

Name: _____

Maths websites you might find useful to help you complete the work and for extra revision

<https://www.bbc.co.uk/bitesize/subjects/z38pycw>

<https://www.skillsworkshop.org/category/numeracy/functional-maths>

<https://www.skillsworkshop.org/maths?q=maths&search=&page=1>

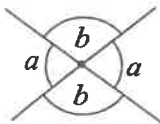
<https://www.bbc.co.uk/teach/skillswise/maths/zfdymfr>

<https://www.tes.com/teaching-resources/gcse-exam-revision/gcse-revision-games>

15.3 Parallel and Intersecting Lines

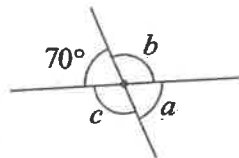
Opposite Angles

When two straight lines **intersect** (or cross), two pairs of **opposite angles** are formed. Opposite angles are equal.



Example 1

Find the values of a , b and c shown in the diagram.



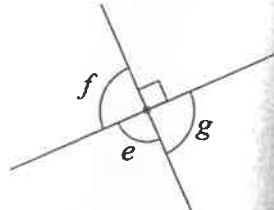
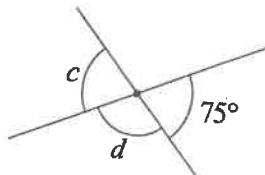
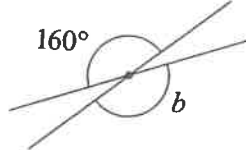
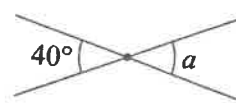
- a and 70° are opposite angles, so they are equal.
- b and the angle marked 70° lie on a straight line, so they add up to 180° .
- c and b are opposite angles, so they are equal.

$$\begin{aligned} a &= 70^\circ \\ 70^\circ + b &= 180^\circ \\ b &= 180^\circ - 70^\circ = 110^\circ \\ c &= b = 110^\circ \end{aligned}$$

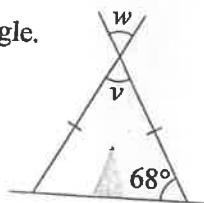
Exercise 1

In Questions 1-4, the angles aren't drawn accurately, so don't try to measure them.

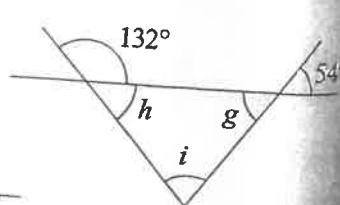
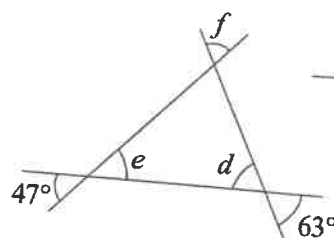
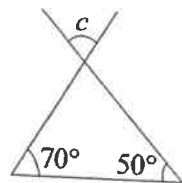
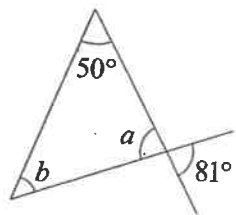
- 1 Find the missing angles marked by letters.



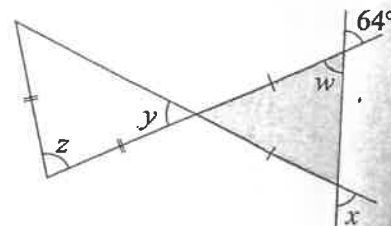
- 2 The diagram on the right shows a teepee in the shape of an isosceles triangle.
- Find the size of angle v .
 - Use your answer to write down the value of w . Give a reason for your answer.



- 3 Find the missing angles marked by letters.

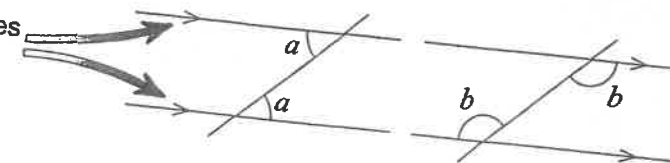


- 4 The diagram on the right shows two isosceles triangles. Find the missing angles marked by letters.



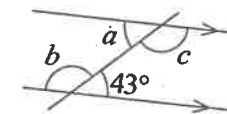
Alternate Angles

When a straight line crosses two parallel lines (shown by the arrows), it forms two pairs of **alternate angles** (in a sort of Z-shape). Alternate angles are equal.



Example 2

Find the values of a , b and c in the diagram.



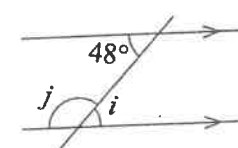
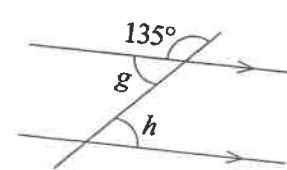
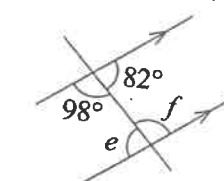
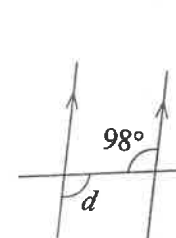
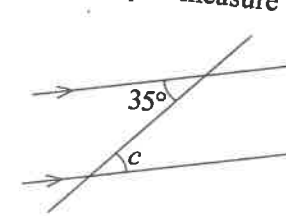
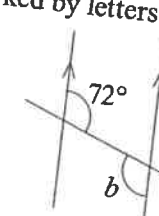
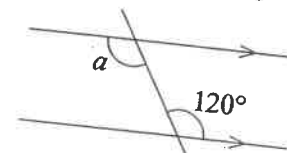
- a and 43° are alternate angles, so they are equal.
- b and the angle marked 43° lie on a straight line, so they add up to 180° .
- c and b are alternate angles, so they are equal.

$$\begin{aligned} a &= 43^\circ \\ 43^\circ + b &= 180^\circ \\ b &= 180^\circ - 43^\circ = 137^\circ \\ c &= b = 137^\circ \end{aligned}$$

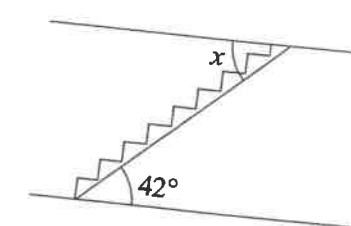
Exercise 2

In Questions 1-3, the angles aren't drawn accurately, so don't try to measure them.

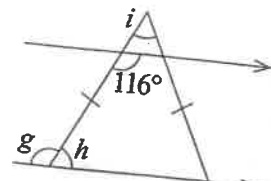
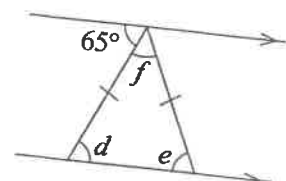
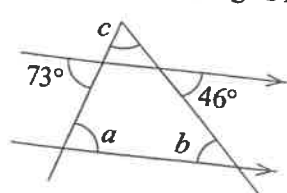
- 1 Find the missing angles marked by letters.



- 2 The diagram on the right shows a staircase between two parallel floors of a building. The staircase makes an angle of 42° with the lower floor.
- Write down the angle that the staircase makes with the upper floor, marked x on the diagram.
 - Give a reason for your answer.



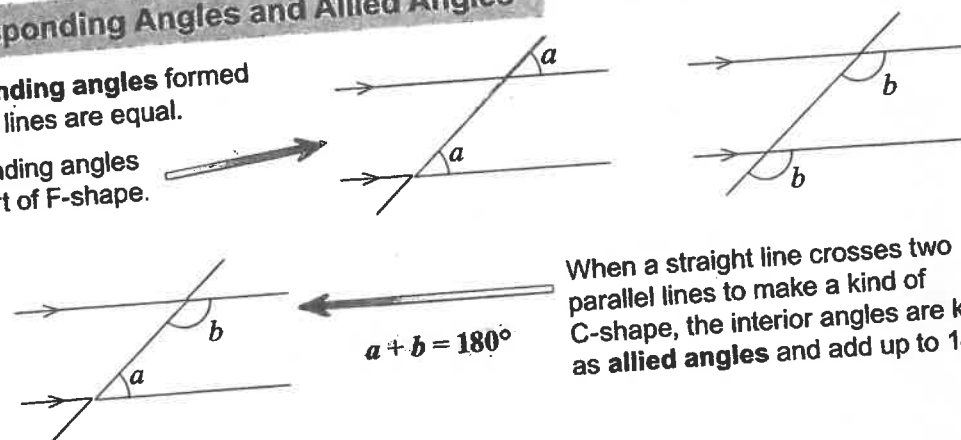
- 3 Find the missing angles marked by letters.



Corresponding Angles and Allied Angles

Corresponding angles formed by parallel lines are equal.

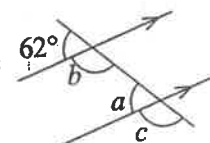
Corresponding angles form a sort of F-shape.



When a straight line crosses two parallel lines to make a kind of C-shape, the interior angles are known as allied angles and add up to 180° .

Example 3

Find the values of a , b and c shown in the diagram.



- a and 62° are corresponding angles, so they are equal.
- b and the angle marked 62° lie on a straight line, so they add up to 180° . (Or you could say a and b are allied angles, so they add up to 180° .)
- c and b are corresponding angles, so they're equal.

$$a = 62^\circ$$

$$62^\circ + b = 180^\circ$$

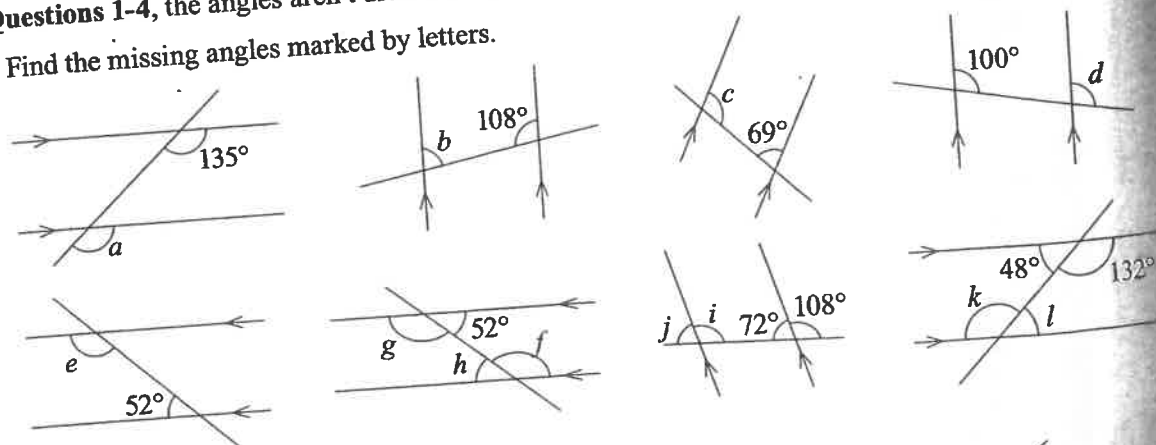
$$b = 180^\circ - 62^\circ = 118^\circ$$

$$c = b = 118^\circ$$

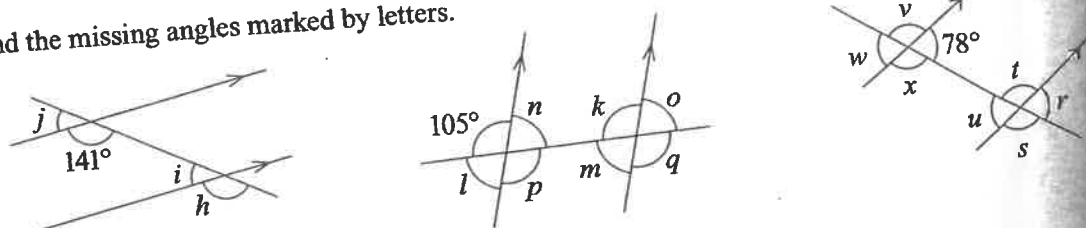
Exercise 3

In Questions 1-4, the angles aren't drawn accurately, so don't try to measure them.

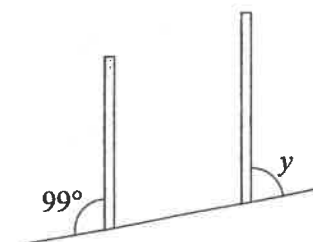
- 1 Find the missing angles marked by letters.



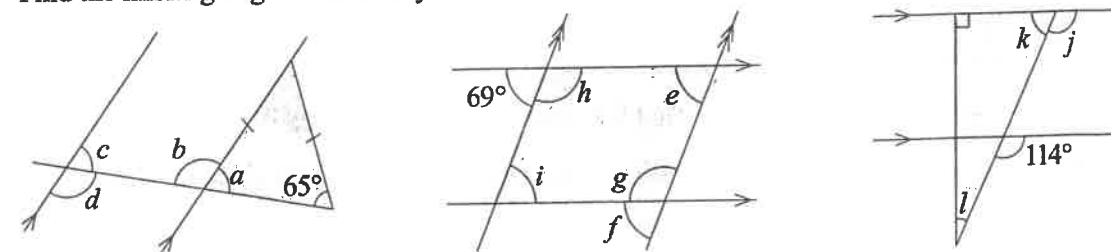
- 2 Find the missing angles marked by letters.



- 3 Two wooden posts stand vertically on sloped ground. The first post makes an angle of 99° with the downward slope, as shown. Find the angle that the second post makes with the upward slope, labelled y on the diagram.



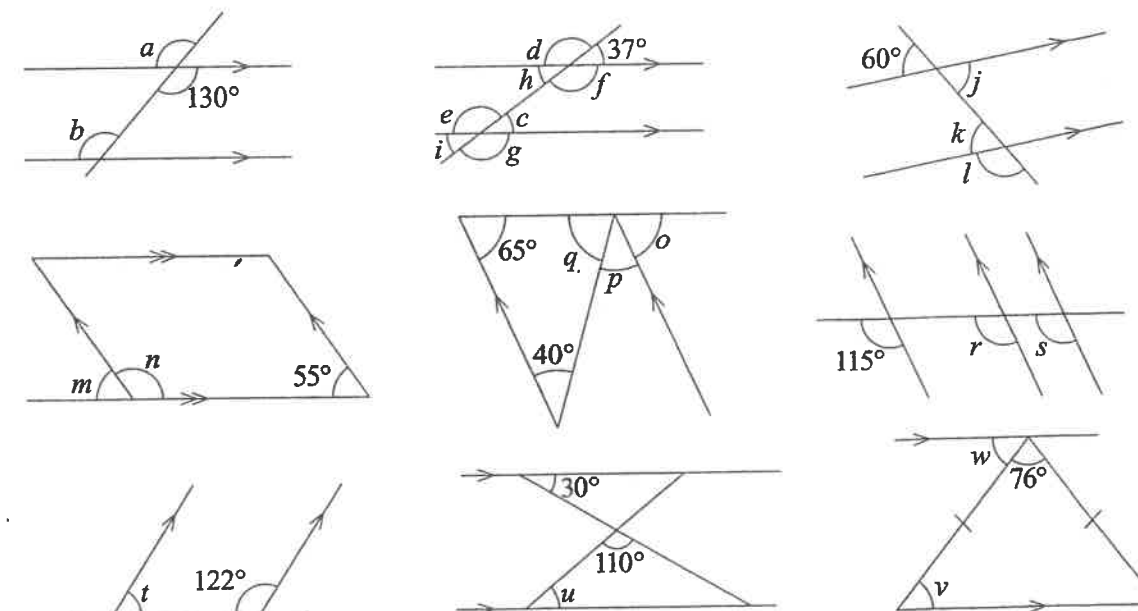
- 4 Find the missing angles marked by letters.



Exercise 4 — Mixed Exercise

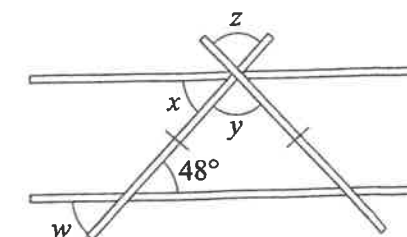
In Questions 1-2, the angles aren't drawn accurately, so don't try to measure them.

- 1 Find the missing angles marked by letters. In each case, give a reason for your answer.



- 2 The diagram on the right shows some scaffolding. The triangle formed between the two horizontal bars is isosceles.

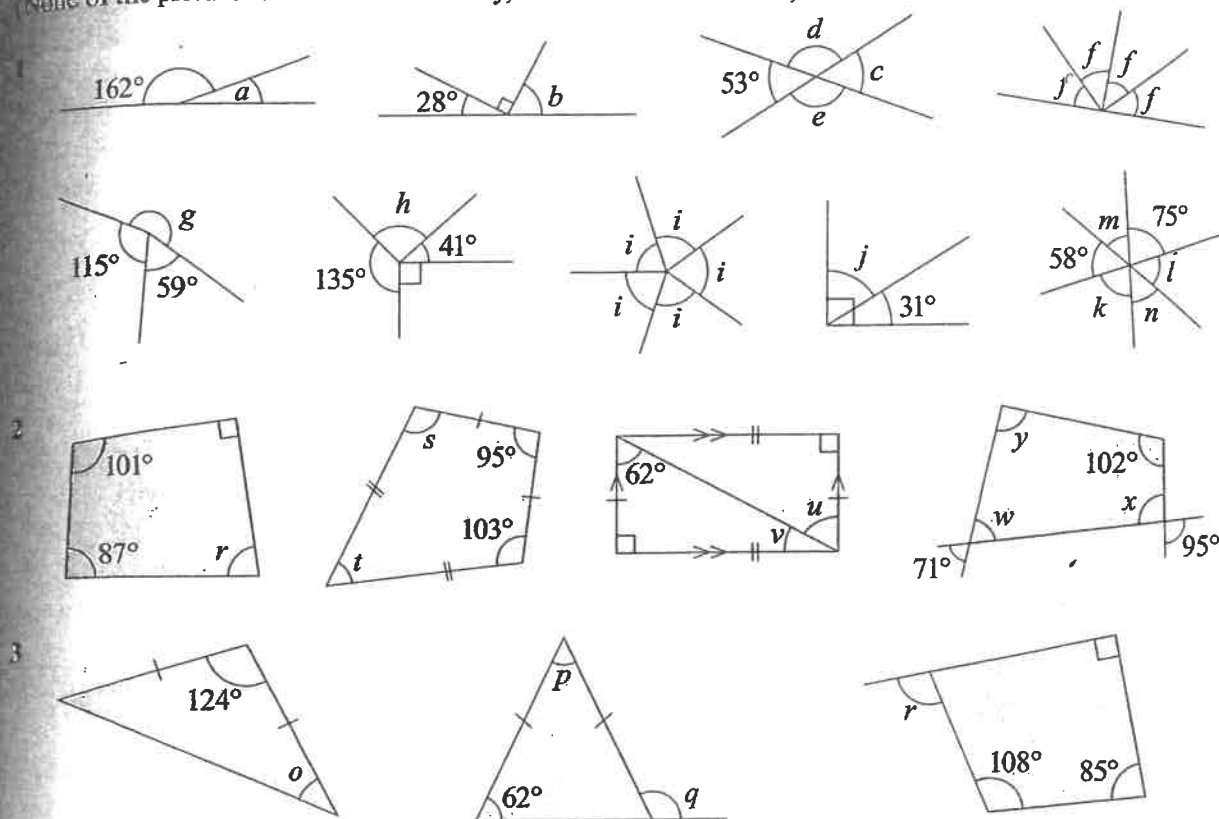
Find the size of angles w , x , y and z , giving a reason for your answer in each case.



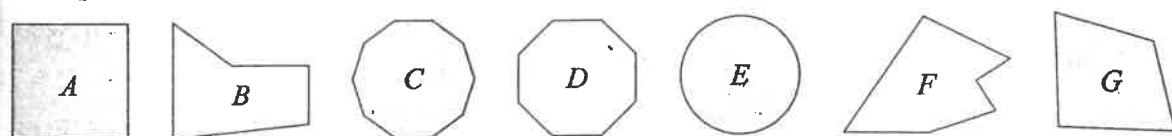
15.7 Angles and 2D Shapes Problems

Exercise 1

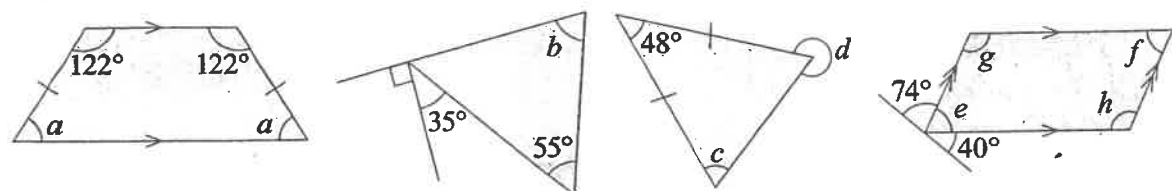
For Questions 1-3, find the missing angles marked with letters.
(None of the pictures are drawn accurately, so don't measure them.)



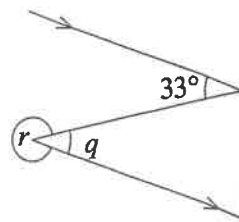
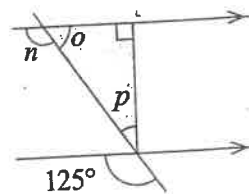
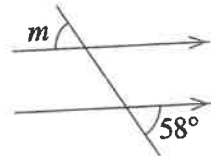
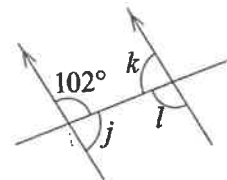
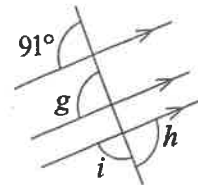
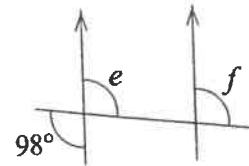
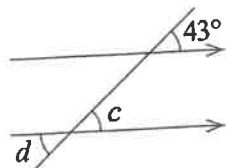
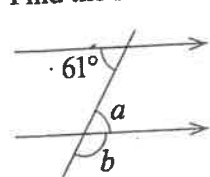
4 This question is about shapes A-G below.



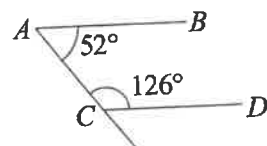
- Which of the shapes are polygons?
 - Which of the shapes are hexagons?
 - Which of the shapes are pentagons?
 - Which of the shapes are quadrilaterals?
- 5 Find the size of the angles marked with letters.



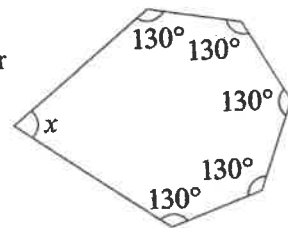
- 6 Find the size of the angles marked with letters.



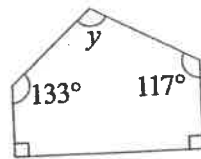
- 7 Are the lines AB and CD parallel to each other? Explain your answer.



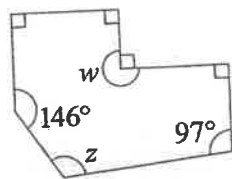
- 8 a) Calculate the sum of the interior angles of a hexagon.
b) Use your answer to find angle x .



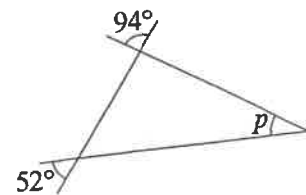
- 9 a) Calculate the sum of the interior angles of a pentagon.
b) Use your answer to find the size of angle y .



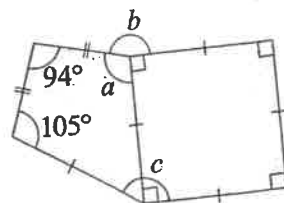
- 10 Find angles w and z .



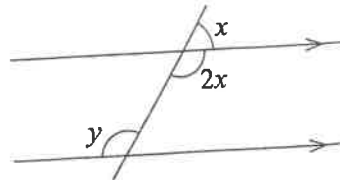
- 11 Find angle p in the diagram below.



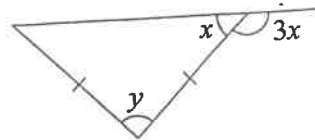
- 12 The diagram below shows a kite and a square.
a) Write down the value of a .
b) Use your answer to find the size of angles b and c .



- 13 a) Find x in the diagram below.
b) Use your answer to find y .

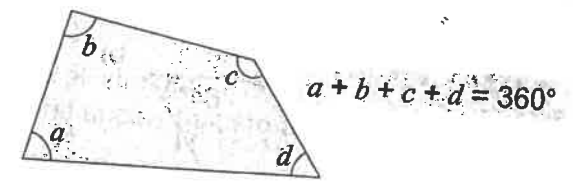


- 14 Find x and y in the diagram below.



Angles in a Quadrilateral

The angles in a quadrilateral add up to 360° .



Example 3

a) Find the size of angle x .

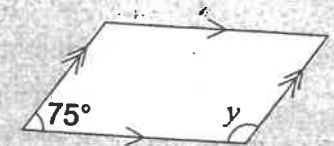
- The angles in a quadrilateral add up to 360° . Use this to write an equation involving x .
- Solve your equation to find x .



$$\begin{aligned} x + 112^\circ + 88^\circ + 93^\circ &= 360^\circ \\ x + 293^\circ &= 360^\circ \\ x &= 360^\circ - 293^\circ = 67^\circ \end{aligned}$$

b) Find the size of angle y .

- Opposite angles in a parallelogram are equal, so there are two 75° angles and two of angle y .
- Now use the fact that angles in a quadrilateral add up to 360° to write an equation involving y .
- Solve your equation to find y .

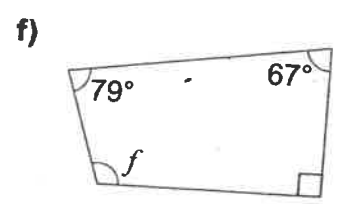
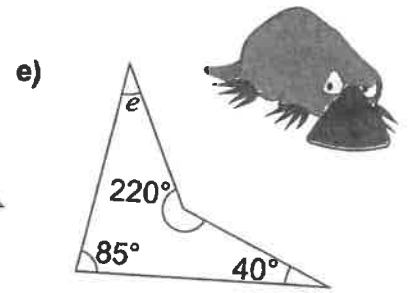
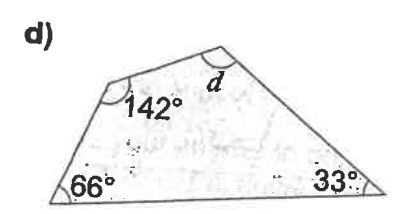
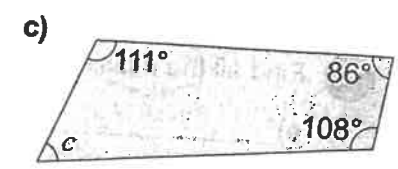
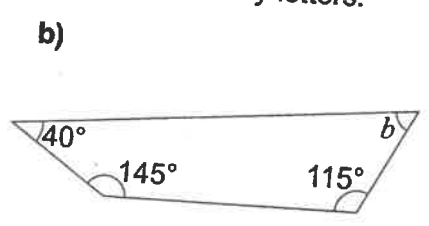
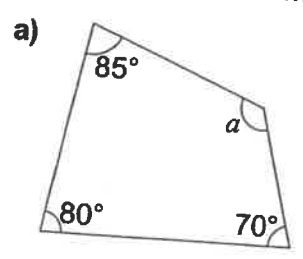


$$\begin{aligned} y + 75^\circ + y + 75^\circ &= 360^\circ \\ 2y + 150^\circ &= 360^\circ \\ 2y &= 360^\circ - 150^\circ = 210^\circ \\ y &= 210^\circ \div 2 = 105^\circ \end{aligned}$$

Exercise 4

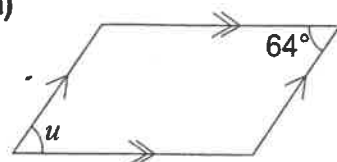
The diagrams in this exercise are **not** drawn accurately, so don't try to measure the angles.

- 1 Find the sizes of the missing angles marked by letters.

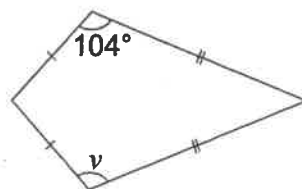


2 Find the sizes of the missing angles marked by letters.

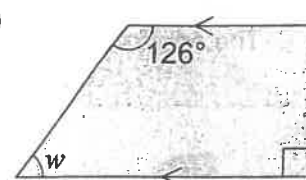
a)



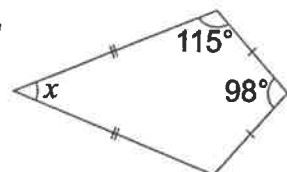
b)



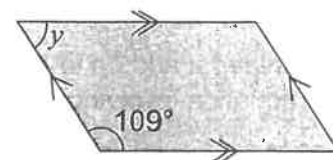
c)



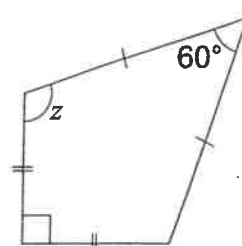
d)



e)



f)

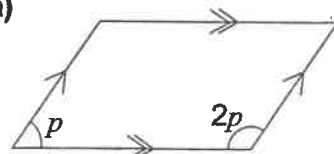


3 One angle of a rhombus measures 27° .
Find the sizes of the other three angles.

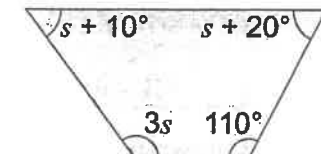


4 Find the value of each letter in the following diagrams.

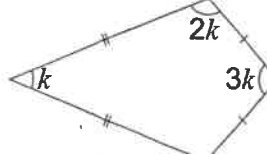
a)



b)

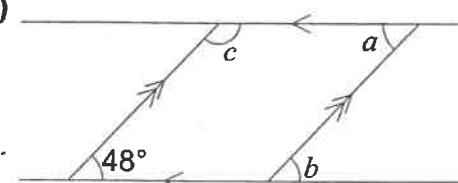


c)

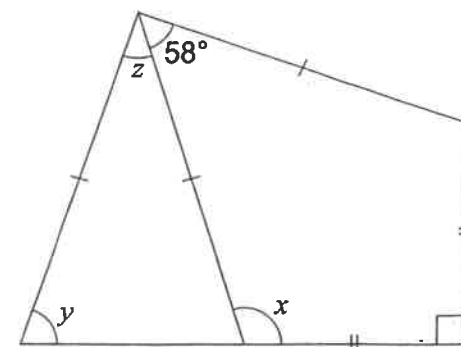


5 Find all the missing angles marked by letters in the diagrams.

a)




b)



Name : _____ Score : _____
Teacher : _____ Date : _____

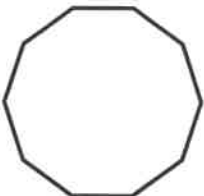
Find the measure of one interior angle, one exterior angle, and the interior angle sum for each polygon. Round your answer to the nearest tenth if necessary.

1)



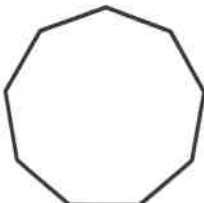
Interior Angle: _____
Exterior Angle: _____
Interior Angle Sum: _____

2)



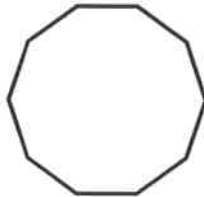
Interior Angle: _____
Exterior Angle: _____
Interior Angle Sum: _____

3)




Interior Angle: _____
Exterior Angle: _____
Interior Angle Sum: _____

4)



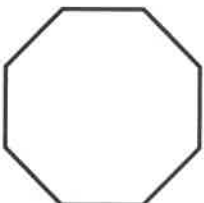
Interior Angle: _____
Exterior Angle: _____
Interior Angle Sum: _____

5)



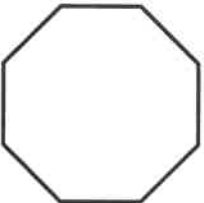
Interior Angle: _____
Exterior Angle: _____
Interior Angle Sum: _____

6)




Interior Angle: _____
Exterior Angle: _____
Interior Angle Sum: _____

7)



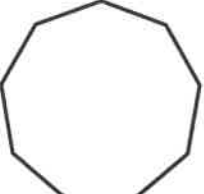
Interior Angle: _____
Exterior Angle: _____
Interior Angle Sum: _____

8)




Interior Angle: _____
Exterior Angle: _____
Interior Angle Sum: _____

9)




Interior Angle: _____
Exterior Angle: _____
Interior Angle Sum: _____

10)



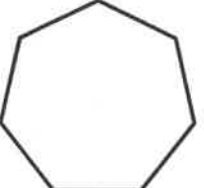
Interior Angle: _____
Exterior Angle: _____
Interior Angle Sum: _____

11)



Interior Angle: _____
Exterior Angle: _____
Interior Angle Sum: _____

12)



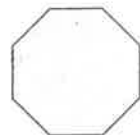
Interior Angle: _____
Exterior Angle: _____
Interior Angle Sum: _____

- 5 A regular heptagon is shown on the right.
 a) Find the sum of the interior angles of a heptagon.
 b) Find the size of each of the interior angles of a regular heptagon.

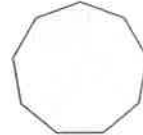


- 6 Find the size of each of the interior angles in the following shapes.

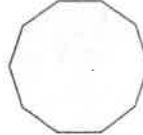
a) Regular octagon
(8 sides)



b) Regular nonagon
(9 sides)



c) Regular decagon
(10 sides)



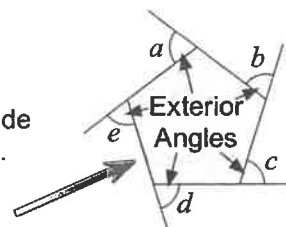
- 7 Four angles of a pentagon are 110° .
 a) Find the size of the fifth angle.
 b) Is this a regular pentagon? Give a reason for your answer.

Exterior Angles

The **exterior angles** of a polygon are the angles between a side and a line that extends out from one of the neighbouring sides.

The sum of the exterior angles of any polygon is 360° .

$$a + b + c + d + e = 360^\circ$$



You can use the **exterior angle** to find the corresponding **interior angle** using this formula:

$$\text{Interior angle} = 180^\circ - \text{Exterior angle}$$

In a **regular n -sided polygon**, all the exterior angles are **equal** and their size is given by:

$$\text{Exterior angle} = \frac{360^\circ}{n}$$



Example 4

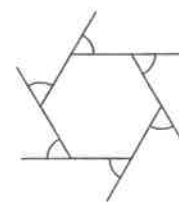
Find the size of each of the exterior angles of a regular hexagon.

1. A hexagon has 6 sides.

2. The hexagon is regular so use the exterior angle formula.

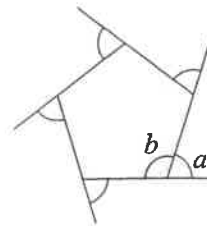
$$360^\circ \div 6 = 60^\circ$$

So each exterior angle is 60°

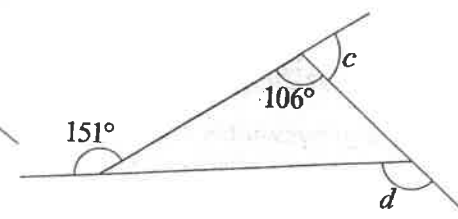
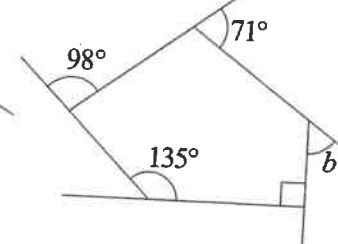
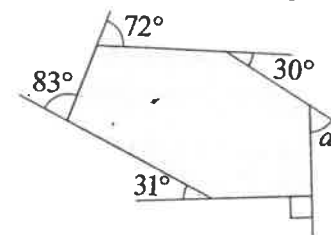


Exercise 2

- 1 The diagram on the right shows a regular pentagon with the exterior angles marked on.
 a) Find a , the size of each of the exterior angles of the pentagon.
 b) Hence find b , the size of each of the interior angles of the pentagon.
- 2 a) Find the size of each of the exterior angles of the following polygons.
 (i) regular heptagon (ii) regular octagon (iii) regular nonagon
 b) Use your answers to part a) to find the size of each of the interior angles of these polygons.



- 3 Find the size of the angles marked by letters in these diagrams.



- 4 Find the size of the unknown exterior angle in a shape whose other exterior angles are:
 a) 100° , 68° , 84° and 55°
 b) 30° , 68° , 45° , 52° , 75° and 50°
 c) 42° , 51° , 60° , 49° , 88° and 35°
 d) 19° , 36° , 28° , 57° , 101° , 57° and 22°

Example 5

A regular polygon has exterior angles of 30° .
 How many sides does the polygon have?

It's a regular polygon so just put 30° into the exterior angle formula.

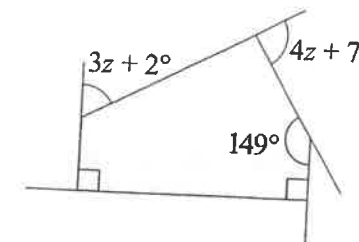
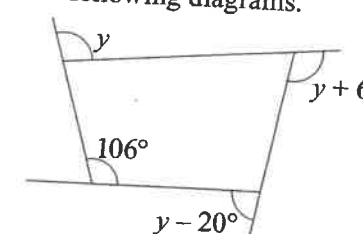
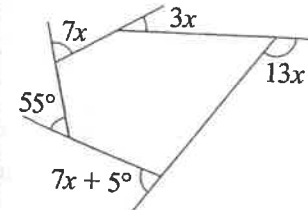
$$30^\circ = \frac{360^\circ}{n}$$

$$n = 360^\circ \div 30^\circ = 12$$

So the regular polygon has **12 sides**.

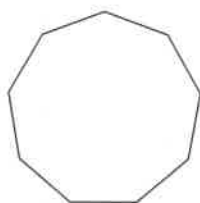
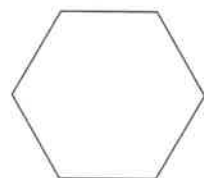
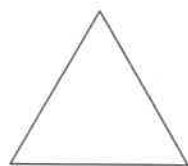
- 5 A regular polygon has exterior angles of 45° .
 a) How many sides does the polygon have? What is the name of this kind of polygon?
 b) Sketch the polygon.
 c) What is the size of each of the polygon's interior angles?
 d) What is the sum of the polygon's interior angles?
- 6 The exterior angles of some regular polygons are given below. For each exterior angle, find:
 (i) the number of sides the polygon has,
 (ii) the size of each of the polygon's interior angles,
 (iii) the sum of the polygon's interior angles.
- | | | | | |
|---------------|---------------|---------------|----------------|---------------|
| a) 60° | b) 90° | c) 40° | d) 120° | e) 10° |
| f) 9° | g) 6° | h) 5° | i) 4° | j) 3° |

- 7 Find the values of the letters in the following diagrams.



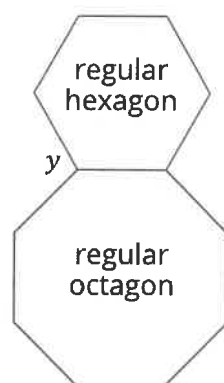
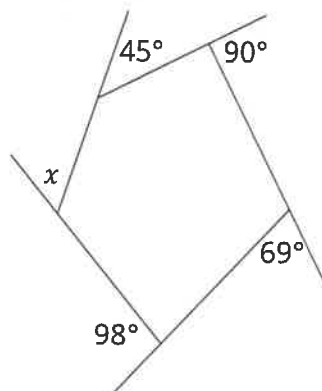
Polygons Exterior Angles

1. By extending the sides of these shapes, produce all of the exterior angles. You should have 18 exterior angles. Measure each and label the angle with its size. What is the sum of the exterior angles in the equilateral triangle? What about the hexagon and the nonagon? What do you notice?



2. What is the size of an exterior angle of a regular octagon?
Use this information to find the size of an interior angle in a regular octagon.
3. What is the size of an exterior angle of a regular decagon?
Use this information to find the size of an interior angle in a regular decagon.
4. A regular polygon has exterior angles of 30° . How many sides does it have?
What is the size of each interior angle in this polygon?
5. A regular polygon has interior angles of 170° . How many sides does it have?

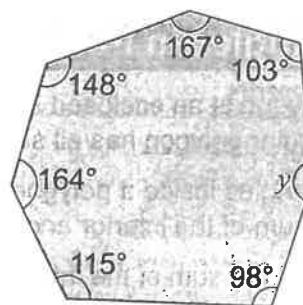
6. Find the value of x and y .



7. Draw a regular hexagon with sides of length 4cm using a protractor, ruler and your knowledge of exterior angles in a hexagon. Do not erase your construction lines.

- 3 a) Find the sum of the interior angles of the polygon shown.

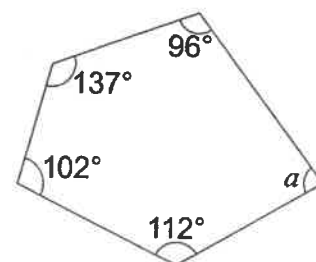
- b) Use your answer to part a) to find the size of the angle marked y .



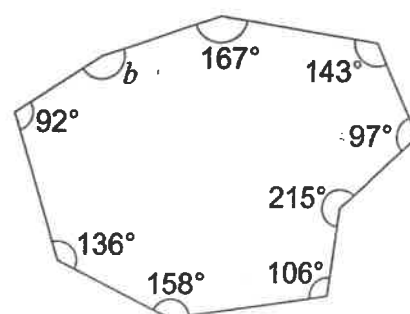
- 4 A hexagon has two angles measuring 130° , two right angles and one angle measuring 175° . Find the size of the missing angle.

- 5 Find the sizes of the missing angles marked by letters in the following shapes:

a)



b)



- 6 In a regular polygon, all the interior angles are the same size.

- a) Find the sum of the interior angles in a dodecagon (a 12-sided shape).

- b) Use your answer to part a) to find the size of one interior angle of a regular dodecagon.

- 7 Find the size of one interior angle in the following shapes:

a) a regular hexagon

b) a regular octagon

c) a regular 15-sided polygon

Investigate — Angles in Polygons

- a) Starting with a quadrilateral and increasing the number of sides up to a decagon (a 10-sided shape), write down the number of sides the polygon has, the number of triangles it can be split into and the sum of the interior angles.
- b) Can you find a link between the number of sides the polygon has and the number of triangles it can be split into? Try and use this link to write a formula to work out the sum of the interior angles for any polygon.

13.3 Angles in Polygons

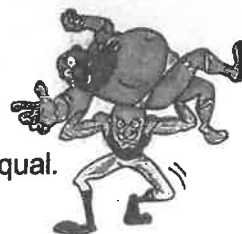
A polygon is an enclosed shape whose sides are all straight.

A regular polygon has all sides of equal length and angles that are all equal.

The angles inside a polygon are called interior angles.

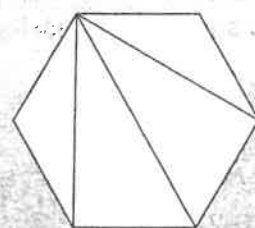
The sum of the interior angles depends on how many sides the polygon has.

To find the sum of the interior angles, divide the polygon up into triangles (by drawing lines from one corner to all the others). Then multiply the number of triangles by 180° (as there are 180° in a triangle) — this is the sum of the interior angles.



Example 1 Find the sum of the interior angles in a hexagon.

1. Draw any hexagon (it doesn't have to be regular).
2. Split the hexagon into triangles by drawing lines from one corner to all the others.
3. Angles in a triangle add up to 180° and there are 4 triangles, so multiply 180° by 4.



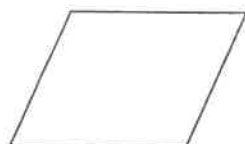
Sum of interior angles:
 $4 \times 180^\circ = 720^\circ$

Exercise 1

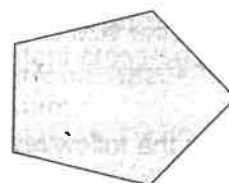
The diagrams in this exercise are **not** drawn accurately, so don't try to measure the angles.

- 1 By dividing each shape into triangles, find the sum of the interior angles of the following:

a)



b)



c)



- 2 Sketch the polygon shown.

- a) Divide the polygon into triangles, starting from the 150° angle.
- b) Find the sum of the interior angles of the polygon.
- c) Use your answer to part b) to find the size of the angle marked x .

