Mathematics Department William Perkin CofE High School



An introduction to A level Mathematics and Further Mathematics

This induction booklet is for students intending to begin studying A-level Maths or A level Further Maths in Year 12 from next September.

It is important that you are able to work at this standard – read the introduction carefully, and spend time working through the exercises before you start in September.

All questions are GCSE standard and it is expected all students are familiar with the techniques covered.

Introduction to A level Maths at William Perkin

Thank you for choosing to study Mathematics in the Sixth Form at William Perkin High School.

If you are studying **Mathematics** on its own, you will sit two internal exams at the end of Y12, which will assess a combination of three areas: Pure maths, Statistics and Mechanics. At the end of Y13 you will sit 3 external exams that will assess content covered during both years with the same combination of Pure maths, Statistics and Mechanics. If you have chosen to study **Further Mathematics** as well, you will sit two internal exams at the end of Y12, which will assess a combination of Pure maths, Decision maths and Mechanics. At the end of Y13 you will sit 3 external exams that will assess content covered during both years with the same combination of Pure maths, Decision maths and Mechanics. At the end of Y13 you will sit 3 external exams that will assess content covered during both years with the same combination of Pure maths, Decision maths and Mechanics.

The Mathematics Department is committed to ensuring that you make good progress throughout your A level course. In order that you make the best possible start to the course, we have placed this booklet on our website. It is important that you spend time working through the topics in this booklet over the summer, as you need to have a good knowledge of these topics <u>before</u> you commence your course in September. You should have met all the topics before at GCSE. The answers to the exercises are at the back of the booklet. You will need to be organised about your approach to this, so keep your work in a folder and note any queries you have, so that you can ask about them at the beginning of term. You will notice that there is a heavy focus on algebraic manipulation in this booklet, as you must have a strong grasp of this in order to cope with the level of difficulty that the Mathematics and Further Mathematics courses demand. There are also sections on indices and surds, which too form an integral part of these courses.

In the second week of term, you will take a test in class to check how well you understand these topics, so it is important that you thoroughly understand the content of the booklet by then. If you do not pass this test, this will be considered a serious concern, and it will beg the question as to whether you should be taking this subject. In particular, please note that there is a practice test provided at the back of this booklet, which is similar in style to the test that you will be given in class, and which you must submit in your first lesson.

Use this introduction to give you a good start to your A level work so that you enjoy, and benefit from, the course. The more effort you put in, right from the start, the better you should do.

Mr V Bhardwaj Head of Mathematics

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Y	12 Maths and Further Maths induction task			
W	Work through all of these questions to ensure you are familiar with all of the content.			
W	Written solutions are provided at the back of this pack for you to mark your work.			
If y	you get stuck on a question please use the answers, but please make sure you re-learn this content.			
Se	ction 1 - Algebra			
1	Solve $20 + w < 3(w + 2)$			
	[3 marks]			
	Answer			
2	Work out the greatest integer value of x that satisfies the inequality $3x + 10 < 1$			
	Answer			
3	Expand and simplify $(v^2 - 5v + 2)(2v - 3)$			
	Answer (3 marks)			

4		Here is a formula.
		5t + 3 = 4w(t + 2)
	(a)	Rearrange the formula to make t the subject.
		Answer (4 marks)
	(b)	Work out the exact value of <i>t</i> when $w = -\frac{1}{8}$
		Give your answer in its simplest form.
		t = (3 marks)

5	Make <i>x</i> the subject of the formula $\frac{a+2x}{a-x} = n$
	Answer
6	Solve $x^2 + 6x + 7 = 0$
	Give your answer in the form $a \pm \sqrt{b}$, where <i>a</i> and <i>b</i> are integers.
	Answer

7	Simplify fully $\frac{4x^2 + 19x - 5}{9x^2 - 16} \div \frac{x + 5}{3x - 4}$
	Answer

0	(2)	Eastorico fully	$2r^2$ $2r$ 10
0	(a)	Facionse lully	$\Delta x = \Delta x = 40$
			Answer
	(h)	Fostarian fully	$(a + a)^2 + (a + a)(2a + Ea)$
	(a)	Factorise fully	$(x + y)^{-} + (x + y)(2x + 5y)$
	_		Answer
9	Solve	the simultaneou	s equations
			$2v - 3r \pm 4$
			2y = 5x + 4
			2x = -3y - 7
	_		
	Do no	ot use trial and in	nprovement.

10	(a)	Factorise fully $3x^2 - 12$	
			[2 marks]
		Answer	
	(1-)	F_{a} staring $F_{a}^{2} + 4m + 42m^{2}$	
	(a)	Factorise $5x^2 + 4xy - 12y^2$	[3 marks]
	_	Answer	
11	Simpl	ify $\frac{x^2 + 4x - 12}{x^2 - 25} \div \frac{x + 6}{x^2 - 5x}$	
		x = 25 $x = 5x$	
			(5
		Answer	(o marks)

12	Express	$2x^2 - 12x - 7$	in the form	$a(x+b)^2 + c$
12		Answer		
13	$(x-5)^2 + a$	$x \equiv x^2 + bx + 28$		
	Work out th	e values of a and b	<i>)</i> .	
		а	=	<i>b</i> = (3 marks)

14	$2x^2 - 2bx + 7a \equiv 2(x - a)^2 + 3$
	Work out the two possible pairs of values of a and b . [6 marks]
	a - b - b
	$u = \dots, v = \dots$
	and
	,
	$a = \dots p = \dots$

15	(a)	Show that	$\frac{4}{r} + \frac{2}{r-1}$	simplifies to	$\frac{6x-4}{r(r-1)}$	
			<i>x x</i> = 1		x(x-1)	
						(2 marks)
	(b)	Hence, or other	wise, solve	$\frac{4}{x} + \frac{2}{x-1} = 3$		
		Give your soluti	ions to 3 signifi	cant figures.		
			Anou/2-			(5 mortes)
			Allawel			

Se	ction 2 – Indices - DO NOT USE A CALCULATOR - YOU ARE EXPECTED TO SHOW YOUR WORKING CL	EARLY
1	Simplify $(2cd^4)^3$	
	Answer	(2 marks)
2	Simplify $(3xy^5)^4$	
	Answer	(2 marks)
3	(a) Simplify $\sqrt{x^5 \times x^9}$	
	Give your answer in the form x^p where p is an integer.	
		[2 marks]
	Answer	
	(b) Solve $y^{-3} = 125$	
		[2 marks]
	v =	
	2	

4	01- 116 6 11	$8c^7$ $6c^2$
	Simplify fully	$\frac{15d^6}{15d^3}$
		$15a = 5a^2$ [3 marks]
		[o mano]
		Answer
F	- 2	
5	Solve $r^{-\frac{1}{3}} = 7^{\frac{1}{3}}$	
	Write vour answer a	s a proper fraction.
	,	
	•••••	
	•••••	
		x = (5 marks)
L		

6	(-)	11 16 9 110	
6	(a)	$a \cdots \times b^{\circ} \times c = a^{\circ} \times b^{\circ \circ}$	
		Write a in terms of a and h	
		Write c in terms of a and b .	
		Give your answer in its simplest form.	
		<i>c</i> =	(3 marks)
	(1-)	x^{-2} $x^{6} \times x^{4}$	
	(a)	$p - = q - \times r$	
		Write win terms of a and a	
		$\begin{array}{c} \text{White } p \text{ in terms of } q \text{ and } r. \\ \text{Cive your ensure in its simplest form} \end{array}$	
		Give your answer in its simplest form.	
		$p = \dots$	(2 marks)



Sec	Section 3 – Surds – DO NOT USE A CALCULATOR - YOU ARE EXPECTED TO SHOW YOUR WORKING CLEARLY						
1	Write $\sqrt{500} - 2\sqrt{45}$ in the form $a\sqrt{5}$ where a is an integer.						
	Answer (2 marks)						
2	Write this ratio in its simplest form						
	$\sqrt{12}$: $\sqrt{48}$: $\sqrt{300}$						
	Answer : (2 morke)						
	Allowel						

3	Rationalise the denominator of $\frac{8}{3-\sqrt{5}}$
	Give your answer in the form $a + b\sqrt{5}$ where <i>a</i> and <i>b</i> are integers. [3 marks]
	Answer
4	Solve $\sqrt{(33+\sqrt{x})}=6$
	$x = \dots \qquad (3 marks)$

5	Solve $y(\sqrt{3} - 1) = 8$
	Give your answer in the form $a + b\sqrt{3}$ where <i>a</i> and <i>b</i> are integers.
	v = (4 marks)
	y =

Se	ction 4 –	Proof
1	(a)	n is a positive integer.
		Write down the next odd number after $2n-1$
		Answer (1 mark)
	(b)	Prove that the product of two consecutive odd numbers is always one less than a multiple of 4.
		(3 marks)
2	Prove is a m	that $(5n+3)(n-1) + n(n+2)$ nultiple of 3 for all integer values of <i>n</i> .
		(4 marks)

3	The n^{th} term of the linear sequence	2	7	12	17		is 5 <i>n</i> – 3
	A new sequence is formed by squaring ea adding 1.	ach te	erm of	f the lin	ear se	quence	and
	Prove algebraically that all the terms in the	ie ne	w seq	uence	are mu	iltiples o	f5.
							(4 marks)

4	(a)	The <i>n</i> th term of a sequence is $n^2 + 12n + 27$
		By factorising, or otherwise, show that the 20th term can be written as the product of two prime numbers.
		[2 marks]
	(b)	The <i>n</i> th term of a different sequence is $n^2 - 6n + 14$
		By completing the square, or otherwise, show that every term is positive. [3 marks]



3	A sketch of $2x + 3v = 12$ is shown.
	(a) Work out the coordinates of <i>R</i> .
	Answer () (1 mark)
	(b) Work out the coordinates of the midpoint of RS.
	Answer () (2 marks)
4	A straight line has gradient -2 and passes through the point $(-3, 10)$.
	Work out the equation of the line. Give your answer in the form $y = mx + c$ [2 marks]
	•••••••••••••••••••••••••••••••••••••••
	Answer

5		OABC is a kite.
		C(0, 4) O O A(12, 0)
	(a)	Work out the equation of AC.
		Answer
	(b)	Work out the coordinates of B.
		Answer (,,

Y12 Maths and Further Maths written solutions

Use these written solutions to check that you have answered the questions correctly, or to offer some hints when you are stuck with a question. **Do not** just copy these solutions into the booklet. You must have a strong grasp of this content before you start the course in September. Please also ensure you revise any of the topics that you may have become less familiar with if necessary.

Section 1 – Algebra

III Algebia	
1) 20	+ W < 3 (W + Z)
20 +	W L 3W +6
	14 L Zw
	$\omega > f$
2) 3x	+10 4 1
	3x 2 -9
	x 4 - 3
	Grentest integer 'is -4
3) 1.2	-5, +2/12, -3)
·) (9	3 10 3 10 3 3 1 15 1
= 24	5 - 10 - 1 4 - 5 - 5 - 6
= 2.	$3^{3} - 13y^{2} + 19y - 6$
× .	
4)~15t +	·3 = 4w(t+2)
25 -	+3 = 4tw + 8w
26	-4+w = 8w - 3
۴(5 - 4w) = 8w - 3
	$t = \underline{8\omega - 3}$
	5-4~
b) w	= - 1/8
+	$= 8(-\frac{1}{2}) - 3$
	5-4(-4)
F	<u> </u>
	= -4 $(\underline{\mu})$
	(2)

5)
$$\frac{A + 2x}{A - x} = n$$

 $a + 2x = n(a - x)$
 $a + 2x = An - nx$
 $2x + nx = an - a$
 $x (2 + n) = an - n$
 $x = \frac{an - n}{2 + n}$
6) $x^{3} + bx + 3 = 0$
 $(x + 3)^{2} - 9 + 3 = 0$
 $(x + 3)^{2} - 9 + 3 = 0$
 $(x + 3)^{2} - 2 = 0$
 $(x + 3)^{2} = 2$
 $x + 3 = \pm J2$
 $x = -3 \pm J2$
7) $\frac{4x^{3} + 19x - 5}{7x^{2} - 16} \div \frac{x + 5}{3x - 4}$
 $= \frac{(4x - 1)(x + 5)}{(3x - 4)(x + 4)} \times \frac{3x - 4}{3x + 5}$
 $= \frac{4x - 1}{3x + 4}$
8) $a) 2x^{2} - 2x - 40$
 $= 2(x^{2} - x - 20)$
 $= 2(x^{2} - x - 20)$
 $= 2(x^{2} - x - 20)$
 $= x(x - 5)(x + 4)$
b) $(x + y)^{2} + (x + y)(2x + 5y)$
 $= (x + y)(x + 5y)$
 $= 3(x + y)(x + 2y)$





$$(5) b) \frac{4}{x} + \frac{2}{x-1} = 3$$

$$\therefore \frac{6x - 4}{x(x-1)} = 3$$

$$6x - 4 = 3x(x-1)$$

$$6x - 4 = 3x^{2} - 3x$$

$$0 = 3x^{2} - 9x + 4$$

$$0 = (3x)(x) x$$

$$x = \frac{9 \pm \sqrt{81 - 4(3)(4)}}{2(3)}$$

$$x = 0.543 \text{ or } x = 2.46$$



Section 3-Surds
i)
$$\sqrt{500} - 2\sqrt{45}$$

= $10\sqrt{5} - 6\sqrt{5}$
= $4\sqrt{5}$
2) $\sqrt{52} + \sqrt{5} + \sqrt{5} + 10\sqrt{5}$
 $2\sqrt{5} + \sqrt{5} + \sqrt{5} + 10\sqrt{5}$
 $2\sqrt{5} + \sqrt{5} + \sqrt{5}$
 $2\sqrt{5} + \sqrt{5} + \sqrt{5}$
 $2\sqrt{5} + \sqrt{5} + \sqrt{5}$
 $3) \frac{8}{(5-55)} \times \frac{(3+55)}{(5+15)}$
 $= \frac{24 + 8\sqrt{5}}{9 - 5}$
 $= \frac{24 + 8\sqrt{5}}{9 - 5}$
 $= \frac{24 + 8\sqrt{5}}{9 - 5}$
 $= \frac{24 + 8\sqrt{5}}{9}$
 $= \frac{24 + 8\sqrt{5}}{9}$
 $= \frac{24 + 8\sqrt{5}}{9}$
 $= \frac{6 + 2\sqrt{5}}{4}$
 $\sqrt{5} + \sqrt{5} + 1$
 $\sqrt{5} - 1$) $= 8$
 $\sqrt{5} - \frac{8}{3} - 1}$
 $\sqrt{5} = \frac{8\sqrt{5} - 8}{2}$
 $\sqrt{5} = 4\sqrt{5} - 4$

1) a)
$$2n + 1$$

b) $(2n-1)(2n + 1)$
 $= 4n^{2} - 1$
 $4n^{3}$ is a multiple of 4 if n an integer
 $\therefore 4n^{3} - 1$ is one fewer then a multiple of 4
2) $(5n+3)(n-1) + n(n+2)$
 $= 5n^{3} - 5n + 3n - 3 + n^{2} + 2n$
 $= 6n^{2} - 2n + 2n - 3$
 $= 6n^{2} - 3$
 $= 3(2n^{2} - 1)$
This is a multiple of 3.
 $\therefore (5n+3)(n-1) + n(n+2)$ is a multiple of 3.
3) $(5n-3)^{2} + 1$
 $= 25n^{2} - 30n + 9 + 1$
 $= 25n^{2} - 30n + 9 + 1$
 $= 25n^{2} - 30n + 10$
 $= 5(5n^{2} - 6n + 2)$
This is a multiple of 5
 \therefore all terms in new sequence as a multiple
of 5.
4) a) $n^{2} + 12n + 27 = (n+3)(n+9)$
 $= 20^{44}$ term $= (20+3)(20+4)$
 $= 73 \times 29$
Both 23 and 24 as prime
b) $n^{2} - 6n + 14 = (n-3)^{2} - 9 + 14$
 $= (n-3)^{2} + 5$
 $(n-3)^{2} \ge 0$
 $\therefore (n-3)^{2} + 5 \ge 5$
 \therefore all terms are positive.

4) m=-	ι		
. y =	- 2x +L		
use (-3,	10)		
10 =	-2(->)+(
10	= 6 + c		
د	= 4		
· · ·	y= -2x +4		

Y 1	12 Maths and Further Maths practice test	
Ple Th	ease prepare full written answers to the following on lined paper, stapled, with your name his must be submitted to your teacher in your first A level Maths lesson.	at the top.
1	Solve $2(3x+1) > 3-4x$	[2 marks]
2	Write $\frac{5\sqrt{2}}{3\sqrt{6}-7}$ in the form $\sqrt{w} + \sqrt{k}$ where w and k	are integers. [5 marks]
3	<i>GH</i> is a straight line. The coordinates of <i>G</i> are $(-2, 8)$ The midpoint of <i>GH</i> is $(5, -3)$ Work out the coordinates of <i>H</i> .	[2 marks]
4	The diagram shows a rectangle with area 9 cm ² x cm $(2x - 1) cm$ Set up and solve an equation to work out the value of x. Give your answer to 3 significant figures.	ot drawn ccurately
5	Simplify fully $\frac{5x}{(x+4)(x-6)} - \frac{3}{(x-6)}$	[5 marks]
6	In the expansion of $(x+2)(x^2+kx-3)$ the coefficient of x^2	is zero.
	 (a) Work out the value of k. (b) Work out the coefficient of x. 	[1 mark] [2 marks]
7	Make <i>y</i> the subject of $\sqrt{\frac{3xy}{x+y}} = 4$	[4 marks]

8	$x^2 + 2ax + b \equiv (x - 5)^2 - a$								
	Work	out the v	alues of a and b					[3 marks]	
9		A straigh	t line with equation	n $y = mx$	c+c	has gradient	m and y-inte	ercept c.	
		Here are the equations of four straight lines, P, Q, R and S.							
		Ρ	2y - 4x = 5		Q	5y = 2x - 4			
		R	2y - 4 = 5x		S	4y = 5 - 2x			
	(a) Circle the line that passes through (7, 2)							[1 mark]	
			Р	Q	F	R	S		
	(b)	Circle the	e line with gradien	t 2 <u>1</u>				[1 mark]	
			Ρ	Q	F	ર	S		
	(c)	(c) Circle the line with <i>y</i> -intercept $2\frac{1}{2}$						[1 mark]	
			Р	Q	F	R	S		
	(d) Circle the line with a negative gradient.						[1 mark]		
			Ρ	Q	F	2	S		
	(e) Circle a pair of perpendicular lines.						[1 mark]		
			Р	Q	F	R	s		

10	The distance between the points $(2, 5p)$ and $(2, -10)$ is 30 units.							
	Work out the two possible values of <i>p</i> .							
11	(a) Expand and simplify $(3w+2y)(w-4y)$	3 marks]						
	(b) Expand and simplify $\frac{3}{x^2}\left(\frac{x}{3}+3x^2-1\right)$	3 markel						
12	Use algebra to prove that the value of $\frac{8c^2 + 16}{3c^2 + 6} + \frac{1}{3}$ is an integer for values of c.	r all						
	[3	3 marks]						
13	Work out the values of a when							
	$2^{a^2} = 8^a \times 16$							
	Do not use trial and improvement. You must show your working. [4 marks]							