Mathematics Department
William Perkin CofE High School

An introduction to

A level Mathematics and Further Mathematics

This induction booklet is for students intending to begin studying A level Maths or A level Further Maths in Year 12 from next September.

It is important that you are able to work at this standard – read the introduction carefully, and spend time working through the exercises before you start in September.

All questions are GCSE standard and it is expected all students are familiar with the techniques covered.

Introduction to A level Maths at William Perkin

Thank you for choosing to study Mathematics in the Sixth Form at William Perkin High School.

If you are studying **Mathematics** on its own, you will sit two internal exams at the end of Y12, which will assess a combination of three areas: Pure maths, Statistics and Mechanics. At the end of Y13 you will sit 3 external exams that will assess content covered during both years with the same combination of Pure maths, Statistics and Mechanics. If you have chosen to study **Further Mathematics** as well, you will sit two internal exams at the end of Y12, which will assess a combination of Pure maths, Decision maths and Mechanics. At the end of Y13 you will sit 3 external exams that will assess content covered during both years with the same combination of Pure maths, Decision maths and Mechanics.

The Mathematics Department is committed to ensuring that you make good progress throughout your A level course. In order that you make the best possible start to the course, we have placed this booklet on our website. It is important that you spend time working through the topics in this booklet over the summer, as you need to have a good knowledge of these topics before you commence your course in September. You should have met all the topics before at GCSE. The answers to the exercises are at the back of the booklet. You will need to be organised about your approach to this, so keep your work in a folder and note any queries you have, so that you can ask about them at the beginning of term. You will notice that there is a heavy focus on algebraic manipulation in this booklet, as you must have a strong grasp of this in order to cope with the level of difficulty that the Mathematics and Further Mathematics courses demand. There are also sections on indices and surds, which too form an integral part of these courses.

In the second week of term, you will take a test in class to check how well you understand these topics, so it is important that you thoroughly understand the content of the booklet by then. If you do not pass this test, this will be considered a serious concern, and it will beg the question as to whether you should be taking this subject. In particular, please note that there is a practice test provided at the back of this booklet, which is similar in style to the test that you will be given in class, and which you must submit in your first lesson.

Use this introduction to give you a good start to your A level work so that you enjoy, and benefit from, the course. The more effort you put in, right from the start, the better you should do.

Mr V Bhardwaj
Head of Mathematics

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Induction Task

Please complete all questions. If you have any difficulty with the content use the support videos available in the induction booklet. The worked solutions are available in the induction booklet.

You must show the completed and marked booklet to your teacher in your first lesson back in half term.

Section 1: algebra

1 Rearrange to make x the subject

a) $\frac{3x+y}{5} = 2x - z$

b) $3x = \frac{2m+n}{xp}$

c) $\frac{a+2x}{a-x} = n$

2 Here is a formula

$$5t + 3 = 4w(t + 2)$$

a) Rearrange the formula to make t the subject

b) Find the exact value of t when $w = -\frac{1}{8}$

Give your answer in its simplest form

3 Expand $(y^2 - 5y + 2)(2y - 3)$

4 Factorise fully

a) $24ab^3 + 16a^2b^2c$

b) $x^2 - 10x - 24$

c) $2y^2 + 5y - 12$

d) $4a^2 - 25$

e) $(x + y)^2 + (x + y)(2x + 5y)$

f) $5x^2 + 4xy - 12y^2$

5 Solve $x^2 + 6x + 7 = 0$

Give your answer in the form $a \pm \sqrt{b}$

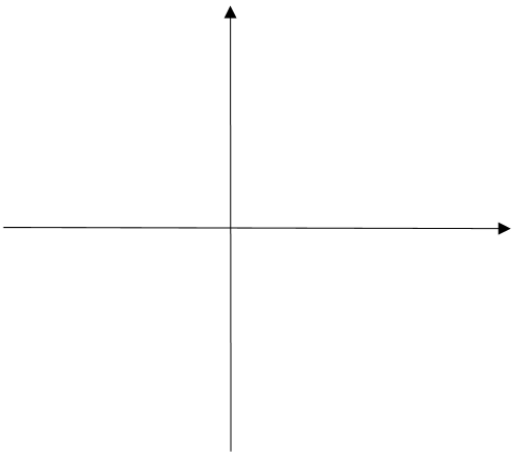
6 a) Express $x^2 - 8x - 3$ in the form $(x + b)^2 + c$

b) Hence write down the coordinates of the minimum point of $y = x^2 - 8x - 3$

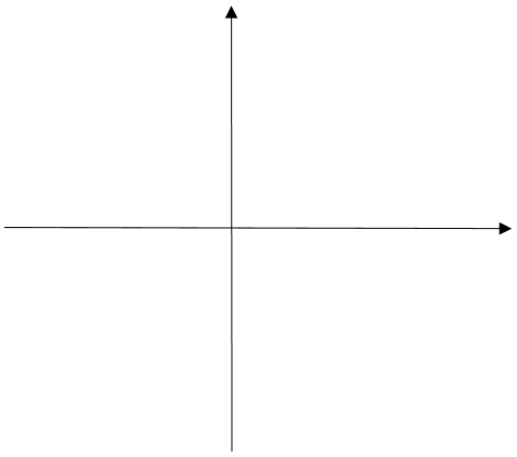
7 Express $2x^2 - 12x - 3$ in the form $a(x + b)^2 + c$

8 Sketch the curves below showing all intercepts with the axes

a) $y = x^2 - 7x + 6$



b) $y = -x^2 - 5x$



9 Simplify

a) $\frac{2x^2 - 14x + 20}{3x^2 - 75}$

b) $\frac{3}{y^2 - y} + \frac{1}{2y - 2}$

c) $\frac{10a+5}{2a^2 - 18} \times \frac{a^2 + a - 6}{2a^2 + a}$

d) $\frac{x^2 + 4x - 12}{x^2 - 25} \div \frac{x + 6}{x^2 - 5x}$

10 Solve $\frac{4}{x} + \frac{2}{x-1} = 3$ give your solutions to three significant figures

11 Solve

a) $20 + w < 3(w + 2)$

b) $2x^2 - 6x \geq 3x^2 + 8$

12 Solve the following pairs of simultaneous equations. Do not use trial and improvement

a) $2y = 3x + 4$
$2x = -3y - 7$

b) $y = x + 6$
$x^2 + y^2 = 50$

13 $f(x) = 2x^2$ $g(x) = 3x - 1$ $h(x) = \frac{1}{x+2}$

find:

a) $f(3)$

b) x when $g(x) = 20$

c) $fg(x)$

c) $hg(x)$

d) $gf(2)$

e) $g^{-1}(x)$

f) $h^{-1}(x)$

g) $hg^{-1}(17)$

Section 2 – Indices and Surds

DO NOT USE A CALCULATOR - YOU ARE EXPECTED TO SHOW YOUR WORKING CLEARLY

1 Simplify

a) $4a^3bc^2 \times 5a^2b^7$

b) $(3xy^5)^4$

c) $\frac{6p^3q^2r}{8p^5q}$

d) $\frac{3m^4n}{10m} \times \frac{15n}{m^{-7}}$

e) $\frac{8c^7}{15d^6} \div \frac{6c^2}{5d^3}$

2 Simplify the following, giving final answer as a sum of powers of x

a) $\sqrt{x}(x+1)$

b) $\sqrt{x^5 \times x^9}$

c) $\frac{x^3+x^2}{x^{-5}}$

3 Solve

a) $y^{-3} = 125$

b) $x^{-\frac{2}{3}} = 7\frac{1}{9}$

c) $8^{x+1} = 4^{2x}$

4 a)

$a^{11} \times b^6 \times c = a^9 \times b^{10}$

Write c in terms of a and b.
Give your answer in its simplest form.

b)

$p^{-2} = q^6 \times r^4$

Write p in terms of q and r.
Give your answer in its simplest form.

5 $x^{\frac{3}{2}} = 8$ where $x > 0$ and $y^{-2} = \frac{25}{4}$ where $y > 0$

Work out the value of $\frac{x}{y}$.

6 Simplify

a) $\sqrt{72}$

b) $\frac{\sqrt{75}}{2\sqrt{12}}$

c) $\sqrt{5}(2\sqrt{20} - \sqrt{125})$

d) $\sqrt{500} - 2\sqrt{45}$

7 Rationalise the denominator

a) $\frac{2}{\sqrt{8}}$

b) $\frac{8}{3-\sqrt{5}}$

c) $\frac{4-\sqrt{48}}{\sqrt{3}+1}$

8 Solve $\sqrt{(33+\sqrt{x})}=6$

9 Solve $y\left(\sqrt{3}-1\right)=8$

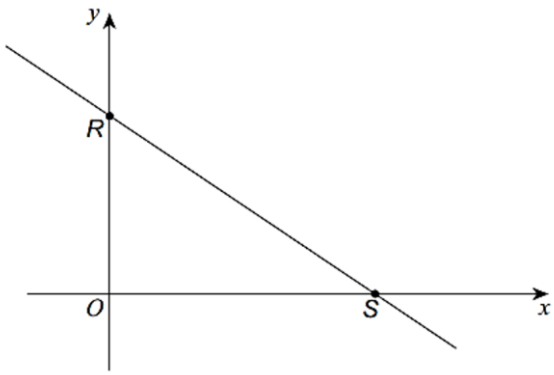
Giving your answer in the form $a+b\sqrt{3}$

Section 3 – Coordinate Geometry

1 a) The line L_1 passes through the points (1,-2) and (4, 17).
Find the equation of line L_1, giving your answer in the form $y = mx + c$

b) The line L_2 passes through the point (-6,5) and is perpendicular to the line $x - 3y = 11$
Find the equation of line L_1, giving your answer in the form $y = mx + c$

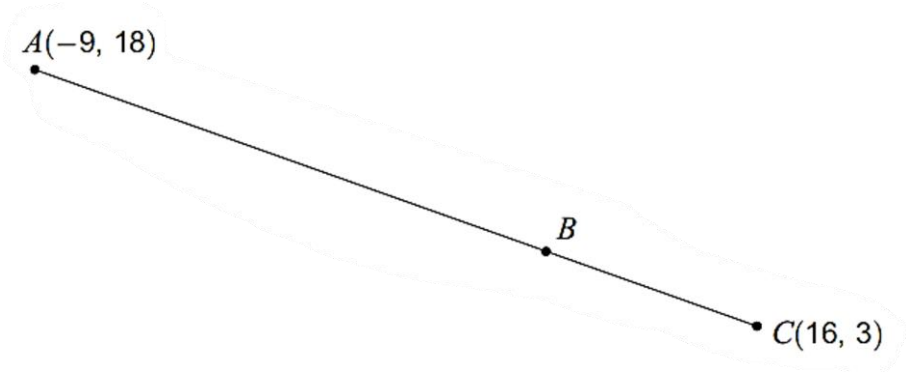
2 A sketch of $2x + 3y = 12$ is shown



a) Write down the coordinates of R

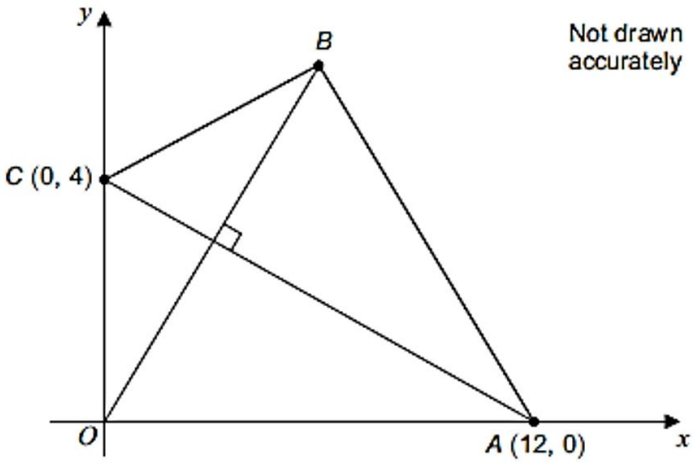
b) Find the coordinates of the midpoint of RS

3 ABC is a straight line.
BC is 20% of AC



Work out the coordinates of B

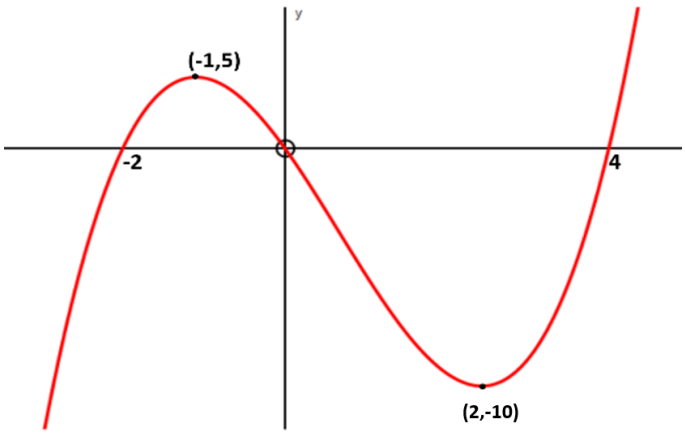
4 OABC is a kite



a) Work out the equation of AC

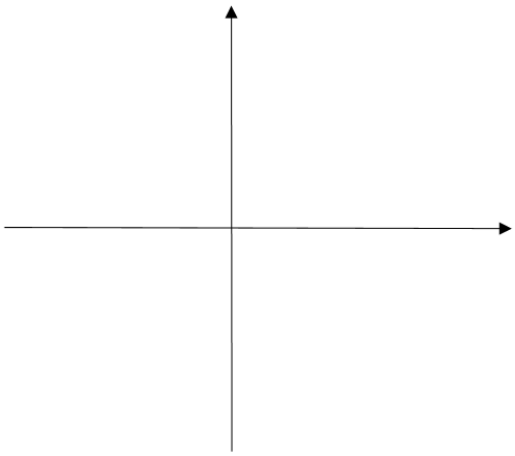
b) Work out the coordinates of B

5 The graph $y = f(x)$ is shown below.

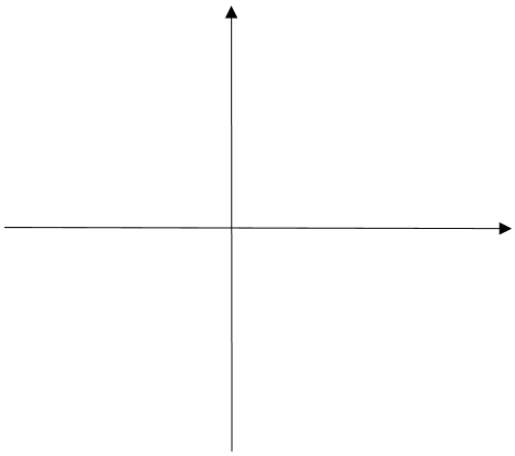


Sketch:

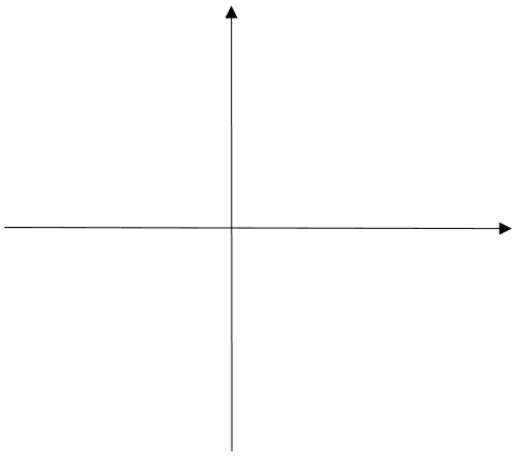
a) $y = f(x - 2)$



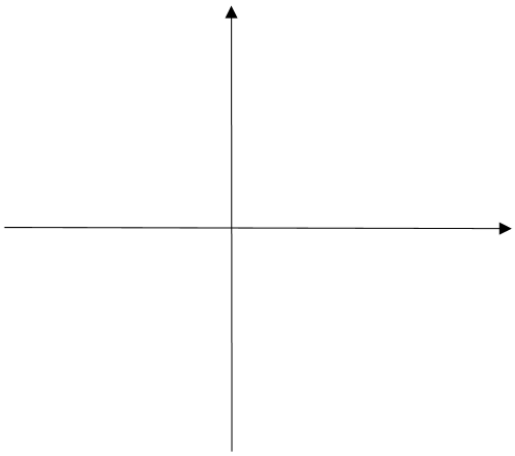
b) $y = f(x) + 1$



c) $y = -f(x)$



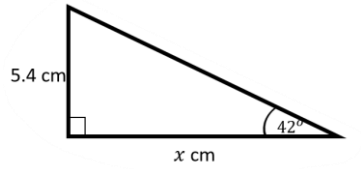
d) $y = f(-x)$



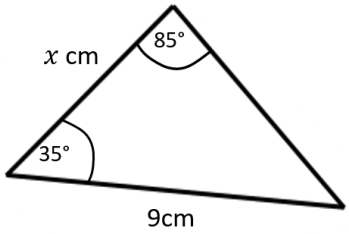
Section 4 – Trigonometry

1 Find the missing sides in the following triangles

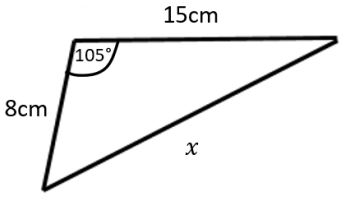
a)



b)

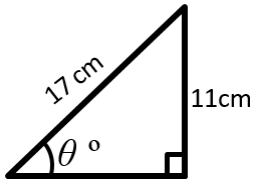


c)

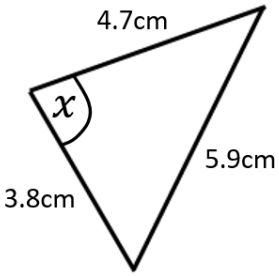


2 Find the missing angles in the following triangles

a)

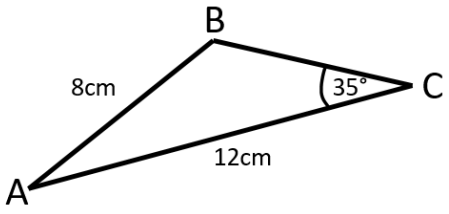


b)



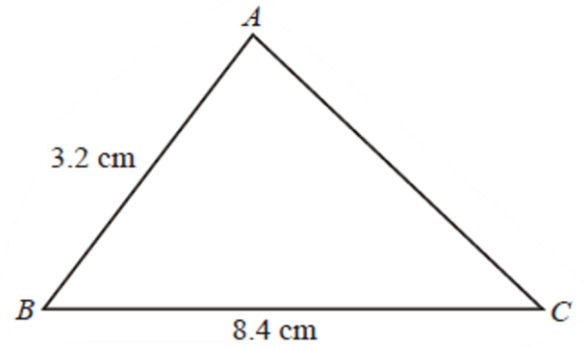
3

Find the size of the **obtuse** angle ABC below.



4 The area of triangle ABC in $10cm^2$

Calculate the perimeter of triangle ABC
Give your answer correct to three significant figures



5 Sketch, for $0^o \leq x \leq 360^o$ the graph of

a) $y = \sin x$

b) $y = \cos x$

6 Solve the following equations giving all answers between 0^o and 360^o

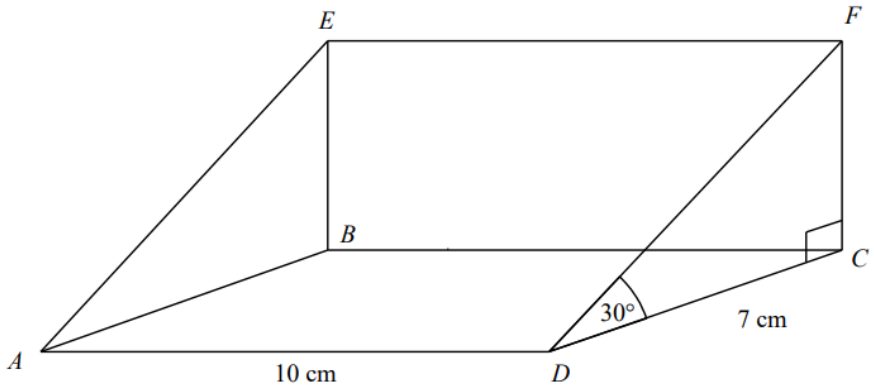
a) $\sin x = 0.6$

b) $3\cos x = 1.2$

c) $5 + \sin x = 4.8$

7 The diagram shows a triangular prism.

$CD = 7$ cm
$AD = 10$ cm
Angle $FDC = 30°$

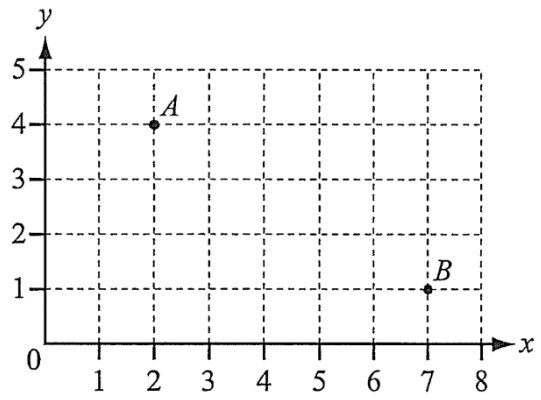


Calculate the size of angle AFC.
Give your answer correct to 1 decimal place.

Section 5 – Vectors

1

Write down the position vector of A



a) Find $\left|\overrightarrow{AB}\right|$

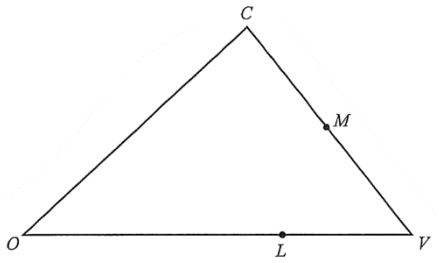
2 $\boldsymbol{p} = \begin{pmatrix}3\\2\end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix}6\\3\end{pmatrix}$

a) Find as a single column vector $\boldsymbol{p} + 2\boldsymbol{q}$

b) Calculate the value of $|\boldsymbol{p} + 2\boldsymbol{q}|$

3

In the diagram $CM = MV$ and $OL = 2LV$
O is the origin. $\overrightarrow{OC} = \boldsymbol{c}$ and $\overrightarrow{OV} = \boldsymbol{v}$



Find, in terms of $\boldsymbol{c}$ and $\boldsymbol{v}$, in their simplest forms

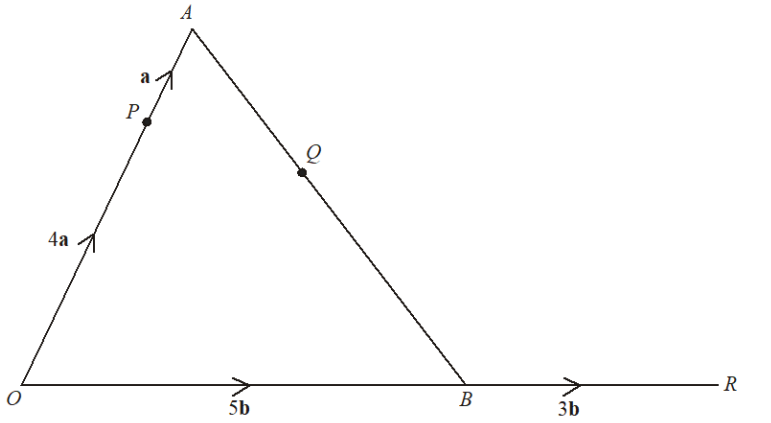
a) $\overrightarrow{CM}$

b) The position vector of M

c) $\overrightarrow{ML}$

4

In the diagram $\overrightarrow{OP}$ = 4**a**, $\overrightarrow{PA}$ = **a**, $\overrightarrow{OB}$ = 5**b**, $\overrightarrow{BR}$ = 3**b** and $\overrightarrow{AQ} = \frac{2}{5}\overrightarrow{AB}$



Not drawn accurately

a) Find, in terms of **a** and **b**, simplifying your answers,

(i) $\overrightarrow{AB}$

(ii) $\overrightarrow{PQ}$

(b) Show clearly that points *P, Q* and *R* lie on a straight line.

Section 6 – Proof

1 a) n is a positive integer

Write down the next odd number after $2n - 1$

b) Prove that the product of two consecutive odd number is always one less than a multiple of 4

2 Prove that $(5n + 3)(n - 1) + n(n + 2)$ is a multiple of 3 for all integer values of n

3 The n^{th} term of the linear sequence 2, 7, 12, 17, … is $5n - 3$

A new sequence is formed by squaring each term of the linear sequence and adding 1.

Prove algebraically that all the terms in the new sequence are multiples of 5

4 a) The n^{th} term of a sequence is $n^2 + 12n + 27$
by factorising or otherwise, show that the 20th term can be written as the product of two prime numbers.

b) The n^{th} term of a different sequence is $n^2 - 6n + 14$
by completing the square or otherwise, show that every term is positive

Y12 Maths and Further Maths written solutions

Use these written solutions to check that you have answered the questions correctly, or to offer some hints when you are stuck with a question. **Do not** just copy these solutions into the booklet. You must have a strong grasp of this content before you start the course in September. Please also ensure you revise any of the topics that you may have become less familiar with if necessary.

Section 1 – Algebra

1 Rearrange to make x the subject

a) $\frac{3x+y}{5} = 2x - z$

$3x + y = 5(2x - z)$

$3x + y = 10x - 5z$

$y + 5z = 7x$

$x = \frac{y + 5z}{7}$

b) $3x = \frac{2m+n}{xp}$

$3x^2p = 2m + n$

$x^2 = \frac{2m+n}{3p}$

$x = \pm\sqrt{\frac{2m+n}{3p}}$

Careful!

c) $\frac{a+2x}{a-x} = n$

$a + 2x = n(a - x)$

$a + 2x = na - nx$

$2x + nx = na - a$

$x(2+n) = na - a$

FACTORISE!

$x = \frac{na - a}{2+n}$

2 Here is a formula $5t + 3 = 4w(t + 2)$

a) Rearrange the formula to make t the subject

$5t + 3 = 4wt + 8w$

$5t - 4wt = 8w - 3$

$t(5 - 4w) = 8w - 3$

$t = \frac{8w - 3}{5 - 4w}$

b) Find the exact value of t when $w = -\frac{1}{8}$

Give your answer in its simplest form

$t = \frac{(8 \times -\frac{1}{8}) - 3}{5 - (4 \times -\frac{1}{8})} = \frac{-1 - 3}{5\frac{1}{2}} = -4 \div \frac{11}{2}$

$= \frac{-8}{11}$

3 Expand $(y^2 - 5y + 2)(2y - 3)$

$2y^3 - 3y^2 - 10y^2 + 15y + 4y - 6$

$= 2y^3 - 13y^2 + 19y - 6$

4 Factorise fully

a) $24ab^3 + 16a^2b^2c$

$8ab^2(3b + 2ac)$

b) $x^2 - 10x - 24$ (+) (×)

$(x - 12)(x + 2)$

c) $2y^2 + 5y - 12$ (+) 5 (×) −24

$2y^2 + 8y - 3y - 12$

$2y(y + 4) - 3(y + 4)$

$(2y - 3)(y + 4)$

d) $4a^2 - 25$ DOTS!

$(2a + 5)(2a - 5)$

e) $(x + y)^2 + (x + y)(2x + 5y)$

$x^2 + 2xy + y^2 + 2x^2 + 5xy + 2xy + 5y^2$

$3x^2 + 9xy + 6y^2$

f) $5x^2 + 4xy - 12y^2$

$(5x - 6y)(x + 2y)$

5 Solve $x^2 + 6x + 7 = 0$

complete the square

Give your answer in the form $a \pm \sqrt{b}$

$(x+3)^2 - 9 + 7 = 0$

$(x+3)^2 - 2 = 0$

$(x+3)^2 = 2$

cont..

$x + 3 = \pm\sqrt{2}$

$x = -3 \pm \sqrt{2}$

6 a) Express $x^2 - 8x - 3$ in the form $(x+b)^2 + c$

Complete the square

$(x-4)^2 - 16 - 3$

$(x-4)^2 - 19$

b) Hence write down the coordinates of the minimum point of $y = x^2 - 8x - 3$

$(4, -19)$

7 Express $2x^2 - 12x - 3$ in the form $a(x+b)^2 + c$

$2(x^2 - 6x) - 3$

$2[(x-3)^2 - 9] - 3$

$2(x-3)^2 - 18 - 3$

$2(x-3)^2 - 21$

8 Sketch the curves below showing all intercepts with the axes

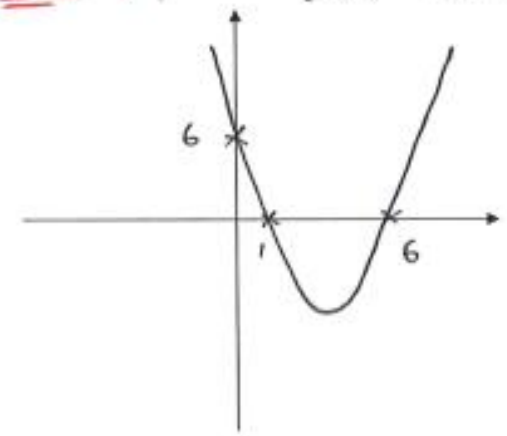
a) $y = x^2 - 7x + 6$

y intercept ⇒ $x = 0$, $y = 6$

x intercept ⇒ $y = 0$

FACTORISE! $(x-6)(x-1) = 0$

$x = 6$ $x = 1$



b) $y = -x^2 - 5x$

negative quadratic

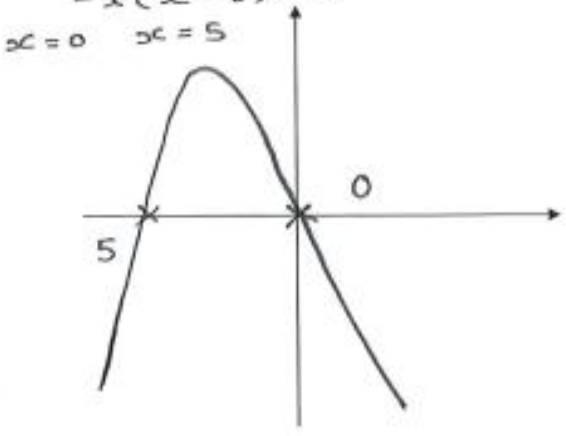
y intercept ⇒ $x = 0$, $y = 0$

x intercept ⇒ $y = 0$

$-x^2 - 5x = 0$

$-x(x-5) = 0$

$x = 0$ $x = 5$



12 Solve the following pairs of simultaneous equations. Do not use trial and improvement

a) $2y = 3x + 4$ ① $\times 3$
$2x = -3y - 7$ ② $\times 2$

$6y = 9x + 12$

$6y - 9x = 12$

$6y + 4x = -14$

$-13x = 26$

$x = -2$

$2y = 3x - 2 + 4$

$2y = -6 + 4$

$y = -1$

$4x = -6y - 14$

b) $y = x + 6$
$x^2 + y^2 = 50$

$x^2 + (x+6)^2 = 50$

$x^2 + (x+6)(x+6) = 50$

$x^2 + x^2 + 12x + 36 = 50$

$2x^2 + 12x - 14 = 0$

$x^2 + 6x - 7 = 0$

$(x+7)(x-1) = 0$

$x = 1 \quad y = 7 \qquad x = -7 \quad y = -1$

$(1, 7)$ $(-7, -1)$

13 $f(x) = 2x^2$ $g(x) = 3x - 1$ $h(x) = \frac{1}{x+2}$

find:

a) $f(3)$

2×3^2

$= 2 \times 9$

$= 18$

b) x when $g(x) = 20$

$3x - 1 = 20$

$3x = 21$

$x = 7$

c) $fg(x)$

$= 2(3x-1)^2$

$= 2(3x-1)(3x-1)$

$= 2(9x^2 - 6x + 1)$

$= 18x^2 - 12x + 2$

c) $hg(x)$

$\frac{1}{(3x-1)+2}$

$= \frac{1}{3x+1}$

d) $gf(2)$

$f(2) = 2 \times 2^2$

$= 8$

$g(8) = 3 \times 8 - 1$

$= 23$

e) $g^{-1}(x)$

$y = 3x - 1$

$y + 1 = 3x$

$\frac{y+1}{3} = x$

$g^{-1}(x) = \frac{x+1}{3}$

f) $h^{-1}(x)$ $y = \frac{1}{x+2}$

$y(x+2) = 1$

$yx + 2y = 1$

$yx = 1 - 2y$

$x = \frac{1-2y}{y}$

$h^{-1}(x) = \frac{1-2x}{x}$

g) $hg^{-1}(17)$ from (e)

$g^{-1}(17) = \frac{17+1}{3}$

$= 9$

$h(9) = \frac{1}{9+2} = \frac{1}{11}$

Section 2 – Indices and Surds

DO NOT USE A CALCULATOR - YOU ARE EXPECTED TO SHOW YOUR WORKING CLEARLY

1 Simplify

a) $4a^3bc^2 \times 5a^2b^7$

$20a^5b^8c^2$

b) $(3xy^5)^4$

$3xy^5 \times 3xy^5 \times 3xy^5 \times 3xy^5$

$= 81x^4y^{20}$

c) $\frac{6p^3q^2r}{8p^5q}$

$\frac{3p^{-2}qr}{4}$ or $\frac{3qr}{4p^2}$

d) $\frac{3m^4n}{10m} \times \frac{15n}{m^{-7}}$

$= \frac{9m^4n^2}{2m^{-6}}$

$= \frac{9m^{4--6}n^2}{2} = \frac{9m^{10}n^2}{2}$

e) $\frac{8c^7}{15d^6} \div \frac{6c^2}{5d^3}$

$\frac{\overset{4}{\cancel{8}c^7}}{\underset{3}{\cancel{15}d^6}} \times \frac{\cancel{5}d^3}{\underset{3}{\cancel{6}}c^2} = \frac{4c^7d^3}{9d^6c^2}$

$= \frac{4c^{7-2}d^{3-6}}{9} = \frac{4c^5d^{-3}}{9}$ or $\frac{4c^5}{9d^3}$

2 Simplify the following, giving final answer as a sum of powers of x

a) $\sqrt{x}(x+1)$

$x^{\frac{1}{2}}(x+1)$

$x^{\frac{3}{2}} + x^{\frac{1}{2}}$

b) $\sqrt{x^5 \times x^9}$

$\sqrt{x^{5+9}}$

$= \sqrt{x^{14}} = (x^{14})^{\frac{1}{2}}$

$= x^7$

c) $\frac{x^3+x^2}{x^{-5}}$

$\frac{x^3}{x^{-5}} + \frac{x^2}{x^{-5}}$

$= x^8 + x^7$

3 Solve

a) $y^{-3} = 125$

$y = 125^{-\frac{1}{3}}$

$y = \frac{1}{\sqrt[3]{125}} = \frac{1}{5}$

b) $x^{-\frac{2}{3}} = 7\frac{1}{9}$

$x^{-\frac{2}{3}} = \frac{64}{9}$

$x^{\frac{2}{3}} = \frac{9}{64}$

$x^{\frac{1}{3}} = \frac{3}{8}$

$x = \frac{27}{512}$

c) $8^{x+1} = 4^{2x}$

$2^{3(x+1)} = (2^2)^{2x}$

$2^{3x+3} = 2^{4x}$

$3x+3 = 4x$

$x = 3$

4 a)

$a^{11} \times b^6 \times c = a^9 \times b^{10}$

Write c in terms of a and b.
Give your answer in its simplest form.

$c = \frac{a^9 \times b^{10}}{a^{11} \times b^6} = \frac{b^4}{a^2}$

b)

$p^{-2} = q^6 \times r^4$

Write p in terms of q and r.
Give your answer in its simplest form.

$p^{-2} = \frac{1}{p^2}$

$\frac{1}{p^2} = q^6 r^4$

$p^2 = \frac{1}{q^6 r^4}$

$p = \frac{1}{q^3 r^2}$

5 $x^{\frac{3}{2}} = 8$ where $x > 0$ and $y^{-2} = \frac{25}{4}$ where $y > 0$

Work out the value of $\frac{x}{y}$.

$x = 8^{\frac{2}{3}}$

$= 2^2$

$= 4$

$y = \left(\frac{25}{4}\right)^{-\frac{1}{2}}$

$= \frac{2}{5}$

So, $\frac{x}{y} = 4 \div \frac{2}{5}$

$= 4 \times \frac{5}{2}$

$= 10$

6 Simplify

a) $\sqrt{72}$

$\sqrt{36} \times \sqrt{2}$

$= 6\sqrt{2}$

b) $\frac{\sqrt{75}}{2\sqrt{12}}$

$\frac{\sqrt{25} \times \sqrt{3}}{2 \times \sqrt{4} \times \sqrt{3}}$

$= \frac{5}{4}$

c) $\sqrt{5}(2\sqrt{20} - \sqrt{125})$

$= 2\sqrt{100} - \sqrt{625}$

$= 2 \times 10 - 25$

$= -5$

d) $\sqrt{500} - 2\sqrt{45}$

$\sqrt{100} \times \sqrt{5} - 2 \times \sqrt{9} \times \sqrt{5}$

$= 10\sqrt{5} - 6\sqrt{5}$

$= 4\sqrt{5}$

7 Rationalise the denominator

a) $\frac{2}{\sqrt{8}}$

$\frac{2}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}}$

$= \frac{2\sqrt{8}}{8} = \frac{2\sqrt{4}\sqrt{2}}{8}$

$= \frac{\sqrt{2}}{2}$

b) $\frac{8}{3-\sqrt{5}}$

$\frac{8}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$

$= \frac{24+8\sqrt{5}}{9-5}$

$= \frac{24+8\sqrt{5}}{4}$

$= 6 + 2\sqrt{5}$

c) $\frac{4-\sqrt{48}}{\sqrt{3}+1}$

$\frac{4-\sqrt{16} \times \sqrt{3}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{(4-4\sqrt{3}) \times (\sqrt{3}-1)}{\sqrt{3}+1 \quad \sqrt{3}-1}$

$= \frac{4\sqrt{3} - 4 - 12 + 4\sqrt{3}}{3-1}$

$= \frac{8\sqrt{3}-16}{2} = 4\sqrt{3} - 8$

8 Solve $\sqrt{(33+\sqrt{x})} = 6$

$33 + \sqrt{x} = 36$

$\sqrt{x} = 3$

$x = 9$

9 Solve $y(\sqrt{3}-1) = 8$

Giving your answer in the form $a + b\sqrt{3}$

$y = \frac{8}{\sqrt{3}-1}$ Rationalise!

$y = \frac{8}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{\sqrt{3}+1}$

$= \frac{8\sqrt{3}+8}{3-1} = \frac{8\sqrt{3}+8}{2} = 4\sqrt{3}+4$

Section 3 – Coordinate Geometry

1 a) The line L_1 passes through the points (1, 2) and (4, 17).
Find the equation of line L_1, giving your answer in the form $y = mx + c$

$m = \frac{17-2}{4-1} = \frac{19}{3}$ $\quad y = \frac{19}{3}x + c$

$2 = \frac{19}{3} \times 1 + c$

$c = \frac{-13}{3}$ $\quad y = \frac{19}{3}x - \frac{13}{3}$

b) The line L_2 passes through the point (-6,5) and is perpendicular to the line $x - 3y = 11$
Find the equation of line L_2, giving your answer in the form $y = mx + c$

↳ $x - 11 = 3y$

$y = \frac{1}{3}x - \frac{11}{3}$

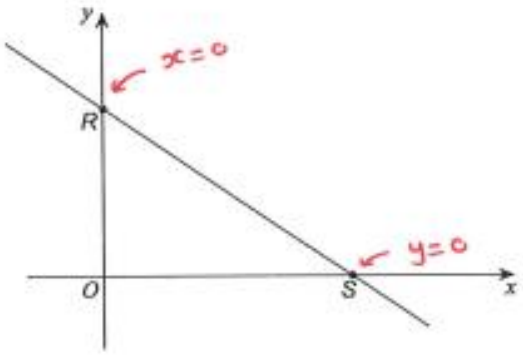
$m_{perp} = -3$

$y = -3x + c$

$5 = -3 \times -6 + c$

$c = -13$ $\quad$ **$y = -3x - 13$**

2 A sketch of $2x + 3y = 12$ is shown



a) Write down the coordinates of R

when $x = 0$ $\quad 2 \times 0 + 3y = 12 \Rightarrow y = 4$ $\quad$ **(0, 4)**

b) Find the coordinates of the midpoint of RS

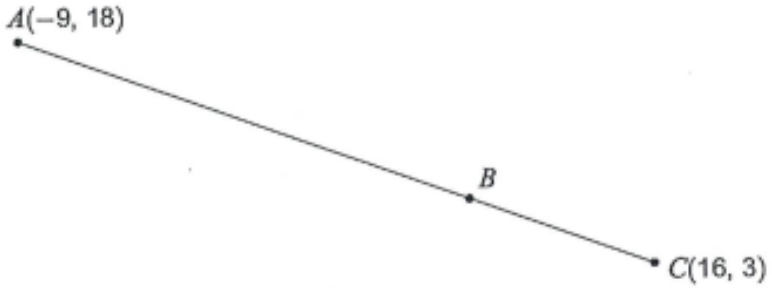
S is where $y = 0$ $\quad 2x + 0 = 12$

$x = 6$ $\quad (6, 0)$

Midpoint $= \left(\frac{6+0}{2}, \frac{4+0}{2}\right)$

$=$ **(3, 2)**

3 ABC is a straight line.
BC is 20% of AC



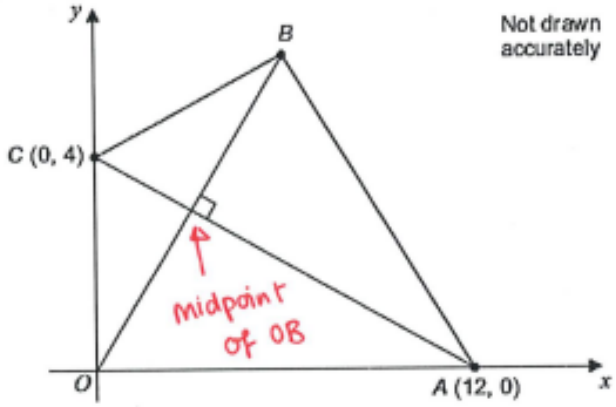
Work out the coordinates of B

$80\% \times (16 - -9) = 20$ $\quad x = 20$

$80\% \times (18 - 3) = 12$ $\quad y = 6$

$(20, 6)$

4 OABC is a kite



a) Work out the equation of AC

$m_{AC} = \frac{0-4}{12-0} = -\frac{4}{12} = -\frac{1}{3}$

$y = -\frac{1}{3}x + 4$

b) Work out the coordinates of B

OB is perpendicular to AC $\Rightarrow m = 3$

$y = 3x$

$3x = -\frac{1}{3}x + 4$

Midpoint of OB is at the intersection

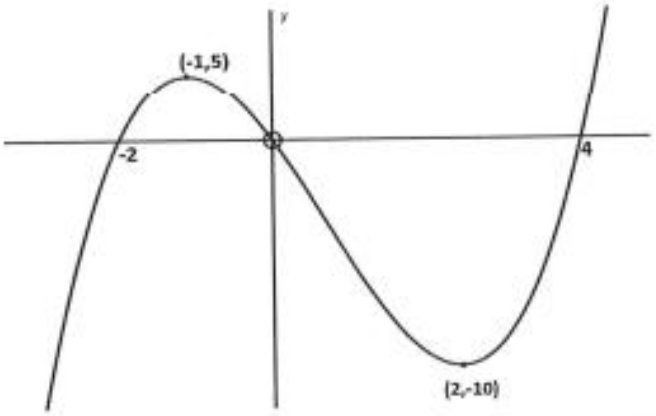
$x = \frac{6}{5}$ so $y = 3 \times \frac{6}{5}$

$= \frac{18}{5}$

$\Rightarrow x = \frac{12}{5}$ $\quad y = \frac{36}{5}$

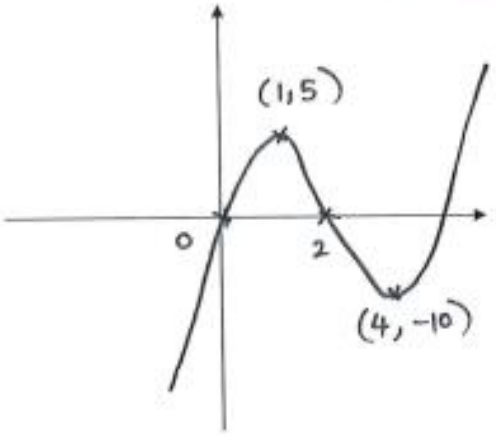
$\left(\frac{12}{5}, \frac{36}{5}\right)$

5 The graph $y = f(x)$ is shown below.

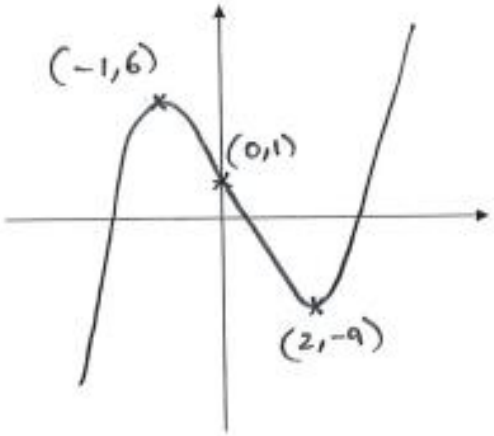


Sketch:

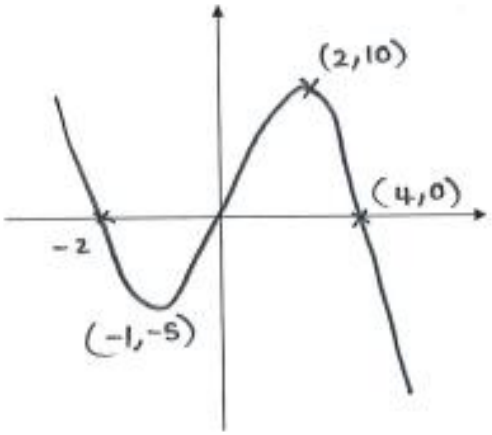
a) $y = f(x-2)$ → Translation in x direction by $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$



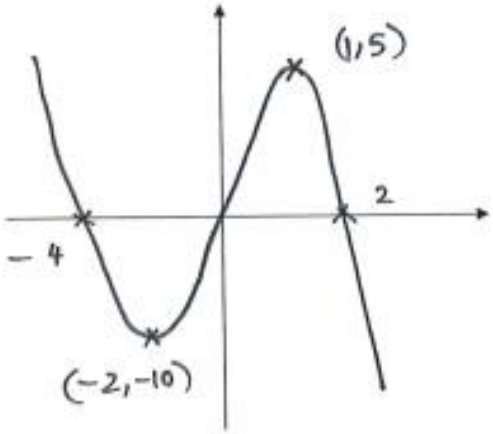
b) $y = f(x) + 1$ → Translation in y direction by $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$



c) $y = -f(x)$ ← reflection in x axis



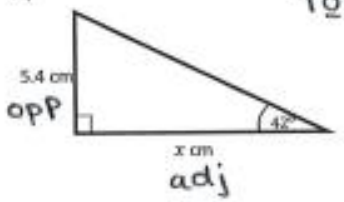
d) $y = f(-x)$ ← reflection in y axis



Section 4 – Trigonometry

1 Find the missing sides in the following triangles

a)

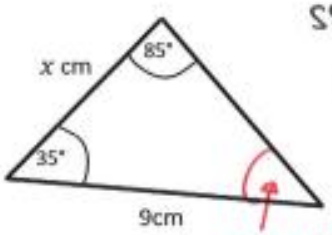


TOA

$\tan 42 = \frac{5.4}{x}$

$x = \frac{5.4}{\tan 42} = 5.997\text{cm}$

b)



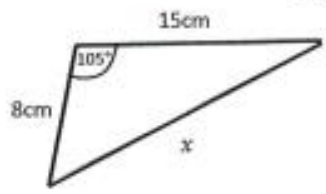
SSAA → sine rule

$180 - (85 + 35) = 60°$

$\frac{9}{\sin 85} = \frac{x}{\sin 60}$

$\frac{9 \sin 60}{\sin 85} = x \qquad x = 7.82\text{cm}$

c)



SSSA → cosine rule

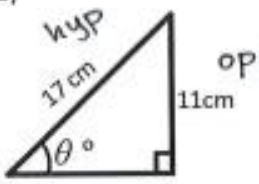
$a^2 = b^2 + c^2 - 2bc\cos A$

$x^2 = 15^2 + 8^2 - 2 \times 15 \times 8 \cos 105$

$x = 18.7\text{cm}$

2 Find the missing angles in the following triangles

a)

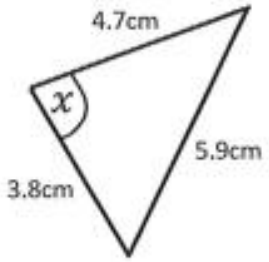


SOH

$\sin\theta = \frac{11}{17}$

$\sin^{-1}\left(\frac{11}{17}\right) = 40.3°$

b)



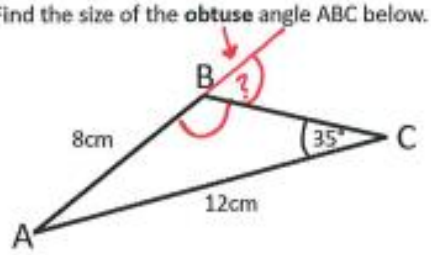
SSSA → cosine rule

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

$\cos A = \frac{3.8^2 + 4.7^2 - 5.9^2}{2 \times 3.8 \times 4.7}$

$x = 87.2°$

3 Find the size of the **obtuse** angle ABC below.



$\frac{8}{\sin 35} = \frac{12}{\sin x}$

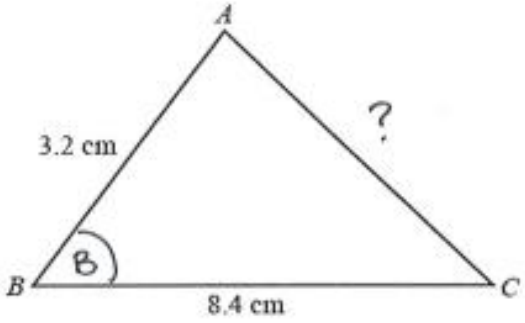
$8\sin x \times \frac{\sin 35}{8} = \frac{\sin x}{12}$

$\frac{12\sin 35}{8} = \sin x$

$x = 59.36°$ $\quad 180 - 59.36 = 120.6°$

4 The area of triangle ABC is $10cm^2$

Calculate the perimeter of triangle ABC
Give your answer correct to three significant figures



Area of triangle $= \frac{1}{2}ab\sin C$

$\frac{1}{2} \times 8.4 \times 3.2 \times \sin B = 10$

$\Rightarrow B = 48.077°$

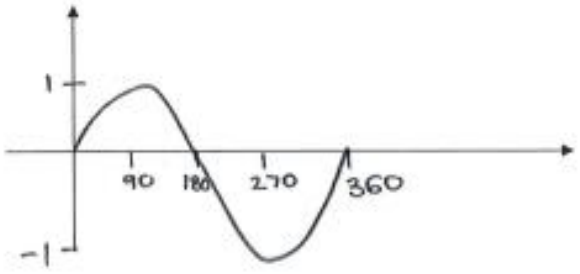
$b^2 = 8.4^2 + 3.2^2 - 2 \times 8.4 \times 3.2 \times \cos B$

$b^2 = 44.88\ldots$

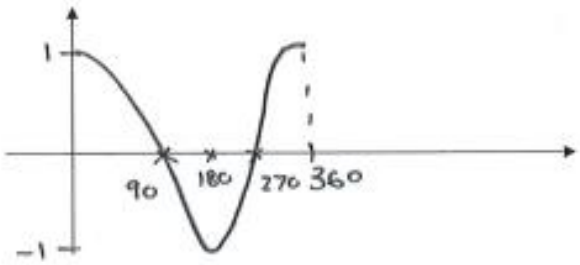
$b = 6.70cm$

5 Sketch, for $0° \leq x \leq 360°$ the graph of

a) $y = \sin x$



b) $y = \cos x$



6 Solve the following equations giving all answers between 0° and 360°

a) $\sin x = 0.6$

$\sin^{-1}(0.6) = 36.9°$

$180 - 36.9° = 143.1°$

b) $3\cos x = 1.2$

$\cos x = \frac{1.2}{3}$

$\cos^{-1}\left(\frac{1.2}{3}\right) = 66.4°$

$-66.4 + 360 = 294°$

c) $5 + \sin x = 4.8$

$\sin x = 4.8 - 5$

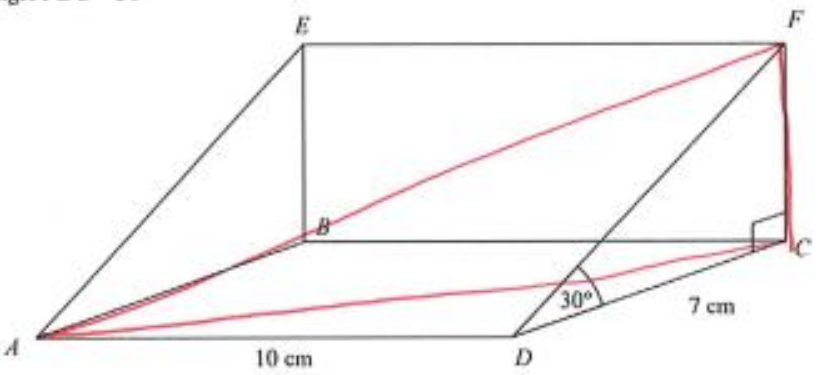
$\sin^{-1}(-0.2) = 191.5°$

$348.5°$

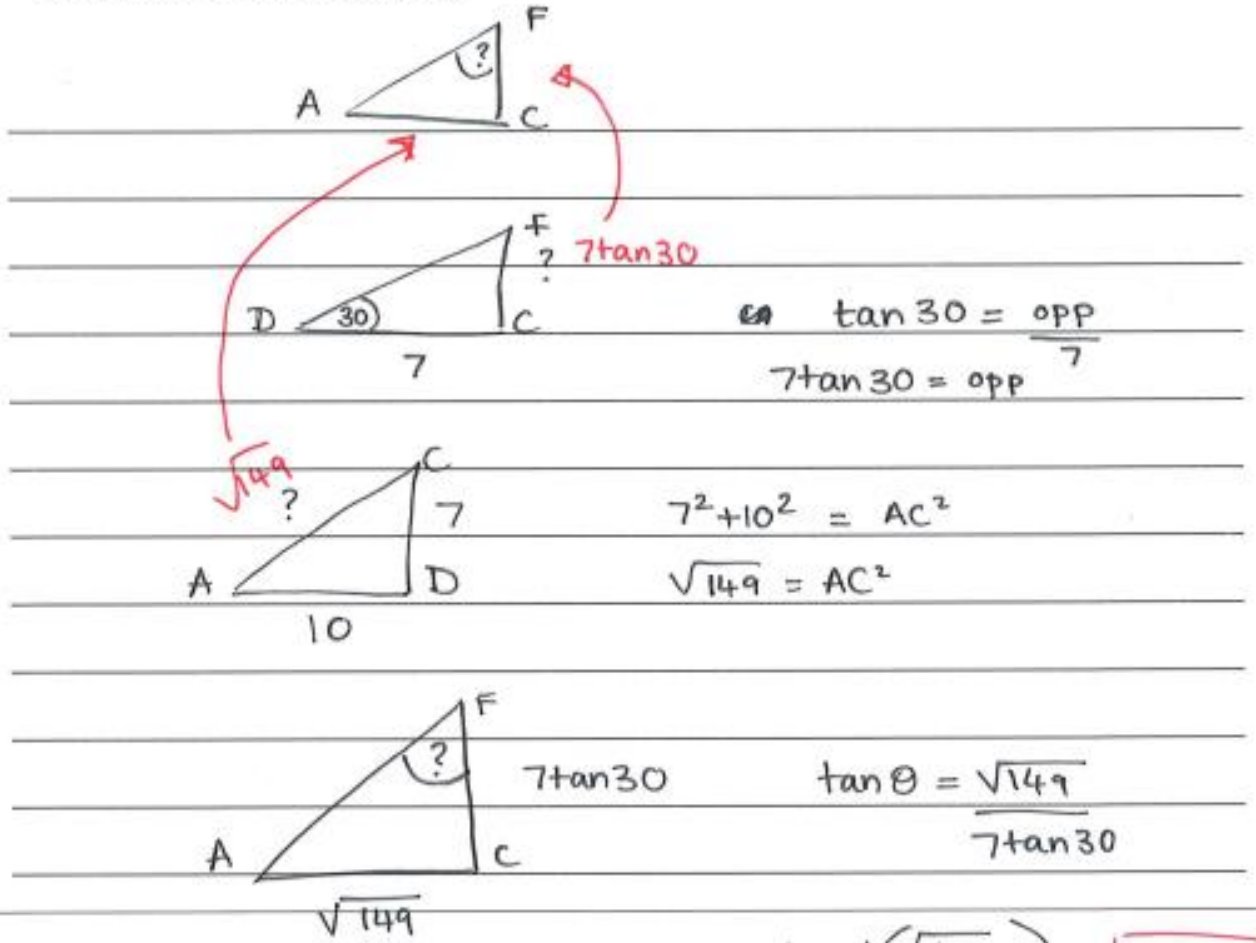
7 The diagram shows a triangular prism.

3D Trigonometry!

$CD = 7$ cm
$AD = 10$ cm
Angle $FDC = 30°$



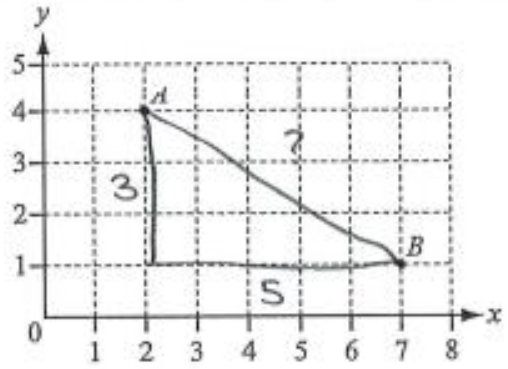
Calculate the size of angle AFC.
Give your answer correct to 1 decimal place.



$\tan^{-1}\left(\frac{\sqrt{149}}{7\tan 30}\right) = 71.7°$

Section 5 – Vectors

1 Write down the position vector of A $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$



a) Find $|\overrightarrow{AB}|$

$\sqrt{3^2 + 5^2} = \sqrt{34}$

2 $\boldsymbol{p} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} 6 \\ 3 \end{pmatrix}$

a) Find as a single column vector $\boldsymbol{p} + 2\boldsymbol{q}$ $\begin{pmatrix} 3 \\ 2 \end{pmatrix} + 2\begin{pmatrix} 6 \\ 3 \end{pmatrix}$

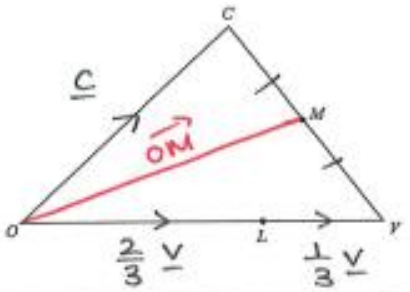
$= \begin{pmatrix} 15 \\ 8 \end{pmatrix}$

b) Calculate the value of $|\boldsymbol{p} + 2\boldsymbol{q}|$

$\sqrt{15^2 + 8^2} = 17$

3 1 : 2

In the diagram $CM = MV$ and $OL = 2LV$
O is the origin. $\overrightarrow{OC} = \boldsymbol{c}$ and $\overrightarrow{OV} = \boldsymbol{v}$



Find, in terms of $\boldsymbol{c}$ and $\boldsymbol{v}$, in their simplest forms

a) $\overrightarrow{CM}$

$\overrightarrow{CM} = \frac{1}{2}\overrightarrow{CV}$

$= \frac{1}{2}(-\underline{c} + \underline{v})$

b) The position vector of M $\overrightarrow{OM}$

$\overrightarrow{OM} = \overrightarrow{OC} + \overrightarrow{CM}$

$= \underline{c} + \frac{1}{2}(-\underline{c} + \underline{v})$

$= \frac{1}{2}\underline{c} + \frac{1}{2}\underline{v}$

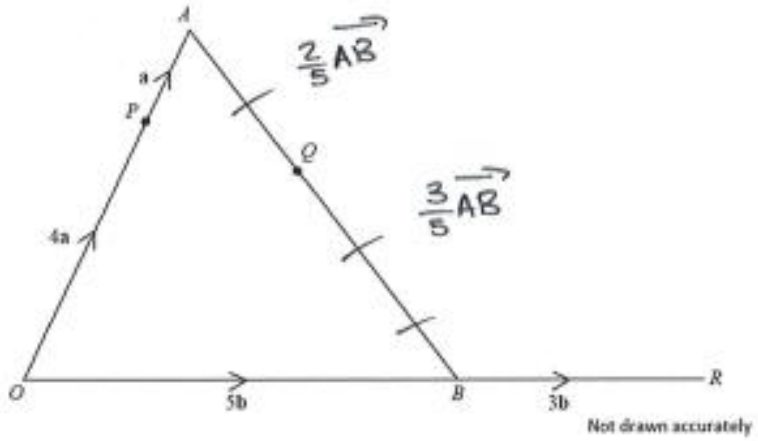
c) $\overrightarrow{ML}$ $\overrightarrow{ML} = \overrightarrow{MC} + \overrightarrow{CO} + \overrightarrow{OL}$

$= \frac{1}{2}\underline{c} - \frac{1}{2}\underline{v} + \underline{c} + \frac{2}{3}\underline{v}$

$= \frac{3}{2}\underline{c} + \frac{1}{6}\underline{v}$

4

In the diagram $\overrightarrow{OP} = 4\mathbf{a}$, $\overrightarrow{PA} = \mathbf{a}$, $\overrightarrow{OB} = 5\mathbf{b}$, $\overrightarrow{BR} = 3\mathbf{b}$ and $\overrightarrow{AQ} = \frac{2}{5}\overrightarrow{AB}$



Not drawn accurately

a) Find, in terms of **a** and **b**, simplifying your answers,

(i) $\overrightarrow{AB}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -5\underline{a} + 5\underline{b}$

(ii) $\overrightarrow{PQ}$

$\overrightarrow{PQ} = \overrightarrow{PA} + \frac{2}{5}\overrightarrow{AB}$

$= \underline{a} + \frac{2}{5}(5\underline{b} - 5\underline{a})$

$= \underline{a} + 2\underline{b} - 2\underline{a}$

$= -\underline{a} + 2\underline{b}$

(b) Show clearly that points *P*, *Q* and *R* lie on a straight line.

$\overrightarrow{PQ} = -\underline{a} + 2\underline{b}$

$\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OB} + \overrightarrow{BR}$

$= 4\underline{a} + 5\underline{b} + 3\underline{b}$

$= 4\underline{a} + 8\underline{b} \qquad = 4(2\underline{b} - \underline{a})$

So $PR = 4PQ \Rightarrow$ parallel

+ share point P

∴ straight line.

Section 6 – Proof

1 a) n is a positive integer

Write down the next odd number after $2n - 1$

$2n+1$

b) Prove that the product of two consecutive odd number is always one less than a multiple of 4

multiplication

$(2n+1)(2n-1) = 4n^2 - 2n + 2n - 1$

$= 4n^2 - 1$

$4n^2$ is a multiple of 4

So, $4n^2 - 1$ is 1 less than a multiple of 4.

2 Prove that $(5n + 3)(n - 1) + n(n + 2)$ is a multiple of 3 for all integer values of n

$5n^2 - 2n - 3 + n^2 + 2n$

$= 6n^2 - 3$

$= 3(2n^2 - 1)$ $\therefore$ has a factor of 3

$\Rightarrow$ it is a multiple of 3.

3 The n^{th} term of the linear sequence 2, 7, 12, 17, ... is $5n - 3$

A new sequence is formed by squaring each term of the linear sequence and adding 1.

Prove algebraically that all the terms in the new sequence are multiples of 5

$(5n-3)^2 + 1$

$= (5n-3)(5n-3) + 1$

$= 25n^2 - 15n - 15n + 9 + 1$

$= 25n^2 - 30n + 10$

$= 5(5n^2 - 6n + 2)$

which has a factor of 5

$\Rightarrow$ multiple of 5

4 a) The n^{th} term of a sequence is $n^2 + 12n + 27$
by factorising or otherwise, show that the 20th term can be written as the product of two prime numbers.

(+) (×)

$n^2 + 12n + 27$

$(n+9)(n+3)$

$u_{20} = (20+9)(20+3)$

$= 29 \times 23$

both prime

b) The n^{th} term of a different sequence is $n^2 - 6n + 14$
by completing the square or otherwise, show that every term is positive

$(n-3)^2 - 9 + 14$

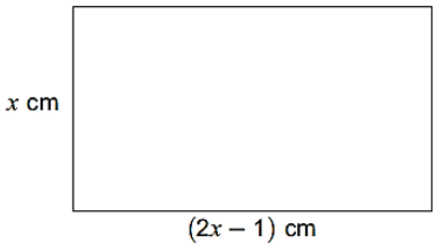
$(n-3)^2 + 5$

$(n-3)^2 \geq 0$ so, $(n-3)^2 + 5 \geq 5 > 0$

Therefore, $n^2 - 6n + 14$ is always positive.

Y12 Maths and Further Maths Practice Test

Please prepare full written answers to the following on lined paper, stapled, with your name at the top.
This must be submitted to your teacher in your first A level Maths lesson.

1	Solve $x^2 + 10x + 14 \geq 2 - x - x^2$	[4]
2	Write $\frac{5\sqrt{2}}{3\sqrt{6}-7}$ in the form $\sqrt{w} + \sqrt{k}$ where w and k are integers	[4]
3	The diagram shows a rectangle with area 9cm^2  Not drawn accurately Set up and solve an equation to work out the value of x. Give your answer to 3 s.f.	[4]
4	Simplify fully $$\frac{5x}{x^2 - 2x - 24} - \frac{3}{x - 6}$$	[4]
5	Make y the subject of $\sqrt{\frac{3xy}{x+y}} = 4$	[3]
6	$$x^2 + 2ax + b \equiv (x - 5)^2 - a$$ a) Find the values of a and b b) Hence write down the coordinates of the minimum point of $y = x^2 + 2ax + b$	[3] [1]
7	Points A and B have coordinates (-7,4) and (5,10). The perpendicular bisector of A and B crosses the x-axis and point P and the y-axis and point Q Find the coordinates of P and Q	[5]
8	Work out the values of a when $$2^{a^2} = 8^a \times 16$$ Do not use trial and improvement You must show your working	[4]
9	$f(x) = x^2 + 1$ $\quad g(x) = \frac{3x-1}{2}$ $\quad h(x) = \frac{1}{2x}$ find a) $hf(x)$ b) $g^{-1}(x)$	[1] [1]
10	Solve simultaneously	[4]

$x + y = 3$
$x^2 + 3y = 27$

11

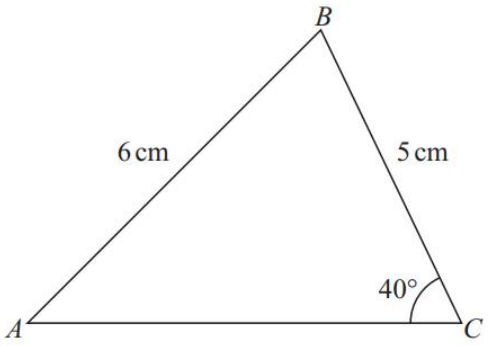
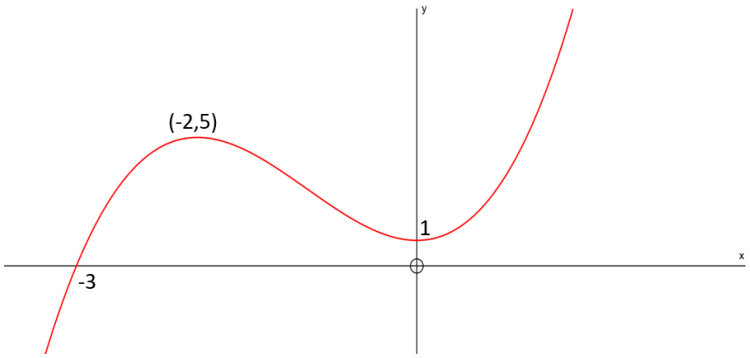


Diagram **NOT** accurately drawn

Calculate the area of triangle ABC.
Give your answer correct to 3 significant figures.

[5]

12 The graph $y = f(x)$ is shown below.



Sketch

[2]

a) $y = f(x - 2) - 1$ [2]

b) $y = -f(x)$ [2]

13 Solve to find θ giving all answers between 0° and 360°

$$\cos\theta = 0.4$$

[2]

14 OPQ is a triangle

R is the midpoint of OP
S is the midpoint of PQ

$\overrightarrow{OP} = \boldsymbol{p}$ and $\overrightarrow{OQ} = \boldsymbol{q}$

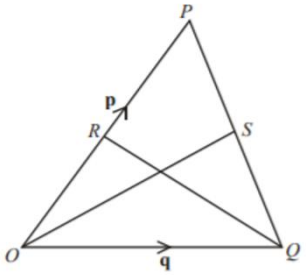


Diagram **NOT** accurately drawn

a) Find $\overrightarrow{OS}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$ [1]

b) Show that $\overrightarrow{RS}$ is parallel to $\overrightarrow{OQ}$ [3]

15 Use algebra to prove that the value of $\frac{8c^2+16}{3c^2+6} + \frac{1}{3}$ is an integer for all values of c [2]