



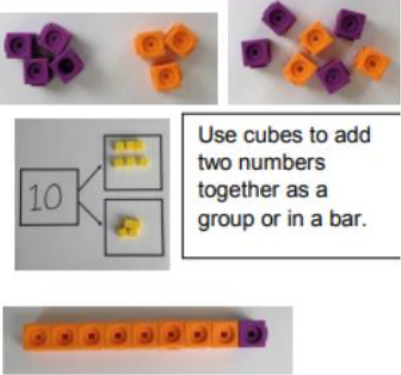
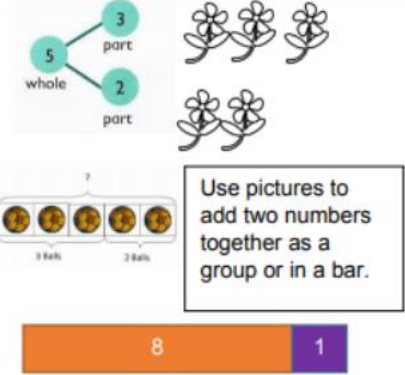

Warton St Paul's

Church of England Primary Academy
A member of **CDARI**

Warton St Paul's CE Primary Academy Progression in Calculations

Progression in Calculations

Addition


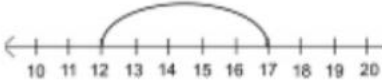

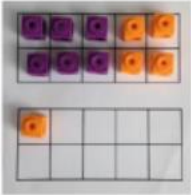

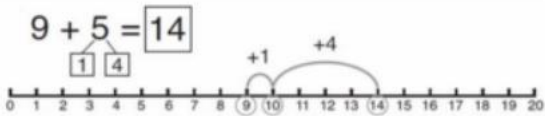
Objective and strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part whole model	 <p>Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	$4 + 3 = 7$ $10 = 6 + 4$  <p>Use the part-part whole diagram as shown above to move into the abstract.</p>



Warton St Paul's

Church of England Primary Academy
A member of **CDARI**

Warton St Paul's CE Primary Academy Progression in Calculations

<p>Starting at the bigger number and counting on</p>	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	<p>$12 + 5 = 17$</p>  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	<p>$5 + 12 = 17$</p> <p>Place the larger number in your head and count on the smaller number to find your answer.</p>
<p>Regrouping to make 10.</p>	 <p>$6 + 5 = 11$</p>  <p>Start with the bigger number and use the smaller number to make 10.</p>	 <p>$3 + 9 =$</p> <p>Use pictures or a number line. Regroup or partition the smaller number to make 10.</p> <p>$9 + 5 = 14$</p> 	<p>$7 + 4 = 11$</p> <p>If I am at seven, how many more do I need to make 10. How many more do I add on now?</p>



Warton St Paul's

Church of England Primary Academy
A member of **CDARI**

Warton St Paul's CE Primary Academy Progression in Calculations

<p>Column method regrouping</p>	<p>Make both numbers on a place value grid.</p> <p>146 + 527</p> <p>Add up the units and exchange 10 ones for one 10</p> <p>146 + 527</p> <p>Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. As children move on to decimals, money and decimal place value counters can be used to support learning.</p>	<p>Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.</p> <p>e.g. $65 + 27$</p> <p>Step 1</p> <p>Step 2</p> <p>Step 3</p>	<p>Start by partitioning the numbers before moving on to clearly show the exchange below the addition.</p> $\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array} \quad \begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$ <p>As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.</p> $\begin{array}{r} 72.8 \\ + 54.6 \\ \hline 127.4 \\ 11 \end{array} \quad \begin{array}{r} 423.59 \\ + 631.14 \\ \hline 1054.73 \end{array}$ <p>Written method</p> <table border="0"> <tr> <td>Step 1</td> <td>Step 2</td> <td>Step 3</td> </tr> <tr> <td> $\begin{array}{r} T \ U \\ 6 \ 5 \\ + 2 \ 7 \\ \hline \end{array}$ </td> <td> $\begin{array}{r} T \ U \\ 6 \ 5 \\ + 2 \ 7 \\ \hline 2 \end{array}$ </td> <td> $\begin{array}{r} T \ U \\ 6 \ 5 \\ + 2 \ 7 \\ \hline 9 \ 2 \\ 1 \end{array}$ </td> </tr> </table>	Step 1	Step 2	Step 3	$\begin{array}{r} T \ U \\ 6 \ 5 \\ + 2 \ 7 \\ \hline \end{array}$	$\begin{array}{r} T \ U \\ 6 \ 5 \\ + 2 \ 7 \\ \hline 2 \end{array}$	$\begin{array}{r} T \ U \\ 6 \ 5 \\ + 2 \ 7 \\ \hline 9 \ 2 \\ 1 \end{array}$
Step 1	Step 2	Step 3							
$\begin{array}{r} T \ U \\ 6 \ 5 \\ + 2 \ 7 \\ \hline \end{array}$	$\begin{array}{r} T \ U \\ 6 \ 5 \\ + 2 \ 7 \\ \hline 2 \end{array}$	$\begin{array}{r} T \ U \\ 6 \ 5 \\ + 2 \ 7 \\ \hline 9 \ 2 \\ 1 \end{array}$							



Warton St Paul's

Church of England Primary Academy
A member of **CDARI**

Warton St Paul's CE Primary Academy Progression in Calculations

Subtraction


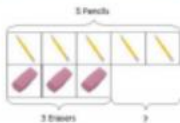
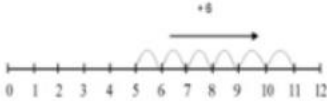
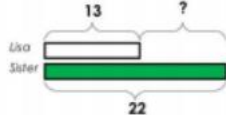
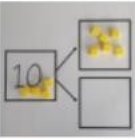
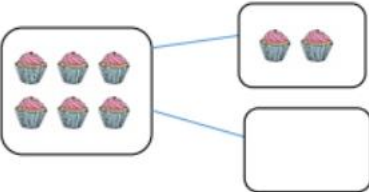


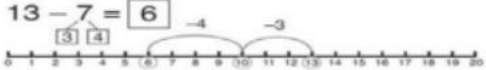
Objective and strategies	Concrete	Pictorial	Abstract
Taking away ones	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p> <p>$6 - 2 = 4$</p> <p>$15 - 3 = 12$</p>	<p>Cross out drawn objects to show what has been taken away.</p> <p>$15 - 3 = 12$</p> <p>$8 - 2 = 6$</p>	<p>$18 - 3 = 15$</p> <p>$8 - 2 = 6$</p>
Counting back	<p>Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones.</p> <p>$13 - 4$</p> <p>Use counters and move them away from the group as you take them away counting backwards as you go.</p>	<p>Count back on a number line or number track</p> <p>$9 \quad 10 \quad 11 \quad 12 \quad 13 \quad 14 \quad 15$</p> <p>Start at the bigger number and count back the smaller number showing the jumps on the number line.</p> <p>$34 \quad 35 \quad 36 \quad 37 \quad 47 \quad 57$</p> <p>This can progress all the way to counting back using two 2 digit numbers.</p>	<p>Put 13 in your head, count back 4. What number are you at? Use your fingers to help.</p> <p>$13 - 4 = 9$</p>



Warton St Paul's

Church of England Primary Academy
A member of **CDARI**

Warton St Paul's CE Primary Academy Progression in Calculations

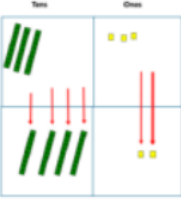
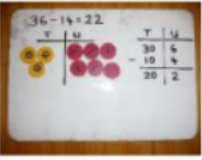
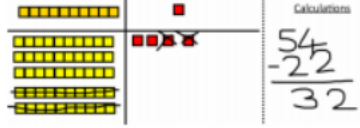
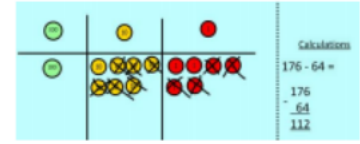
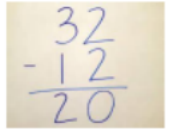

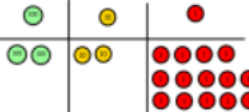

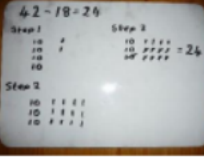
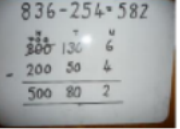
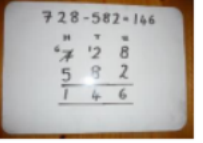
<p>Find the difference</p>	<p>Compare amounts and objects to find the difference.</p>  <p>Use cubes to build towers or make bars to find the difference</p>  <p>Use basic bar models with items to find the difference</p>	 <p>Count on to find the difference.</p> <p>Comparison Bar Models</p> <p>Draw bars to find the difference between 2 numbers.</p> <p>Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.</p> 	<p>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</p> <p>$23 - 15 = 8$</p>
<p>Part Part Whole Model</p>	<p>Link to addition- use the part whole model to help explain the inverse between addition and subtraction.</p>  <p>If 10 is the whole and 6 is one of the parts. What is the other part?</p> <p>$10 - 6 =$</p>	<p>Use a pictorial representation of objects to show the part part whole model.</p> 	 <p>Move to using numbers within the part whole model.</p>
<p>Make 10</p>	<p>$14 - 9 =$</p>  <p>Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.</p>	<p>$13 - 7 = 6$</p>  <p>Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.</p>	<p>$16 - 8 =$ How many do we take off to reach the next 10? How many do we have left to take off?</p>



Warton St Paul's

Church of England Primary Academy
A member of **CDARI**

Warton St Paul's CE Primary Academy Progression in Calculations

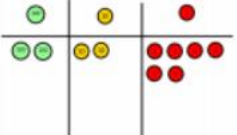
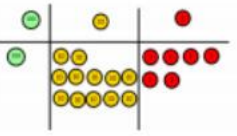
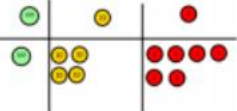
<p>Column method without regrouping</p>	<p>Use Base 10 to make the bigger number then take the smaller number away.</p>  <p>Show how you partition numbers to subtract. Again make the larger number first.</p> 	 <p>Draw the Base 10 or place value counters alongside the written calculation to help to show working.</p> 	$47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$ <p>This will lead to a clear written column subtraction.</p> 
<p>Column method with regrouping</p>	<p>Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.</p> <p>Make the larger number with the place value counters</p>  <p>Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.</p> 	 <p>Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.</p> <p>When confident, children can find their own way to record the exchange/regrouping.</p>  <p>Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.</p>	 <p>Children can start their formal written method by partitioning the number into clear place value columns.</p>  <p>Moving forward the children use a more compact method.</p>



Warton St Paul's

Church of England Primary Academy
A member of **CDARI**

Warton St Paul's CE Primary Academy Progression in Calculations

	<p>Now I can subtract my ones.</p>  <p>Calculations 234 - 88</p> <p>Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.</p>  <p>Calculations 234 - 88</p> <p>Now I can take away eight tens and complete my subtraction</p>  <p>Calculations 234 - 88 146</p> <p>Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.</p>		<p>This will lead to an understanding of subtracting any number including decimals.</p> $ \begin{array}{r} 5 \quad 12 \quad 1 \\ 2 \quad \cancel{6} \quad \cancel{3} \quad . \quad 0 \\ - 2 \quad 6 \quad . \quad 5 \\ \hline 2 \quad 3 \quad 6 \quad . \quad 5 \end{array} $
--	---	--	---

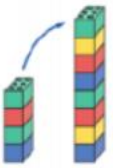

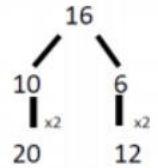
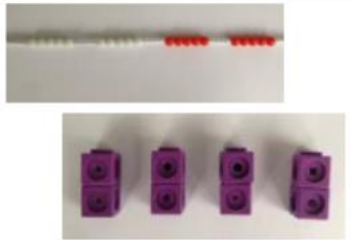
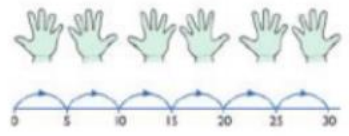



Warton St Paul's

Church of England Primary Academy
A member of **CDARI**

Warton St Paul's CE Primary Academy Progression in Calculations

Multiplication

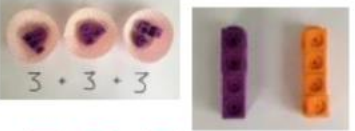

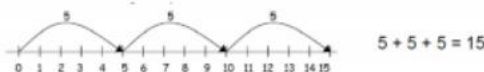



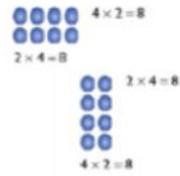
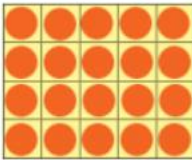

Objective and strategies	Concrete	Pictorial	Abstract
Doubling	<p>Use practical activities to show how to double a number.</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double a number.</p> <p>Double 4 is 8</p> 	 <p>Partition a number and then double each part before recombining it back together.</p>
Counting in multiples	 <p>Count in multiples supported by concrete objects in equal groups.</p>	 <p>Use a number line or pictures to continue support in counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>



Warton St Paul's

Church of England Primary Academy
A member of **CDARI**

Warton St Paul's CE Primary Academy Progression in Calculations

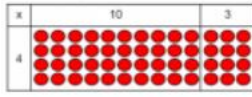

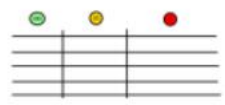
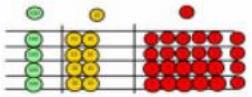
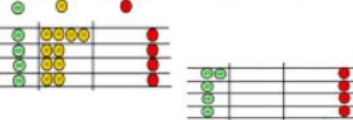
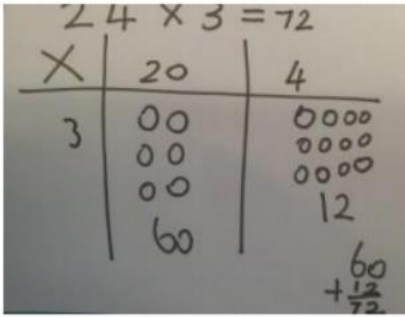
<p>Repeated addition</p>	 <p>Use different objects to add equal groups.</p>	<p>There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?</p>   <table border="1" data-bbox="985 694 1355 805"> <tbody> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> </tbody> </table>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	<p>Write addition sentences to describe objects and pictures.</p> 
1	2	3	4	5	6	7	8	9	10																								
11	12	13	14	15	16	17	18	19	20																								
21	22	23	24	25	26	27	28	29	30																								
<p>Arrays showing commutative multiplication</p>	<p>Create arrays using counters/ cubes to show multiplication sentences. e.g. 4 rows of 6 = 24</p>  	<p>Draw arrays in different rotations to find commutative multiplication sentences.</p>   <p>Link arrays to area of rectangles.</p>	<p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p> $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$ </p>																														



Warton St Paul's

Church of England Primary Academy
A member of **CDARI**

Warton St Paul's CE Primary Academy Progression in Calculations

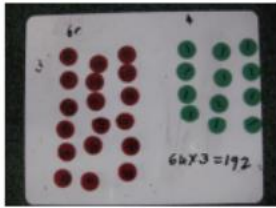
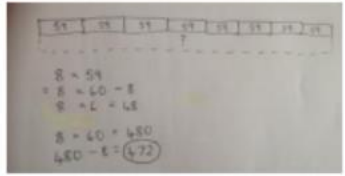
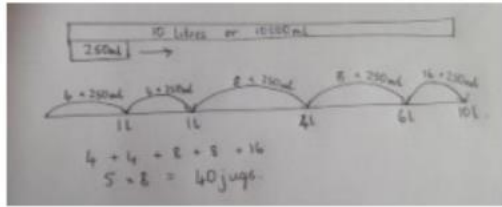
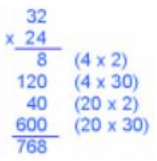
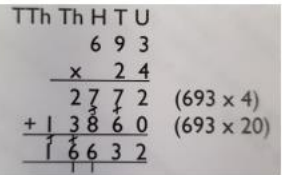
<p>Grid Method</p>	<p>Show the link with arrays to first introduce the grid method.</p>  <p>4 rows of 10 4 rows of 3</p> <p>Move on to using Base 10 to move towards a more compact method.</p>  <p>4 rows of 13</p> <p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.</p>  <p>Calculations 4 x 126</p> <p>Fill each row with 126.</p>  <p>Calculations 4 x 126</p> <p>Add up each column, starting with the ones making any exchanges needed.</p>  <p>Then you have your answer.</p>	<p>Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1624 494 1848 566"> <tr> <td>x</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p>$210 + 35 = 245$</p> <p>Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</p> <table border="1" data-bbox="1624 782 1848 933"> <tr> <td></td> <td>10</td> <td>8</td> </tr> <tr> <td>10</td> <td>100</td> <td>80</td> </tr> <tr> <td>3</td> <td>30</td> <td>24</td> </tr> </table> <table border="1" data-bbox="1612 957 1848 1069"> <tr> <td>x</td> <td>1000</td> <td>300</td> <td>40</td> <td>2</td> </tr> <tr> <td>10</td> <td>10000</td> <td>3000</td> <td>400</td> <td>20</td> </tr> <tr> <td>8</td> <td>8000</td> <td>2400</td> <td>320</td> <td>16</td> </tr> </table>	x	30	5	7	210	35		10	8	10	100	80	3	30	24	x	1000	300	40	2	10	10000	3000	400	20	8	8000	2400	320	16
x	30	5																															
7	210	35																															
	10	8																															
10	100	80																															
3	30	24																															
x	1000	300	40	2																													
10	10000	3000	400	20																													
8	8000	2400	320	16																													



Warton St Paul's

Church of England Primary Academy
A member of **CDARI**

Warton St Paul's CE Primary Academy Progression in Calculations

Column multiplication	Children can continue to be supported by place value counters at the stage of multiplication.	Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.	Start with long multiplication, reminding the children about lining up their numbers clearly in columns. If it helps, children can write out what they are solving next to their answer.
	 <p>It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.</p>	 	  <p>This moves to the more compact method.</p>

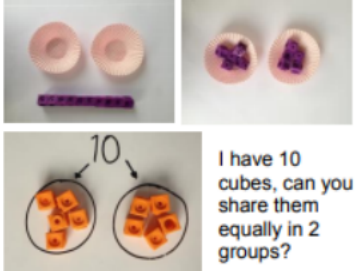

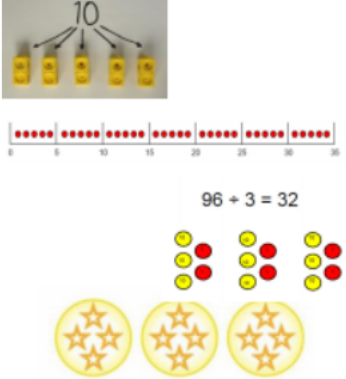
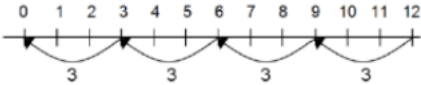
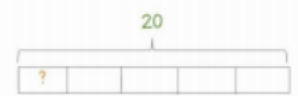


Warton St Paul's

Church of England Primary Academy
A member of **CDARI**

Warton St Paul's CE Primary Academy Progression in Calculations

Division


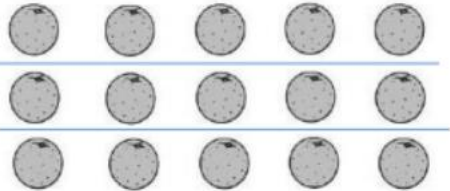
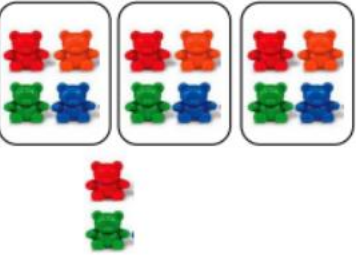
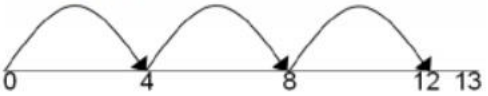

Objective and strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  $8 \div 2 = 4$	<p>Share 9 buns between three people.</p> $9 \div 3 = 3$
Division as grouping	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p>  $96 \div 3 = 32$	<p>Use a number line to show jumps in groups. The number of jumps equals the number of groups.</p>  <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>  $20 \div 5 = ?$ $5 \times ? = 20$	$28 \div 7 = 4$ <p>Divide 28 into 7 groups. How many are in each group?</p>



Warton St Paul's

Church of England Primary Academy
A member of **CDARI**

Warton St Paul's CE Primary Academy Progression in Calculations

<p>Division within arrays</p>	 <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$</p>	 <p>Draw an array and use lines to split the array into groups to make multiplication and division sentences.</p>	<p>Find the inverse of multiplication and division sentences by creating four linking number sentences.</p> <p>$7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$</p>
<p>Division with a remainder</p>	<p>$14 \div 3 =$ Divide objects between groups and see how much is left over</p> 	<p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.</p>  <p>Draw dots and group them to divide an amount and clearly show a remainder.</p> 	<p>Complete written divisions and show the remainder using r.</p> <p>$29 \div 8 = 3 \text{ REMAINDER } 5$</p> <p>↑ ↑ ↑ ↑ dividend divisor quotient remainder</p>



Warton St Paul's

Church of England Primary Academy
A member of **CDARI**

Warton St Paul's CE Primary Academy Progression in Calculations

Division by
Subtraction
(Chunking)

$48 \div 4 = 12$ (groups of 4)

$48 \div 4 = 10$ (groups of 4) + 2 (groups of 4)
 $= 12$ (groups of 4)

$4 \overline{) 48}$
 $\underline{- 40}$ 10x
 8
 $\underline{- 8}$ 2x
 0
 Answer: 12

Children should write their answer above the calculation to make it easy for them and the teacher to distinguish.

Progressing to:

$28 \overline{) 6367}$ 227r11

$\underline{- 5600}$ 200x
 767
 $\underline{- 560}$ 20x
 207
 $\underline{- 140}$ 5x
 67
 $\underline{- 56}$ 2x
 11

If children have full conceptual understanding and are proficient with chunking method, they may be taught short (bus stop) and long division.



Warton St Paul's

Church of England Primary Academy
A member of **CDARI**

Warton St Paul's CE Primary Academy Progression in Calculations




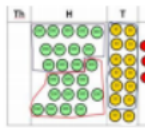

Short division	<div data-bbox="638 400 929 582" data-label="Diagram"> </div> <div data-bbox="593 598 974 662" data-label="Text"> <p>Use place value counters to divide using the bus stop method alongside</p> </div> <div data-bbox="593 670 940 805" data-label="Diagram"> </div> <div data-bbox="593 813 974 965" data-label="Text"> <p>$42 \div 3 =$ Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p> </div> <div data-bbox="593 973 907 1117" data-label="Diagram"> </div> <div data-bbox="593 1133 974 1228" data-label="Text"> <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p> </div> <div data-bbox="593 1236 862 1316" data-label="Diagram"> </div> <div data-bbox="593 1324 974 1372" data-label="Text"> <p>We look how much in 1 group so the answer is 14.</p> </div>	<div data-bbox="996 400 1500 486" data-label="Text"> <p>Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p> </div> <div data-bbox="1008 534 1276 638" data-label="Diagram"> </div> <div data-bbox="996 654 1467 710" data-label="Text"> <p>Encourage them to move towards counting in multiples to divide more efficiently</p> </div>	<div data-bbox="1534 400 1926 454" data-label="Text"> <p>Begin with divisions that divide equally with no remainder</p> </div> <div data-bbox="1534 470 1736 582" data-label="Equation-Block"> $\begin{array}{r} 218 \\ 4 \overline{) 872} \end{array}$ </div> <div data-bbox="1534 614 1926 646" data-label="Text"> <p>Move onto divisions with a remainder.</p> </div> <div data-bbox="1534 654 1780 758" data-label="Equation-Block"> $\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \end{array}$ </div> <div data-bbox="1534 774 1926 837" data-label="Text"> <p>Finally move into decimal places to divide the total accurately.</p> </div> <div data-bbox="1534 885 1780 973" data-label="Equation-Block"> $\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$ </div>
----------------	--	---	--



Warton St Paul's

Church of England Primary Academy
A member of **CDARI**

Warton St Paul's CE Primary Academy Progression in Calculations

Long division	<p data-bbox="607 336 963 422">  $2544 \div 12$ How many groups of 12 thousands do we have? None </p> <p data-bbox="607 438 929 462">Exchange 2 thousand for 20 hundreds.</p> <p data-bbox="607 470 884 566">  $12 \overline{) 2544}$ </p> <p data-bbox="607 598 963 686"> How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one. </p> <p data-bbox="607 694 918 805">  $12 \overline{) 2544}$ $\underline{24}$ 1 </p> <p data-bbox="607 837 963 901">Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2</p> <p data-bbox="607 917 918 1045">  $12 \overline{) 2544}$ $\underline{24}$ 14 $\underline{12}$ 2 </p> <p data-bbox="607 1061 963 1125">Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2</p> <p data-bbox="607 1125 918 1252">  $12 \overline{) 2544}$ $\underline{24}$ 14 $\underline{12}$ 24 $\underline{24}$ 0 </p>	<p data-bbox="992 336 1467 422">Instead of using physical counters, children can draw the counters and circle the groups on a whiteboard or in their books.</p> <p data-bbox="992 454 1500 574">Use this method to explain what is happening and as soon as they have understood what move on to the abstract method as this can be a time consuming process.</p>	<p data-bbox="1534 343 1780 646"> $\begin{array}{r} 0318 \text{ r}5 \\ 20 \overline{) 6365} \\ \underline{-60} \\ 36 \\ \underline{-20} \\ 165 \\ \underline{-160} \\ 5 \end{array}$ </p>