-22
32

Draw the Base 10 or place value counters alongside the written calculation to help to show working.



Calculations

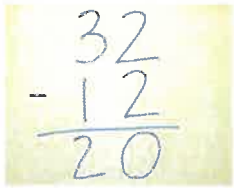
176 - 64 =

176
- 64
112

47 - 24 = 23

40 + 7
- 20 + 4
20 + 3

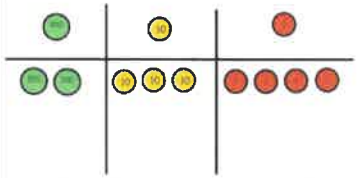
This will lead to a clear written column subtraction.



Column method with regrouping

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

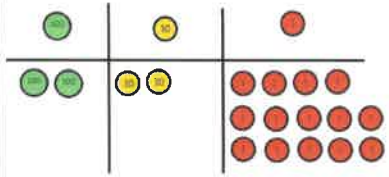
Make the larger number with the place value counters



Calculations

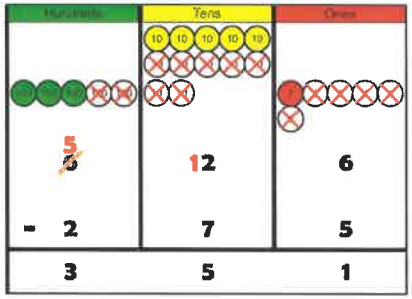
234
- 88

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.

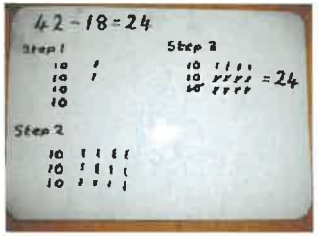


Calculations

234
- 88

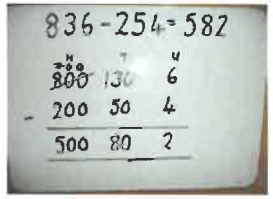


Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

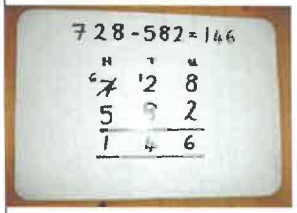


When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

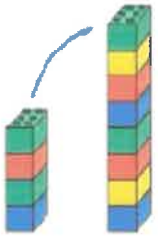
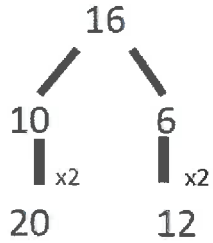
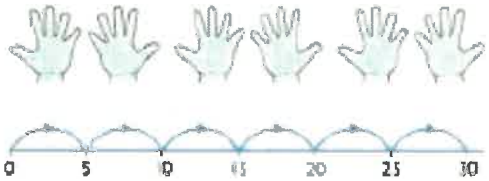


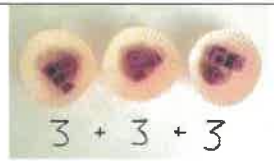
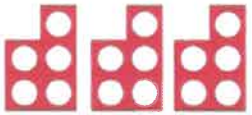
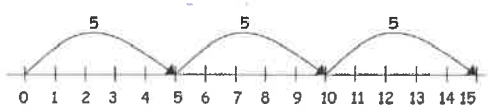
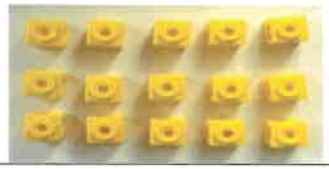
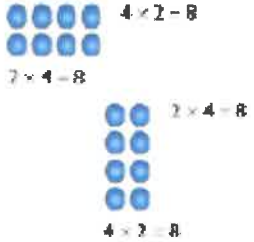
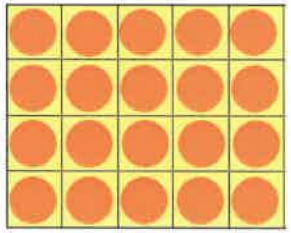
Children can start their formal written method by partitioning the number into clear place value columns.



Moving forward the children use a more compact method.

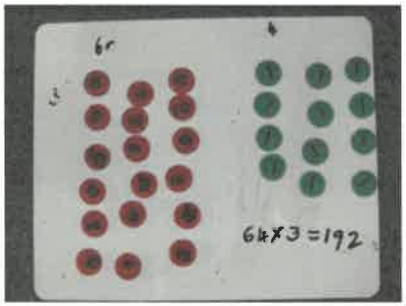
Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.  double 4 is 8 4 x 2 = 8	Draw pictures to show how to double a number. Double 4 is 8 	 Partition a number and then double each part before recombining it back together.
Counting in multiples	  Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30

Repeated addition	   Use different objects to add equal groups.	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10
Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.  	Draw arrays in different rotations to find **commutative** multiplication sentences.   Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

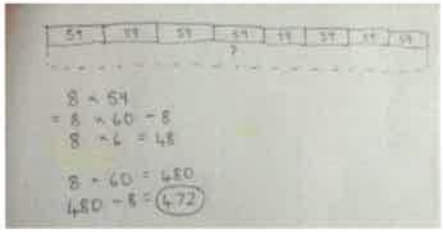
Column multiplication

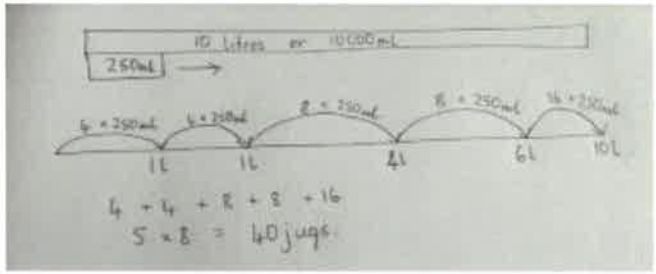
Children can continue to be supported by place value counters at the stage of multiplication.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.





Start with long multiplication, reminding the children about lining up their numbers clearly in columns.

If it helps, children can write out what they are solving next to their answer.

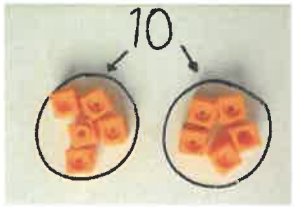
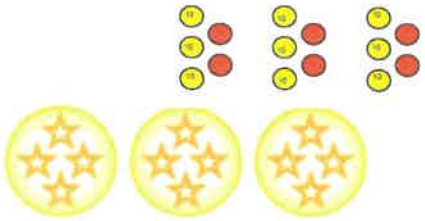
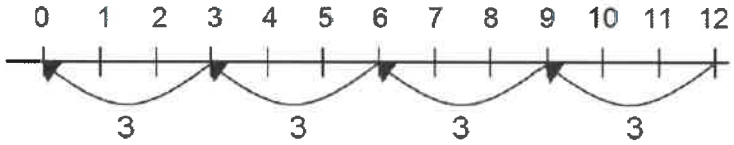
```
  32
x 24
   8   (4 x 2)
 120   (4 x 30)
  40   (20 x 2)
 600   (20 x 30)
 768
```

```
   74
 x 63
  222
 296
+420
 4662
```

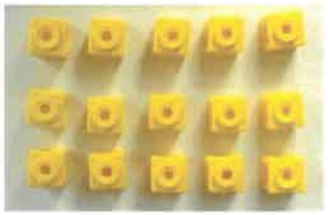
This moves to the more compact method.

```
  2 3 1
  1342
x   18
13420
10736
24156
  1
```

Division

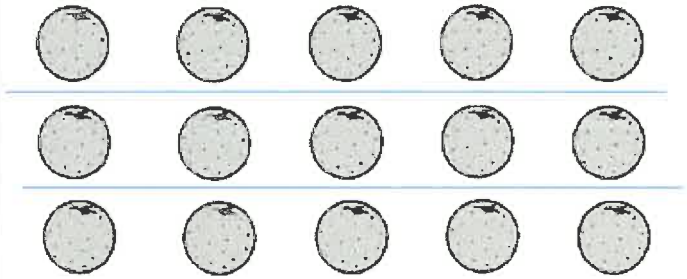
Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 ÷ 2 = 4	Share 9 buns between three people. 9 ÷ 3 = 3
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   96 ÷ 3 = 32 	Use a number line to show jumps in groups. The number of jumps equals the number of groups.  Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.  20 ÷ 5 = ? 5 x ? = 20	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?

Division within arrays



Link division to multiplication by creating an array and thinking about the number sentences that can be created.

Eg 15 ÷ 3 = 5 5 x 3 = 15
15 ÷ 5 = 3 3 x 5 = 15



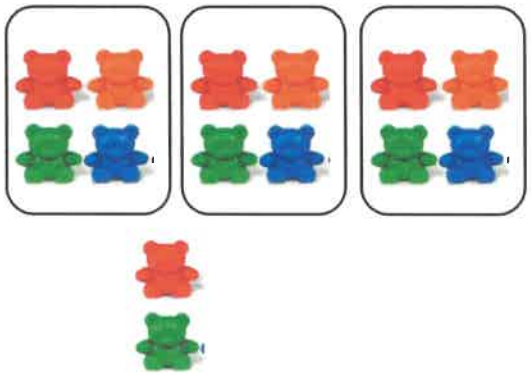
Draw an array and use lines to split the array into groups to make multiplication and division sentences.

Find the inverse of multiplication and division sentences by creating four linking number sentences.

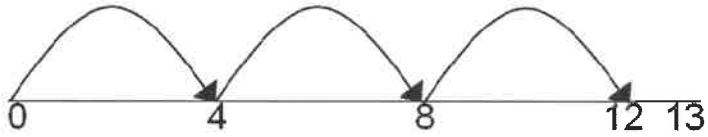
7 x 4 = 28
4 x 7 = 28
28 ÷ 7 = 4
28 ÷ 4 = 7

Division with a remainder

14 ÷ 3 =
Divide objects between groups and see how much is left over



Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.



Draw dots and group them to divide an amount and clearly show a remainder.

Complete written divisions and show the remainder using r.

29 ÷ 8 = 3 REMAINDER 5
↑ dividend ↑ divisor ↑ quotient ↑ remainder